

L10 - 05/09/2024



‘Dirghachakustrayakshyanaha rajuh
pashyavami tiyagamani ch
yat prithag bhoote kurutidubhaya
karoti’

Hypotenuse

Adjacent sides

• Pythagoras Thm in number theory

Determine $(a, b, c) \in \mathbb{Z}_{\geq 0}^3$ s.t
they form sides of a right-
angled triangle with c as
hypotenuse

$$\Rightarrow a^2 + b^2 = c^2 \quad - (E)$$

eg - $(3, 4, 5)$, $(5, 12, 13)$, $(7, 24, 25)$,
 $(8, 15, 17)$, $(12, 35, 37)$, etc.

NOTE - (a, b, c) satisfies E

$\Rightarrow (ka, kb, kc)$ satisfies E

$$\forall k \in \mathbb{Z}_{\geq 0}$$

Solⁿs s.t. $\text{GCD}(a, b, c) = 1$ are called primitive solⁿs.

There are infinitely many such primitive solⁿs.

Non-trivial examples in Sautramani

Altar - $(5\sqrt{3}, 12\sqrt{3}, 13\sqrt{3})$
 $(15\sqrt{2}, 36\sqrt{2}, 39\sqrt{2})$

• General Solⁿ -

(NOT in Sulvasutras.

By Brahmagupta)

$$a = (m^2 - n^2)$$

$$b = 2mn$$

$$c = (m^2 + n^2), \quad \text{GCD}(m, n) = 1$$

• Connection to Modern Math

Study of integer solⁿs to polynomial is an active and open discipline.

eg - Fermat's last Thm

$$a^n + b^n = c^n$$

has no non-trivial integer solⁿs for $n \geq 3$

Geometric Constructions

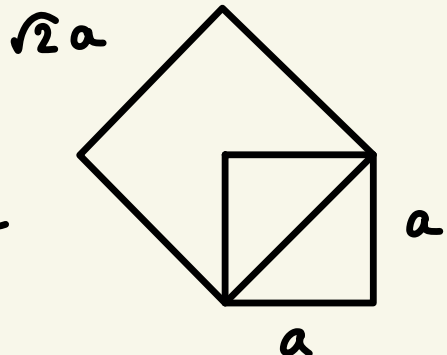
- Only compass & straight edge available for construction
- Area axiom - Area of rect. with sides of lengths a & b is ab .

Problems

1. Given a sq. of side a and a natural no. n , construct a sq. with area na^2

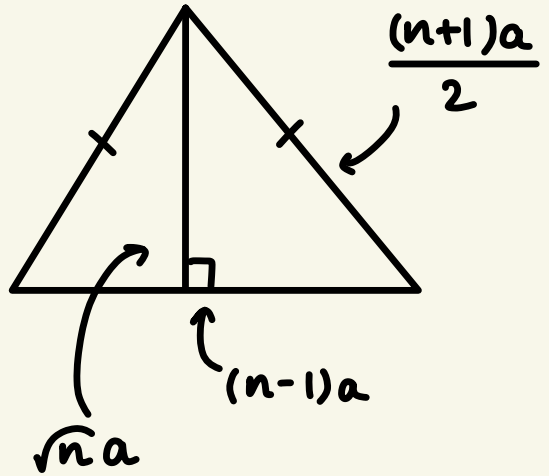
A. 11 For $n=2$

Construct a sq. on with the diagonal of original sq. as side.



1.2 General case

Construct an isosceles triangle with base $(n-1)a$ & equal sides $(n+1)a/2$



So, the altitude will be

$$\sqrt{\frac{(n+1)^2}{4}a^2 - \frac{(n-1)^2}{4}a^2} = \sqrt{n}a$$

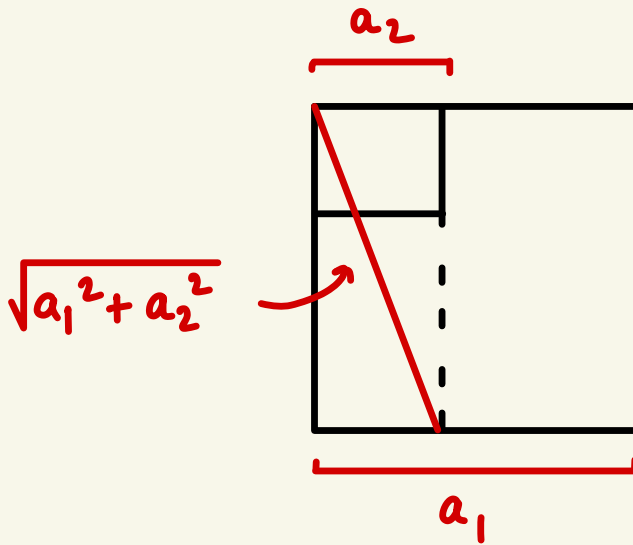
Now construct a sq. with this altitude as the side.

2. Given 2 sq. S_1 & S_2 ,
construct a sq. with area
which is

2.1 sum of areas of given sq.

2.2 diff of areas of given sq.

A. 2.1



2.2

