

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 1

Posted Monday 13 January 2014. Due 1pm Monday 27 January 2014.

1. [25 points]

Let $f(x) = 1$ for $x \in [0, 1]$. Also, let

$$\phi_n(x) = \sin(n\pi x)$$

and let

$$a_n = \frac{\int_0^1 f(x)\phi_n(x)dx}{\int_0^1 (\phi_n(x))^2 dx}$$

for $n = 1, 2, 3, \dots$. We can approximate f as a linear combination of the periodic functions ϕ_n for $n = 1, 2, 3, \dots, N$. We call the approximation f_N which is defined as

$$f_N(x) = \sum_{n=1}^N a_n \phi_n(x)$$

for $x \in [0, 1]$. In this course, you will learn in what sense a_n are the correct coefficients to best approximate f as a linear combination of the ϕ_n for $n = 1, 2, 3, \dots, N$.

- (a) Compute the coefficients a_n symbolically using the MATLAB functions **syms**, **int**, and **simplify** as needed.
- (b) Use the fact that n is an integer to simplify the formula that you obtained for a_n .
- (c) Simplify your formula for a_n further by considering the cases when n is odd and n is even separately.
- (d) On the same figure plot $f_N(x)$ for $N = 4, 16, 64, 256$. Make sure to:
 - Use a different color for each value of N ;
 - label the axes and provide a title;
 - create an accurate legend for the figure;
 - adjust the text sizes if necessary to make everything easily legible;
 - use the LATEX interpreter to make your labels, titles, and legend look stylish.