CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 21

Posted Monday 17 February 2014. Due 1pm Friday 28 February 2014.

21. [25 points]

Let the inner product $(\cdot,\cdot):\ C[-1,1]\times C[-1,1]\to\mathbb{R}$ be defined by

$$(u,v)=\int_{-1}^{1}u(x)v(x)\,dx.$$

Let the linear operator $P_e:\ C[-1,1]\to C[-1,1]$ be defined by

$$(P_e f)(x)=\frac{1}{2}\left(f(x)+f(-x)\right)$$

and let the linear operator $P_o:\ C[-1,1]\to C[-1,1]$ be defined by

$$(P_o f)(x)=\frac{1}{2}\left(f(x)-f(-x)\right).$$

Note that P_e and P_o project functions onto their even and odd parts, respectively.

(a) Verify that P_e and P_o are projections.

(b) For all $f\in C[-1,1]$, verify that $P_e f$ and $P_o f$ are orthogonal with respect to the inner product $(\cdot,\cdot)$.

(c) Is P_e+P_o a projection? Note that $P_e+P_o:\ C[-1,1]\to C[-1,1]$ is defined by

$$(P_e+P_o)f=P_e f+P_o f.$$

(d) Let $a,b\in\mathbb{R}$ be such that $a<b$. Let $\phi\in C[a,b]$ be defined by $\phi(x)=1$ and let the inner product $B(\cdot,\cdot):C[a,b]\times C[a,b]\to\mathbb{R}$ be defined by

$$B(u,v)=\int_a^b u(x)v(x)\,dx.$$

Let the linear operator $P:C[a,b]\to C[a,b]$ be defined by

$$Pf=\frac{1}{b-a}B(f,\phi)\phi.$$

Determine whether or not P is a projection.