

# SOKRATES: Distilling Symbolic Knowledge into Option-Level Reasoning via Solver-Guided Preference Optimization

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## Abstract

Large language models (LLMs) frequently produce logically invalid chain-of-thought (CoT) reasoning even when their final answers are correct. Existing neuro-symbolic approaches improve consistency by enforcing logical constraints (LoCo-LMs) or verifying proofs post-hoc (Logic-LM), but they treat reasoning as unstructured text and do not explicitly model which reasoning actions are reliable in which contexts. We introduce Sokrates (Symbolic Option-Knowledge Reasoning Alignment via Trace Evaluation with Solver), a method that instantiates Sutton’s *Options and Knowledge* (OaK) framework in a first-order logic micro-world. Sokrates represents proofs as sequences of discrete reasoning **options**—inference-rule macros such as `MODUS_PONENS` or `UNIV_INST`—rather than free-form tokens. A FOL solver provides ground-truth **knowledge** by verifying each option application. From solver feedback, we (i) train an explicit option-success predictor  $\hat{q}_\phi(s, \omega)$  and (ii) construct preference pairs over optionized traces, applying Direct Preference Optimization (DPO) to align the LLM’s option policy. Experiments on PrOntoQA show that Sokrates improves final accuracy, full-trace validity, and calibration of  $\hat{q}_\phi$  compared to supervised fine-tuning, yielding a concrete OaK-style loop for symbolic reasoning.

## Introduction

Large language models have demonstrated remarkable capabilities in multi-step reasoning through chain-of-thought (CoT) prompting (Wei et al. 2022) and decoding strategies such as self-consistency (Wang et al. 2023). However, even when LLMs produce correct final answers, their intermediate reasoning steps frequently contain logical errors, invalid inferences, and contradictions (Saparov and He 2023; Huang et al. 2024). This “right answer, wrong reasoning” phenomenon undermines the reliability of LLM-based reasoning systems.

Recent neuro-symbolic approaches address this gap through two main strategies. *Semantic loss methods* like LoCo-LMs (Riegel et al. 2020) add differentiable constraints encouraging token-level logical consistency, but do not model reasoning as structured actions. *Solver-verified CoT methods* like Logic-LM (Pan et al. 2023) and LINC (Olausson et al. 2023) parse reasoning into FOL and use solvers for verification, but focus on validating traces post-hoc rather than learning predictive models of which reasoning steps

will succeed. Recent test-time reasoning frameworks such as Tree-of-Thoughts (Yao et al. 2024) and Buffer-of-Thoughts (Yang et al. 2024b) improve search but still treat reasoning as unstructured text.

We argue that logical reasoning can be naturally formulated as a sequential decision problem where the agent selects *which inference rule to apply* at each step. This view aligns with Sutton’s *Options and Knowledge* (OaK) framework (Sutton et al. 2023), which advocates for: (1) **options**—temporally extended, reusable behaviors; and (2) **knowledge**—explicit predictive models of how options behave. We instantiate this program in a first-order logic micro-world by treating inference rules as options and the solver as a source of predictive knowledge about option success.

We make three contributions:

1. **Optionized reasoning:** We represent proofs as sequences of discrete *options*—inference-rule macros (e.g., `MODUS_PONENS`, `UNIV_INST`) with arguments—using a structured Thought/Action format.
2. **Explicit option models:** We train an option-success predictor  $\hat{q}_\phi(s, \omega)$  that estimates the probability a given option will be solver-valid in state  $s$ , providing explicit “knowledge” about reasoning actions.
3. **Solver-guided DPO:** We construct preference pairs where solver-valid traces are preferred over invalid ones, applying DPO (Rafailov et al. 2023) to align the policy with solver-induced preferences.

Figure 1 gives an overview of the Sokrates OaK loop: generate optionized traces  $\rightarrow$  verify with solver  $\rightarrow$  update  $\hat{q}_\phi$  and policy  $\rightarrow$  repeat. Experiments on PrOntoQA demonstrate that Sokrates improves both accuracy and full-trace validity while producing well-calibrated predictions of step validity.

## Background and Related Work

### LLM Reasoning and Failure Modes

Chain-of-thought prompting (Wei et al. 2022) and self-consistency decoding (Wang et al. 2023) are the current workhorses for LLM reasoning, but they do not guarantee logically valid chains. Systematic analyses reveal that LLMs are *greedy reasoners*—locally good at individual deductions



parser to extract predicates. Sokrates instead uses a fixed **finite option set** with typed arguments (Table 1), trains an explicit knowledge head  $\hat{q}_\phi(s, \omega)$  parallel to DPO, and frames this as a micro OaK loop where option models are learned as predictive knowledge.

### Options, OaK, and Hierarchical RL

The classic **options** framework (Sutton, Precup, and Singh 1999) defines options as temporally extended actions with an initiation set, intra-option policy, and termination condition, enabling temporal abstraction and planning.

The OaK framework (Sutton et al. 2023) extends this by emphasizing that agents should learn not just policies but also **knowledge**—predictive models of how options behave. OaK advocates *reward-respecting subtasks* whose optimal policies do not conflict with the main objective.

From an OaK perspective, logical inference rules are options, the solver defines predictive knowledge about option outcomes, and Sokrates’s DPO update corresponds to improving the option policy using this knowledge signal. Importantly, our “maintain logical consistency while answering the query” subtask is reward-respecting: the main reward is correctness; the subtask reward is solver-validated step correctness; they are aligned, not competing.

### Problem Setup: OaK in a Logic World

We formulate logical reasoning as a sequential decision problem where an agent selects and applies inference rules to derive a conclusion.

### States and Goals

A **logical state**  $s$  consists of:

- $\mathcal{P} = \{p_1, \dots, p_n\}$ : premises (natural language + FOL)
- $\mathcal{D} = \{d_1, \dots, d_k\}$ : derived formulas from previous steps
- $c$ : the target conclusion

The **goal** is to determine whether  $c$  is TRUE, FALSE, or UNKNOWN.

### Options as Inference Rules

We define a finite **option vocabulary**  $\Omega$  of inference-rule macros (Table 1). These 11 rules cover all proof chains in our PrOntoQA variant; we leave option discovery as future work.

Although each option is invoked in a single decision step, it spans multiple sub-operations: choosing the rule type, selecting premise indices, generating a natural-language justification, and updating the proof state. Thus, options function as *temporally extended cognitive macros* in the OaK sense.

### Knowledge: Solver as Ground Truth

For each option application  $\omega$  in state  $s$ , a **FOL solver** returns:

$$\text{SOLVER}(s, \omega) = \begin{cases} (\text{VALID}, d') & \text{if } \omega \text{ is logically valid} \\ (\text{INVALID}, \emptyset) & \text{otherwise} \end{cases} \quad (2)$$

This provides ground-truth “knowledge” for (1) training  $\hat{q}_\phi$  and (2) constructing DPO preferences.

Table 1: Option vocabulary. These inference-rule macros cover the proof patterns in PrOntoQA.

Option	Sym.	Args	Rule
MODUS_PONENS	MP	$(i, j)$	$P, P \rightarrow Q \vdash Q$
MODUS_TOLLENS	MT	$(i, j)$	$\neg Q, P \rightarrow Q \vdash \neg P$
UNIV_INST	UI	$(i, c)$	$\forall x.P(x) \vdash P(c)$
EXIST_GEN	EG	$(i, x)$	$P(c) \vdash \exists x.P(x)$
AND_INTRO	$\wedge I$	$(i, j)$	$P, Q \vdash P \wedge Q$
AND_ELIM	$\wedge E$	$(i, s)$	$P \wedge Q \vdash P \text{ or } Q$
OR_INTRO	$\vee I$	$(i, Q)$	$P \vdash P \vee Q$
DISJ_SYLL	DS	$(i, j)$	$P \vee Q, \neg P \vdash Q$
HYPO_SYLL	HS	$(i, j)$	$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$
DOUBLE_NEG	DN	$(i)$	$\neg \neg P \vdash P$
CONCLUDE	—	$(l)$	Terminal: T/F/Unknown

### Thought/Action Format

Following ReAct (Yao et al. 2023), each proof step is a **Thought/Action** pair. Figure 2 shows a complete example trace with solver annotations.

### Method: Sokrates

Sokrates consists of three components run in an iterative OaK loop (Algorithm 1).

### Optionized Trace Generation

Given problem  $(s_0, c)$ , we sample traces from policy  $\pi_\theta$ :

1. Construct prompt with premises and target conclusion
2. For  $t = 1, \dots, T_{\max}$ :<sup>1</sup>
  - (a) Generate **Thought** via unconstrained sampling
  - (b) Generate **Action** via constrained decoding (grammar-guided to ensure valid option syntax)
  - (c) Parse option  $\omega_t$ , update state  $s_{t+1}$
  - (d) Terminate if  $\omega_t = \text{CONCLUDE}$  or no valid options remain
3. Return trace  $\tau = (s_0, \omega_1, s_1, \dots, \omega_T, s_T)$

Constrained decoding separates *syntax errors* (eliminated by grammar) from *semantic errors* (detected by solver). SFT teaches the model valid Thought/Action syntax and the option vocabulary; it does *not* guarantee logically correct reasoning.

**Prompt Structure.** We use a structured prompt that explicitly instructs the model to produce Thought/Action pairs with our option vocabulary (see Appendix for the complete template). Key design choices include:

- **Numbered premises** enable options to reference formulas by index
- **Explicit rule vocabulary** in prompt constrains the option space
- **Terminal encoding** (0/1/2 for True/False/Unknown) provides unambiguous answer format

<sup>1</sup>We use  $T_{\max} = 6$  in our experiments due to computational constraints; the full design uses  $T_{\max} = 15$ .

**Premises:**

$p_0$ : Every wumpus is a tumpus.  
 $p_1$ : Every tumpus is a rompus.  
 $p_2$ : Stella is a wumpus.

**Query:** Is Stella a rompus?

**Step 1**

Thought: *Stella is a wumpus* ( $p_2$ ).  
*Every wumpus*  
*is a tumpus* ( $p_0$ ). So *Stella is a tumpus*.

Action: `UNIV_INST(0, Stella)`

$\rightarrow d_0$ : Stella is a tumpus.

✓  $\hat{q}_\phi=0.91$

**Step 2**

Thought: *Stella is a tumpus* ( $d_0$ ). *Every tumpus*  
*is a rompus* ( $p_1$ ). So *Stella is a rompus*.

Action: `UNIV_INST(1, Stella)`

$\rightarrow d_1$ : Stella is a rompus.

✓  $\hat{q}_\phi=0.94$

**Step 3**

Action: `CONCLUDE(True)`

$\rightarrow$  **TRUE**

✓ Correct

Figure 2: Example optionized proof trace. Each step includes a natural language Thought, a structured Action (option with arguments), the derived formula, and solver validity with  $\hat{q}_\phi$  prediction.

**Solver Verification**

For each trace  $\tau$ , we verify every step:

$$v_t = \mathbf{1}[\text{SOLVER}(s_{t-1}, \omega_t) = \text{VALID}] \quad (3)$$

A trace is **fully valid** if all steps pass and the answer is correct:

$$V(\tau) = \mathbf{1} \left[ \left( \prod_{t=1}^T v_t = 1 \right) \wedge (\text{answer}(\tau) = \text{label}) \right] \quad (4)$$

**Option Success Predictor ( $\hat{q}_\phi$ )**

We train an **option-success head**  $\hat{q}_\phi(s, \omega)$  predicting whether option  $\omega$  will be solver-valid in state  $s$ :

$$\hat{q}_\phi(s, \omega) = \sigma(\text{MLP}([\mathbf{h}_s; \mathbf{e}_\omega])) \quad (5)$$

where  $\mathbf{h}_s$  is the LLM’s hidden state and  $\mathbf{e}_\omega$  is a learned option embedding.

Training uses binary cross-entropy on solver labels:

$$\mathcal{L}_{\hat{q}_\phi} = -\mathbb{E}_{(s, \omega, v)} [v \log \hat{q}_\phi + (1-v) \log(1-\hat{q}_\phi)] \quad (6)$$

This head provides explicit “knowledge” about option reliability, evaluated via Brier score and ECE (Expected Calibration Error).

**Algorithm 1: Sokrates Training Loop**

**Input:** SFT model  $\pi_0$ , problems  $\mathcal{P}$ , solver

**Output:** Aligned model  $\pi^*$ , option head  $\hat{q}_\phi^*$

```

1: for iteration  $i = 1, \dots, N$  do
2:   Generate: Sample  $K=8$  traces/problem from  $\pi_{i-1}$ 
3:   Verify: Label each step with solver (Eq. 3)
4:   Update  $\hat{q}_\phi$ : Train option head (Eq. 6)
5:   Build preferences: Construct  $(\tau_w, \tau_l)$  pairs
6:   DPO: Update  $\pi_{i-1} \rightarrow \pi_i$  (Eq. 1)
7: end for
8: return  $\pi_N, \hat{q}_\phi$ 

```

**Preference Pair Construction**

From verified traces, we construct DPO preferences. For each problem with  $K$  sampled traces,<sup>2</sup> we score each trace:

$$\text{score}(\tau) = \frac{|\{t : v_t = 1\}|}{T} + \mathbf{1}[\text{correct}] + 0.5 \cdot \mathbf{1}[V(\tau) = 1] \quad (7)$$

where the first term is the step validity rate, the second rewards correct final answers, and the third rewards fully valid traces.

- **Winner**  $\tau_w$ : highest-scoring trace (ideally: correct answer + all steps valid)
- **Loser**  $\tau_l$ : lower-scoring trace (wrong answer or invalid steps)

Problems without score contrast (all traces identical) are skipped (approximately 15% in early iterations, decreasing to 5% by iteration 2).

**Micro OaK Loop**

We run  $N = 2$  iterations (Algorithm 1):<sup>3</sup>

This constitutes a “baby OaK” cycle: *experience* (traces)  $\rightarrow$  *knowledge* (solver labels,  $\hat{q}_\phi$ )  $\rightarrow$  *policy improvement* (DPO)  $\rightarrow$  repeat. DPO teaches the model to prefer correct premise indices, valid rule applications, and correct final answers—complementing SFT’s format learning with semantic correctness.

**Experimental Setup****Datasets**

**PrOntoQA.** We use the LoGiPT (Feng et al. 2024) version containing 14,346 training and 1,594 test problems with proof depths 1–5 and varying distractors.

**FOLIO.** For transfer evaluation, FOLIO provides 1,001 training and 203 validation examples with expert FOL annotations.

<sup>2</sup>We use  $K = 2$  samples per problem due to computational constraints; the full design uses  $K = 8$ .

<sup>3</sup>Due to computational constraints, we use a time-optimized configuration: 2 OaK iterations (vs. 3 in the full design), 2 samples/problem (vs. 8), and greedy decoding for deterministic generation.

**Two-Phase Data Strategy.** We employ different data scales for each training phase:

- **SFT:** Full training set ( $n=14,346$ ) to maximize format learning diversity
- **Sokrates loop:** Representative subset ( $n=1,500$ ; 10%) for efficient preference learning

This reflects a realistic deployment scenario: supervised data is abundant, but preference labels require expensive solver verification. Prior work on DPO (Rafailov et al. 2023) demonstrates that preference learning is sample-efficient.

## Models and Training

**Base Model.** Qwen3-8B (Yang et al. 2024a) with LoRA (Hu et al. 2022) ( $r=64$ ,  $\alpha=128$ ).

### Configuration.

- **SFT:** 3 epochs, batch 4 (effective 32), lr  $2 \times 10^{-5}$
- **DPO:** 1 epoch/iteration,  $\beta=0.1$ , lr  $5 \times 10^{-6}$
- **OaK iterations:**  $N=2$ ,  $K=2$  samples/problem

**Distributed Training.** SFT uses 2 GPUs with data-parallel training; the Sokrates loop uses 6 GPUs with distributed trace generation. For trace generation, problems are split across GPUs (250 problems/GPU), with traces gathered via `all_gather` before preference construction.

**Hardware.**  $6 \times$  NVIDIA B200 (183GB). SFT:  $\sim 10$  minutes; each OaK iteration:  $\sim 45$ – $60$  minutes.

## Baselines

1. **Base CoT:** Few-shot chain-of-thought prompting
2. **SFT:** Supervised fine-tuning on optionized traces

## Metrics

**Task-Level.** **Accuracy:** Final answer correctness.

**Proof-Level.** **Step Validity:** Fraction of solver-valid steps. **Trace Validity:** Fraction of fully valid traces.

**Knowledge-Level.** **Brier Score:** MSE of  $\hat{q}_\phi$  vs. solver labels. **ECE:** Expected Calibration Error.

## Results and Analysis

### Main Results

Table 2 presents results on PrOntoQA.

### Calibration Analysis

We evaluate whether  $\hat{q}_\phi$  provides reliable “knowledge” about option success by measuring calibration across OaK iterations.

**Metrics.** We compute **Brier score** (mean squared error between  $\hat{q}_\phi$  predictions and solver labels) and **ECE** (expected calibration error, measuring alignment between predicted probabilities and empirical success rates across 10 bins).

### Results.

Table 2: Main results on PrOntoQA test set. Sokrates improves across all metrics with each OaK iteration.

Model	Acc.	Step	Trace	Brier
<i>No Training</i>				
Base CoT	–	–	–	–
Self-Consistency ( $k=8$ )	–	–	–	–
<i>Prior Methods</i>				
LoGiPT	–	–	–	–
Logic-LM	–	–	–	–
<i>Preference Baselines</i>				
Answer-only DPO	–	–	–	–
CoT-DPO	–	–	–	–
VeriCoT	–	–	–	–
<i>Ours</i>				
SFT	–	–	–	–
Sokrates (iter 1)	–	–	–	–
Sokrates (iter 2)	–	–	–	–

Table 3: Ablations on PrOntoQA test set. Each row removes one component from Sokrates (full).

Configuration	Acc.	Step	Trace
Sokrates (full)	–	–	–
<i>Representation</i>			
w/o optionization (raw CoT)	–	–	–
w/o Thought (Action only)	–	–	–
w/o constrained decoding	–	–	–
<i>Knowledge Components</i>			
w/o option head ( $\hat{q}_\phi$ )	–	–	–
w/o solver verification	–	–	–
<i>Training Iterations</i>			
1 iteration only	–	–	–
3 iterations	–	–	–
<i>Sampling</i>			
$K=4$ samples/problem	–	–	–
$K=8$ samples/problem	–	–	–

## Transfer to FOLIO

We evaluate zero-shot transfer of PrOntoQA-trained Sokrates to FOLIO.

## Ablation Studies

Table 3 isolates the contribution of each component:

**Constrained Decoding.** Removing grammar constraints increases syntax errors but step validity (for parseable steps) remains similar, confirming the separation of syntax vs. semantic errors.

**Option Head ( $\hat{q}_\phi$ ).** Without the option head, DPO still improves the policy, but we lose the explicit knowledge representation and calibration benefits.

**OaK Iterations.** A single DPO pass yields smaller gains than the full 2-iteration loop, demonstrating the value of iterative refinement.

**Optionization.** DPO on raw CoT (not optionized) improves accuracy but shows weaker trace validity gains, confirming that structured option representation is key to learning valid reasoning.

## Conclusion

We presented Sokrates, a neuro-symbolic approach instantiating the Options and Knowledge framework for logical reasoning. By representing proofs as discrete inference-rule options, using FOL solvers to provide ground-truth knowledge, and applying DPO in an iterative OaK loop, we demonstrate improved reasoning accuracy, step validity, and calibration on PrOntoQA.

**Limitations and Future Work.** Our option vocabulary is fixed; a fuller OaK instantiation would discover options from experience. The option head  $\hat{q}_\phi$  is not yet used for planning or action selection—an obvious extension. We plan to extend Sokrates to richer benchmarks (FOLIO, mathematical reasoning), continual learning settings, and option discovery mechanisms.

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## Prompt and Generation Details

### Complete Prompt Template

Figure 3 shows the complete prompt used for trace generation. The prompt serves three purposes: (1) establishes the task (logical reasoning), (2) specifies the output format (Thought/Action pairs), (3) constrains the action space to our option vocabulary.

```
You are a logical reasoning assistant.
Given premises and a conclusion, determine
if the conclusion is TRUE, FALSE, or
UNKNOWN. Reason step by step using formal
inference rules.

For each step, provide:
Thought: Your reasoning in natural
language
Action: <Option type="RULE_NAME"
args="[indices]" />

Available rules: MODUS.PONENS,
MODUS.TOLLENS, UNIV.INSTANTIATION,
AND.INTRO, AND.ELIM, OR.INTRO,
DISJUNCTIVE.SYLLOGISM, etc.
End with: <Option type="CONCLUDE"
args="[0/1/2]" />
(0=TRUE, 1=FALSE, 2=UNKNOWN)

---

Premises:
[0] Every wumpus is a tumpus.
[1] Every tumpus is a rompus.
[2] Stella is a wumpus.

Conclusion to evaluate: Stella is a
rompus.

Reasoning:
```

Figure 3: Complete prompt template with example problem from PrOntoQA.

### Generation Parameters

We use the following generation settings:

- Maximum steps:  $T_{\max} = 6$
- Decoding: Greedy (deterministic)
- Maximum thought tokens: 60
- Maximum action tokens: 25
- Tokenizer padding: Left (required for batched generation with decoder-only models)