CAAM 336 · DIFFERENTIAL EQUATIONS

Recitation Example 3

Will be worked through on 23 September 2013.

3. Let $\mathcal{V}$ and $\mathcal{W}$ be real vector spaces, and suppose $f : \mathcal{V} \to \mathcal{W}$ is a linear operator.
   The *range* of f is the set of all vectors in $\mathcal{W}$ that can be written in the form $f(v)$ for some $v \in \mathcal{V}$:

   $$\mathcal{R}(f) = \{f(v) : v \in \mathcal{V}\}.$$

   Show that $\mathcal{R}(f)$ is a subspace of $\mathcal{W}$.

   (The *range* generalizes the notion of *column space* from matrix theory.)