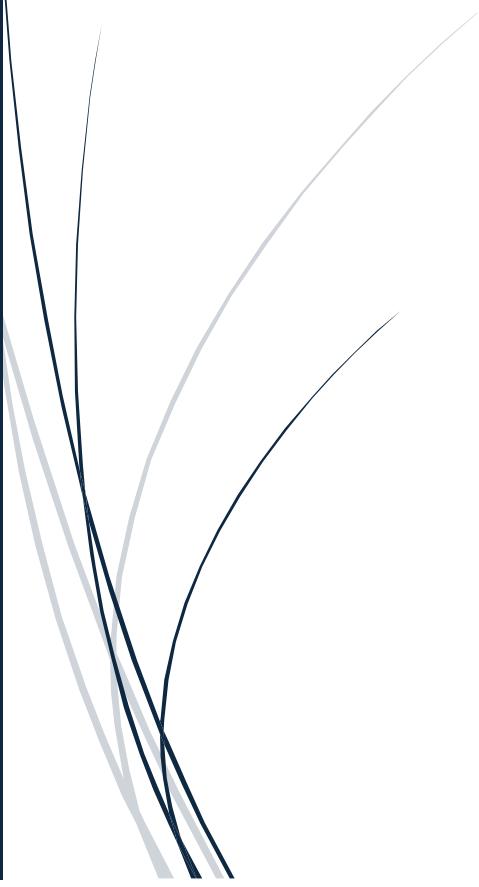


6/7/2025

ASSET ALLOCATIONS FOR SUSTAINABLE TRUST FUND PORTFOLIO

Analysis and Recommendations



Aarti Garaye
PSTAT 177: FINANCIAL RISKS
HAL PEDERSEN

INVESTMENT PROPOSAL FOR
Charitable Trust

TABLE OF CONTENTS

TABLE OF CONTENTS-----	2
EXECUTIVE SUMMARY -----	3
INTRODUCTION-----	4
Assumptions and Methodology-----	4
Business Problem -----	4
DATA DISCUSSION-----	5
Introduction -----	5
Historical Data -----	5
Summary Statistics-----	5
Meeting the Threshold -----	5
Correlation -----	6
Wealth Factor-----	7
ANALYSIS-----	8
The 60/40 Approach-----	8
Overview-----	8
Benchmark Values-----	8
Pros and Cons -----	8
Metrics to Focus On-----	9
Economic Scenario Generator -----	9
CONCLUSION-----	10
First Allocation: 62% Equities, 2.5% Money Market, and 35.5% Blended Fixed -----	10
Second Allocation: 55% in Equities, 0% in Money Market, and 45% in Blended Fixed -----	11
Third Allocation: 55% in Equities, -30% in Money Market, and 75% in Blended Fixed. -----	13
SPENDING RATE-----	14
SUMMARY-----	15

INVESTMENT PROPOSAL FOR
Charitable Trust

EXECUTIVE SUMMARY

In December 2024, the board of directors of a legacy trust fund entrusted us with the responsibility to evaluate their traditional 60/40 allocation — 60% in US large cap equities and 40% in blended fixed income, 65% intermediate US treasury bonds and 35% long term corporate bonds — and to assess its ability to meet the trust's long-term goals. The trust distributes 3% of its beginning-of-the-year balance to support its charitable missions. The board's primary concern is to have a simple allocation that preserves purchasing power over the 30-year horizon, assuming an inflation rate of 2.5%.

Using an economic scenario generator, we simulated a 1000 paths of possible portfolio performance over different market conditions. We evaluated multiple asset allocations across key metrics: probability of losing purchasing power, conditional expectation of percent loss in purchasing power given purchasing power, minimum and maximum real returns, average real returns, volatility, median real returns, and the 25th and 75th percentile paths.

Although the benchmark 60/40 allocation performs well but leaves room for improvements, our analysis showed that there are better allocations that adhere to the trusts' goals. One of our recommended allocations was 62% equities, 2.5% in money market, and 35.5% in blended fixed. This allocation decreases the probability of losing purchasing power while keep the volatility almost the same. This also helps the trust to grow faster.

Our second recommendation was on a conservative side, investing 55% in equities, 0 in money market, and 45% in the blended fixed. This reduces probability of losing the purchasing power as well as volatility. However, the trust grows at a much slower rate. And we had a third unconventional recommendation of borrowing 30% from the money market and take the risk of paying it back with interest. This significantly reduces the probability of losing purchasing power and increases the minimum, maximum and average real returns. However, it comes at a cost of increased volatility by 0.4%. Given this allocation's performance, we do not recommend increasing the 3% annual spending rate. Preserving long-term purchasing power remains achievable at 3%, but any increase would raise the probability of shortfall risk across economic conditions.

INVESTMENT PROPOSAL FOR Charitable Trust

INTRODUCTION

Assumptions and Methodology

Please note all information shown is based on assumptions and simulated data created by using an economic generator. We estimate the performance of an asset class or strategy by analyzing current market conditions and historical market trends.

This report outlines a strategic review of the charitable trust's legacy portfolio, with the goal of recommending a long-term investment allocation that preserves the purchasing power of the trust while continuing to support its annual 3% spending mandate.

Business Problem

As of December 31, 2024, the trust has followed a traditional 60/40 approach — with 60% in US large cap and 40% in a blended fixed income fund. The blended fixed income is managed to mimic a constant asset mix of 65% intermediate-term US treasury bonds and 35% long-term corporate bonds. However, due to recent market conditions the fund has experienced a lot more volatility than it's comfortable with.

While this approach has served as a balanced strategy in prior decades, it has exhibited more volatility in recent years raising concerns about the sustainable of the trust with the ongoing strategy.

Each year, the trust distributed 3% of its beginning-of-the-year balance to support its charitable missions. The board's top priority is to retain the purchasing power of the trust at the end of the 30-year period, assuming an average annual inflation rate of 2.5%. While real growth is welcomed and appreciated, the primary mandate is to avoid the loss of purchasing power in inflation-adjusted terms. The board favors simple strategies and has requested to really emphasize the preservation of the spending rate.

The report aims to recommend investment strategies adhering to the clients well laid out requirements. Investment options are restricted to being built up from three fund classes: money market, blended fixed income, and US large cap equities. Furthermore, the report will provide recommendations about the spending rate and whether it can sustain an increase in the purchasing power with the provided inflation rate.

INVESTMENT PROPOSAL FOR Charitable Trust

DATA DISCUSSION

Introduction

This section provides a detailed overview of how the data was obtained and some preliminary analysis of the data. The goal is to better understand the behavior and risk-return characteristics of the available classes — US large cap equities and the blended fixed income.

Two main sources provided by my assistant Ronny Destin:

1. Historical returns (1926-2024) from file *Project 2 Historical Data.xlsx*. This includes the annual and monthly return data for the asset classes.
2. Simulated return paths from the AIRG Economic Scenario Generator (ESG), provided in the file *Trust Investment Calculations (Annual) MRP650 Sim with MONEY.xlsx*. This file contained 1000 simulated paths projecting possible future economic scenarios to test the long-term portfolio outcomes under different market conditions.

Historical Data

Summary Statistics

To evaluate potential asset allocation for the trust we would want to look at the trust's performance with the current strategy to see what could be changed. To do this we will be looking at different metrics and values. Below is the summary statistics of historical data (1926-2024).

	Large Cap (alpha=0.6) TR	Blended Fixed TR	Portfolio Returns
Min	-0.43337	-0.13263	-0.26864
Max	0.53990	0.33809	0.34322
Average	0.12291	0.05390	0.09531
St. Dev	0.19668	0.06501	0.12331

Table 1: Summary Statistics of Historical Data (1926-2024) of 60/40 Annual Returns

Meeting the Threshold

Since these are the returns before being adjusted for inflation, we would want to check for how many years, did the portfolio return was above 5.5%. 5.5% is the minimum threshold needed to cover the 3% annual spending of the charity and the 2.5% of annual inflation.

INVESTMENT PROPOSAL FOR Charitable Trust

Number of Years Portfolio return was higher than 5.5%	Percentage of Years that exceeded the 5.5% hurdle
64	64.6464646%

Table 2: Number of Years (out of 98 total years) where the portfolio returns were higher than 5.5%.

As we can see from the table the traditional approach only resulted in 64.6% of success rate of preserving the purchasing power. The figure below shows which years the strategy was successful such that the portfolio met the threshold of 5.5% (blue bars), and the years where it was unsuccessful in meeting the threshold (orange bars).

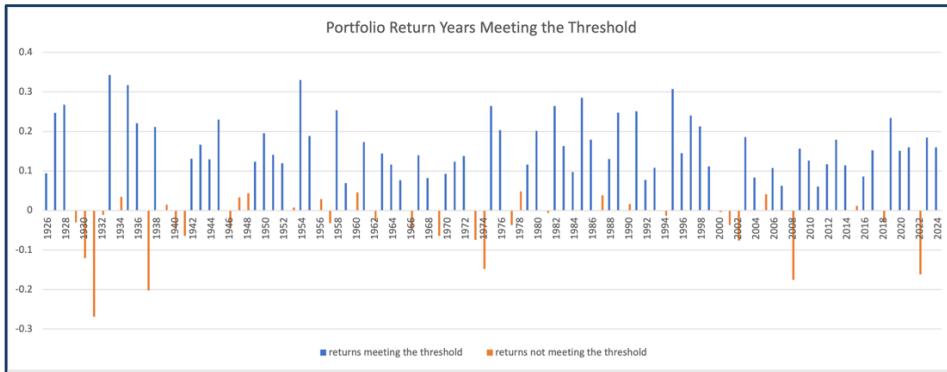


Figure 1: Portfolio Annual Returns Meeting the Threshold Before Inflation.

From the table above and the charts, we can confirm that U.S. large-cap equities delivered the highest average returns but with significant year-to-year volatility. Fixed income provided more stable returns but was not without risk, particularly during periods of rising interest rates.

Correlation

To see how the large-cap equities and bonds affected the portfolio return, we can look at their returns together. The graph shows that every time the large cap equities had a negative return, the portfolio had a negative return. This further highlights the strong correlation between equities and portfolio returns. Let's look at the correlation matrix:

	US Return	Fixed Return	Portfolio Return
US Return	1	0.098534177	0.977735949
Fixed Return	0.098534177	1	0.305158281
Portfolio Return	0.977735949	0.305158281	1

Table 3: Correlation matrix between Large-cap Equities TR, fixed TR, and portfolio TR.

the correlation between fixed income and large-cap equities is positive but relatively weak, which still offers some diversification benefits. While a negative correlation would be ideal from a diversification standpoint—helping to offset losses in one asset class with gains in another—a low positive correlation is still beneficial. It implies that these two asset classes do not move

INVESTMENT PROPOSAL FOR Charitable Trust

closely together, reducing overall portfolio volatility. Maintaining this low correlation between equities and fixed income is important in preserving long-term purchasing power while managing downside risk. The historic data so far has shown some promising statistics.

The graph below shows the historical trend of total returns of the two asset classes and our portfolio.

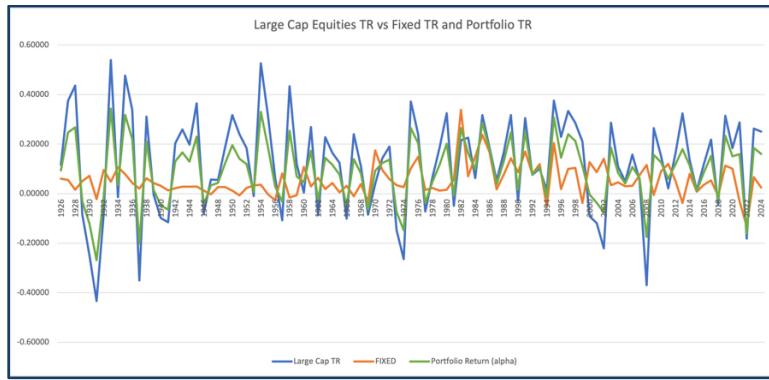


Figure 2: The chart shows volatility among the portfolio returns over the past years

Wealth Factor

While rolling returns provided valuable insight into the consistency and variability of the portfolio's total return over the 10-year period, they don't capture the impact of annual spending and show the growth of the portfolio itself.

To address this, we analyzed the wealth factor. The wealth factor measures how the portfolio's value evolves across the investment horizon. Since we start with a wealth factor of 1, a wealth factor greater than 1 indicates the portfolio has preserved or increased purchasing power in real terms, while a value below 1 signals erosion of purchasing power. Overall, we should see an increasing trend.

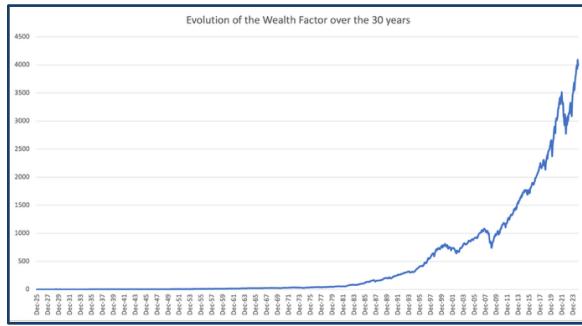


Figure 3: Evolution of the wealth factor over 30 years.

As we can see, the strategy has done well up until very recently where it took a hit during the COVID-19 pandemic. There is a positive trend indicating that the portfolio's value has been increasing.

INVESTMENT PROPOSAL FOR Charitable Trust

ANALYSIS

The 60/40 Approach

The traditional 60/40 approach allocates 60% to US large-cap equities and 40% to the fixed income (blended bonds). This approach is popular due to its balance between growth potential and risk management.

Overview

In the 60/40 approach there is nothing in the Money Market. Money Market is a low-risk investment option. Low-risk investments help stabilize the volatility of the portfolio. Since our client has been experiencing more volatility than they are comfortable with, I would recommend allocating some percent to Money markets. That being said, the primary goal is to preserve the purchasing power.

Benchmark Values

With the 60/40 allocation, 3% spending rate, and a 2.5% average annual inflation the table shows the benchmark metric.

60/40 allocation	Real Balance	% change in PP	Real Return
Min	26.05	-73.95%	-0.0438
Max	933.59	833.59%	0.0773
Average	195.04	95.04%	0.0173

Table 4: Benchmark metrics for the 60/40 Allocation

Additional metrics for the 60/40 allocations are:

Probability loss in PP	E[% Loss PP Loss PP]	Volatility	Paths preserving PP (out of 1000)	Median Real Return	25th percentile RR	75th percentile RR
0.181	-0.2512	0.1069976	819	0.0177	0.0034	0.0298

Table 5: Additional metrics for the Benchmark 60/40 Strategy

Pros and Cons

Pros	Cons
<ul style="list-style-type: none">US large cap equities have historically provided strong long-term returns. They are the primary source of capital appreciation for the portfolio.These are critical for offsetting the inflation and increasing wealth/value of the portfolio over time. Equities	<ul style="list-style-type: none">Equities can experience significant volatility, and they usually are the first to collapse during economic downturns and market crashes.A high positive correlation between equities and the portfolio returns suggest that portfolio is very sensitive

INVESTMENT PROPOSAL FOR Charitable Trust

<p>tend to outpace inflation over the long-term</p> <ul style="list-style-type: none">• The fixed income tends to lower volatility and provide steady income. In an interest rate rising environment Fixed income securities become less sensitive to interest rate changes• This is a good mix for diversification of the portfolio, especially since the correlation matrix showed very weak positive correlation between equities and fixed income securities.	<p>to economic downturns and market crashes.</p> <ul style="list-style-type: none">• As interest rate rises, bond prices fall, and it may result in change in the bond duration.• When we are investing in corporate bonds, we are taking on default risk.• Fixed income generally offers lower long-term returns than equities, potentially limiting portfolio growth.
--	---

Table 6: Pros and Cons of the 60/40 Allocation

Metrics to Focus On

Our primary goal is to preserve the purchasing power. Therefore, our priority would be to make sure the probability of losing purchasing power doesn't go below the benchmark probability of 18.1%. If anything, we would like to focus on an allocation that would reduce this probability. Next in the priority list would be to have an allocation that reduces volatility, while preserving purchasing power. On the lower side of the list would be the conditional expectation and other factors such as the percentile paths.

Economic Scenario Generator

To combat this problem, we used the well-known AIRG Economic Generator which simulated 1000 paths of possible economic conditions. We want an allocation such that most of the paths result in preserving the purchasing power.

INVESTMENT PROPOSAL FOR
Charitable Trust

CONCLUSION

First Allocation: 62% Equities, 2.5% Money Market, and 35.5% Blended Fixed

To focus directly on preserving the trust's purchasing power, we explored multiple constant allocation strategies using the AIRG Economic Scenario Generator. One allocation that stood out was 62% US Large Cap Equities, 35.5% Blended Fixed Income, and 2.5% Money Market.

This mix modestly increases exposure to equities compared to the benchmark 60/40 strategy, while retaining a substantial allocation to fixed income and a small buffer in money markets for liquidity and volatility protection. Comparison metrics are given below:

62/2.5/35.5 allocation	Real Balance	% change in PP	Real Geo return
Min	25.54	-74.46%	-0.0445
Max	911.96	811.96%	0.0765
Average	196.44	96.44%	0.0173

Table 7: Comparison metric for the 62/2.5/35.5 allocation

Additional metrics are:

Probability loss in PP	E[Loss PP Loss PP]	Volatility	Paths preserving PP	Median RR	25th percentile RR	75th percentile RR
0.178	-0.2643	0.1087656	822	0.0177	0.0033	0.0299

Table 8: Additional metric for the 62/2.5/35.5 allocation

Compared to the benchmark metrics, we reduce our probability of losing purchasing power which goes hand-in-hand with increasing the number of paths preserving the purchasing power. This aligns directly with the board's top priority. Our median real return, the minimum, maximum, and average has increased as well.

The tradeoff to this is an increase in volatility by 0.2% which is very similar to the original volatility. We also increase the conditional expectation of losing purchasing power given that we are lose the purchasing power.

The graph below shows the total return over the last 5 years with this strategy. The red dot is the original 60/40 approach. As we can see the 62/2.5/35.5 is essentially keeping the same volatility but increasing the mean return. It's also decreasing the probability of losing purchasing power.

Furthermore, since we will be investing in equities an extra 2%, it will grow the trust's balance while still meeting essentially all of the primary goals.

INVESTMENT PROPOSAL FOR Charitable Trust

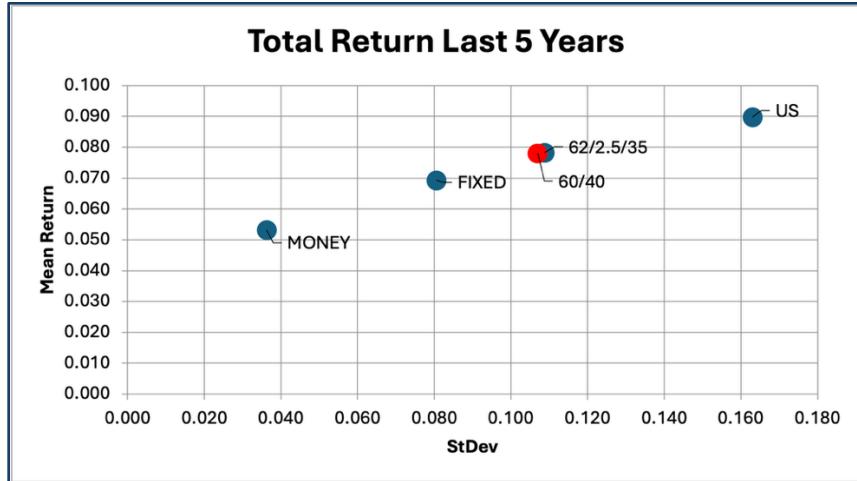


Figure 4: Total returns over the past 5 years

Next, we will be presenting the nominal balances along the path at the 25th percentile and the 75th percentile to see how these quantile paths perform under the new allocation:

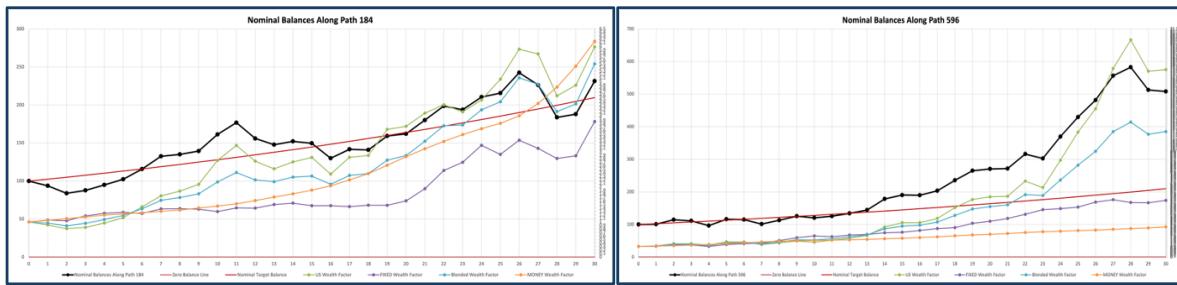


Figure 5: Path 184 is the path at the 25th percentile. Figure 6: Path 596 is the 75th percentile path

This shows that the portfolio will do better than the nominal target balance. The 75th percentile path does even better.

From these path configurations, we can see now the allocation is behaving. Majority of the time the portfolio is above the nominal target balance path (in red).

Overall, this allocation strikes a strong balance between risk control and returns while reducing the probability of losing the purchasing power. The increased weight in US large cap equities will result in accelerated growth of the trust fund. The small exposure to money market provides some liquidity but does not dilute the returns. Most importantly, it essentially exceeds all the boards' expectations and requirements.

Second Allocation: 55% in Equities, 0% in Money Market, and 45% in Blended Fixed

While working on different allocations, one that stood out was investing 55% in US large cap, 0% in money market, and 45% in the blended fixed income. Reducing the percentage in US

INVESTMENT PROPOSAL FOR Charitable Trust

equities by 5% will result in reducing the volatility. Furthermore, this allocation also results in reduction of the probability of losing the purchasing power compared to the original 60/40 allocation.

55/0/45 allocation	Real Balance	% change in PP	Real Geo return
Min	27.25	-72.75%	-0.0424
Max	947.11	847.11%	0.0778
Average	187.76	87.76%	0.0164

Table 9: Metrics for the 50/0/45 allocation.

The minimum, maximum, and average has all changed for the better. This allocation gives higher minimum, maximum, and average real balances. The minimum % change in purchasing power is lower than the benchmark resulting in the worst-case scenario loss would still be better than the original 60/40 allocation. Let's see how the probability of losing purchasing power and volatility changes under this allocation:

Probability loss in PP	E[Loss PP Loss PP]	Volatility	Paths preserving PP	Median RR	25th percentile RR	75th percentile RR
0.18	-0.2394	0.1009306	820	0.0166	0.0034	0.0285

Table 10: Additional metrics for the 55/0/45 allocation.

There is an 18% chance of losing purchasing power, slightly less than the 60/40 benchmark where we lose purchasing power 18.1%. However, this is a much more conservative approach. By reducing the allocation in the US large cap equities, we are decreasing the volatility, but the tradeoff is that the trust fund won't grow as much as it did in the 60/40 allocation. We also slightly increase the paths preserving the purchasing power, the median real return and the percentile real returns.



Figure 7: The last 5 years returns mean and standard deviation

The red dot is the benchmark 60/40 allocation, and the second allocation focuses on reducing the volatility. Furthermore, these preservers the purchasing power. This allocation is a strong

INVESTMENT PROPOSAL FOR Charitable Trust

candidate if the board is looking to go more conservative. This prioritizes capital preservation and risk minimization over aggressive growth. It delivers the best outcome in terms of minimizing the probability of purchasing power loss, which is the board's primary goal. While it may not offer the highest upside potential, it excels in protecting the real value of the trust over time, even under less favorable economic scenarios.

Third Allocation: 55% in Equities, -30% in Money Market, and 75% in Blended Fixed.

An unconventional alternate and more aggressive strategy would involve leveraging the portfolio by allocating 30% to the Money Market, effectively borrowing funds to invest 75% in the Blended Fixed Income and 55% in US Large Cap Equities. This allocation brings the total invested capital to 130% of the trust's balance, aiming to enhance returns by utilizing cheap borrowing to amplify exposure to income-generating and growth assets. This effectively reduces the probability of losing purchasing power to 15.8%, but the tradeoff is an increase in volatility.

55/-30/75 allocation	Real Balance	% change in PP	Real Geo return
Min	27.45	-72.55%	-0.0422
Max	1140.93	1040.93%	0.0845
Average	206.61	106.61%	0.0190

Table 11: Metrics for the 55/-30/75 allocation

The table above shows that the minimum, maximum, and average real balances have increased than the benchmark strategy. Furthermore, the % change in purchasing power for minimum and maximum and average has also increased and so has the real geometric return.

Probability loss in PP	E[Loss PP Loss PP]	Volatility	Paths preserving PP	Median RR	25th percentile RR	75th percentile RR
0.158	-0.2579	0.1126733	842	0.0189	0.0058	0.0324

Table 12: Additional metrics for 55/-30/75 allocation

As we can see the probability of losing purchasing power is significantly lower, conversely the paths preserving the purchasing power also increase. The median and percentile values of real returns increase, however, the volatility and conditional expectation increase. As suggested in the tables, this is more aggressive approach.

INVESTMENT PROPOSAL FOR Charitable Trust

SPENDING RATE

An increase in spending would result in increase in probability of loss in the purchasing power. The volatility would be the same since spending power is not contributing to the trend of US large cap equities or the blended fixed. Every allocation would result in an increase probability of loss in the purchasing power and an increase in the conditional expectation.

Analysis suggests that an increase to this rate would likely work in opposition to the fund's primary objective of preserving purchasing power over the next 30 years. If anything, the board is encouraged to consider lowering the purchasing rate or staying with the same one.



Figure 8: Spending rate vs. Probability of Loss in Purchasing Power

INVESTMENT PROPOSAL FOR Charitable Trust

SUMMARY

In December 2024, we were presented with analyzing the 60/40 allocation of the trust where 60% was invested in US large cap equities and 40% in the blended fixed income. The boards' primary goal was for the trust to preserve its 3% purchasing power to support its charitable missions.

In our analysis we found three different allocations that not only preserve but reduce the probability of losing the purchasing power. Unfortunately, there are some tradeoffs for each strategy, the first allocation is investing 62% in US large cap equities, 2.5% in the money market and 35.5% in the blended fixed income. This results in a slight increase in volatility but also an increase in the number of the paths preserving the purchasing power. Furthermore, this allocation also increases the rate at which the fund grows.

The second allocation is investing 55% in equities, 0 in money market, and 45% in the blended fixed income. Since we are decreasing the allocation in equities, which are the primary source of increased volatility, our volatility decreases and so does our probability of losing purchasing power. However, the fund doesn't grow as well as it did with the benchmark allocation of 60/40.

Lastly, an unconventional and aggressive strategy to increase the number of paths preserving the purchasing power is to borrow 3% from the money market and take the risk of pay it back with interest. This results in an allocation of 55% in equities, -3% in money market, and 75% in the blended fixed income. It significantly decreases the probability of losing purchasing power and the conditional expectation but increases the volatility.

It's up to the board of directors to assess these allocations and go forward with the one they are most comfortable with. Our recommendation would be the moderately conservative allocation of 62% in equities, 2.5% in money market, and 35.5% in the blended fixed income. It effectively preserves purchasing power, maintains acceptable volatility, and avoids unnecessary complications. Given this allocation's performance, we do not recommend increasing the 3% annual spending rate. Preserving long-term purchasing power remains achievable at 3%, but any increase would raise the probability of shortfall risk across economic conditions.