L17 - 08/10/2024

Ancient Greek Mathematics

Stilwell: Mathematics & Reference its History

Rational pts on a Cucle

This is related to enumerating Pythagorean triples.

$$a^2 + b^2 = c^2$$
, $a_1b_1c \in \mathbb{Z}_{>0}$
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$$QCD(a,b,c) = 1$$

General soln -
$$a = p^2 - q^2$$

 $b = 2pq$
 $c = p^2 + q^2$

But, there's a geometric approach to the same problem.

for non-trivial solns,
$$c \neq 0$$
.

$$\Rightarrow \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

Hence, consider the eqn:
$$x^2 + y^2 = 1$$

$$S = \left\{ (x_1)y_1 : x_2 + y_2 = 1 \right\}$$

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