CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 46

Posted Wednesday 9 April 2014. Due 1pm Friday 25 April 2014.

46. [25 points]

Let the norm $\|\cdot\|:\ \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$\|\mathbf{y}\| = \sqrt{\mathbf{y}\cdot\mathbf{y}}.$$

Let the timestep $\Delta t \in \mathbb{R}$ be such that $\Delta t > 0$ and let $t_k = k\Delta t$ for $k = 0, 1, 2, \ldots$. Let

$$\mathbf{A} = \begin{bmatrix} -50 & 49 \\ 49 & -50 \end{bmatrix}$$

and consider the problem of finding $\mathbf{x}(t)$ such that

$$\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t), \quad t \geq 0$$

and

$$\mathbf{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

(a) Compute $\mathbf{x}(t)$.

(b) How does $\|\mathbf{x}(t)\|$ behave as $t \to \infty$?

(c) For $k = 0, 1, 2, \ldots$, let $\mathbf{x}_k$ be the approximation to $\mathbf{x}(t_k)$ obtained using the forward Euler method. What choice of the timestep $\Delta t > 0$ will result in $\|\mathbf{x}_k\| \to 0$ as $k \to \infty$?

(d) For $k = 0, 1, 2, \ldots$, let $\mathbf{x}_k$ be the approximation to $\mathbf{x}(t_k)$ obtained using the backward Euler method. What choice of the timestep $\Delta t > 0$ will result in $\|\mathbf{x}_k\| \to 0$ as $k \to \infty$?