CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 5 · Solutions

Posted Friday 17 January 2014. Due 1pm Friday 31 January 2014.

5. [25 points]

Let u be the solution of

$$\begin{aligned} -u''(x) &= f(x), \quad 0 < x < 1; \\ -u'(0) &= \alpha; \\ u(1) &= \beta; \end{aligned}$$

where $\alpha, \beta \in \mathbb{R}$ and

$$f(x) = 12x^2 - 24x + 4.$$

(a) Use integration and the boundary conditions to compute the solution u.

(b) Plot u in the case when $\alpha = \beta = 0$.

(c) Plot u in the case when $\alpha = -1$ and $\beta = 1$.

Solution.

(a) [15 points] Since

$$-u''(x) = f(x)$$

and

$$f(x) = 12x^2 - 24x + 4$$

we have that

$$u''(x) = -12x^2 + 24x - 4.$$

Integrating yields that

$$u'(x) = -4x^3 + 12x^2 - 4x + A$$

where A is a constant. From this we have that $-u'(0) = -A$ and so $-u'(0) = \alpha$ when $A = -\alpha$ and so

$$u'(x) = -4x^3 + 12x^2 - 4x - \alpha.$$

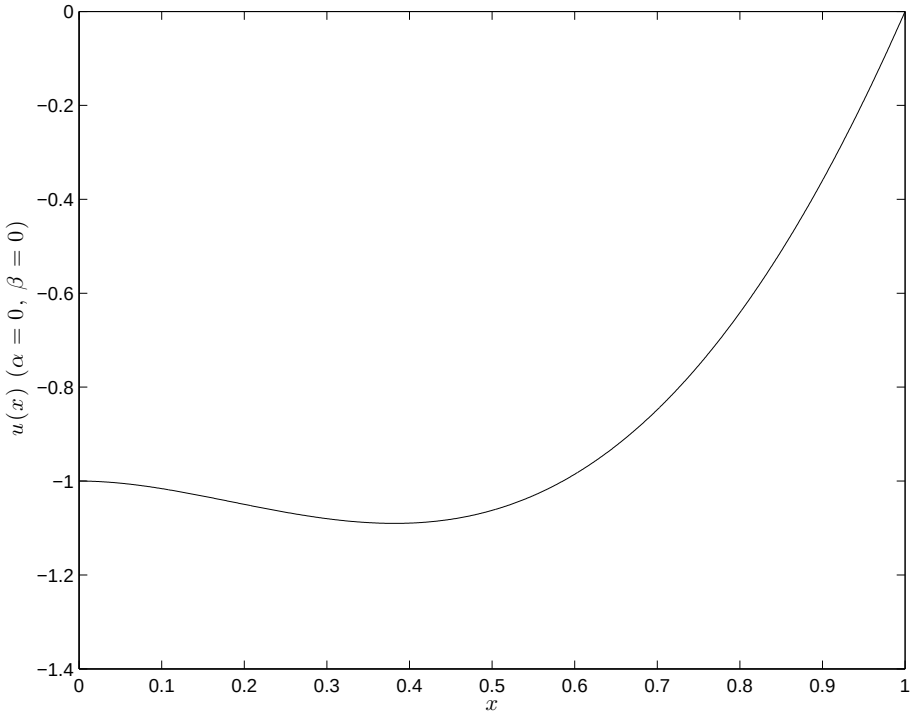
Integrating again then yields that

$$u(x) = -x^4 + 4x^3 - 2x^2 - \alpha x + B$$

where B is a constant. From this we have that $u(1) = -1 + 4 - 2 - \alpha + B = 1 - \alpha + B$ and so $u(1) = \beta$ when $B = \alpha + \beta - 1$ and so

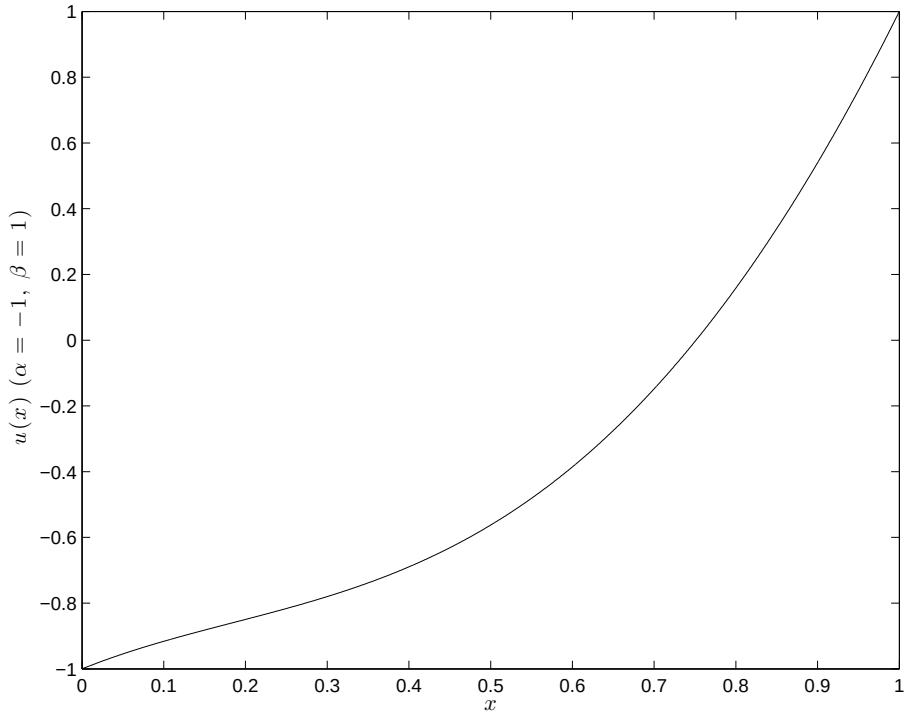
$$u(x) = -x^4 + 4x^3 - 2x^2 - \alpha x + \alpha + \beta - 1.$$

(b) [5 points] The plot and code used to create it are below.




```
clear
clc
alpha=0;
beta=0;
figure(1)
clf
x=linspace(0,1,1000);
u=-x.^4+4*x.^3-2*x.^2-alpha*x+alpha+beta-1;
plot(x, u, '-k')
xlabel('$x$','interpreter','latex','FontSize',12)
ylabel('$u(x)$ ($\alpha=0$, $\beta=0$)','interpreter','latex','FontSize',12)
saveas(figure(1),'hw5b.eps','epsc')
```

(c) [5 points] The plot and code used to create it are below.




```
clear
clc
alpha=-1;
beta=1;
figure(1)
clf
x=linspace(0,1,1000);
u=-x.^4+4*x.^3-2*x.^2-alpha*x+alpha+beta-1;
plot(x, u, '-k')
xlabel('$x$','interpreter','latex','FontSize',12)
ylabel('$u(x)$ ($\alpha=-1$, $\beta=1$)','interpreter','latex','FontSize',12)
saveas(figure(1),'hw5c.eps','epsc')
```