

# CAAM 336 · DIFFERENTIAL EQUATIONS

## Homework 8

Posted Wednesday 29 January 2014. Due 1pm Friday 7 February 2014.

8. [25 points]

(a) Consider the forward difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x)}{h}.$$

For  $u(x) = \exp(2x)$ , compute (in MATLAB) the error

$$\left| u'(1/2) - \frac{u(1/2+h) - u(1/2)}{h} \right|,$$

for  $N = 2, 4, 8, 16, 32, 64, 128, 256, 512$  (powers of 2) with  $h = 1/(N+1)$ . When  $h$  is small enough, this error should be proportional to  $h$ . Present your results in a table like the one below but with the missing entries filled in.

$N$	error
2	2.2920610
4	
8	
16	
32	
64	
128	
256	
512	

(b) Consider the centered difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x-h)}{2h}.$$

For  $u(x) = \exp(2x)$ , compute (in MATLAB) the error

$$\left| u'(1/2) - \frac{u(1/2+h) - u(1/2-h)}{2h} \right|$$

for  $N = 2, 4, 8, 16, 32, 64, 128, 256, 512$  with  $h = 1/(N+1)$ . When  $h$  is small enough, this error should be proportional to  $h^2$ . Present your results in a table like the one below but with the missing entries filled in.

$N$	error
2	0.4117528
4	
8	
16	
32	
64	
128	
256	
512	

- (c) Use MATLAB's `loglog` command to produce a plot of the error for the approximations considered in part (a) and part (b) for  $N = 2, 4, 8, 16, 32, 64, 128, 256, 512$ . Use the `hold on` command so that the plot showing the errors from part (a) is on the same figure as the plot showing the errors from part (b).
- (d) By inspecting the plot you have created, estimate the value of  $N$  that you need to approximate  $u'(1/2)$  to an error of  $10^{-2}$  using the approximations in part (a) and part (b).