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Section - AI and DS

Tutorial - 04

Ques.1. $T(n) = 3T(n/2) + n^2$

→ $a = 3, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

$\therefore n^{\log_b a} < f(n)$

$\therefore \underline{T(n) = \theta(n^2)}$

Ques.2. $T(n) = 4T(n/2) + n^2$

→ $a = 4, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 4} = n^{\log_2 2^2}$

$= n^2$

$\therefore n^{\log_b a} = f(n)$

$\therefore \underline{T(n) = \theta(n^2 \log n)}$

Ques.3. $T(n) = T(n/2) + 2^n$

→ $a = 1, b = 2, f(n) = 2^n$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$n^{\log_b a} < 2^n$

$\therefore T(n) = \theta(2^n)$

Ques.4. $T(n) = 2^n + T(n/2) + n^n$

$\therefore a$ is a function.

⇒ Master's theorem is not possible.

Ques.5. $T(n) = 16T(n/4) + n$

→ $a = 16, b = 4, f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^2$

$\therefore n^{\log_b a} \geq f(n)$

$\therefore \underline{T(n) = \theta(n^2)}$

Ques.6. $T(n) = 2T(n/2) + n\log n$

→ $a = 2,\ b = 2,\ f(n) = n\log n$

$n^{\log_b a} = n\log_2 2 = n$

$f(n) > n^{\log_b a}$

$\therefore T(n) = \Theta(n\log n)$

Ques.7. $T(n) = 2T(n/2) + n/\log n$

→ $a = 2,\ b = 2,\ f(n) = n/\log n$

$n^{\log_b a} = n\log_2 2 = n$

$\therefore n^{\log_b a} < f(n)$

$\therefore T(n) = \Theta(n)$

Ques.8. $T(n) = 2T(n/4) + n^{0.51}$

→ $a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n\log_4 2 = n^{0.5}$

$\therefore n^{\log_b a} < f(n)$

$\therefore T(n) = \Theta(n^{0.51})$

Ques.9. $T(n) = 0.5T(n/2) + 1/n$

$\because a < 1$

$\therefore$ Master's theorem not applicable.

Ques.10. $T(n) = 16T(n/4) + n!$

→ $a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^2$

$\because n^2 < n!$ i.e. $n^{\log_b a} < f(n)$

$\therefore T(n) = \Theta(n!)$

Ques.11. $T(n) = 4T(n/2) + \log n$

→ $a = 4,\ b = 2,\ f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \Theta(n^2)$

Ques.12. $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$\because a$ is not constant

$\therefore$ Master's theorem not applicable.

Ques.13 $T(n) = 3T(n/2) + n$

→ $a=3, b=2, f(n)=n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n^{1.58})$

Ques.14. $T(n) = 3T(n/3) + \sqrt{n}$

→ $a=3, b=3, f(n)=\sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$\therefore n^{\log_b a} > f(n)$

$\Rightarrow T(n) = \theta(n)$

Ques.15. $T(n) = 4T(n/2) + cn$

→ $a=4, b=2, f(n)=c*n$

$n^{\log_b a} = n^{\log_2 4} = n^{\log_2 2^2} = n^2$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n^2)$

Ques.16. $T(n) = 3T(n/4) + n\log n$

→ $a=3, b=4, f(n)=n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$\because n^{\log_b a} < f(n)$

$\therefore T(n) = \theta(n\log n)$

Ques17. $T(n) = 3T(n/3) + n/2$

→ $a=3, b=3, f(n)=n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n)$

Ques.18. $T(n) = 6T(n/3) + n^2\log n$

→ $a=6, b=3, f(n)=n^2\log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{\log_b a} < f(n) \Rightarrow T(n) = \theta(n^2\log n)$

Ques.19. $T(n) = 4T(n/2) + n/\log n$

→ $a=4, b=2, f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^{\log_b a} > f(n)$

$\Rightarrow T(n) = \theta(n^2)$

Ques.20. $T(n) = 64T(n/8) - n^2 \log n$

→ Master's theorem is not applied as $f(n)$ is not increasing function.

Ques.21. $T(n) = 7T(n/3) + n^2$

→ $a = 7,\ b = 3,\ f(n) = n^2$

$$n^{\log_b a} = n^{\log_3 7}$$
$$= n^{1.3}$$
$$\therefore n^{\log_b a} < f(n)$$
$$\Rightarrow T(n) = \Theta(n^2)$$

Ques.22. $T(n) = T(n/2) + n(2 - \cos n)$

→ Master's theorem is not applied.

∵ regularity condition is isolated in case 3.