CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 6

Posted Wednesday 22 January 2014. Due 1pm Friday 31 January 2014.

6. [25 points]

Consider a bar of metal alloy manufactured such that its thermal conductivity is $\kappa(x) = e^{x/\ell}$ for $0 \leq x \leq \ell$.

(a) Use the homogeneous steady-state heat equation

$$\frac{d}{dx}\left(\kappa(x)u'(x)\right) = 0, \quad 0 \leq x \leq \ell$$

to determine a general formula for the steady-state temperature distribution $u(x)$ of this bar. The solution should include two arbitrary constants.

(b) Determine the steady-state temperature distribution $u(x)$ satisfying the boundary conditions

$$-\kappa\left(0\right)u'\left(0\right) = -K\left(u\left(0\right) - T\right)$$

and

$$u(\ell) = \gamma.$$

The condition at the left end $x = 0$ is called a Robin boundary condition. In the context of this problem it arises when the bar is in contact with air moving at constant speed. The moving air carries heat away from the bar, and this is known as convection. The temperature of the air is T and K is the convection coefficient.