CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 4 · Solutions

Posted Wednesday 28 August 2013. Due 5pm Wednesday 5 September 2013.

4. [25 points]

(a) Is $v(x) = 1/x^2$ a solution of
$$\frac{dv}{dx} + \frac{2}{x}v = 0?$$

(b) Is $v(x,t) = t(t+x)$ a solution of
$$\frac{\partial v}{\partial t} - 3\frac{\partial v}{\partial x} = x - t?$$

(c) Is $u(x,t) = xe^t$ a solution of
$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(2u\frac{\partial u}{\partial x}\right) = 0?$$

Solution.

(a) [8 points] $v(x) = 1/x^2$ is a solution of (1.1).
To plug $v(x) = 1/x^2$ into the left-hand side of (1.1), we compute $dv/dx = d(x^{-2})/dx = -2x^{-3}$. Substituting this formula, the left-hand side of (1.1) becomes
$$-2x^{-3} + 2x^{-1}x^{-2} = 0.$$
This agrees with the right-hand side of (1.1), so this v is a solution.

(b) [8 points] $v(x,t) = t(t+x)$ is a solution of (1.2).
We compute $\partial v/\partial t = 2t + x$ and $\partial v/\partial x = t$. Thus the left-hand side of (1.2) becomes
$$(2t + x) - 3(t) = x - t.$$
This agrees with the right-hand side of (1.2), so this v is a solution.

(c) [9 points] $u(x,t) = xe^t$ is *not* a solution of (1.3).
We compute $\partial u/\partial t = xe^t$ and $\partial u/\partial x = e^t$. From this it follows that
$$\frac{\partial}{\partial x}\left[2u\frac{\partial u}{\partial x}\right] = \frac{\partial}{\partial x}2xe^{2t} = 2e^{2t}.$$
Thus the left-hand side of (1.3) is
$$xe^t - 2e^{2t},$$
which is nonzero in general, in disagreement with the right-hand side of (1.3).
