

# CAAM 336 · DIFFERENTIAL EQUATIONS

## Homework 29

Posted Friday 21 March 2014. Due 1pm Friday 28 March 2014.

29. [25 points]

Let  $N$  be a positive integer, let  $h = \frac{1}{N+1}$  and let  $x_k = kh$  for  $k = 0, 1, \dots, N+1$ . Let the continuous piecewise quadratic functions  $\phi_j \in C[0, 1]$  be such that

$$\phi_j(x) = \begin{cases} \frac{(x - x_{j-1})(2x - x_{j-1} - x_j)}{h^2} & \text{if } x \in [x_{j-1}, x_j], \\ \frac{(x_j + x_{j+1} - 2x)(x_{j+1} - x)}{h^2} & \text{if } x \in [x_j, x_{j+1}], \\ 0 & \text{otherwise,} \end{cases}$$

for  $j = 1, \dots, N$  and let the continuous piecewise quadratic bubble functions  $\psi_j \in C[0, 1]$  be such that

$$\psi_j(x) = \begin{cases} \frac{4(x - x_{j-1})(x_j - x)}{h^2} & \text{if } x \in [x_{j-1}, x_j], \\ 0 & \text{otherwise,} \end{cases}$$

for  $j = 1, \dots, N+1$ .

- Write a MATLAB function for  $\phi_j(x)$ . It should take in as input  $x$ ,  $j$ , and  $N$ . It should return the value  $\phi_j(x)$ . It should also be able to take in a vector for  $\mathbf{x} = (\hat{x}_1, \dots, \hat{x}_m)$  and return the vector  $\phi_j(\mathbf{x}) = (\phi_j(\hat{x}_1), \dots, \phi_j(\hat{x}_m))$ .
- Write a MATLAB function for  $\psi_j(x)$ . It should take in as input  $x$ ,  $j$ , and  $N$ . It should return the value  $\psi_j(x)$ . It should also be able to take in a vector for  $\mathbf{x} = (\hat{x}_1, \dots, \hat{x}_m)$  and return the vector  $\psi_j(\mathbf{x}) = (\psi_j(\hat{x}_1), \dots, \psi_j(\hat{x}_m))$ .
- For  $N = 3$ , plot  $\phi_1(x), \phi_2(x), \phi_3(x), \psi_1(x), \psi_2(x), \psi_3(x), \psi_4(x)$  on the same figure. If you experience problems with displaying  $\psi$  using the latex interpreter then you may use the command `legend('phi_1(x)', 'phi_2(x)', 'phi_3(x)', 'psi_1(x)', 'psi_2(x)', 'psi_3(x)', 'psi_4(x)')` to produce the legend.