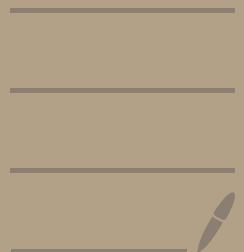


L20 - 17/10/2024

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Suppose  $E$  has a non-t'vl int. sol<sup>n</sup>  $(x_0, y_0)$

$$\text{let } s = N((x_0, y_0)) = x_0^2 + y_0^2$$

$\therefore$  The set of int. pts. with size  $\leq s$  is finite

$\therefore$  The set of non-t'vl int. sol<sup>n</sup>s with size atmost  $s$  is also finite.

So,  $\exists$  a pt. with min. size.

Let  $(a_0, b_0)$  be such a pt.

But, as shown prev.,  $(b_0, a_0/2)$  is also a non-t'vl int. sol<sup>n</sup>.

However,  $N((b_0, a_0/2)) < N((a_0, b_0))$  which is a contd<sup>n</sup>.

## Proof based on Aristotle's Prior Analytics

If  $\exists$  a non-trivial int. sol<sup>n</sup>, wlog,  
we can assume it is primitive.

$$\text{i.e. } \gcd(x_0, y_0) = 1$$

$$x_0^2 = 2y_0^2$$

$$x_0^2 \text{ is even} \Rightarrow x_0 \text{ is even}$$

$$\text{So, } x_0 = 2k_0 \text{ for some } k_0$$

$$\Rightarrow (2k_0)^2 = 2y_0^2$$

$$\Rightarrow 2k_0^2 = y_0^2$$

$$y_0^2 \text{ is even} \Rightarrow y_0 \text{ is even}$$

which is a contd<sup>n</sup>

The result shocked the ancient Greeks.  
They did not accept  $\sqrt{2}$  as a no.

So, they tried to approximate  $\sqrt{2}$  in  
terms of rational nos.

(Diophantine approximations)

## Pythagoras

- Very little is known for certain.
- None of the documents from his times have seemingly survived.
- Rely on stories passed on through generations
- Birth : Samos , Greek Island  
near Turkey ~ 580 BC
- Travelled to Miletus to learn  
Math from Thales ~ 624 - 547 BC  
(founder of  
Greek Math)
- Also travelled to Egypt & Babylon  
& picked up more Math.

- Settled in Croton ~ 540 BC  
(a Greek colony in Italy)
- Here, he founded a school, now called the Pythagoreans.
- Philosophy: 'All is number'  
To bring all human endeavour including science, religion, philosophy in the realm of Math.
- The word 'Mathematics' is attributed to this school.
- Strict conduct of conduct:  
Secrecy, vegetarianism, taboo against eating beans, sought numerical laws governing orbits of planets

- Highlights : Explanation of musical harmony of whole - no. ratios.

- Death : 497 BC

Archaeological sites . Pythagoreion , Samos

# Greek Geometry

## 1. The method of deduction (Proof)

The method involves demonstrating the validity of a statement using prev. established statements using principles of logic.

These established statements are called postulates or axioms - 'Self-evident' or 'intuitive' statements

Hence, all statements trace back to the axioms.