## **CAAM 336 · DIFFERENTIAL EQUATIONS**

## Homework 14

Posted Friday 7 February 2014. Due 1pm Friday 14 February 2014.

## 14. [25 points]

Determine whether or not each of the following mappings is an inner product on the real vector space  $\mathcal{V}$ . If not, show all the **properties** of the inner product that are violated.

(a) 
$$(\cdot,\cdot): \mathcal{V} \times \mathcal{V} \to \mathbb{R}$$
 defined by  $(u,v) = \int_0^1 u(x)v'(x) dx$  where  $\mathcal{V} = C^1[0,1]$ .

(b) 
$$(\cdot,\cdot): \mathcal{V} \times \mathcal{V} \to \mathbb{R}$$
 defined by  $(u,v) = \int_0^1 |u(x)| |v(x)| \, dx$  where  $\mathcal{V} = C[0,1]$ .

(c) 
$$(\cdot,\cdot): \mathcal{V} \times \mathcal{V} \to \mathbb{R}$$
 defined by  $(u,v) = \int_0^1 u(x)v(x)e^{-x} dx$  where  $\mathcal{V} = C[0,1]$ .

(d) 
$$(\cdot,\cdot): \mathcal{V} \times \mathcal{V} \to \mathbb{R}$$
 defined by  $(u,v) = \int_0^1 (u(x) + v(x)) dx$  where  $\mathcal{V} = C[0,1]$ .

(e) 
$$(\cdot,\cdot): \mathcal{V} \times \mathcal{V} \to \mathbb{R}$$
 defined by  $(u,v) = \int_{-1}^{1} xu(x)v(x) dx$  where  $\mathcal{V} = C[-1,1]$ .