

Step-02: Arrange all the items in decreasing order of their value / weight ratio.

Step-03: Start putting the items into the knapsack beginning from the item with the highest ratio. Put as many items as you can into the knapsack.

Example:

Find the optimal solution for the fractional knapsack problem making use of greedy approach. Consider-

$$n = 5$$

$$w = 60 \text{ kg}$$

$$(w_1, w_2, w_3, w_4, w_5) = (5, 10, 15, 22, 25)$$

$$(b_1, b_2, b_3, b_4, b_5) = (30, 40, 45, 77, 90)$$

Solution-

Step-01:

Compute the value / weight ratio for each item-

Items	Weight	Value	Ratio
1	5	30	6
2	10	40	4
3	15	45	3
4	22	77	3.5
5	25	90	3.6

Step-02:

Sort all the items in decreasing order of their value / weight ratio-

I1 I2 I5 I4 I3

(6) (4) (3.6) (3.5) (3)

Step-03:

Start filling the knapsack by putting the items into it one by one.

Knapsack Weight	Items in Knapsack	Cost
60	Ø	0
55	I1	30
45	I1, I2	70
20	I1, I2, I5	160

Now,

- Knapsack weight left to be filled is 20 kg but item-4 has a weight of 22 kg.
- Since in fractional knapsack problem, even the fraction of any item can be taken.
- So, knapsack will contain the following items-

< I1 , I2 , I5 , (20/22) I4 >

Total cost of the knapsack

$$= 160 + (20/27) \times 77$$

$$= 160 + 70$$

$$= 230 \text{ units}$$

Algorithm- Fractional knapsack

• Greedy-fractional-knapsack (w, v, W)

1. for $i = 1$ to n
2. do $x[i] = 0$
3. weight = 0
4. while weight < W
5. do $i =$ best remaining item
6. if weight + $w[i] \leq W$
7. then $x[i] = 1$
8. weight = weight + $w[i]$
9. else
10. $x[i] = (W - \text{weight}) / w[i]$
11. weight = W
12. return x

Time Complexity-

- The main time taking step is the sorting of all items in decreasing order of their value / weight ratio.
- If the items are already arranged in the required order, then while loop takes $O(n)$ time.
- The average time complexity of Quick Sort is $O(n \log n)$.
- Therefore, total time taken including the sort is $O(n \log n)$.

Viva Questions:

1. What is Greedy Approach?
2. Explain concept of fractional knapsack
3. Difference between Fractional and 0/1 Knapsack.

Conclusion: In this way concept of Fractional Knapsack is explained using greedy method.