MATH-640 project proposal

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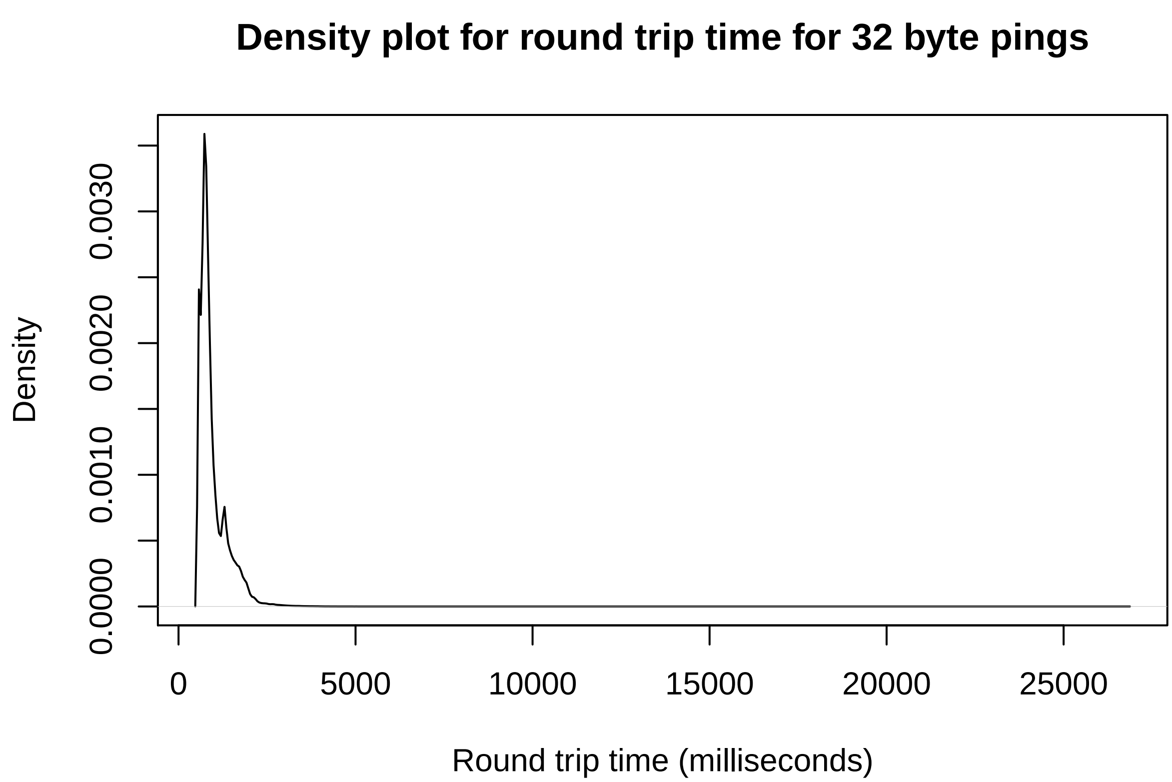
Introduction

Round trip times on a communication link are typically long tailed and can be modeled using the Pareto Type 1 distribution. Data is collected from a high delay bandwidth network for round trip time measurements over a one month period. This data is then modeled using a Pareto Type 1 distribution with a known minimum r_m and a shape parameter θ. A conjugate prior $Gamma(\alpha, \beta)$ is placed on θ, such that the hyper parameters α and β themselves are drawn from two independant Gamma distributions $Gamma(a_1, b_1)$ and $Gamma(a_2, b_2)$.

A Gibbs-MH sampler is used to draw samples from the joint posterior $p(\theta, \alpha, \beta|R)$ where R represents the dataset of round trip times. A particular test statistic of intrest is the 75th percentile of the round trip time, denoted by T_{75}. Summaries are created for $T_{75}|\theta^{(b)}, \alpha^{(b)}, \beta^{(b)}$ and the adequacy of fit of the model is tested by comparing the observed value of T_{75} with the created summaries.

Methods

Here is a density plot of the observed data.



The Pareto likelihood can be expressed as follows

$\mathcal{L}(R|\theta, r_m) = \prod_{i=1}^{n} \frac{\theta}{r_m} \left(\frac{r_m}{r_i}\right)^{\theta+1}$. $p(r_i) = \frac{\alpha}{\lambda}(\frac{\lambda}{r_i})^{\alpha+1}$

$\mathcal{L}(R|\alpha, \lambda) = \prod_{i=1}^{n} \frac{\alpha}{\lambda}(\frac{\lambda}{r_i})^{\alpha+1}$

$\mathcal{L}(R|\alpha, \lambda) = \alpha^n \lambda^{n\alpha} \prod_{i=1}^{n}(r_i)^{-(\alpha+1)}$

$\propto \alpha^n \lambda^{n\alpha} \prod_{i=1}^{n} exp[log((r_i)^{-(\alpha+1)})]$

$= \alpha^n \lambda^{n\alpha} \prod_{i=1}^{n} exp[-(\alpha+1)log(r_i)]$

$= \alpha^n \lambda^{n\alpha} exp[-(\alpha+1)\sum_{i=1}^{n} log(r_i)]$

$= \alpha^n \lambda^{n\alpha} exp[-\alpha \sum_{i=1}^{n} log(r_i)]$

Joint Posterior, assuming $\pi(\alpha, \lambda) \propto (\sigma^2)^{-\frac{1}{2}} exp[-\frac{\lambda-\lambda_0}{2\sigma^2}]$

$p(\alpha, \lambda|R) \propto \alpha^n \lambda^{n\alpha} exp[-\alpha \sum_{i=1}^{n} log(r_i)] \times 1 \times (\sigma^2)^{-\frac{1}{2}} exp[-\frac{\lambda-\lambda_0}{2\sigma^2}]$

$p(\alpha, \lambda|R) \propto \alpha^n \lambda^{n\alpha} exp[-\alpha \sum_{i=1}^{n} log(r_i)]\ exp[-\frac{\lambda-\lambda_0}{2\sigma^2}]$