L12 - 12/09/2024

4. Circling a sq.

A. From our knowledge,

$$\pi R^2 = a^2 \Rightarrow R = \frac{a}{\sqrt{\pi}}$$

But, if R is constructible by ruler & compass, so is π.

And we know from Galois Theory that π isn't constructible.

Hence, such an R is <u>not</u> constructible.

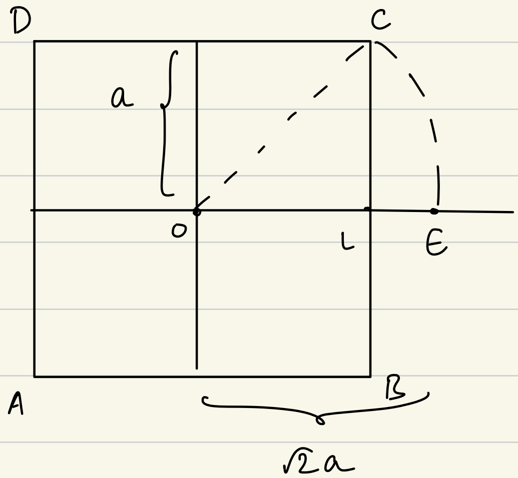
Since the problem can't be solved exactly by ruler & compass

Thus, we seek approximate sol^{n}s.

Note The no. π isn't defined explicitly in the Sulva Sutras.

Approx. Sol^n -

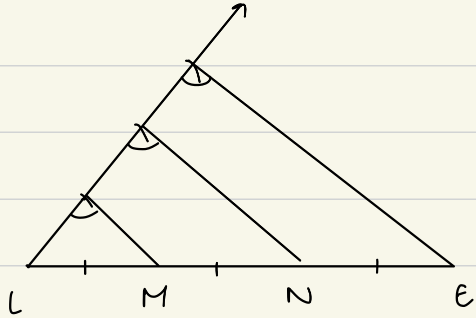
Consider a sq. of length $2a$



1. Mark E on OL s.t $OC = OE$

2. Trisect LE

3. Circle Ω with radius OM gives approximate sol^n the problem



Trisection of line segment

$$ar(\Omega) = \pi(OM)^2$$
$$= \pi(OE - ME)^2 = \pi\left(\sqrt{2}a - \tfrac{2}{3}(\sqrt{2}-1)a\right)^2$$
$$= \pi a^2\left(\frac{\sqrt{2}+2}{3}\right)^2$$
$$\sim 1.29\, a^2$$

This gives us an approximation of π.

$$\left.\begin{aligned} \text{Expected ar.} &= 4a^2 \\ \text{Approx. ar.} &= \pi\left(\frac{\sqrt{2}+2}{3}\right)^2 \end{aligned}\right\} \Rightarrow \pi\left(\frac{\sqrt{2}+2}{3}\right)^2 \sim 4a^2$$

$$\Rightarrow \pi \sim 18(3-2\sqrt{2})$$
$$\sim 3.088$$

For squaring a circle one can try to reverse the constr^n.

Brick Constructions

Altar consists of various layers of bricks. Each layer consisted of bricks of diff. sizes.

This led to mathematical problems involving integer sol^{n}s to poly. eq^{n}s.

eg - 1. Garhipatyagni : 5 layers of 21 bricks each.

Each layer has same ar. - 1 sq. vyam

There are 2 types of bricks of length 1/m & 1/n vyam.

Let x : # type I bricks

y : # type II bricks

<u>Constraints</u> :

$$x + y = 21$$

$$\frac{x}{m^2} + \frac{y}{n^2} = 1 \quad , \quad \begin{matrix} x, y \geq 0 \\ m > n \end{matrix}$$

$$\Rightarrow \quad \frac{x}{m^2} + \frac{(21-x)}{n^2} = 1$$

$$\Rightarrow \quad x = \frac{m^2(21-n^2)}{(m^2-n^2)}$$

$$y = \frac{n^2(21-m^2)}{(n^2-m^2)}$$

$\because$ $m > n$

$\therefore$ For $x \geq 0$, $n^2 < 21$ $\Rightarrow$ $n = 1, 2, 3, 4$

By sub^n n & finding corresponding m, we get.

$$(m, n) = \underbrace{(6,3)}_{\text{Config 1}}, \underbrace{(6,4)}_{\text{Config 2}}$$

$$\Rightarrow (x, y) = (16, 5), (9, 12)$$

Thus, we get 3 types of bricks – 1/3, 1/4, 1/6.

For stability of layers placed atop others, the following constrⁿ is suggested -

L5 - Config 1

L4 - Config 2

L3 - Config 1

L2 - Config 2

L1 - Config 1

2. <u>Garud Chayan</u> : 5 layers

Each layer consists of 200 bricks & has area 15/2 sq. vyam

There are 4 types of bricks with area $1/a_i$ for type i

Let x_i : # bricks of type i

<u>Constⁿ</u> :

$$\sum x_i = 200$$

$$\sum x_i/a_i = 15/2$$

<u>Solⁿ</u> : $(a_1, a_2, a_3, a_4) = (16, 25, 36, 100)$

$(x_1, x_2, x_3, x_4) = (2, 120, 36, 20); (12, 125, 63, 0)$