CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 9

Posted Monday 9 September 2013. Due 5pm Wednesday 18 September 2013.

9. [25 points]
Suppose $N \geq 1$ is an integer and define $h = 1/(N+1)$ and $x_j = jh$ for $j = 0, \ldots, N+1$.

We can approximate the differential equation

$$\frac{d^2}{dx^2} u = f(x), \quad x \in (0,1),$$

with homogeneous Dirichlet boundary conditions $u(0) = u(1) = 0$ by the matrix equation

$$\frac{1}{h^2} \begin{bmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & 1 & -2 & \ddots & & \\ & & \ddots & \ddots & 1 & \\ & & & 1 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_{N-1}) \\ f(x_N) \end{bmatrix},$$

where $u_j \approx u(x_j)$. (Entries of the matrix that are not specified are zero.)

(a) Suppose that $f(x) = 25\pi^2 \cos(5\pi x)$.
Compute and plot the approximate solutions obtained when $N = 8, 16, 32, 64, 128$.
To solve the linear systems, you may use MATLAB's 'backslash' command: `u = A\f`.

(b) For each value of N used in part (a) compute the maximum error $|u_j - u(x_j)|$, given that the true solution is

$$u(x) = 1 - 2x - \cos(5\pi x).$$

Plot this error using a `loglog` plot with error on the vertical axis and N on the horizontal axis.

(c) Explain what adjustments to the right hand side of the matrix equation are necessary to accommodate the inhomogeneous Dirichlet boundary conditions

$$u(0) = 1, \quad u(1) = 2.$$

(d) Compute and plot the approximate solutions obtained when

$$u(0) = 1, \quad u(1) = 2$$

with $N = 8, 32, 128$.