CAAM 336 · DIFFERENTIAL EQUATIONS

Homework 24

Posted Monday 7 October 2013. Due 1pm Friday 18 October 2013.

24. [25 points] Let the inner product $(\cdot,\cdot) : C[0,1] \times C[0,1] \to \mathbb{R}$ be defined by

$$(v, w) = \int_0^1 v(x)w(x)\,dx.$$

Consider the linear operator $L : C^2_m[0,1] \to C[0,1]$ defined by

$$Lu = -u''$$

where

$$C^2_m[0,1] = \left\{u \in C^2[0,1] : u'(0) = u(1) = 0\right\}.$$

(a) Is L symmetric?

(b) What is the null space of L?

(c) Show that $(Lu, u) \geq 0$ for all $u \in C^2_m[0,1]$ and explain why this and the answer to part (b) mean that $\lambda > 0$ for all eigenvalues λ of L.

(d) Find the eigenvalues and eigenfunctions of L.