L25 - 05/11/2024

Calculus

- Marks a shift in the focus from geometry to algebra ~ 16 th century

- Allowed for systematic treatment of areas, volumes, tangents, etc.

- These were considered by the Greeks. Their method is called the 'method of exhaustion'.

<u>Key</u>: Approximating shapes by simpler ones

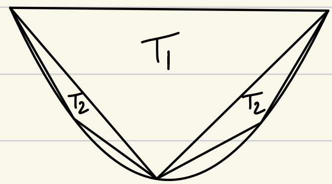
<u>eg</u> - Using regular n-sided polygon to approx circle.

- This method was tedious & hence, Calculus developed as a system of shortcuts.

— <u>Types of Problems</u>

- Area, Volume — Integral Calculus
- Tangent — Differential Calculus

— <u>Archimedes</u>



Method of Exhaustion
to calculate area of parabolic segment.

— More generally, the Greeks tried finding area under $y = x^k$, $k \in \mathbb{Z}_{\geq 0}$

This leads to the sum
$$1^k + 2^k + \dots + n^k$$
which the Arabs calculated for $K = 1, 2, 3, 4$
(956 – 1039 AD)

- Cavalieri (1635 AD) conjectured

$$\int_0^a x^k \, dx = \frac{a^{k+1}}{k+1}$$

Later, Fermat & Descartes established it for integral k $(k \neq -1)$

- Archimedes tried to calculate the tangents to pts in a spiral $r = \theta$

Fermat (1629)

- one of the founders of Calculus

- Introduced limits

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

for polynomials f (one & two variables)
Used this to calculate maxima, minima, tangents ~ 1625 (published 1679)

- This involved a 'sleight of hand' with an infinitesimal element.

i.e. introduce e, divide by e, simplify, and omit e as if it were 0.

eg - $$\frac{(x+e)^2 - x^2}{e} = \frac{2xe + e^2}{e} = 2x + e \sim 2x$$

This confused Philosophers.

- Followed by Descartes in his book 'La Géometrie'

eg – $P(x, y) = 0$

$x = x(t), \quad y = y(t)$

$$\frac{dy}{dx} = -\frac{P_x}{P_y} \quad \text{where} \quad P_x = \partial P / \partial x$$

$$P_y = \partial P / \partial y$$

Newton

Most imp. discovery ~ 1665–1666

Studied works of Descartes, Wallis, Viete

Works : De Analysi, De Methodis
~ 1669 ~ 1671

- Contributions to differentiation are 'minor' except Chain Rule

- Misleading to consider him the founder of calculus, unless one sees it as algebra of infinite series

Key : Manipulation of infinite series

Diff & Int carried out term by term.

eg - $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$

$$\frac{d}{dx}(\sin(x)) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots$$

$$= \cos(x)$$

- De Methodis

'Since the operations for computing with nos. & variables are so similar ... amazed that no one (except Mercator) recognized that the doctrine recently established for decimal nos. can also be carried to variables'