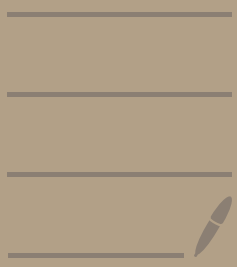
L23 – 29/10/2024



Conic Sections

Std. cone : $x^2 + y^2 = z^2$

Parameters – Axis of revolution
Angle of cone (θ)

When a plane inclined at diff. angles to the axis intersect the cone, conic sections are obtained

Conic Section	$\angle$ b/w axis & normal of plane
Circle	0
Ellipse	$< \theta$
Parabola	$= \theta$
Hyperbola	$> \theta$

Attributed to Menaechmus ~ 300 BC
(contemporary of Alexander)

- Doubling the cube using conic sections –
  Intersection of the parabola $y = \frac{1}{2}x^2$ with the hyperbola $xy = 1$

- Kepler later built on this in his theory of elliptical orbits of planets. Newton derived it from his gravitation law.

Euclid

- Less is known about Euclid than Pythagoras

- Taught in Alexandria, Egypt ~ 300 BC

- Told Ptolemy I : 'There is no royal road to geometry'

- Student : 'Gain from Math ?'
  Euclid : Gives him a coin

- Known for 'Elements' ~ 12 vols.

- Not all of it was original
  The following was already known :-
  1. Elementary ppts. of lines & circles
  2. Irrationality (Eudoxus ~ 400 - 347 BC)
  3. Thory of regular polyhedra (Theaetetus ~ 413 - 309 BC)

- Stands out for organisation, dissemination of Math.
  Core of Math education in West and at the heart of Western culture for > 2000 years.

- Influenced : Lincoln, Russell

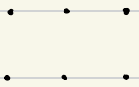
Greek Number Theory

Comparison b/w Geometry & Number Theory

- Geometry allows for a systematic theory compared to Number Theory.
- NT has many open problems with unknown theoretical framework.
- Both are almost as old with NT being slightly older.
- Recently, connections b/w them have emerged.

Prime Nos.

- Rect. nos. — eg. 6



Primes : Non-rect. nos.

- Prime no. — A natural no. $n = pq$ s.t atleast one of p or q is 1.

- Divisibility — m divides n if $\exists\ k \in \mathbb{Z}$ s.t $n = km$

Notⁿ: $m \mid n$

Here, m is called a divisor of n

P: Every int. has a prime divisor.

Euclid's Thm – There are infinitely many primes.

Pf – Assume finitely many primes $\{p_1, \dots, p_n\}$

Consider $M = \prod_{i=1}^{n} p_i + 1$

Note $p_i \nmid M$, $1 \leq i \leq n$

which is a contdⁿ to prev. ppⁿ □