

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 8

Posted Friday 6 September 2013. Due 5pm Wednesday 18 September 2013.

8. [25 points]

(a) In class we considered the forward difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x)}{h}.$$

For $u(x) = \exp(2x)$, compute (in MATLAB) the error

$$\left| u'(1/2) - \frac{u(1/2+h) - u(1/2)}{h} \right|,$$

for $N = 2, 4, 8, 16, 32, \dots, 512$ (powers of 2) with $h = 1/(N+1)$. In class, we showed that, when h is small enough, this error should be proportional to h . Present your results in a table like the one below but with the missing entries filled in.

N	error
2	2.2920610
4	
8	
16	
32	
64	
128	
256	
512	

(b) Consider the centered difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x-h)}{2h}.$$

For $u(x) = \exp(2x)$, compute (in MATLAB) the error

$$\left| u'(1/2) - \frac{u(1/2+h) - u(1/2-h)}{2h} \right|$$

for $N = 2, 4, 8, 16, 32, \dots, 512$ (powers of 2) with $h = 1/(N+1)$. When h is small enough, this error should be proportional to h^2 . Present your results in a table like the one below but with the missing entries filled in.

N	error
2	0.4117528
4	
8	
16	
32	
64	
128	
256	
512	

- (c) Use MATLAB's `loglog` command to produce a plot of N versus the corresponding error for the approximations considered in part (a) and part (b). Use the `hold on` command so that the plot showing the errors from part (a) is on the same figure as the plot showing the errors from part (b).
- (d) By inspecting the plot you have created, estimate the value of N that you need to approximate $u'(1/2)$ to an error of 10^{-2} using the approximations in part (a) and part (b).