CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 7

Posted Wednesday 4 September 2013. Due 5pm Wednesday 11 September 2013.

7. [25 points]
   Consider the temperature function

$$u(x,t) = e^{-\kappa\theta^2 t/(\rho c)} \sin(\theta x)$$

for constant κ, ρ, c, and θ.

(a) Show that this function $u(x,t)$ is a solution of the homogeneous heat equation

$$\rho c \frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2}, \quad \text{for } 0 < x < \ell \text{ and all } t.$$

(b) For which values of θ will u satisfy homogeneous Dirichlet boundary conditions at $x = 0$ and $x = \ell$?

(c) Suppose $\kappa = 2.37$ W/(cm K), $\rho = 2.70$ g/cm^3, and $c = 0.897$ J/(g K) (approximate values for aluminum found on Wikipedia), and that the bar has length $\ell = 10$ cm. Let θ be such that $u(x,t)$ satisfies homogeneous Dirichlet boundary conditions as in part (b) and $u(x,t) \geq 0$ for $0 \leq x \leq \ell$ and all t.

Use MATLAB to plot the solution $u(x,t)$ for $0 \leq x \leq \ell$ and time $0 \leq t \leq 20$ sec.
You may choose to do this in one of the following ways: (1) Plot the solution for $0 \leq x \leq \ell$ at times $t = 0, 4, 8, \ldots, 20$ sec., superimposing all six plots on the same axis (helpful commands: `linspace`, `plot`, `hold on`); (2) Create a three-dimensional plot of the data using `surf`, `mesh`, or `waterfall`. In either case, be sure to produce an attractive, well-labeled plot.