Table 2: Skew-Gaussian coefficients used for rescaling procedure described in Eqs.(5.1-5.3).. Here, $a_1 = -41.64, b_1 = -3.26, c_1 = 1.481, a_2 = 8.888 \times 10^{15}, b_2 = -70.3, c_2 = 11.79$. These have been tested for Knudsen numbers from $10^{-1.5}$ to 10^2.

Coefficient	g_{MC}	g_{LTE}
A	$a_1 e^{-\left(\frac{\ln K - b_1}{c_1}\right)} + a_2 e^{-\left(\frac{\ln K - b_2}{c_2}\right)}$	$27.17K$
x_0	0.18	0.17
σ	0.34	0.35
b	$-0.03658K^{-1.818} - 3.227$	-4.35

To construct the scaling function, we fit the luminosity curves obtained from the GR-style Monte Carlo simulations (SHO potential, constant density and linear temperature gradient) to a skew Gaussian function $g_{MC}(r)$, and fit the LTE luminosity L_{LTE} to a second skew-Gaussian $g_{LTE}(r)$.

$$g(r) = Ae^{-\chi^2/2}\left(1 + \mathrm{Erf}\left(b\frac{\chi}{\sqrt{2}}\right)\right), \tag{5.1}$$

with

$$\chi \equiv \frac{\log_{10}(r/r_T) - x_0}{\sigma}, \tag{5.2}$$

where r_T is the radius at which the plasma temperature $T(r_T) = T_w$. For the GR simulations, $r_T \equiv 1$. The mean value x_0, width σ and skewness b of each of these distributions are functions of the Knudsen number K only. These coefficients are provided as functions of K in Table 2.

By construction,

$$L(r) \equiv \frac{g_{MC}(r)}{g_{LTE}(r)} L_{LTE}(r). \tag{5.3}$$

To verify that this prescription works independently of the solar profile, we compare the predictions of (5.3) with the luminosities obtained using our realistic Monte Carlo simulations **AV: refer to section**, which include a SHO potential but implement real solar target density and temperature profiles. Further, whereas the coefficients for $g(r)$ were obtained with $\mu = 1$, our realistic simulations implement a different target mass.

The prescription is thus:

1. Compute the LTE luminosity as usual, using α and κ tables from Gould & Raffelt [10] and [30].
2. compute g_{MC} and g_{LTE} using the respective tables of coefficients A, x_0, b and σ.

Results are shown in Fig. 10. We plot luminosities on the left, and their derivatives (proportional to transported energy ϵ_χ) on the right, for three representative Knusen numbers, ranging from local $\sigma = 5 \times 10^{-35}$ cm^2, $K \simeq 0.1$, to nonlocal $\sigma = 1 \times 10^{-36}$ cm^2, $K \simeq 6.5$ and $\sigma = 1 \times 10^{-37}$ cm^2, $K \simeq 65$. In all cases, the nonlocal parametrisation (Spergel & Press, dashed orange), overestimates the luminosity by a factor of 3 to 10, predictably faring worse in the LTE limit. This confirms the conclusions of Ref. [10, 31], and urges caution when implementing and interpreting results using this formalism. The Gould & Raffet formalism using the standard radial and Knudsen suppression factors fares well in the LTE regime