CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 11

Posted Monday 3 February 2014. Due 1pm Friday 14 February 2014.

11. [25 points]

Demonstrate whether or not each of the following is a linear operator.

(a) $f : \mathbb{R}^n \to \mathbb{R}^m$ defined by $f(\mathbf{u}) = \mathbf{A}\mathbf{u} + \mathbf{b}$ for a fixed matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ and fixed nonzero vector $\mathbf{b} \in \mathbb{R}^m$.

(b) $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(\mathbf{x}) = \mathbf{x}^T\mathbf{x}$.

(c) $f : \mathbb{R}^{n \times n} \to \mathbb{R}^{n \times n}$ defined by $f(\mathbf{X}) = \mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{B}$ for fixed matrices $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$.

(d) $L : C^1[0,1] \to C[0,1]$ defined by $(Lu)(x) = u(x)u'(x)$.

(e) $L : C^2[0,1] \to C[0,1]$ defined by $(Lu)(x) = u''(x) - \sin(x)u'(x) + \cos(x)u(x)$.