CAAM 336 · DIFFERENTIAL EQUATIONS

Recitation Example 1

Will be worked through on 9 September 2013.

1. Consider a bar of metal alloy manufactured such that its thermal conductivity is $\kappa(x) = 1 + \alpha x$ for constant α and $0 \leq x \leq \ell$.

   (a) Determine a general formula for the steady-state temperature distribution of this bar, which should include two free constants. You may assume the homogeneous heat equation for a non-uniform bar:
   $$c(x)\rho(x)u_t(x,t) = \frac{\partial}{\partial x}\Big(\kappa(x)u_x(x,t)\Big).$$

   (b) Find formulas for these free constants in the case that the ends of the bar are submerged in ice baths of γ degrees on the left and δ degrees on the right.

   (c) Now find formulas for the free constants in the case that the left end has a fixed *heat flux* equal to γ (measured in $\mathrm{J}/(\mathrm{m}^2 \cdot \mathrm{sec})$) and the right end is submerged in an ice bath of δ degrees.