L3 - 09/08/2024

Add^n

1. $0 + m := m$
2. $(n{+}{+}) + m = (n+m){+}{+}$

For proving commutativity

$$n + m = m + n$$

we first need to prove the following 2 lemmas using ind^n.

1. $n + 0 = n$
2. $n + (m{+}{+}) = (n+m){+}{+}$

Q. If a is positive & b is a natural no., show that $(a+b)$ is positive

Pf – Fix a positive no. a.
Let $P(b)$ be the ppt.
$(a+b)$ is positive

BC – $(a+0) = a$ which is positive by defn.

So, $P(0)$ is true.

IH – Given $(a+b)$ is positive,
PT $a+(b++)$ is positive.

$a+(b++) = (a+b)++$

By Axiom 2, $(a+b)++$ is a natural no.

By Axiom 3, $(a+b)++ \neq 0$

So, $P(b) \Rightarrow P(b++)$

$\therefore$ By PMI, $P(b)$ is true $\forall$ natural nos. b $\square$