

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 38

Posted Wednesday 13 November 2013. Due 5pm Wednesday 20 November 2013.

38. [25 points] Parts (a) and (c) of this question should be done by hand.

Let

$$f(x, t) = \begin{cases} 2x & \text{if } x \in \left[0, \frac{1}{2}\right), \\ 2 - 2x & \text{if } x \in \left[\frac{1}{2}, 1\right]. \end{cases}$$

- (a) Use the spectral method to obtain a series solution to the problem of finding the solution $\tilde{u}(x, t)$ to the heat equation

$$\tilde{u}_t(x, t) - \tilde{u}_{xx}(x, t) = f(x, t), \quad 0 < x < 1, \quad t > 0$$

with Dirichlet boundary conditions

$$\tilde{u}(0, t) = \tilde{u}(1, t) = 0, \quad t \geq 0$$

and initial condition

$$\tilde{u}(x, 0) = 0, \quad 0 < x < 1.$$

- (b) Plot the approximations to $\tilde{u}(x, t)$ obtained by replacing the upper limit of the summation in your series solution with 20 for $t = 0, 0.1, 0.2, 0.3, 0.5, 1, 2$.
- (c) By shifting the data and then using the spectral method, obtain a series solution to the problem of finding the solution $u(x, t)$ to the heat equation

$$u_t(x, t) - u_{xx}(x, t) = f(x, t), \quad 0 < x < 1, \quad t > 0$$

with Dirichlet boundary conditions

$$u(0, t) = 0, \quad t \geq 0$$

and

$$u(1, t) = 1, \quad t \geq 0$$

and initial condition

$$u(x, 0) = x^3, \quad 0 < x < 1.$$

- (d) Plot the approximations to $u(x, t)$ obtained by replacing the upper limit of the summation in your series solution with 20 for $t = 0, 0.1, 0.2, 0.3, 0.5, 1, 2$.