CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 19

Posted Friday 14 February 2014. Due 1pm Friday 28 February 2014.

19. [25 points]

All parts of this question should be done by hand.

Let

$$C_z^1[-1,1] = \left\{ v \in C^1[-1,1] : \int_{-1}^{1} v(x)\,dx = 0 \right\}.$$

Let $v_1 \in C_z^1[-1,1]$, $v_2 \in C_z^1[-1,1]$, and $f \in C_z^1[-1,1]$ be defined by

$$v_1(x) = \frac{\sqrt{3}}{\sqrt{2}}x,$$

$$v_2(x) = \frac{\sqrt{3}}{\sqrt{2}}(3x^2 - x - 1),$$

and

$$f(x) = \frac{\sqrt{2}}{\sqrt{3}}\cos(\pi x)$$

for all $x \in [-1,1]$. Let the inner product $(\cdot,\cdot) : C_z^1[-1,1] \times C_z^1[-1,1] \to \mathbb{R}$ be defined by

$$(u,v) = \int_{-1}^{1} u(x)v(x)\,dx$$

and let the norm $\|\cdot\| : C_z^1[-1,1] \to \mathbb{R}$ be defined by

$$\|u\| = \sqrt{(u,u)}.$$

Also, let the inner product $a(\cdot,\cdot) : C_z^1[-1,1] \times C_z^1[-1,1] \to \mathbb{R}$ be defined by

$$a(u,v) = \int_{-1}^{1} (2+x)u'(x)v'(x)\,dx$$

and let the norm $\|\cdot\|_a : C_z^1[-1,1] \to \mathbb{R}$ be defined by

$$\|u\|_a = \sqrt{a(u,u)}.$$

Moreover, let the inner product $B(\cdot,\cdot) : C_z^1[-1,1] \times C_z^1[-1,1] \to \mathbb{R}$ be defined by

$$B(u,v) = a(u,v) + (u,v)$$

and the norm $\|\cdot\|_B : C_z^1[-1,1] \to \mathbb{R}$ be defined by

$$\|u\|_B = \sqrt{B(u,u)}.$$

Note that $(v_1,v_1) = 1$; $(v_2,v_2) = \frac{17}{5}$; $(f,v_1) = 0$; $(f,v_2) = -\frac{12}{\pi^2}$; $a(v_1,v_1) = 6$; $a(v_2,v_2) = 66$; $a(f,v_1) = -2$ and $a(f,v_2) = -22$.

(a) Use the fact that $(\cdot,\cdot)$ and $a(\cdot,\cdot)$ are inner products on $C_z^1[-1,1]$ to verify that $B(\cdot,\cdot)$ is an inner product on $C_z^1[-1,1]$.

(b) What is the best approximation to f from $\text{span}\{v_1\}$ with respect to the norm $\|\cdot\|$?

(c) What is the best approximation to f from $\text{span}\{v_1\}$ with respect to the norm $\|\cdot\|_a$?

(d) What is the best approximation to f from $\text{span}\{v_1\}$ with respect to the norm $\|\cdot\|_B$?

(e) What is the best approximation to f from $\text{span}\{v_1, v_2\}$ with respect to the norm $\|\cdot\|_a$?

(f) What is the best approximation to f from $\text{span}\{v_1, v_2\}$ with respect to the norm $\|\cdot\|$?