CAAM 336 · DIFFERENTIAL EQUATIONS
Homework 1

Posted Monday 26 August 2013. Due 5pm Wednesday 4 September 2013.

1. [25 points]

    Let $f(x) = 1$ for $x \in [0, 1]$. Also, let

    $$\phi_n(x) = \sin(n\pi x)$$

    and let

    $$a_n = \frac{\displaystyle\int_0^1 f(x)\phi_n(x)dx}{\displaystyle\int_0^1 (\phi_n(x))^2 dx}$$

    for $n = 1, 2, 3, \ldots$. We can approximate f as a linear combination of the periodic functions ϕ_n for $n = 1, 2, 3, \ldots, N$. We call the approximation f_N which is defined as

    $$f_N(x) = \sum_{n=1}^N a_n \phi_n(x)$$

    for $x \in [0, 1]$. In this course, you will learn in what sense a_n are the correct coefficients to best approximate f as a linear combination of the ϕ_n for $n = 1, 2, 3, \ldots, N$.

    (a) Compute the coefficients a_n symbolically using the MATLAB functions **syms**, **int**, and **simplify** as needed.

    (b) Use the fact that n is an integer to simplify the formula that you obtained for a_n.

    (c) Simplify your formula for a_n further by considering the cases when n is odd and n is even separately.

    (d) On the same figure plot $f_N(x)$ for $N = 4, 16, 64, 256$. Make sure to:
    - Use a different color for each value of N;
    - label the axes and provide a title;
    - create an accurate legend for the figure;
    - adjust the text sizes if necessary to make everything easily legible;
    - use the LATEX interpreter to make your labels, titles, and legend look stylish.