

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 24

Posted Monday 7 October 2013. Due 1pm Friday 18 October 2013.

24. [25 points] Let the inner product $(\cdot, \cdot) : C[0, 1] \times C[0, 1] \rightarrow \mathbb{R}$ be defined by

$$(v, w) = \int_0^1 v(x)w(x) dx.$$

Consider the linear operator $L : C_m^2[0, 1] \rightarrow C[0, 1]$ defined by

$$Lu = -u''$$

where

$$C_m^2[0, 1] = \{u \in C^2[0, 1] : u'(0) = u(1) = 0\}.$$

- (a) Is L symmetric?
- (b) What is the null space of L ?
- (c) Show that $(Lu, u) \geq 0$ for all $u \in C_m^2[0, 1]$ and explain why this and the answer to part (b) mean that $\lambda > 0$ for all eigenvalues λ of L .
- (d) Find the eigenvalues and eigenfunctions of L .