

CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 3 · Solutions

Posted Wednesday 15 January 2014. Due 1pm Monday 27 January 2014.

3. [25 points]

For each of the following equations, (a) specify whether it is an ODE or a PDE; (b) determine its order; and (c) specify whether it is linear or nonlinear. For those that are linear, specify whether they (d) are homogeneous or inhomogeneous; and (e) have constant or variable coefficients.

$$(1.1) \quad \frac{dv}{dx} + \frac{2}{x}v = 0$$

$$(1.2) \quad \frac{\partial v}{\partial t} - 3\frac{\partial v}{\partial x} = x - t$$

$$(1.3) \quad \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left(2u \frac{\partial u}{\partial x} \right) = 0$$

$$(1.4) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

$$(1.5) \quad \frac{d^2 y}{dx^2} - 7(1 - y^2) \frac{dy}{dx} + y = 0$$

$$(1.6) \quad \frac{d^2}{dx^2} \left(x^2 \frac{d^2 u}{dx^2} \right) = \sin(x)$$

Solution.

(1.1) [5 points] ODE, first order, linear, homogeneous, variable coefficient

The $2/x$ factor in front of the v is the variable coefficient.

(1.2) [5 points] PDE, first order, linear, inhomogeneous, constant coefficient

The $x - t$ term on the right, which does not involve v , makes the equation inhomogeneous.

(1.3) [3 points] PDE, second order, nonlinear

Using the product rule for partial derivatives, we can write this equation in the equivalent form

$$\frac{\partial u}{\partial t} - 2 \left(\frac{\partial u}{\partial x} \right)^2 - 2u \left(\frac{\partial^2 u}{\partial x^2} \right) = 0.$$

The second and third terms on the left hand side make this equation nonlinear.

(1.4) [3 points] PDE, third order, nonlinear

The $u(\partial u / \partial x)$ term makes this equation nonlinear. This is a version of the famous Korteweg-de Vries (KdV) equation that describes shallow water waves.

(1.5) [3 points] ODE, second order, nonlinear

The $(1 - y^2)(dy/dx)$ term makes this ODE nonlinear.

(1.6) [6 points] ODE, fourth order, linear, inhomogeneous, variable coefficient

Using the product rule for partial derivatives, we can write this equation in the equivalent form

$$2 \frac{d^2 u}{dx^2} + 4x \frac{d^3 u}{dx^3} + x^2 \frac{d^4 u}{dx^4} = \sin(x),$$

hence we can see that it is fourth order.
