One-Time Pad & Basics of Provable Security (I)

601.642/442: Modern Cryptography

Fall 2020

Today's Agenda

- Private Communication via One-Time Pads
- Basics of Provable Security Approach

Before we begin...

A few remarks

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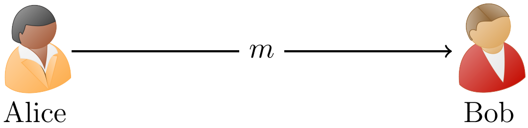
  - What matters is the underlying idea. Always ask "why?"
- View definitions as constructs. They are meant to capture the "rules" of the "game". Think about alternatives.
- Sometimes, the intuition may seem to not align with the proof. But eventually it will, once we make the intuition *robust*.

Private Communication



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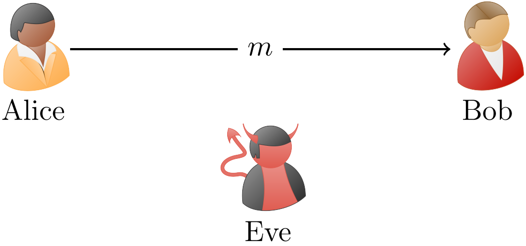

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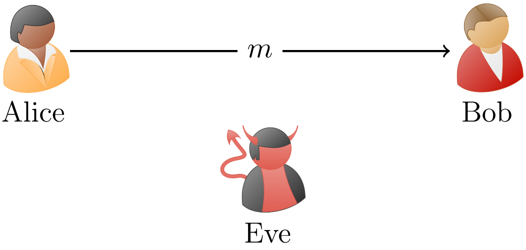

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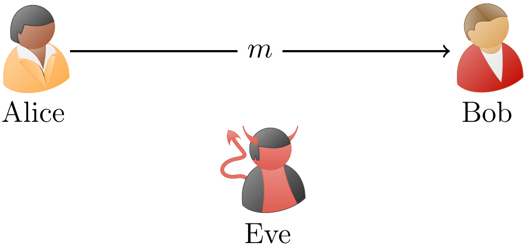
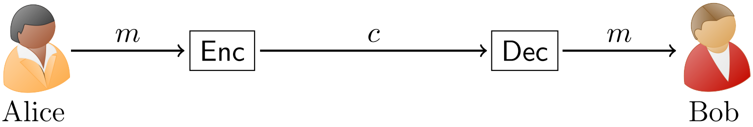


- Alice wants to send a message m to Bob.
- How can Alice convey this message to Bob, while keeping it hidden from an eavesdropper Eve?

Encryption



- **Encryption:** Alice transforms (encrypts) the message m – also called the "plaintext" – into a "ciphertext" c.
- **Communication:** Alice sends c to Bob.
- **Decryption:** Bob recovers (decrypts) the original m from c.

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- Inventing good encryption algorithms is not an easy task.

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- The *length* of the secret key is called the **security parameter**.

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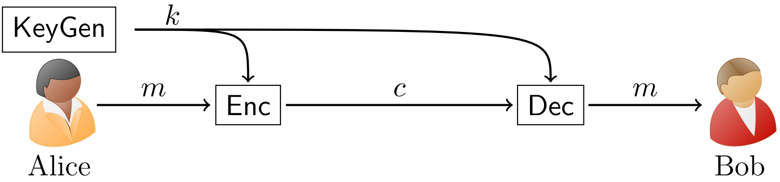

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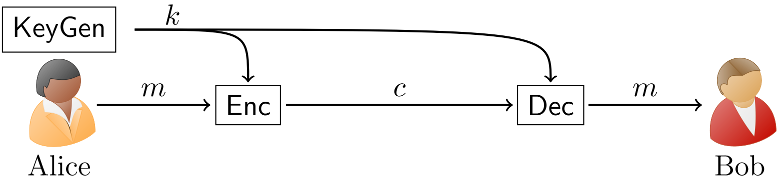

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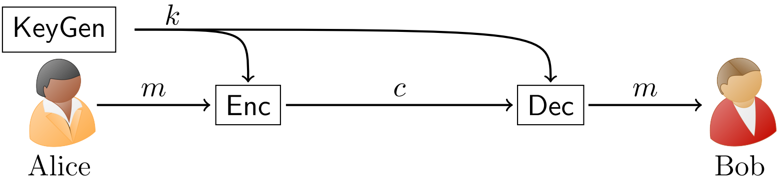

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Encryption: Syntax

An **encryption scheme** consists of the following three algorithms:

- $\mathsf{KeyGen}(1^n) \to k$
- $\mathsf{Enc}(k, m) \to c$
- $\mathsf{Dec}(k, c) \to m$

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- We are assuming that all users have the ability to generate random bits. Will discuss randomness generation and usage in more detail later in the course.

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One-Time Pad: Construction

- $\mathsf{KeyGen}(1^n) := k \xleftarrow{\$} \{0,1\}^n$
- $\mathsf{Enc}(k, m) := c = k \oplus m$
- $\mathsf{Dec}(k, c) := m = k \oplus c$

- Recall: $k \xleftarrow{\$} \{0,1\}^n$ refers to sampling k uniformly at random from the set of all n-bit strings.

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Proof. For all $k, m \in \{0,1\}^n$, we have:

$$\begin{aligned} \mathsf{Dec}(k, \mathsf{Enc}(k, m)) &= \mathsf{Dec}(k, (k \oplus m)) \\ &= k \oplus (k \oplus m) \\ &= 0^n \oplus m \\ &= m \end{aligned}$$

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Example

Let $n = 3$, $m = 010$. What is the value of c for every possible value of k?

One-Time Pad: Security

Pr	k	$c = k \oplus 010$
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1/8	001	011
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1/8	011	001
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- This holds for each $m \in \{0,1\}^3$, and not just $m = 010$.

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  2. No, because Eve's view does include the key.

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- There are two types of properties:
  1. Ones that should hold in the absence of an attacker. E.g., correctness.
  2. Ones that specify what can happen to a system in the presence of an attacker. E.g., security.
- **Eventual Goal:** How to formally define any *secure* system, in a way that captures all the properties that we need from that system.

Encryption: Correctness

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Does $\mathsf{Enc}(k, m) = 0^n$ satisfy correctness? Is it a useful encryption scheme?

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An encryption scheme is a good one if its ciphertexts look like random values to Eve, when each key is secret and used to encrypt only one plaintext, even when Eve chooses the plaintexts.

Encryption: One-Time Uniform Ciphertext Security

One-Time Uniform Ciphertext Security

We say that an encryption scheme is one-time uniform ciphertext secure if $\forall m \in \mathcal{M}$ chosen by Eve, the ciphertext is uniformly distributed (over the ciphertext space $\mathcal{C}$), i.e., the following distributions are identical:

1. $\mathcal{D}_1 := \left\{ c := \mathsf{Enc}(k, m); k \leftarrow \mathsf{KeyGen}(1^\lambda) \right\}$
2. $\mathcal{D}_2 := \left\{ c \xleftarrow{\$} \mathcal{C} \right\}$

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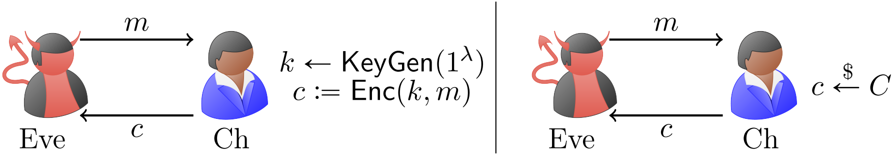
For $m = 0^n$,

$$\Pr[c = 0^n | \mathcal{D}_1] = 1$$
$$\Pr[c = 0^n | \mathcal{D}_2] = 1/2^n$$

Clearly the two distributions are not identical in this case.

Encryption: One-Time Uniform Ciphertext Security

Consider the following two interactions between Eve and a challenger.



- Interaction with a *challenger* helps us model what Eve can see during encryption, and what remains hidden.
- We say that an encryption scheme is secure if for any m chosen by Eve, the above two scenarios seem identical to Eve.