CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 8

Posted Wednesday 29 January 2014. Due 1pm Friday 7 February 2014.

8. [25 points]

(a) Consider the forward difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x)}{h}.$$

For $u(x) = \exp(2x)$, compute (in MATLAB) the error

$$\left| u'(1/2) - \frac{u(1/2+h) - u(1/2)}{h} \right|,$$

for $N = 2, 4, 8, 16, 32, 64, 128, 256, 512$ (powers of 2) with $h = 1/(N+1)$. When h is small enough, this error should be proportional to h. Present your results in a table like the one below but with the missing entries filled in.

N	error
2	2.2920610
4	
8	
16	
32	
64	
128	
256	
512	

(b) Consider the centered difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x-h)}{2h}.$$

For $u(x) = \exp(2x)$, compute (in MATLAB) the error

$$\left| u'(1/2) - \frac{u(1/2+h) - u(1/2-h)}{2h} \right|$$

for $N = 2, 4, 8, 16, 32, 64, 128, 256, 512$ with $h = 1/(N+1)$. When h is small enough, this error should be proportional to h^2. Present your results in a table like the one below but with the missing entries filled in.

N	error
2	0.4117528
4	
8	
16	
32	
64	
128	
256	
512	

(c) Use MATLAB's `loglog` command to produce a plot of the error for the approximations considered in part (a) and part (b) for $N = 2, 4, 8, 16, 32, 64, 128, 256, 512$. Use the `hold on` command so that the plot showing the errors from part (a) is on the same figure as the plot showing the errors from part (b).

(d) By inspecting the plot you have created, estimate the value of N that you need to approximate $u'(1/2)$ to an error of 10^{-2} using the approximations in part (a) and part (b).