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Study on Inertia Wheel Pendulum Applied to Self-Balancing Electric Motorcycle

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Abstract - ***Self-balancing electric motorcycles have been recently developed by high-tech corporations to reaffirm their vision of "enjoying the freedom of mobility", i.e. riders will not have to put their feet on the ground when they stop. A stabilization system based on Inertia Wheel Pendulum (IWP) is a potential approach to this new concept motorcycle. The paper focuses on exploring effects of design parameters on dynamic performance of IWP. A prototype of IWP is implemented. The understanding of IWP is validated through simulation results as well as experimental results. The findings have shown that the prototype has been capable of self-balancing under a PID control algorithm.***

Keywords: ***Inertia Wheel Pendulum; Reaction Wheel Inverted Pendulum; Self-Balancing Electric Motorcycle***

I. INTRODUCTION

Nowadays, together with cutting-edge technologies, small electric portable transport devices have been developed. Figure 1 illustrates I2 SE invented by Segway. This device has a weight of 48kg, be able to speed up 20km/h, and transports up to a distance of 34km. The dynamics of the Segway are similar to a classic control problem, the inverted pendulum. It uses brushless DC electric motors in each wheel powered by lithium-ion batteries with balance achieved using tilt sensors, and gyroscopic sensors. The wheels are driven forward or backward as needed to return its pitch to upright.

Figure 1. *Self-balancing scooter I2 SE by Segway [1]*

With the advantage of compact, flexible, two-wheeled balancing vehicle has been widely used in life. However, it still faces some backlashes of low speed, short distance, potential dangers posting to inexperienced users. Overall, the two-wheeled balancing vehicle is only suitable in small areas such as park, workshop, etc.

The limitations of the two-wheeled balancing vehicle was quickly resolved with self-balancing electric motorcycle in which riders do not have to put their feet on the ground when they stop. In recent years, automotive corporations have introduced many prototypes of self-balancing electric motorcycle, typically from Honda, Lit Motors as demonstrated in Fig. 2.

a) In 2016 by Lit Motors [2]

b) In 2017 by Honda [3]

Figure 2. *Self-balancing electric motorcycles*

For the self-balancing electric motorcycles, there are three approaches using steering control, gyroscopic stabilization, or reaction wheel to create the balance required. In these methods, a self-balancing bicycle robot with a reaction wheel has the same concept as an inverted pendulum that is also known as Inertia Wheel Pendulum (IWP). In comparison with the first two approaches of stabilization, IWP has some advantages of simple modelling and structure that have attracted interests from researchers.

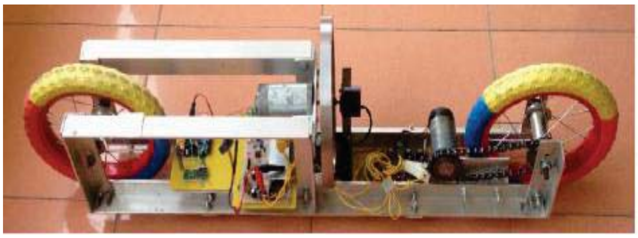

Figure 3. *Prototype of Self-Balancing Electric Motorcycle [4]*

Figure 3 illustrates its prototype developed by Mai [4]. Most researchers mainly focused on control aspects for the IWP, e.g. using LQR and MPC controller by Kanjanawanishkul [5], studying a two-loop control architecture by Nguyen [6]. However, effects of design parameters on dynamic performance of IWP have not been much explored. In this paper, we mainly focus on studying the effects of pendulum length, wheel mass as well as wheel inertial moment on responses of IWP. This facilitates the determination of proper design parameters when applying the IWP to given electric motorcycles. The organization of this paper is as follows: Section II formulates mathematical

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models of the motor and the IWP. This is followed by studying dynamic performance through responses of inertia wheel motion and pendulum angle in Section III. A simple control approach of PID algorithm is applied to the IWP in Section IV that is verified through simulation results in Matlab Simulink and validated through experimental results on an IWP prototype is presented. The paper is summarized in Section V.

II. SYSTEM MODELING

A self-balancing electric motorcycle based on the concept of an inverted pendulum as illustrated in Fig. 4, is an unstable and nonlinear system. To stabilize the system, the following three main components are required, i.e. (1) an IMU sensor that detects the tilt angle of the frame, (2) a controller that is used to control motion of an inertia wheel through an electric motor, and (3) an inertia wheel that is employed to produce reactionary torque to balance.

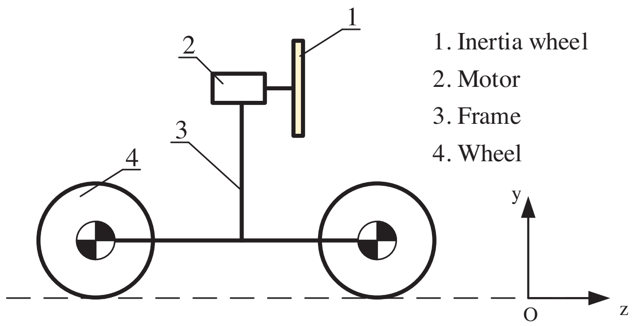


Figure 4. *Structure of a self-balancing electric motorcycle*

In order to study its dynamic response as well as to implement proper control algorithm to this self-balancing electric motorcycle, mathematical models of the motor and the inverted pendulum should be derived.

A. Modeling DC motor

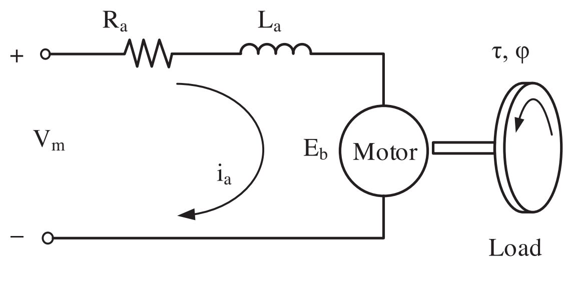


Figure 5. DC motor equivalent model

The equivalent electric circuit of the DC motor and the free-body diagram of the load are shown in Fig. 5. Using Kirchhoff's second law leads to:

$$V_m = i_a R_a + L_a \frac{di_a}{dt} + E_b \quad (1)$$

The back electromotive force (emf) is related to the angular velocity $\dot{\varphi}$ by the back-emf constant K_b as:

$$E_b = K_b \dot{\varphi} \quad (2)$$

Suppose that the effect of inductance is very small ($L_a \ll R_a$), Eqn. (1) leads to

$$i_a = \frac{V_m - K_b \dot{\varphi}}{R_a} \quad (3)$$

The motor torque τ is related to armature current i_a by a motor torque constant K_t as:

$$\tau = K_t i_a = K_t \frac{V_m - K_b \dot{\varphi}}{R_a} \quad (4)$$

B. Modeling Inertia Wheel Pendulum

The IWP system consists of a pendulum and an inertia wheel as shown in Fig. 6. The pendulum can rotate freely about the Oz axis perpendicular to the Oxy plane. The inertia wheel is driven by the DC motor to rotate about an axis in parallel to the Oz axis. The pendulum can balance by reaction force generated by the inertia wheel. The parameter notations of the IWP are presented in Table 1.

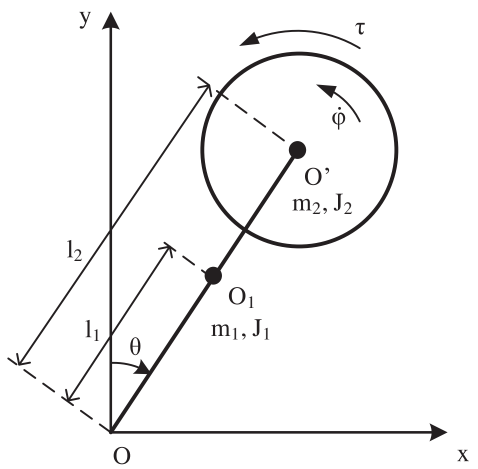


Figure 6. Inertial Wheel Pendulum model

TABLE 1. PARAMETER NOTATIONS OF THE IWP

Symbol	Unit	Definition
θ	rad	Angle of pendulum
φ	rad	Angle of inertia wheel
J_1	kg.m^2	Inertia moment of pendulum including motor stator
J_2	kg.m^2	Inertia moment of wheel including motor rotor
c_1	N.m.s/rad	Friction factor of pendulum
c_2	N.m.s/rad	Friction factor of wheel
m_1	kg	Mass of pendulum and stator
m_2	kg	Mass of wheel and rotor
l_1	m	Length from origin to COG of pendulum
l_2	m	Length from origin to COG of wheel
K_b	V/(rad/s)	Back-emf constant
K_t	N.m/A	Motor torque constant
R_a	Ohm	Armature winding resistance

Mathematical model of IWP is established by Lagrange method. Energy of system consists of kinetic and potential energy.

- Translational kinetic:
$$T_1 = \frac{1}{2}m_1(l_1\dot{\theta})^2 + \frac{1}{2}m_2(l_2\dot{\theta})^2 \quad (6)$$
- Rotational kinetic:
$$T_2 = \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}J_2(\dot{\theta}+\dot{\varphi})^2 \quad (7)$$

Kinetic energy of the system:

$$T = T_1 + T_2 \quad (8)$$
$$= \frac{1}{2}(m_1l_1^2 + m_2l_2^2 + J_1 + J_2)\dot{\theta}^2 + J_2\dot{\theta}\dot{\varphi} + \frac{1}{2}J_2\dot{\varphi}^2$$

Potential energy origin at O. So that, potential energy:

$$V = (m_1l_1 + m_2l_2)g\cos(\theta) \quad (9)$$

From equations (8) and (9), Lagrange function of IWP:

$$L = T - V = \frac{1}{2}(m_1l_1^2 + m_2l_2^2 + J_1 + J_2)\dot{\theta}^2 + J_2\dot{\theta}\dot{\varphi} + \frac{1}{2}J_2\dot{\varphi}^2 - (m_1l_1 + m_2l_2)g\cos(\theta) \quad (10)$$

Dissipated energy:

$$R = \frac{1}{2}c_1\dot{\theta}^2 + \frac{1}{2}c_2\dot{\varphi}^2 \quad (11)$$

Moment at rotating axis of θ from Lagrange's equation as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) + \frac{\partial R}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = 0 \quad (12)$$

$$\Leftrightarrow (m_1l_1^2 + m_2l_2^2 + J_1 + J_2)\ddot{\theta} + J_2\ddot{\varphi} + c_1\dot{\theta} - (m_1l_1 + m_2l_2)g\sin(\theta) = 0 \quad (13)$$

Moment at rotating axis of φ from Lagrange's equation as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) + \frac{\partial R}{\partial\dot{\varphi}} - \frac{\partial L}{\partial\varphi} = 0 \quad (14)$$

$$\Leftrightarrow J_2(\ddot{\theta} + \ddot{\varphi}) + c_2\dot{\varphi} = \tau \quad (15)$$

τ is the torque motor. From (4) and (15), it has:

$$J_2(\ddot{\theta} + \ddot{\varphi}) + c_2\dot{\varphi} = K_t\frac{V_m - K_b\dot{\varphi}}{R_a} \quad (16)$$

From (13) and (16), the nonlinear model of IWP is:

$$\begin{bmatrix} m_1l_1^2 + m_2l_2^2 + J_1 + J_2 & J_2 \\ J_2 & J_2 \end{bmatrix}\begin{bmatrix}\ddot{\theta} \\ \ddot{\varphi}\end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & \frac{K_tK_b}{R_a} + c_2 \end{bmatrix}\begin{bmatrix}\dot{\theta} \\ \dot{\varphi}\end{bmatrix} + \begin{bmatrix} -(m_1l_1 + m_2l_2)g\sin(\theta) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{K_t}{R_a} \end{bmatrix}V_m \quad (17)$$

III. EVALUATION OF SYSTEM PERFORMANACE

In this section, the performance of the IWP is explored by observing its responses when applying step inputs and changing the value of certain parameters, e.g. l_2, J_2 and m_2. The values of the other parameters are kept unchanged as shown in Table 2.

TABLE 2. COEFFICIENT VALUES IN THE IWP

Symbol	Value	Symbol	Value
J_1	0.01186 (kg.m^2)	l_1	0.1053 (m)
J_2	0.0005711 (kg.m^2)	l_2	0.14 (m)
c_1	0.04 (N.m.s/rad)	K_b	0.0987 (V/(rad/s))
c_2	0.0001 (N.m.s/rad)	K_t	0.0987 (N.m/A)
m_1	0.826 (kg)	R_a	1.5562 (Ω)
m_2	0.583 (kg)		

The nonlinear model of IWP in Eqn. (17) is implemented into a Simulink model as illustrated in Fig .7 in which:

- The voltage input V_m is the control variable
- The four state variables are θ, $\dot{\theta}$, φ, and $\dot{\varphi}$
- The output torque τ is calculated from Eqn. (4)

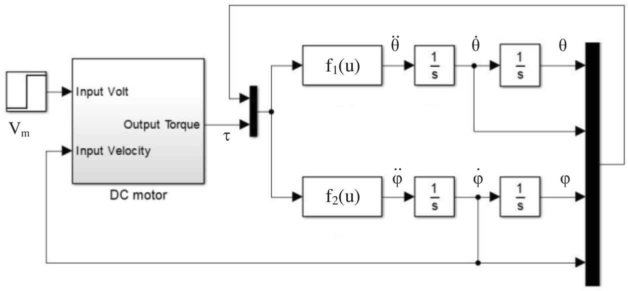


Figure 7. *Simulink model of IWP*

A. Responses under the step input

From the Simulink model, the voltage input V_m is applied in step of 10V, 15V, and 20V. The voltage input results in motions of the wheel as well as the pendulum shown in Fig. 8.

The simulation results show that:

- In Fig. 8.a, velocity of the wheel gets the steady state at a same time about 0,5s. In this time, DC motor generates a moment on wheel as in Fig. 8.b and inertia wheel rotates with an acceleration as in Fig. 8.c. Pendulum is swigged by a torque and oscillates around equilibrium position. As we can see from Fig. 8.d, V_m higher makes amplitude of pendulum angle higher which demonstrates higher force generated on pendulum. However, frequency oscillation of pendulum is the same for all situation, it means V_m does not affect to frequency oscillation of pendulum.
- When velocity of wheel enters steady state, motor cannot maintain rotor torque for a long time as in Fig. 8.b. In this state, this torque will equal to friction torque, acceleration of wheel is zero, pendulum is affected only by gravity force and oscillates gradually back to equilibrium position.

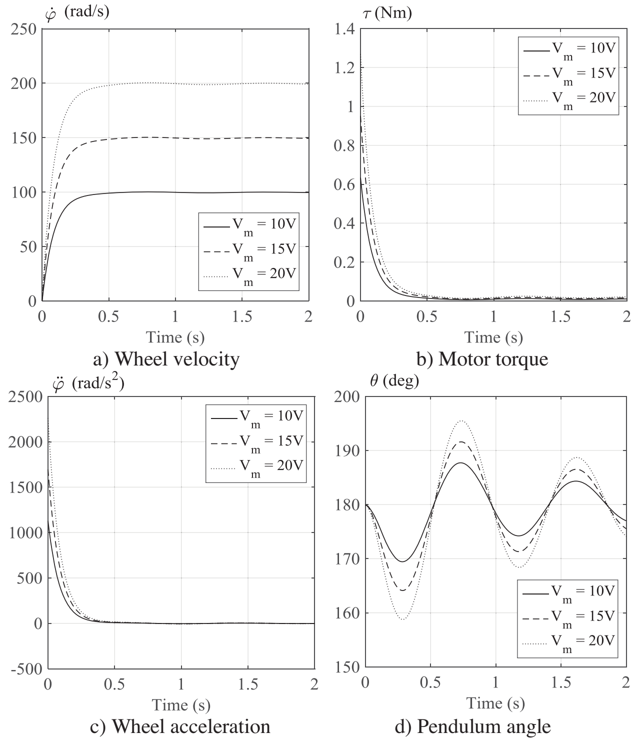


Figure 8. Responses of IWP under the step input V_m

B. Effect of the length from origin to COG of wheel (l_2)

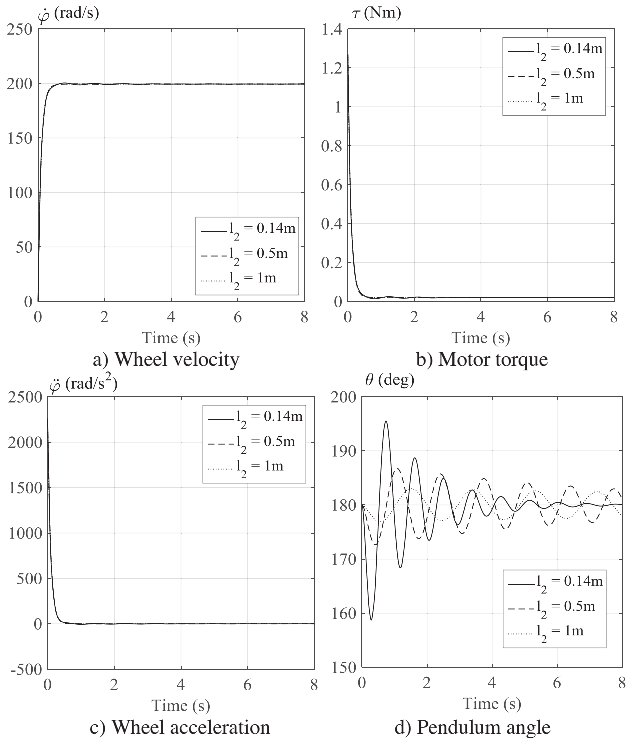


Figure 9. Responses of IWP in case changing l_2

Length from origin to COG of wheel l_2 is one of important parameters to design IWP. To examine the response of system, a signal V_m = 20V is supplied to the DC motor, the value of l_2 is changed into 0.14m, 0.5m, and 1m. The other coefficient values are fixed.

- In Fig. 9.a, 9.b, 9.c, all situations get the same response which means l_2 does not affect on responses of inertia wheel motion. However, pendulum angle is different when l_2 changed. With the same voltge to motor, l_2 higher get amplitude of pendulum angle smaller which means smaller sum of force on pendulum. In addition, frequency oscillate is smaller and stable longer.
- Length of pendulum affects to force on pendulum. It is useful to choose motor fit with l_2. Large value of l_2 makes negative effect on pendulum angle response.

C. Effect of the inertia moment of wheel (J_2)

Another important parameter of IWP is inertia moment of wheel J_2. The same as situation with l_2, a step input V_m = 20V will be supplied to DC motor. Value of J_2 is changed between 0.0005kg.m^2, 0.002kg.m^2, and 0.005kg.m^2. The other remain parameters will be unchanged from Table 2.

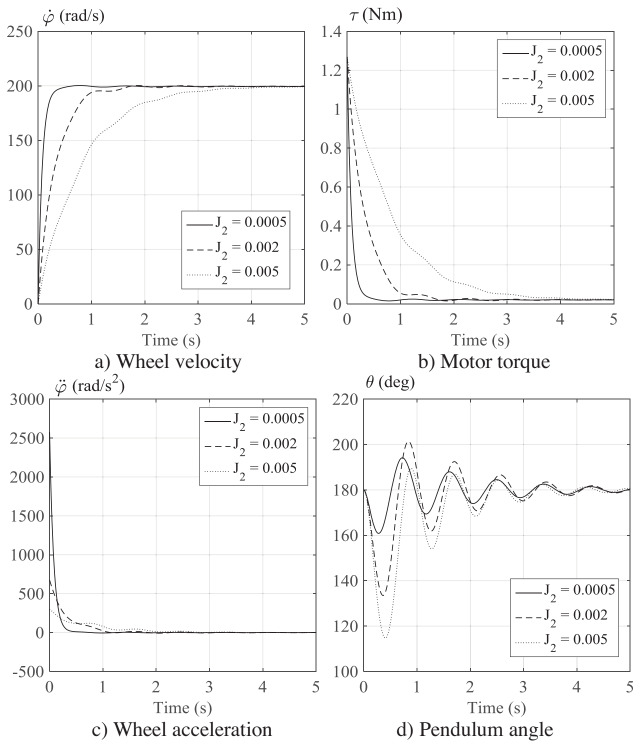


Figure 10. Responses of IWP in case changing J_2

The simulation results show that:

- In Fig. 10.a, settling time of wheel velocity is longer when J_2 higher and steady state value is the same for all cases. In short time condition, torque motor is equal at 0s then extends longer in case J_2 higher as in Fig. 10.b. In Fig. 10.c, acceleration of wheel is different at 0s: inertia moment of wheel smaller has acceleration at start moment higher but more quickly converging to zero than other.
- As a result, in Fig. 10.d, amplitude of pendulum angle is higher in case J_2 higher which ease to understand that because of torque motor affect more longer.

- J_2 affects positively to accelerated process of motor so J_2 higher is better to remain moment on pendulum but mass of system increases.

D. Effect of the mass of wheel (m_2)

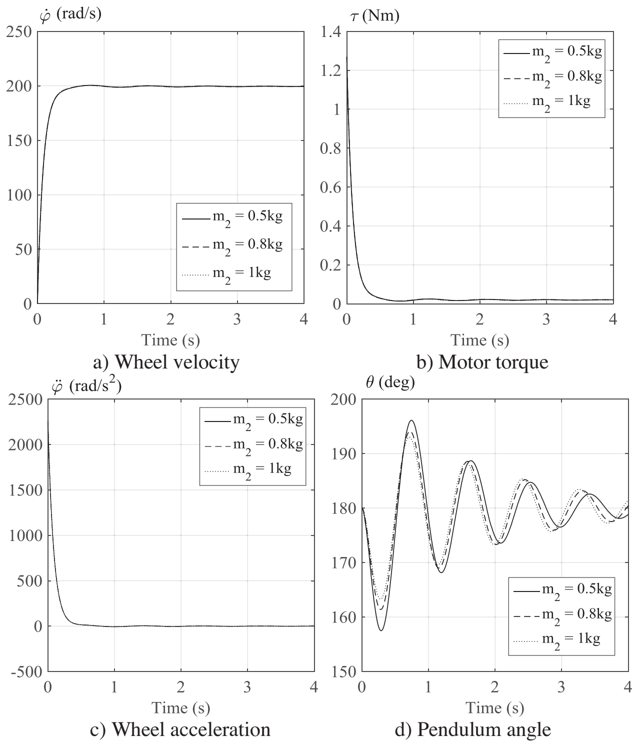


Figure 11. *Responses of IWP in case changing m_2*

The last parameter to simulate is the mass of wheel m_2. Like l_2, J_2, a step input V_m = 20V is supplied to DC motor and value of m_2 is changed between 0.5kg, 0.8kg, and 1kg. The other parameters are unchanged.

The simulation results show that:

- From Fig. 11.a, 11.b, 11.c, velocity, acceleration, torque motor on inertia wheel is the same for all cases of m_2. Therefore, mass of wheel does not affect on responses of above physical parameters.
- From Fig. 11.d, m_2 higher gets amplitude of pendulum angle smaller because of effect of gravity.
- m_2 affects negatively to performance of system.

IV. CONTROL IMPLEMENTATION

In normal state, IWP always stable on bottom so it needs a controller to control torque on origin to balance with torque produced by gravity. Steady state of system is pendulum angle is zero, velocity of wheel is zero in order to speed up and get a maximum torque as well. Therefore, a simple PD controller is designed as in Fig. 12. In the first loop, the pendulum angle is controlled by a PD controller. In the second loop, the wheel velocity is controlled by a P controller. Sum of the two controller outputs is V_m voltage supplied to DC motor.

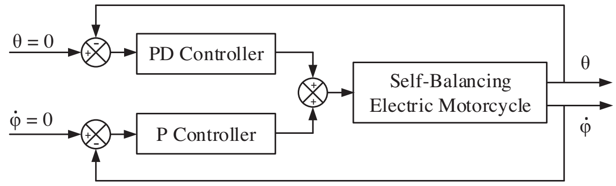


Figure 12. *Control strategy for balancing IWP*

A. Simulation Results

Figure 13. *3D Model of Self-Balancing Electric Motorcycle*

A 3D model of self-balancing motorcycle is designed in Solidworks as illustrated in Fig. 13. Then, the 3D model is embedded into Matlab environment through SimMechanics Link. That results in a Simulink block diagram as shown in Fig. 14, including:

- "Environment" block, which is under the gravity
- "Ground" block representing the road
- "Body Axis" block representing wheel contact line with the road
- "Body" block representing the body of motorcycle
- "Wheel Axis" block representing the motor axis
- "Wheel" block representing the inertia wheel

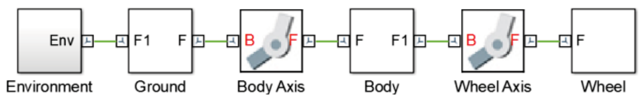


Figure 14. *Simulink block diagram of self-balancing motorcycle*

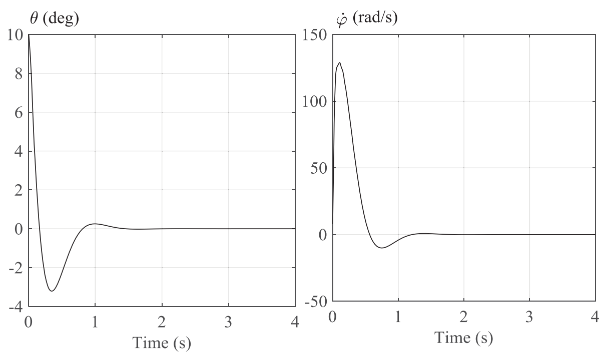


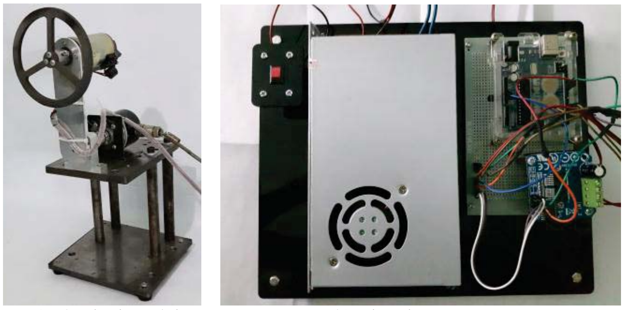
Figure 15. *Simulation results*

$(K_{P,\theta} = 900, K_{D,\theta} = 150, K_{P,\dot{\varphi}} = 1.7)$

Simulation result of PD controller is illustrated as Fig. 15. Pendulum angle θ balances at 0^0 from the angle deviation 10^0. Wheel velocity increases from zero to 130rad/s then enters steady state zero.

B. Experimental Results

Figure 16 is a prototype of IWP. System main parts consist of base, encoder to measure angle of pendulum, wheel connect with DC motor. The electric part to control IWP consists of Arduino Uno, Driver BTS7960, adapter 24V-10A and a switch.



a) Physical model b) Electric components

Figure 16. *Prototype of IWP*

In normal state, pendulum stable at bottom so to move to upper position, swing up process is controlled by a ON-OFF controller with V_s = 15V as in Fig. 17. Results of swing up and balance controller are shown in Fig. 18.

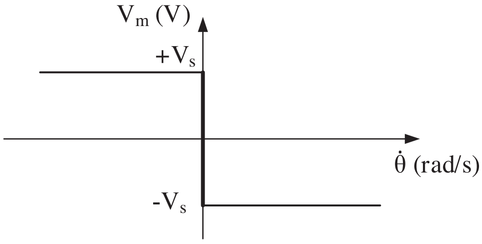
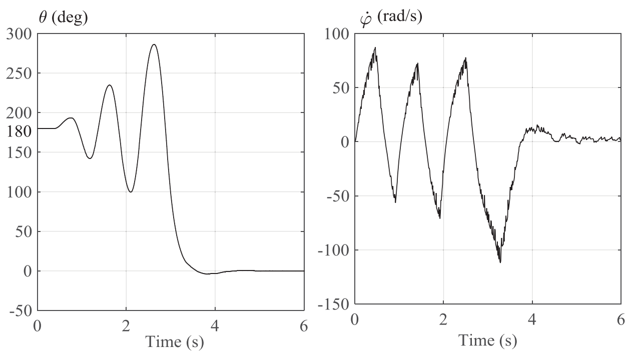


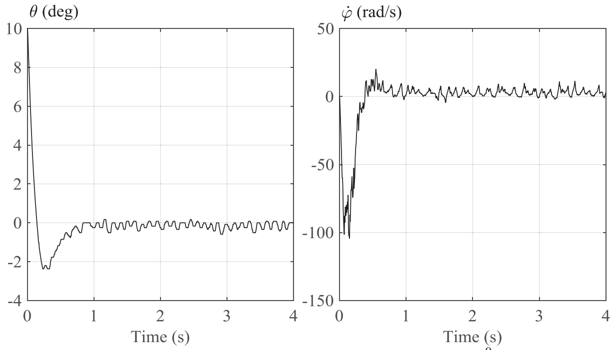
Figure 17. *Control strategy for swing up process*



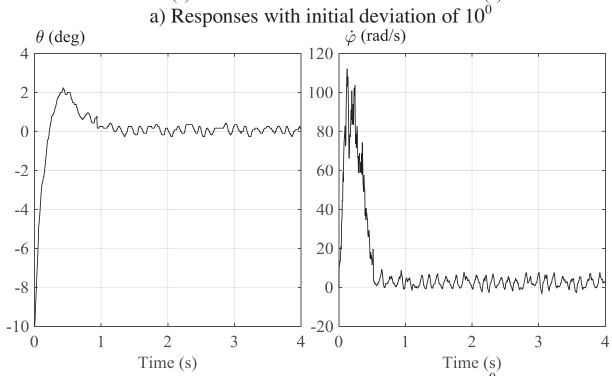
a) Pendulum angle b) Wheel velocity

Figure 18. *Real hardware performance for swing up and balance*

The IWP is able to balance from $\pm 10^0$ and enters the steady state as shown in Fig. 19.



a) Responses with initial deviation of 10^0



b) Responses with initial deviation of -10^0

Figure 19. *Dynamic performance of IWP to maintain its upward balance*

V. CONCLUSION

This paper has explored the principal to balance the IWP. It has been found that the pendulum is only activated under the acceleration of the wheel. In addition, the effects of pendulum length, wheel mass as well as wheel inertial moment on responses of IWP have also been studied. The understanding of IWP responses are very helpful to determine the stable region as well as the motor torque needed to achieve the desired balance. Simple control of PID algorithm has been successfully implemented to IWP. Future work will be more focused on control when the IWP is implemented on self-balancing electric motorcycles.

ACKNOWLEDGMENT

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