

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 33

Posted Friday 21 March 2014. Due 1pm Friday 11 April 2014.

33. [25 points]

Let $f \in C[0, 1]$, let $\alpha \in \mathbb{R}$ and let $\beta \in \mathbb{R}$. Let $p \in C^2[0, 1]$ be such that

$$-p''(x) = f(x), \quad 0 < x < 1;$$

$$-p'(0) = \alpha$$

and

$$p(1) = \beta.$$

Let $u \in C^2[0, 1]$ be such that

$$-4u''(x) + 9u(x) = f(x), \quad 0 < x < 1;$$

$$-4u'(0) = \alpha$$

and

$$4u'(1) = \beta.$$

Let $C_D^2[0, 1] = \{w \in C^2[0, 1] : w(1) = 0\}$.

(a) It can be shown that

$$\int_0^1 p'(x)v'(x) dx = q(f, \alpha, v) \text{ for all } v \in C_D^2[0, 1].$$

Obtain a formula for $q(f, \alpha, v)$.

(b) It can be shown that

$$\int_0^1 (4u'(x)v'(x) + 9u(x)v(x)) dx = g(f, \alpha, \beta, v) \text{ for all } v \in C^2[0, 1].$$

Obtain a formula for $g(f, \alpha, \beta, v)$.