

# CAAM 336 · DIFFERENTIAL EQUATIONS

## Homework 4 · Solutions

Posted Wednesday 15 January 2014. Due 1pm Monday 27 January 2014.

4. [25 points]

(a) Is  $v(x) = 1/x^2$  a solution of

$$\frac{dv}{dx} + \frac{2}{x}v = 0?$$

(b) Is  $v(x, t) = t(t + x)$  a solution of

$$\frac{\partial v}{\partial t} - 3\frac{\partial v}{\partial x} = x - t?$$

(c) Is  $u(x, t) = xe^t$  a solution of

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( 2u \frac{\partial u}{\partial x} \right) = 0?$$

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**Solution.**

(a) [8 points]  $v(x) = 1/x^2$  is a solution of (1.1).

To plug  $v(x) = 1/x^2$  into the left-hand side of (1.1), we compute  $dv/dx = d(x^{-2})/dx = -2x^{-3}$ . Substituting this formula, the left-hand side of (1.1) becomes

$$-2x^{-3} + 2x^{-1}x^{-2} = 0.$$

This agrees with the right-hand side of (1.1), so this  $v$  is a solution.

(b) [8 points]  $v(x, t) = t(t + x)$  is a solution of (1.2).

We compute  $\partial v / \partial t = 2t + x$  and  $\partial v / \partial x = t$ . Thus the left-hand side of (1.2) becomes

$$(2t + x) - 3(t) = x - t.$$

This agrees with the right-hand side of (1.2), so this  $v$  is a solution.

(c) [9 points]  $u(x, t) = xe^t$  is *not* a solution of (1.3).

We compute  $\partial u / \partial t = xe^t$  and  $\partial u / \partial x = e^t$ . From this it follows that

$$\frac{\partial}{\partial x} \left[ 2u \frac{\partial u}{\partial x} \right] = \frac{\partial}{\partial x} 2xe^{2t} = 2e^{2t}.$$

Thus the left-hand side of (1.3) is

$$xe^t - 2e^{2t},$$

which is nonzero in general, in disagreement with the right-hand side of (1.3).

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