

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 15

Posted Wednesday 18 September 2013. Due 5pm Wednesday 25 September 2013.

15. [25 points]

Suppose \mathcal{V} is a vector space with an associated inner product $(\cdot, \cdot) : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}$ and norm $\|\cdot\| : \mathcal{V} \rightarrow \mathbb{R}$ defined by $\|w\| = \sqrt{(w, w)}$. The angle $\angle(u, v)$ between $u \in \mathcal{V}$ and $v \in \mathcal{V}$ is defined via the equation

$$(u, v) = \|u\| \|v\| \cos \angle(u, v).$$

Let $\mathcal{V} = C[0, 1]$ and let the inner product $(\cdot, \cdot) : C[0, 1] \times C[0, 1] \rightarrow \mathbb{R}$ be defined by $(u, v) = \int_0^1 u(x)v(x) dx$. Let m and n be nonnegative integers.

- (a) Compute (x^n, x^m) .
- (b) Compute $\|x^m\|$ and $\|x^n\|$.
- (c) Compute $\cos \angle(x^n, x^m)$ between x^n and x^m .
- (d) Compute $\cos \angle(x^n, x^{n+1})$ between x^n and x^{n+1} .
- (e) What happens to $\angle(x^n, x^{n+1})$ as $n \rightarrow \infty$?