CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 20

Posted Friday 14 February 2014. Due 1pm Friday 28 February 2014.

20. [25 points]

All parts of this question should be done by hand.

Let $\phi_1 \in C[-1,1]$, $\phi_2 \in C[-1,1]$, $f_1 \in C[-1,1]$, and $f_2 \in C[-1,1]$ be defined by

$$\phi_1(x) = \frac{1}{\sqrt{2}},$$

$$\phi_2(x) = \frac{\sqrt{3}}{\sqrt{2}}x,$$

$$f_1(x) = \sin(\pi x),$$

and

$$f_2(x) = \cos(\pi x),$$

for all $x \in [-1,1]$. Note that $\{\phi_1, \phi_2\}$ is linearly independent. Let the inner product $(\cdot,\cdot) : C[-1,1] \times C[-1,1] \to \mathbb{R}$ be defined by

$$(u,v) = \int_{-1}^{1} u(x)v(x)\,dx$$

and let the norm $\|\cdot\| : C[-1,1] \to \mathbb{R}$ be defined by

$$\|u\| = \sqrt{(u,u)}.$$

Note that $\{\phi_1, \phi_2\}$ is orthonormal with respect to the inner product $(\cdot,\cdot)$. Also, let $\psi_1 \in C[0,1]$, $\psi_2 \in C[0,1]$, $g_1 \in C[0,1]$, and $g_2 \in C[0,1]$ be defined by

$$\psi_1(x) = \frac{1}{\sqrt{2}},$$

$$\psi_2(x) = \frac{\sqrt{3}}{\sqrt{2}}x,$$

$$g_1(x) = \sin(\pi x),$$

and

$$g_2(x) = \cos(\pi x),$$

for all $x \in [0,1]$. Note that $\{\psi_1, \psi_2\}$ is linearly independent. Let the inner product $B(\cdot,\cdot) : C[0,1] \times C[0,1] \to \mathbb{R}$ be defined by

$$B(u,v) = \int_{0}^{1} u(x)v(x)\,dx$$

and let the norm $\|\cdot\|_B : C[0,1] \to \mathbb{R}$ be defined by

$$\|u\|_B = \sqrt{B(u,u)}.$$

Note that

$$(f_1, \phi_2) = \frac{\sqrt{6}}{\pi},$$

$$B(g_1, \psi_1) = \frac{\sqrt{2}}{\pi},$$

$$B(g_1, \psi_2) = \frac{\sqrt{6}}{2\pi},$$

$$B(g_2, \psi_2) = -\frac{\sqrt{6}}{\pi^2},$$

and

$$(f_1, \phi_1) = (f_2, \phi_1) = (f_2, \phi_2) = B(g_2, \psi_1) = 0.$$

(a) Construct the best approximation to f_1 from $\text{span}\{\phi_1, \phi_2\}$ with respect to the norm $\|\cdot\|$.

(b) Construct the best approximation to f_2 from $\text{span}\{\phi_1, \phi_2\}$ with respect to the norm $\|\cdot\|$.

(c) Construct the best approximation to g_1 from $\text{span}\{\psi_1, \psi_2\}$ with respect to the norm $\|\cdot\|_B$.

(d) Construct the best approximation to g_2 from $\text{span}\{\psi_1, \psi_2\}$ with respect to the norm $\|\cdot\|_B$.