CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 29 · Solutions

Posted Monday 21 October 2013. Due 5pm Wednesday 30 October 2013.

29. [25 points]

Let N be a positive integer, let $h = \dfrac{1}{N+1}$ and let $x_k = kh$ for $k = 0, 1, \ldots, N+1$. Let the continuous piecewise quadratic functions $\phi_j \in C[0,1]$ be such that

$$\phi_j(x) = \begin{cases} \dfrac{(x - x_{j-1})(2x - x_{j-1} - x_j)}{h^2} & \text{if } x \in [x_{j-1}, x_j), \\ \dfrac{(x_j + x_{j+1} - 2x)(x_{j+1} - x)}{h^2} & \text{if } x \in [x_j, x_{j+1}), \\ 0 & \text{otherwise,} \end{cases}$$

for $j = 1, \ldots, N$ and let the continuous piecewise quadratic bubble functions $\psi_j \in C[0,1]$ be such that

$$\psi_j(x) = \begin{cases} \dfrac{4(x - x_{j-1})(x_j - x)}{h^2} & \text{if } x \in [x_{j-1}, x_j), \\ 0 & \text{otherwise,} \end{cases}$$

for $j = 1, \ldots, N+1$.

(a) Write a MATLAB function for $\phi_j(x)$. It should take in as input x, j, and N. It should return the value $\phi_j(x)$. It should also be able to take in a vector for $\mathbf{x} = (\hat{x}_1, \ldots, \hat{x}_m)$ and return the vector $\phi_j(\mathbf{x}) = (\phi_j(\hat{x}_1), \ldots, \phi_j(\hat{x}_m))$.

(b) Write a MATLAB function for $\psi_j(x)$. It should take in as input x, j, and N. It should return the value $\psi_j(x)$. It should also be able to take in a vector for $\mathbf{x} = (\hat{x}_1, \ldots, \hat{x}_m)$ and return the vector $\psi_j(\mathbf{x}) = (\psi_j(\hat{x}_1), \ldots, \psi_j(\hat{x}_m))$.

(c) For $N = 3$, plot $\phi_1(x), \phi_2(x), \phi_3(x), \psi_1(x), \psi_2(x), \psi_3(x), \psi_4(x)$ on the same figure. If you experience problems with displaying ψ using the latex interpreter then you may use the command `legend('\phi_1(x)','\phi_2(x)','\phi_3(x)','\psi_1(x)','\psi_2(x)','\psi_3(x)','\psi_4(x)')` to produce the legend.

Solution.

(a) [5 points] One way of coding the function is:

```
function phi_j=cpq1(x,j,N)

h=1/(N+1);
xj=j*h;
xjm1=xj-h;
xjp1=xj+h;
hs=h^2;

phi_j=((x>=xjm1)&(x<xj)).*(x-xjm1).*(2*x-xjm1-xj)/hs+...
    ((x>=xj)&(x<xjp1)).*(xj+xjp1-2*x).*(xjp1-x)/hs;
```

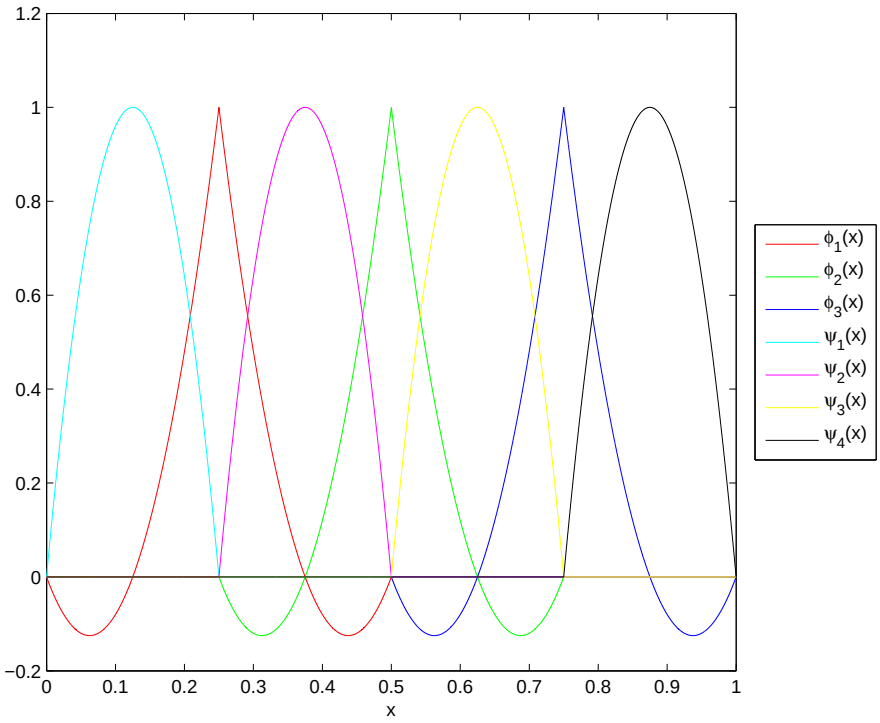
(b) [5 points] One way of coding the function is:

```
function psi_j=cpq2(x,j,N)

h=1/(N+1);
xj=j*h;
xjm1=xj-h;
fohs=4/h^2;

psi_j=((x>=xjm1)&(x<xj)).*(x-xjm1).*(xj-x)*fohs;
```

(c) [15 points]
The plot and code used to create it are below.




```
clear
clc
N=3;
x=linspace(0,1,10000);

figure(1)
clf
colors='rgbcmyk';
for k=1:3
    plot(x,cpq1(x,k,N),colors(k))
    hold on
end
for k=1:4
    plot(x,cpq2(x,k,N),colors(k+3))
end

legend('\phi_1(x)','\phi_2(x)','\phi_3(x)','\psi_1(x)','\psi_2(x)','\psi_3(x)',...
    '\psi_4(x)','Location','EastOutside')
xlabel('x');
saveas(figure(1),'hw29c.eps','psc2')
```