

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 21

Posted Wednesday 25 September 2013. Due 5pm Wednesday 9 October 2013.

21. [25 points] Let the inner product $(\cdot, \cdot) : C[-1, 1] \times C[-1, 1] \rightarrow \mathbb{R}$ be defined by

$$(u, v) = \int_{-1}^1 u(x)v(x) \, dx.$$

Let the linear operator $P_e : C[-1, 1] \rightarrow C[-1, 1]$ be defined by

$$(P_e f)(x) = \frac{f(x) + f(-x)}{2}$$

and let the linear operator $P_o : C[-1, 1] \rightarrow C[-1, 1]$ be defined by

$$(P_o f)(x) = \frac{f(x) - f(-x)}{2}.$$

Note that P_e and P_o project functions onto their even and odd parts, respectively.

- (a) Verify that P_e and P_o are projections.
- (b) For all $f \in C[-1, 1]$, verify that $P_e f$ and $P_o f$ are orthogonal with respect to the inner product (\cdot, \cdot) .
- (c) Is $P_e + P_o$ a projection? Note that $P_e + P_o : C[-1, 1] \rightarrow C[-1, 1]$ is defined by $(P_e + P_o)f = P_e f + P_o f$.