CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 3 · Solutions

Posted Wednesday 28 August 2013. Due 5pm Wednesday 4 September 2013.

3. [25 points]
For each of the following equations, (a) specify whether it is an ODE or a PDE; (b) determine its order; and (c) specify whether it is linear or nonlinear. For those that are linear, specify whether they (d) are homogeneous or inhomogeneous; and (e) have constant or variable coefficients.

(1.1) $\dfrac{dv}{dx} + \dfrac{2}{x}v = 0$ (1.2) $\dfrac{\partial v}{\partial t} - 3\dfrac{\partial v}{\partial x} = x - t$

(1.3) $\dfrac{\partial u}{\partial t} - \dfrac{\partial}{\partial x}\left(2u\dfrac{\partial u}{\partial x}\right) = 0$ (1.4) $\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \dfrac{\partial^3 u}{\partial x^3} = 0$

(1.5) $\dfrac{d^2y}{dx^2} - 7(1-y^2)\dfrac{dy}{dx} + y = 0$ (1.6) $\dfrac{d^2}{dx^2}\left(x^2\dfrac{d^2u}{dx^2}\right) = \sin(x)$

Solution.

(1.1) [5 points] ODE, first order, linear, homogeneous, variable coefficient
The $2/x$ factor in front of the v is the variable coefficient.

(1.2) [5 points] PDE, first order, linear, inhomogeneous, constant coefficient
The $x - t$ term on the right, which does not involve v, makes the equation inhomogeneous.

(1.3) [3 points] PDE, second order, nonlinear
Using the product rule for partial derivatives, we can write this equation in the equivalent form

$$\frac{\partial u}{\partial t} - 2\left(\frac{\partial u}{\partial x}\right)^2 - 2u\left(\frac{\partial^2 u}{\partial x^2}\right) = 0.$$

The second and third terms on the left hand side make this equation nonlinear.

(1.4) [3 points] PDE, third order, nonlinear
The $u(\partial u/\partial x)$ term makes this equation nonlinear. This a version of the famous Korteweg-de Vries (KdV) equation that describes shallow water waves.

(1.5) [3 points] ODE, second order, nonlinear
The $(1-y^2)(dy/dx)$ term makes this ODE nonlinear.

(1.6) [6 points] ODE, fourth order, linear, inhomogeneous, variable coefficient
Using the product rule for partial derivatives, we can write this equation in the equivalent form

$$2\frac{d^2u}{dx^2} + 4x\frac{d^3u}{dx^3} + x^2\frac{d^4u}{dx^4} = \sin(x),$$

hence we can see that it is fourth order.
