

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 2 · Solutions

Posted Monday 13 January 2014. Due 1pm Monday 27 January 2014.

2. [25 points]

Suppose $N \geq 1$ is an integer and define $h = 1/(N + 1)$ and $x_k = kh$ for $k = 0, \dots, N + 1$. Consider the $N + 2$ hat functions, defined for $x \in [0, 1]$ as

$$\phi_j(x) = \begin{cases} \frac{(x - x_{j-1})}{h} & \text{if } x \in [x_{j-1}, x_j), \\ \frac{(x_{j+1} - x)}{h} & \text{if } x \in [x_j, x_{j+1}), \\ 0 & \text{otherwise,} \end{cases}$$

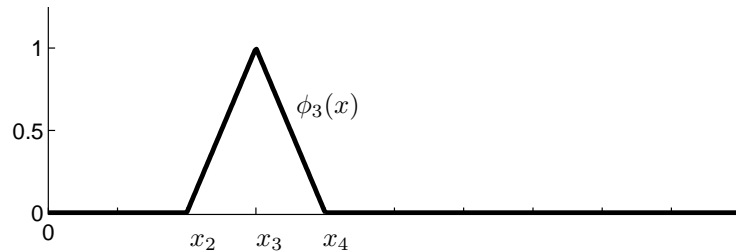
for $j = 1, \dots, N$, with

$$\phi_0(x) = \begin{cases} \frac{x_1 - x}{h} & \text{if } x \in [x_0, x_1), \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\phi_{N+1}(x) = \begin{cases} \frac{x - x_N}{h} & \text{if } x \in [x_N, x_{N+1}], \\ 0 & \text{otherwise.} \end{cases}$$

We call these piecewise linear functions *hat functions* because of their shape. They will be important functions later in the course. For example, when $N = 9$ and $j = 3$, ϕ_j takes the following form.



- (a) Write a MATLAB function for $\phi_j(x)$. It should take in as input x , j , and N . It should return the value $\phi_j(x)$. It should also be able to take in a vector for $\mathbf{x} = (\hat{x}_1, \dots, \hat{x}_m)$ and return the vector $\phi_j(\mathbf{x}) = (\phi_j(\hat{x}_1), \dots, \phi_j(\hat{x}_m))$.
- (b) Let $N = 9$. Plot $\phi_0(x), \phi_4(x), \phi_5(x), \phi_6(x), \phi_{10}(x)$ on the same figure. Make sure to:
 - plot each function with a different color;
 - label the axes and provide a title;
 - create an accurate legend for the figure;
 - adjust the text sizes if necessary to make everything easily legible;
 - use the LATEX interpreter to make your labels, titles, and legend look stylish.

Solution.

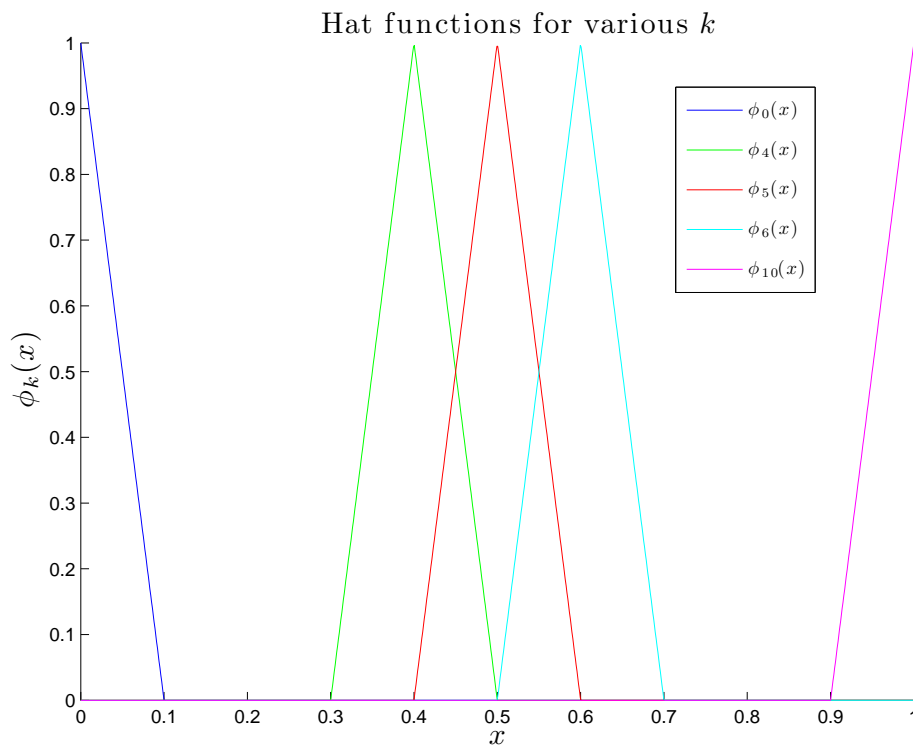
- (a) [12 points] A sample implementation of the hat function follows below. Student solutions should vary widely.

```

function phi_k = hat(x,k,N)
% function phi_k = hat(x,k,N)
%
% evaluates the hat function phi_k(x), where N denotes the
% size of the mesh, so that phi_k is non?zero on ((k?1)*h,(k+1)*h
% with h = 1/(N+1).
h = 1/(N+1);
xk = [0:N+1]*h;
if k==0
    phi_k = ((x>=xk(1))&(x<xk(2))).*((xk(2)-x)/h);
elseif k==N+1
    phi_k = ((x>=xk(N+1))&(x<=xk(N+2))).*((x-xk(N+1))/h);
else
    phi_k = ((x>=xk(k))&(x<xk(k+1))).*((x-xk(k))/h) ...
        + ((x>=xk(k+1))&(x<xk(k+2))).*((xk(k+2)-x)/h);
end

```

(b) [13 points] The requested plot is below.



The MATLAB code used to create the plot is given below.

```

%% Code for plotting hat functions
N=9;
k=[0 4 5 6 10]; % hat function indices
colors='bgrcmyk';
x=linspace(0,1,1000);
figure; hold on;
ct=0; % initializing counter for loop
for j=k
    ct=ct+1;
    plot(x,hat(x,j,N),colors(ct));
    legendStr{ct}=['$\phi_{'$ num2str(j) '}'(x)$'];
end
xlabel('$x$', 'interpreter', 'latex', 'fontsize', 16);
ylabel('$\phi_k(x)$', 'interpreter', 'latex', 'fontsize', 16);

```

```
title('Hat functions for various  $k$ ','interpreter','latex','fontsize',16);  
legend(legendStr,'interpreter','latex');
```
