PH110: Tutorial Sheet 5 (Quantum Mechanics)

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* marked problems will be solved in the Wednesday tutorial class.

Free particle

1. *Show that
$$\psi(x) = A\,\sin(kx) + B\,\cos(kx)$$
and
$$\psi(x) = Ce^{ikx} + De^{-ikx}$$
are equivalent solutions of TISE of a free particle. A, B, C and D can be complex numbers.

2. Show that
$$\Psi(x,t) = A\sin(kx - \omega t) + B\cos(kx - \omega t)$$
does not obey the time-dependant Schroedinger's equation for a free particle.

3. The wave function for a particle is given by,
$$\phi(x) = Ae^{ikx} + Be^{-ikx}$$
where A and B are real constants. Show that $\phi(x)^*\phi(x)$ is always a positive quantity.

4. * A free proton has a wave function given by
$$\Psi(x,t) = Ae^{i(5.02*10^{11}x - 8{:}00*10^{15}t)}$$
The coefficient of x is inverse meters, and the coeffcient of t is inverse seconds. Find its momentum and energy.

5. A particle moving in one dimension is in a stationary state whose wave function,
$$\Psi(x) = \begin{cases} 0, & x < -a \\ A\left(1 + \cos\frac{\pi x}{a}\right), & -a \le x \le a \\ 0, & x > a \end{cases}$$
where A and a are real constants.
(a) Is this a physically acceptable wave function? Explain.
(b) Find the magnitude of A so that $\psi(x)$ is normalized.

(c) Evaluate Δx and Δp. Verify that $\Delta x \Delta p \geq \hbar/2$.
(d) Find the classically allowed region.

6. * Consider the 1-dimensional wave function of a particle of mass m, given by

$$\psi(x) = A\left(\frac{x}{x_0}\right)^n e^{-\frac{x}{x_0}}$$

where, A, n and x_0 are real constants.
(a)Find the potential $V(x)$ for which $\psi(x)$ is a stationary state (It is known that $V(x) \rightarrow 0$ as $x \rightarrow \infty$).
(b)What is the energy of the particle in the state $\psi(x)$?

Particle in a Box:

1. * For a particle in a 1-D box of side L, show that the probability of finding the particle between $x = a$ and $x = a + b$ approaches the classical value b/L, if the energy of the particle is very high.

2. Consider a particle confined to a 1-D box. Find the probability that the particle in its ground state will be in the central one-third region of the box.

3. Consider a particle of mass m moving freely between $x = 0$ and $x = a$ inside an infinite square well potential.
(a) Calculate the expectation values $\langle \hat{X} \rangle_n, \langle \hat{P} \rangle_n, \left\langle \hat{X}^2 \right\rangle_n$, and $\left\langle \hat{P}^2 \right\rangle_n$, and compare them with their classical counterparts.
(b) Calculate the uncertainties product $\Delta x_n \Delta p_n$.
(c) Use the result of (b) to estimate the zero-point energy.

4. Consider a one dimensional infinite square well potential of length L. A particle is in $n = 3$ state of this potential well. Find the probability that this particle will be observed between $x = 0$ and $x = L/6$. Can you guess the answer without solving the integral? Explain how.

5. * Consider a one-dimensional particle which is confined within the region $0 \leq x \leq a$ and whose wave function is $\Psi(x, t) = \sin(\pi x/a) \exp(-i\omega t)$.
(a) Find the potential $V(x)$.
(b) Calculate the probability of finding the particle in the interval $a/4 \leq x \leq 3a/4$.

6. An electron is moving freely inside a one-dimensional infinite potential box with walls at $x = 0$ and $x = a$. If the electron is initially in the ground state ($n = 1$) of the box and if we suddenly quadruple the size of the box (i.e., the right-hand side wall is moved instantaneously from $x = a$ to $x = 4a$), calculate the probability of finding the electron

in:

(a) the ground state of the new box and

(b) the first excited state of the new box.

7. Solve the time independent Schrodinger equation for a particle in a 1-D box, taking the origin at the centre of the box and the ends at $\pm L/2$, where L is the length of the box.

8. * Consider a particle of mass m in an infinite potential well extending from $x = 0$ to $x = L$. Wave function of the particle is given by

$$\psi(x) = A\left[\sin\left(\frac{\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right)\right]$$

where A is the normalization constant

(a) Calculate A

(b) Calculate the expectation values of x and x^2 and hence the uncertainty Δx.

(c) Calculate the expectation values of p and p^2 and hence the uncertaintyΔp.

(d) What is the probability of finding the particle in the first excited state, if an energy measurement is made?

(given, $\int_0^L x\cos\left(\frac{n\pi x}{L}\right)dx = 0$, $\int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)dx = 0$, for all n)
