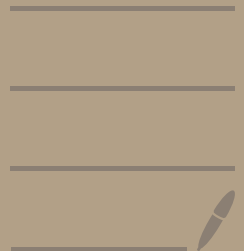


L3 - 09/08/2024



## Add<sup>n</sup>

1.  $0 + m := m$

2.  $(n++) + m = (n+m)++$

For proving commutativity

$$n + m = m + n$$

we first need to prove the following 2 lemmas using ind<sup>n</sup>.

1.  $n + 0 = n$

2.  $n + (m++) = (n+m)++$

Q. If  $a$  is positive &  $b$  is a natural no., show that  $(a+b)$  is positive

Pf - Fix a positive no.  $a$ .

Let  $P(b)$  be the ppt.

$(a+b)$  is positive

BC -  $(a+0) = a$  which is positive by def<sup>n</sup>.

So,  $P(0)$  is true.

IH - Given  $(a+b)$  is positive,  
PT  $a+(b++)$  is positive.

$$a+(b++) = (a+b)++$$

By Axiom 2,  $(a+b)++$  is a natural no.

By Axiom 3,  $(a+b)++ \neq 0$

So,  $P(b) \Rightarrow P(b++)$

$\therefore$  By PMI,  $P(b)$  is true  $\forall$  natural nos.  $b$   $\square$