CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 39

Posted Wednesday 13 November 2013. Due 5pm Wednesday 20 November 2013.

39. [25 points] Parts (a) and (c) of this question should be done by hand.

Let

$$f(x,t) = \begin{cases} 2x & \text{if } x \in \left[0, \dfrac{1}{2}\right), \\ 2 - 2x & \text{if } x \in \left[\dfrac{1}{2}, 1\right]. \end{cases}$$

(a) Use the spectral method to obtain a series solution to the problem of finding the solution $\tilde{u}(x,t)$ to the heat equation

$$\tilde{u}_t(x,t) - \tilde{u}_{xx}(x,t) = f(x,t), \quad 0 < x < 1, \quad t > 0$$

with Neumann boundary conditions

$$\tilde{u}_x(0,t) = \tilde{u}_x(1,t) = 0, \quad t \geq 0$$

and initial condition

$$\tilde{u}(x,0) = 0, \quad 0 < x < 1.$$

(b) Plot the approximations to $\tilde{u}(x,t)$ obtained by replacing the upper limit of the summation in your series solution with 20 for $t = 0, 0.1, 0.2, 0.3, 0.5, 1, 2$.

(c) By shifting the data and then using the spectral method, obtain a series solution to the problem of finding the solution $u(x,t)$ to the heat equation

$$u_t(x,t) - u_{xx}(x,t) = f(x,t), \quad 0 < x < 1, \quad t > 0$$

with Neumann boundary conditions

$$u_x(0,t) = 0, \quad t \geq 0$$

and

$$u_x(1,t) = 2, \quad t \geq 0$$

and initial condition

$$u(x,0) = x^2, \quad 0 < x < 1.$$

(d) Plot the approximations to $u(x,t)$ obtained by replacing the upper limit of the summation in your series solution with 20 for $t = 0, 0.1, 0.2, 0.3, 0.5, 1, 2$.