CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 2 · Solutions

Posted Monday 26 August 2013. Due 5pm Wednesday 4 September 2013.

2. [25 points]
Suppose $N \geq 1$ is an integer and define $h = 1/(N+1)$ and $x_j = jh$ for $j = 0, \ldots, N+1$. Consider the $N+2$ hat functions, defined for $x \in [0,1]$ as

$$\phi_k(x) = \begin{cases} (x - x_{k-1})/h, & x \in [x_{k-1}, x_k); \\ (x_{k+1} - x)/h, & x \in [x_k, x_{k+1}); \\ 0, & \text{otherwise;} \end{cases}$$

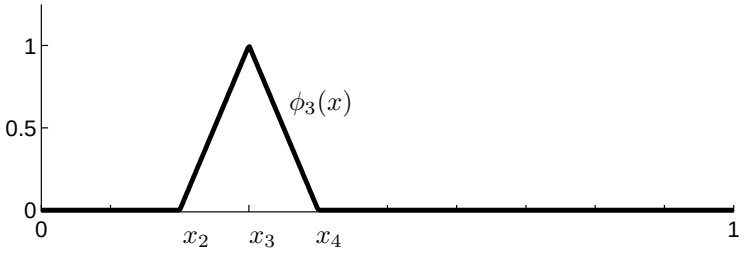
for $k = 1, \ldots, N$, with

$$\phi_0(x) = \begin{cases} (x_1 - x)/h, & x \in [x_0, x_1); \\ 0, & \text{otherwise;} \end{cases}$$

and

$$\phi_{N+1}(x) = \begin{cases} (x - x_N)/h, & x \in [x_N, x_{N+1}]; \\ 0, & \text{otherwise.} \end{cases}$$

We call these piecewise linear functions *hat functions* because of their shape. They will be important functions later in the course. For example, when $N = 9$ and $k = 3$, this function takes the following form.



(a) Write a MATLAB function for $\phi_k(x)$. It should take in as input x, k, and N. It should return the value $\phi_k(x)$. It should also be able to take in a vector for $\mathbf{x} = (\hat{x}_1, \ldots, \hat{x}_m)$ and return the vector $\phi_k(\mathbf{x}) = (\phi_k(\hat{x}_1), \ldots, \phi_k(\hat{x}_m))$.

(b) Let $N = 9$. Plot $\phi_0(x), \phi_4(x), \phi_5(x), \phi_6(x), \phi_{10}(x)$ on the same figure. Make sure to:

- plot each function with a different color;
- label the axes and provide a title;
- create an accurate legend for the figure;
- adjust the text sizes if necessary to make everything easily legible;
- use the LATEX interpreter to make your labels, titles, and legend look stylish.

Solution.

(a) [12 points]
A sample implementation of the hat function follows below. Student solutions should vary widely.

```
function phi_k = hat(x,k,N)
% function phi_k = hat(x,k,N)
%
% evaluates the hat function phi_k(x), where N denotes the
% size of the mesh, so that phi_k is non?zero on ((k?1)*h,(k+1)*h
```

```
% with h = 1/(N+1).
h = 1/(N+1);
xk = [0:N+1]*h;
if k==0
    phi_k = ((x>=xk(1))&(x<xk(2))).*((xk(2)-x)/h);
elseif k==N+1
    phi_k = ((x>=xk(N+1))&(x<=xk(N+2))).*((x-xk(N+1))/h);
else
    phi_k = ((x>=xk(k))&(x<xk(k+1))).*((x-xk(k))/h) ...
        + ((x>=xk(k+1))&(x<xk(k+2))).*((xk(k+2)-x)/h);
end
```

(b) [13 points]

The MATLAB code to create the plot is given below.

```
%% Code for plotting hat functions
N=9;
k=[0 4 5 6 10]; % hat function indices
colors='bgrcmyk';
x=linspace(0,1,1000);
figure; hold on;
ct=0; % initializing counter for loop
for j=k
    ct=ct+1;
    plot(x,hat(x,j,N),colors(ct));
    legendStr{ct}=['$\phi_{' num2str(j) '}(x)$'];
end
xlabel('$x$','interpreter','latex','fontsize',16);
ylabel('$\phi_k(x)$','interpreter','latex','fontsize',16);
title('Hat functions for various $k$','interpreter','latex','fontsize',16);
legend(legendStr,'interpreter','latex');
```