CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 18

Posted Wednesday 25 September 2013. Due 5pm Wednesday 2 October 2013.

18. [25 points] Consider the polynomials $\phi_1(x) = 1$, $\phi_2(x) = x$, and $\phi_3(x) = 3x^2 - 1$, which form a basis for the set of all quadratic polynomials. These polynomials are orthogonal with respect to the inner product $(\cdot, \cdot) : \ C[-1, 1] \times C[-1, 1] \to \mathbb{R}$ defined by

$$(u, v) = \int_{-1}^{1} u(x)v(x)\, dx.$$

Let the norm $\|\cdot\| : \ C[-1, 1] \to \mathbb{R}$ be defined by

$$\|u\| = \sqrt{(u, u)}.$$

Let $f(x) = \cos(\pi x)$.

(a) By hand, construct the best approximation f_1 to f from $\text{span}\{\phi_1\}$ with respect to the norm $\|\cdot\|$.

(b) By hand, construct the best approximation f_2 to f from $\text{span}\{\phi_1, \phi_2\}$ with respect to the norm $\|\cdot\|$.

(c) By hand, construct the best approximation f_3 to f from $\text{span}\{\phi_1, \phi_2, \phi_3\}$ with respect to the norm $\|\cdot\|$.

(d) Produce a plot that superimposes your best approximations from parts (a), (b), and (c) on top of a plot of $f(x)$.