CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 47

Posted Wednesday 9 April 2014. Due 1pm Friday 25 April 2014.

47. [25 points]

Let the norm $\|\cdot\|: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$\|\mathbf{y}\| = \sqrt{\mathbf{y} \cdot \mathbf{y}}.$$

Let the timestep $\Delta t \in \mathbb{R}$ be such that $\Delta t > 0$ and let $t_k = k\Delta t$ for $k = 0, 1, 2, \ldots$ Let

$$\mathbf{A} = \left[\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right]$$

and consider the problem of finding $\mathbf{x}(t)$ such that

$$\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t), \quad t \ge 0$$

and

$$\mathbf{x}(0) = \left[\begin{array}{c} 1 \\ 1 \end{array} \right].$$

(a) Compute $\mathbf{x}(t)$. Note that for real numbers t,

$$e^{it} = \cos(t) + i\sin(t)$$

and

$$e^{-it} = \cos(t) - i\sin(t).$$

- (b) How does $\|\mathbf{x}(t)\|$ behave as t increases?
- (c) For k = 0, 1, 2, ..., let \mathbf{x}_k be the approximation to $\mathbf{x}(t_k)$ obtained using the forward Euler method. For all choices of the timestep $\Delta t > 0$, how will $\|\mathbf{x}_k\|$ behave as $k \to \infty$?
- (d) For k = 0, 1, 2, ..., let \mathbf{x}_k be the approximation to $\mathbf{x}(t_k)$ obtained using the backward Euler method. For all choices of the timestep $\Delta t > 0$, how will $\|\mathbf{x}_k\|$ behave as $k \to \infty$?