L22 - 24/10/2024

In contemporary Math, proof are used to

- verify that a statement is true
- explain 'why' a statement is true.

'Deep' thms usually have multiple proofs.

eg – quadratic reciprocity

See – 'Proofs from the Book' by Ziegler
(coined by Erdos)

attributed to Theatetus

Regular Polyhedra (Platonic Solids)

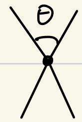
		V	E	F
Tetrahedron	-	(4,	6,	4)
Cube	-	(8,	12,	6)
Octahedron	-	(6,	12,	8)
Dodecahedron	-	(20,	30,	12)
Icosahedron	-	(12,	30,	20)

Regular solid - A solid $\subseteq \mathbb{R}^3$ is called regular if all the faces are congruent to each other & are regular polygons and the no. of faces incident on each vertex is the same.

Why only these ?

- Fix a vertex of the solid S.

- $N \cdot \theta < 2\pi$



$\uparrow$ (no. of faces incident on each vertex) $N \geq 3$

- Fix the no. of sides of regular polygon M

1. $M = 3 \Rightarrow \theta = \pi/3$

$N \cdot \theta < 2\pi \Rightarrow N \cdot (\pi/3) < 2\pi$

$\Rightarrow N < 6$

$N = 1 \longrightarrow \times$

$2 \longrightarrow \times$

$3 \longrightarrow$ Tetrahedron

$4 \longrightarrow$ Octahedron

$5 \longrightarrow$ Icosahedron

2. $M = 4 \Rightarrow \theta = \pi/2$

$N \cdot \theta < 2\pi \Rightarrow N \cdot (\pi/2) < 2\pi$

$\Rightarrow N < 4$

$N = 1 \longrightarrow \times$

$2 \longrightarrow \times$

$3 \longrightarrow$ Cube

3. $M = 5 \Rightarrow \theta = 3\pi/5$

$$N \cdot \theta < 2\pi \Rightarrow N \cdot (3\pi/5) < 2\pi$$
$$\Rightarrow N < 10/2$$

$$N = 1 \longrightarrow \times$$
$$2 \longrightarrow \times$$
$$3 \longrightarrow \text{Dodecahedron}$$

4. $M = 6$ onwards $\Rightarrow \theta = \frac{(s-2)}{s}\pi$

$$N \cdot \theta < 2\pi \Rightarrow N \cdot \frac{(s-2)}{s}\pi < 2\pi$$

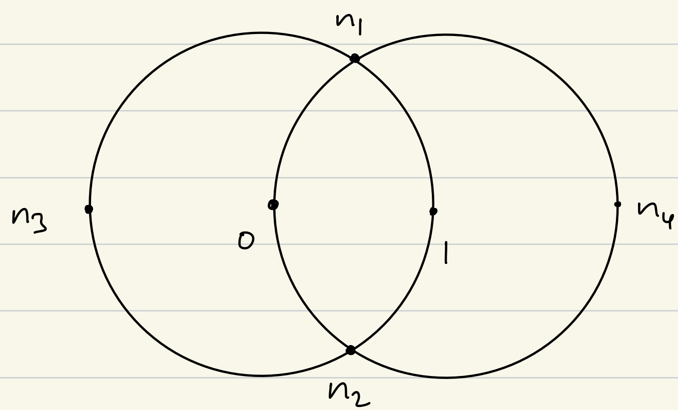
$$\Rightarrow N < \frac{2s}{(s-2)}$$

$\forall s \geq 6, \quad N \in (2, 3)$

<u>Euler's formula</u> – $V - E + F = 2$

Ruler Compass Construction

Recall : constructible nos.



Construct nos. n_1, n_2, n_3, n_4 from $0, 1$.

Inductively repeat to define the set of constructible nos.

Remark – The set of constructible nos. forms a field.

The Greeks tried constructing $\sqrt[3]{2}$, π as well as trisecting the angle.

They couldn't do it & eventually accepted the impossibility & a sol^n by more advanced methods.

<u>Sol^n to</u> –

$\sqrt[3]{2}$ & trisecting angle – Wantzel, Galois (1837)

π – Lindemann

<u>Open</u> – Which n-sided regular polygons are constructible?