CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 14

Posted Wednesday 18 September 2013. Due 5pm Wednesday 25 September 2013.

14. [25 points]
Determine whether or not each of the following mappings is an inner product on the real vector space $\mathcal{V}$. If not, show **all the properties** of the inner product that are violated.

(a) $(\cdot,\cdot):\ \mathcal{V}\times\mathcal{V}\to\mathbb{R}$ defined by $(u,v)=\displaystyle\int_0^1 u(x)v'(x)\,dx$ where $\mathcal{V}=C^1[0,1]$.

(b) $(\cdot,\cdot):\ \mathcal{V}\times\mathcal{V}\to\mathbb{R}$ defined by $(u,v)=\displaystyle\int_0^1 |u(x)||v(x)|\,dx$ where $\mathcal{V}=C[0,1]$.

(c) $(\cdot,\cdot):\ \mathcal{V}\times\mathcal{V}\to\mathbb{R}$ defined by $(u,v)=\displaystyle\int_0^1 u(x)v(x)e^{-x}\,dx$ where $\mathcal{V}=C[0,1]$.

(d) $(\cdot,\cdot):\ \mathcal{V}\times\mathcal{V}\to\mathbb{R}$ defined by $(u,v)=\displaystyle\int_0^1 u(x)+v(x)\,dx$ where $\mathcal{V}=C[0,1]$.