

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 17

Posted Friday 14 February 2014. Due 1pm Friday 21 February 2014.

17. [25 points]

Let $\phi_1 \in C[-1, 1]$, $\phi_2 \in C[-1, 1]$, $\phi_3 \in C[-1, 1]$, and $f \in C[-1, 1]$ be defined by

$$\phi_1(x) = 1,$$

$$\phi_2(x) = x,$$

$$\phi_3(x) = 3x^2 - 1,$$

and

$$f(x) = \cos(\pi x),$$

for all $x \in [-1, 1]$. Let the inner product $(\cdot, \cdot) : C[-1, 1] \times C[-1, 1] \rightarrow \mathbb{R}$ be defined by

$$(u, v) = \int_{-1}^1 u(x)v(x) dx.$$

Let the norm $\|\cdot\| : C[-1, 1] \rightarrow \mathbb{R}$ be defined by

$$\|u\| = \sqrt{(u, u)}.$$

Note that $\{\phi_1, \phi_2, \phi_3\}$ is orthogonal with respect to the inner product (\cdot, \cdot) .

- (a) By hand, construct the best approximation f_1 to f from $\text{span}\{\phi_1\}$ with respect to the norm $\|\cdot\|$.
- (b) By hand, construct the best approximation f_2 to f from $\text{span}\{\phi_1, \phi_2\}$ with respect to the norm $\|\cdot\|$.
- (c) By hand, construct the best approximation f_3 to f from $\text{span}\{\phi_1, \phi_2, \phi_3\}$ with respect to the norm $\|\cdot\|$.
- (d) Produce a plot that superimposes your best approximations from parts (a), (b), and (c) on top of a plot of $f(x)$.