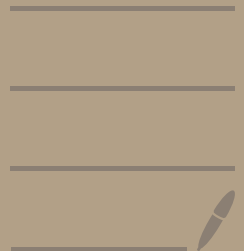


L1 - 31/07/2024



Instructor - Prof. Ronnie Sebastain

Reference book - Analysis I by
Terence Tao
(first 8 chapters)

Mandatory attendance

Quiz - one per week - Friday
(40% weightage)

Mid Sem & End Sem - 30% each

Q. Show $\varphi: \mathbb{N} \rightarrow \mathbb{Z}_{\geq 0}$, $\varphi(n) = n-1$
is injective & surjective

Proof : 1. Injectivity

$$\varphi(n_1) = \varphi(n_2) \quad ; \quad n_1, n_2 \in \mathbb{N}$$

$$\Rightarrow n_1 - 1 = n_2 - 1$$

$$\Rightarrow n_1 = n_2$$

$$\therefore \varphi(n_1) = \varphi(n_2) \Rightarrow n_1 = n_2$$

$\therefore \varphi$ is injective.

2. Surjectivity

Consider $m \in \mathbb{Z}_{\geq 0}$ & $n = m+1$

Clearly $n \in \mathbb{N}$.

$$\varphi(n) = n-1 = (m+1)-1 = m$$

$$\therefore \forall m \in \mathbb{Z}_{\geq 0}, \exists n \in \mathbb{N} \text{ s.t.}$$

$$\varphi(n) = m$$

$\therefore \varphi$ is surjective. \square

Q. Prove $\nexists \alpha \in \mathbb{Q}$ s.t. $\alpha^2 = 2$.

Proof - Let $\alpha = \frac{p}{q}$; $p, q \in \mathbb{Z}$,
 $q \neq 0$

s.t. $\text{GCD}(p, q) = 1$

$$\begin{aligned} \text{By def}^n, \quad \alpha^2 = 2 &\Rightarrow \left(\frac{p}{q}\right)^2 = 2 \\ &\Rightarrow p^2 = 2q^2 \end{aligned}$$

$$\begin{aligned} \text{Now, } 2 \mid \text{RHS} &\Rightarrow 2 \mid \text{LHS} \Rightarrow 2 \mid p^2 \\ &\Rightarrow 2 \mid p \end{aligned}$$

$$\Rightarrow \exists k \in \mathbb{Z} \text{ s.t. } p = 2k$$

Substituting back, $p^2 = 2q^2$
 $\Rightarrow (2k)^2 = 2q^2$
 $\Rightarrow 2k^2 = q^2$

Now, $2 \mid \text{LHS} \Rightarrow 2 \mid \text{RHS} \Rightarrow 2 \mid q^2$
 $\Rightarrow 2 \mid q$

$\therefore 2 \mid p \text{ \& } 2 \mid q$

$\therefore 2 \mid \text{GCD}(p, q) \Rightarrow \underline{2 \mid 1}$
Contdⁿ

$\therefore \nexists \alpha \in \mathbb{Q} \text{ s.t. } \alpha^2 = 2 \quad \square$