

CAAM 336 · DIFFERENTIAL EQUATIONS

Recitation Example 4

Will be worked through on 30 September 2013.

4. Suppose $N \geq 1$ is an integer and define $h = 1/(N + 1)$ and $x_j = jh$ for $j = 0, \dots, N + 1$. Consider the $N + 2$ hat functions, defined for $x \in [0, 1]$ as

$$\phi_k(x) = \begin{cases} (x - x_{k-1})/h, & x \in [x_{k-1}, x_k]; \\ (x_{k+1} - x)/h, & x \in [x_k, x_{k+1}); \\ 0, & \text{otherwise;} \end{cases}$$

for $k = 1, \dots, N$, with

$$\phi_0(x) = \begin{cases} (x_1 - x)/h, & x \in [x_0, x_1]; \\ 0, & \text{otherwise;} \end{cases}$$

and

$$\phi_{N+1}(x) = \begin{cases} (x - x_N)/h, & x \in [x_N, x_{N+1}]; \\ 0, & \text{otherwise.} \end{cases}$$

In this question, we are interested in computing

$$\int_0^1 \phi_j(x) \phi_k(x) dx$$

for $j, k = 0, \dots, N + 1$.

- (a) Compute by hand

$$\int_0^1 \phi_0(x) \phi_0(x) dx.$$

- (b) Compute by hand

$$\int_0^1 \phi_{N+1}(x) \phi_{N+1}(x) dx.$$

- (c) Explain why in order to compute

$$\int_0^1 \phi_j(x) \phi_k(x) dx$$

for $j, k = 0, \dots, N + 1$, there are only 4 different cases to consider:

- i. $j = k, j = 1, \dots, N$;
- ii. $j = k, j = 0, N + 1$;
- iii. $j = k + 1, j = 1, \dots, N + 1$;
- iv. $|j - k| > 1$.

- (d) Let N be arbitrary. Write code to compute

$$\int_0^1 \phi_j(x) \phi_k(x) dx$$

for $j, k = 0, \dots, N + 1$ symbolically using the MATLAB functions **syms**, **int**, and **simplify** as needed.