Multiple Regression

(adapted in part from Dr. Andy Field)

Goals

- When To Use Multiple Regression.
- The multiple regression equation and what the betas represent.
- Different Methods of Regression Model Building
- How to Interpret multiple regression.
- Assumptions of Multiple Regression and how to test them.

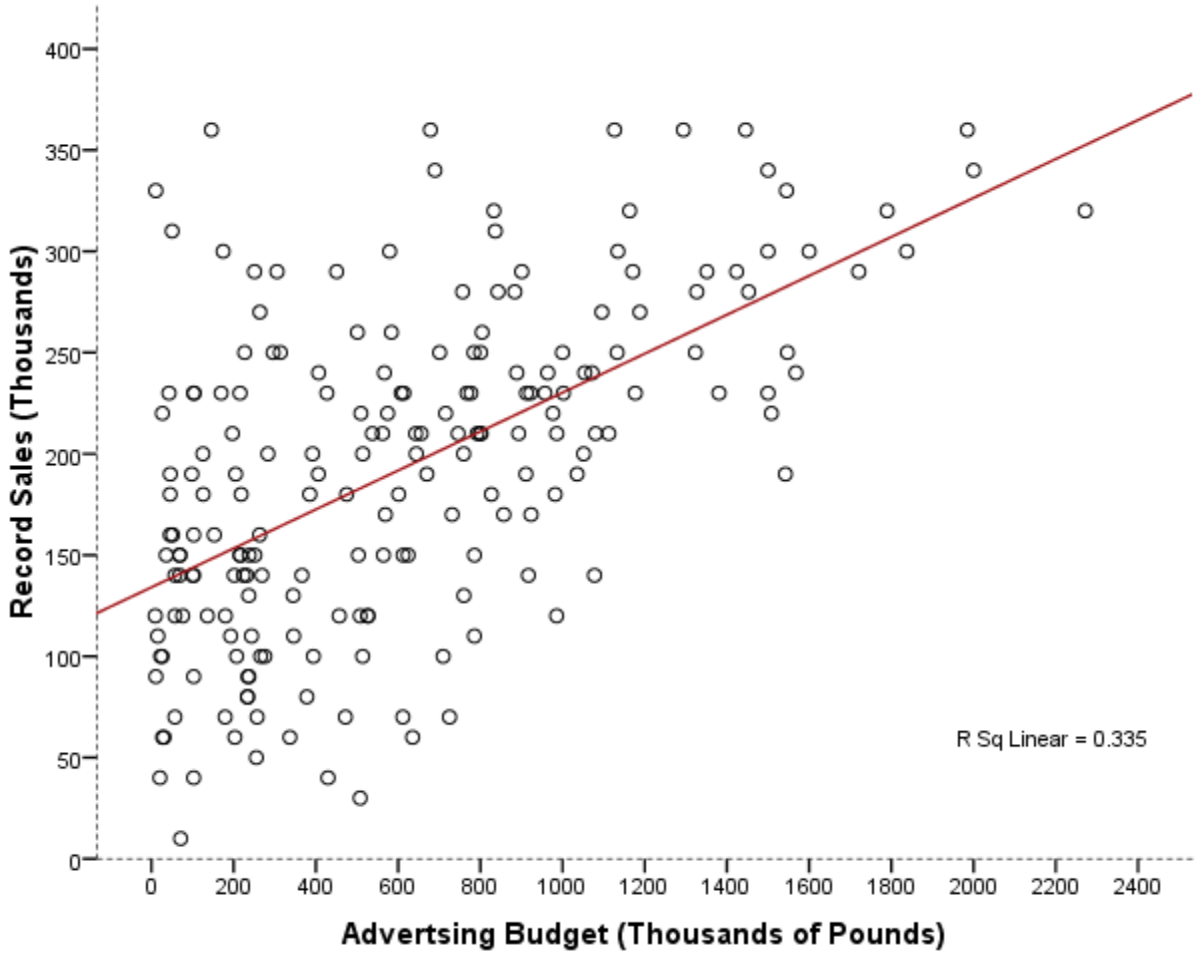
What is Multiple Regression?

- Linear Regression is a model to predict the value of one variable from another.
- Multiple Regression is a natural extension of this model:
  - We use it to predict values of an outcome from *several* predictors.
  - It is a hypothetical model of the relationship between several variables.

Regression: An Example

- A record company boss was interested in predicting record sales from advertising.
- Data
  - 200 different album releases
- Outcome variable:
  - Sales (CDs and Downloads) in the week after release
- Predictor variables
  - The amount (in £s) spent promoting the record before release
  - Number of plays on the radio
  - Attractiveness of band

The Model with One Predictor



Multiple Regression as an Equation

- With multiple regression the relationship is described using a variation of the equation of a straight line.

$$y = b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n + \varepsilon_i$$

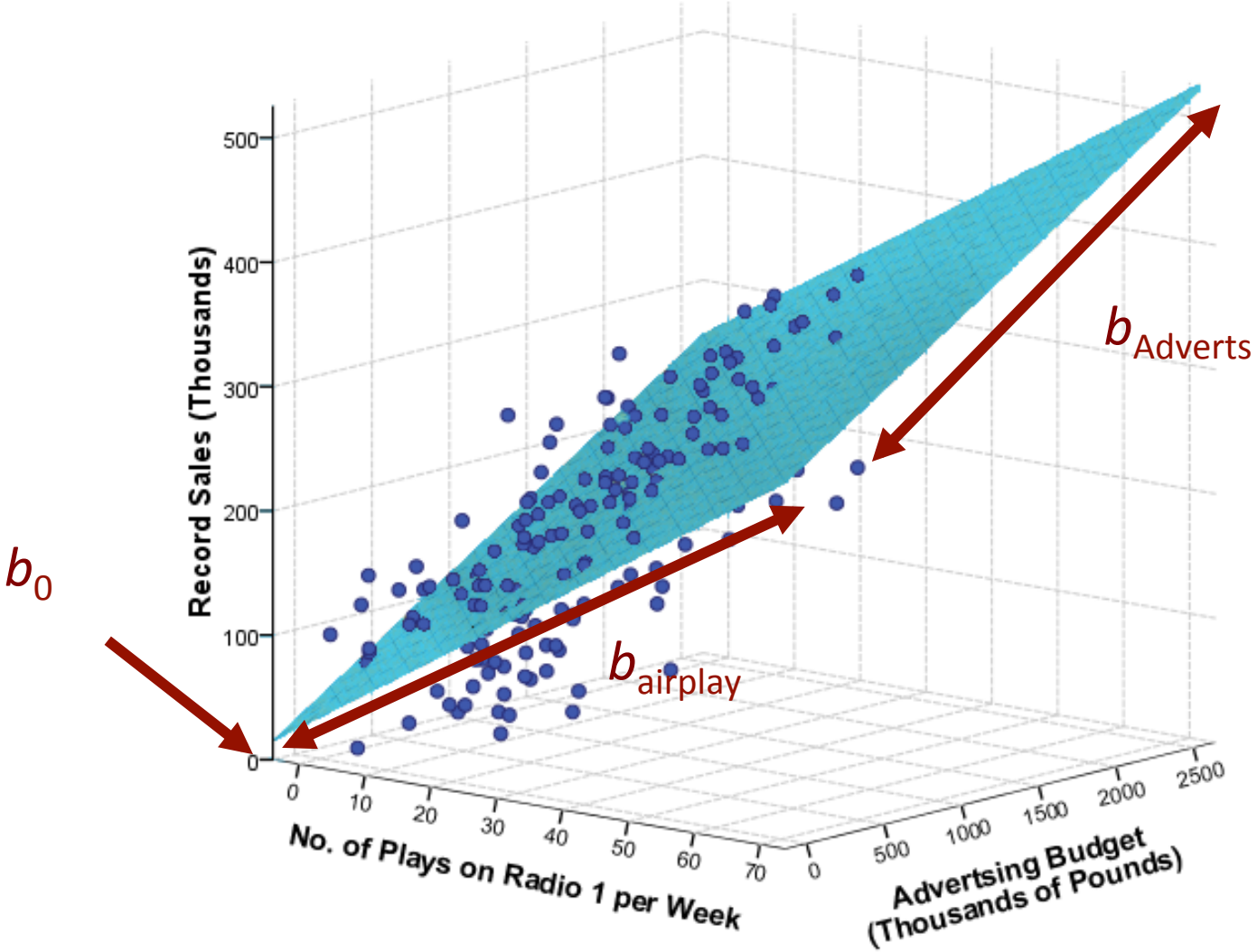
b_0

- b_0 is the intercept.
- The intercept is the value of the Y variable when all Xs = 0.
- This is the point at which the regression plane crosses the Y-axis (vertical).

Beta Values

- b_1 is the regression coefficient for variable 1.
- b_2 is the regression coefficient for variable 2.
- b_n is the regression coefficient for n^{th} variable.

The Model with Two Predictors



Methods of Regression

- Stepwise:
  - Predictors are selected using their semi-partial correlation with the outcome.
- Forced Entry:
  - All predictors are entered simultaneously.
- Hierarchical:
  - Experimenter decides the order in which variables are entered into the model.


Which method of regression should I use?

Stepwise Regression

- Variables are entered into the model based on mathematical criteria.
- Computer selects variables in steps.

Problems with Stepwise Methods

- Rely on a mathematical algorithm.
- Should be used only for exploration

Forced Entry Regression

- All variables are entered into the model simultaneously.
- The results obtained depend on the variables entered into the model.
  - It is important, therefore, to have good theoretical reasons for including a particular variable.

Hierarchical Regression

- Known predictors (based on past research) are entered into the regression model first.
- New predictors are then entered in a separate step/block.
- Experimenter makes the decisions.

Hierarchical Regression

- Based on theory testing:
  - You can see the unique predictive influence of a new variable on the outcome because known predictors are held constant in the model.
- Bad Point:
  - Relies on the experimenter knowing what they're doing!

Output: Model Summary

Model Summary[c]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics					Durbin-Watson
					R Square Change	F Change	df1	df2	Sig. F Change	
1	.578[a]	.335	.331	65.9914	.335	99.587	1	198	.000	
2	.815[b]	.665	.660	47.0873	.330	96.447	2	196	.000	1.950

a. Predictors: (Constant), Advertising Budget (thousands of pounds)

b. Predictors: (Constant), Advertising Budget (thousands of pounds), Attractiveness of Band, No. of plays on Radio 1 per week

c. Dependent Variable: Record Sales (thousands)

R and R^2

- R
  - The correlation between the observed values of the outcome, and the values predicted by the model.
- R^2
  - Yhe proportion of variance accounted for by the model.
- Adj. R^2
  - An estimate of R^2 in the population (*shrinkage*).

Output: ANOVA

ANOVA[c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	433687.833	1	433687.833	99.587	.000[a]
	Residual	862264.167	198	4354.870		
	Total	1295952.0	199			
2	Regression	861377.418	3	287125.806	129.498	.000[b]
	Residual	434574.582	196	2217.217		
	Total	1295952.0	199			

a. Predictors: (Constant), Advertising Budget (thousands of pounds)

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Analysis of Variance: ANOVA

- The F-test
  - looks at whether the variance explained by the model (SS_M) is significantly greater than the error within the model (SS_R).
  - It tells us whether using the regression model is significantly better at predicting values of the outcome than using the mean.

Output: betas

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	134.140	7.537		17.799	.000	119.278	149.002
	Advertsing Budget (thousands of pounds)	.096	.010	.578	9.979	.000	.077	.115
2	(Constant)	-26.613	17.350		-1.534	.127	-60.830	7.604
	Advertsing Budget (thousands of pounds)	.085	.007	.511	12.261	.000	.071	.099
	No. of plays on Radio 1 per week	3.367	.278	.512	12.123	.000	2.820	3.915
	Attractiveness of Band	11.086	2.438	.192	4.548	.000	6.279	15.894

a. Dependent Variable: Record Sales (thousands)

How to Interpret Beta Values

- Beta values:
  - the change in the outcome associated with a unit change in the predictor.
- Standardised beta values:
  - tell us the same but expressed as standard deviations.

Beta Values

- $b_1 = 0.085$.
  - So, as advertising increases by £1,000, record sales increase by 0.085 units (thousands of records).
- $b_2 = 3.367$.
  - So, each time (per week) a song is played on radio 1 its sales increase by 3.367 units (thousands of records).

Beta Values

- $b_3 = 11.086$.
  - So, as attractiveness of band increases by 1 point on the scale, record sales increase by 11.086 units (thousands of records).

Predicting Outcomes

$$y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3$$

2	(Constant)	-26.613
	Advertsing Budget (thousands of pounds)	.085
	No. of plays on Radio 1 per week	3.367
	Attractiveness of Band	11.086

a. Dependent Variable: Record Sales (thousands)

- What is the predicted outcome if:
  - £11 million advertising budget
  - 15 plays on Radio One per week
  - Band attractiveness is a "2"

Standardised Beta Values

- β_1= 0.511
- β_2= 0.512
- β_3= 0.192

- Which X is the best predictor of Y?

Reporting the Model

TABLE 7.2 How to report multiple regression

	B	*SE B*	β
Step 1			
Constant	134.14	7.54	
Advertising Budget	0.10	0.01	.58*
Step 2			
Constant	–26.61	17.35	
Advertising Budget	0.09	0.01	.51*
Plays on BBC Radio 1	3.37	0.28	.51*
Attractiveness	11.09	2.44	.19*

Predictors as "Controls"

- Very often, but not always, you are looking at the relationship between a key independent variable and a dependent variable.

Predictors as “Controls”

- You may need to include other independent variables to ‘control for’ other known correlates with the outcome
- Thus, some independent variables are called ‘controls’

Generalization

- When we run regression, we hope to be able to generalize the sample model to the entire population.
- To do this, several assumptions must be met.
- Violating these assumptions stops us generalizing conclusions to our target population.

Straightforward Assumptions

- Variable Type / Measurement:
  - Outcome must be continuous
  - Predictors can be continuous or dichotomous.
- Non-Zero Variance:
  - Predictors must not have zero variance.
- Linearity:
  - The relationship we model is, in reality, linear.
- Independence:
  - All values of the outcome should come from a different person.

Sample Size

- N should be greater than: 10 * k (k= the number of independent variables

The More Tricky Assumptions

- Models should be **parsimonious**:
  - or, as 'lean' as possible.
  - Don't add variables that have no theoretical basis.
- No Multicollinearity:
  - Predictors must not be highly correlated (Pearson's R of 0.7 or higher).
- No circularity:
  - Or, no variables that predict Y because they are in some way a representation of Y (Pearson's R of 0.8 or higher).

The gravest Regression error

- Omitted variable bias
  - Theory is most important here
- Remedies
  - More data collection
  - Careful survey design
  - A proxy for the omitted variable

What will be on the final exam?

- **Bivariate Hypothesis Tests**
  - Two-Sample t-tests
  - ANOVA
  - Chi-Square
  - Simple Regression
- **Multivariate Hypothesis Test**
  - Multiple Regression
- **Measures of Association/Correlation**
- **Key concepts of statistics**

Format of the final exam

- Minimal calculations/formulas
- Interpretation of R output
- Cases: research ‘situations’ presented, and the answer will be an essay on the approach to take, which variables to use, which hypothesis test, and what the results would tell you
- Definition of key concepts of statistics