

# Round-Optimal Secure Multiparty Computation with Honest Majority

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Technology

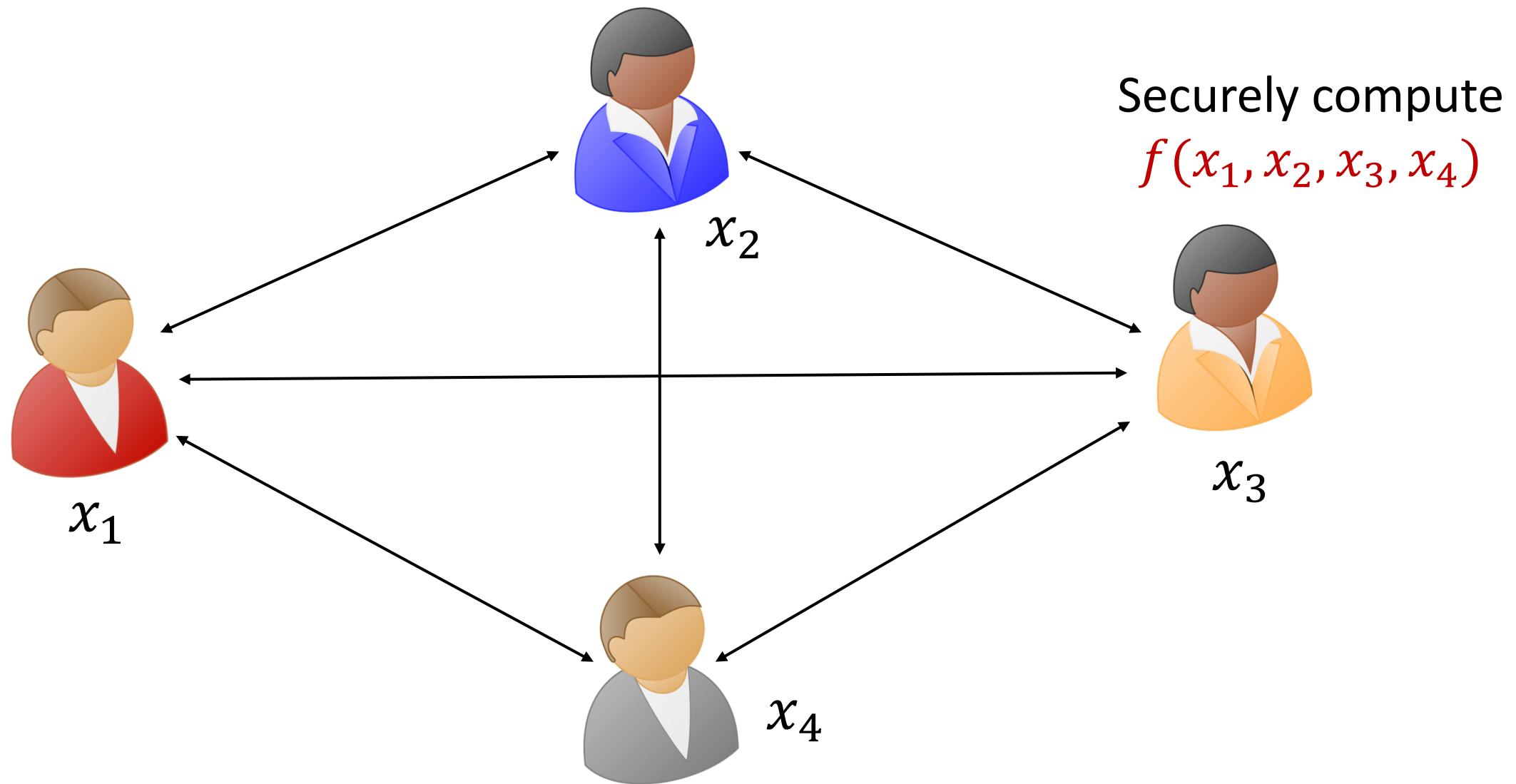


JOHNS HOPKINS  
UNIVERSITY

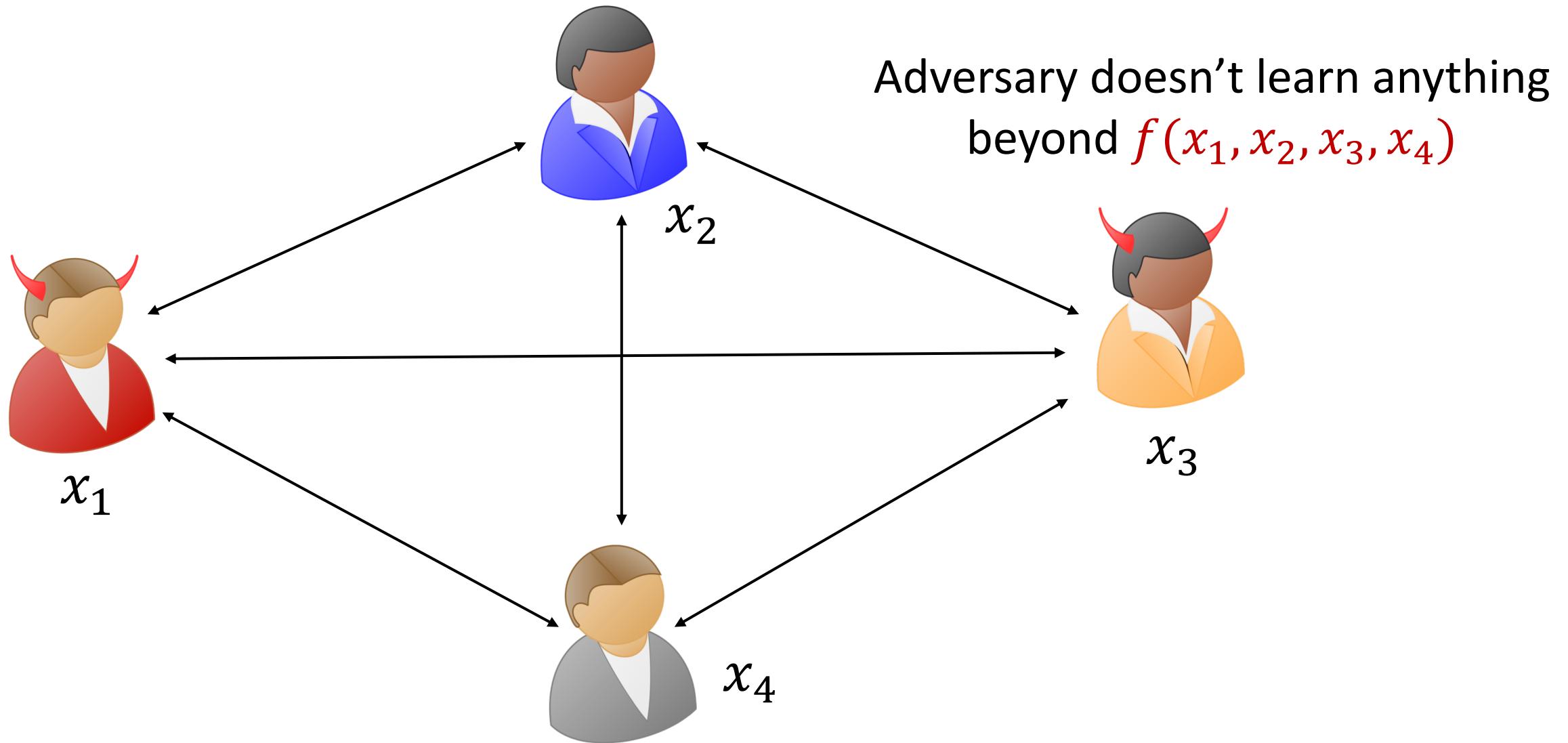
# Secure Multiparty Computation

 $x_1$  $x_4$  $x_2$  $x_3$

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Necessary for dishonest majority [Kil88].
- **Fairness** and **Guaranteed output delivery** can be achieved.
- **UC security** without external trusted setups
- **Round complexity** lower bounds of dishonest majority do not apply.  
4 rounds necessary for dishonest majority in the plain model [Garg-Mukherjee-Pandey-Polychroniadou16]

# Problem Statement

What is the exact **round complexity** of honest majority MPC in the plain model?

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Guaranteed output delivery  $\Rightarrow$  Fairness

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Guaranteed output delivery  $\Rightarrow$  **Fairness**

**Goal:** Develop round optimal protocols in these settings.

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- [Benhamouda-Lin17, Garg-Srinivasan17]:  $t < N$  semi-honest corruptions based on OT. Malicious corruptions in the CRS model.

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Impossible for dishonest majority [Garg- Mukherjee-Pandey-Polychroniadou16]

Open even in **semi-honest** case from assumptions weaker than OT.

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- [Gordon-Liu-Shi'15]: Impossibility of two-round broadcast channel protocols against fail-stop corruptions.

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Does there exist a **three round** MPC protocol secure against  $t < N/2$  malicious corruptions in the plain model?

Both questions open regardless of assumptions.

# Our Results: Security with Abort

Two round MPC for general functionalities in the plain model, assuming one-way functions.

# Our Results: Guaranteed Output Delivery

**Fail-Stop Corruptions:** Two round MPC for general functions:

Broadcast channel protocol in the bare-public-key model, assuming PKE.

Point-to-point channel protocol in the plain model, assuming OT.

# Our Results: Guaranteed Output Delivery

## Fail-Stop Corruptions:

Broadcast channel protocol in the bare-public-key model, assuming PKE.

Three round MPC from one-way functions in the plain model.

OT.

# Our Results: Guaranteed Output Delivery

**Fail-Stop Corruptions:** Two round MPC for general functions:

Broadcast channel protocol in the bare-public-key model, assuming PKE.

Point-to-point channel protocol in the plain model, assuming OT.

**Malicious Corruptions:** Three round MPC for general functions:

Broadcast channel protocol in the plain model, assuming Zaps and PKE.

# Security with Abort against Malicious Adversaries

[Garg-Srinivasan17]

A compiler from any polynomial round MPC protocol to a two round protocol using two round UC secure OT.

[Garg-Srinivasan17]

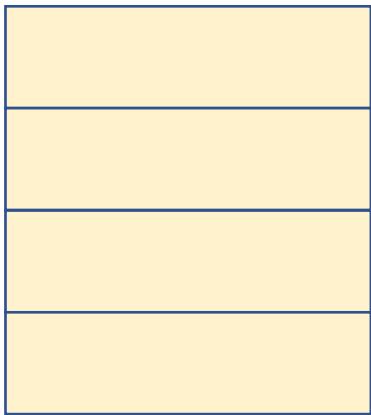
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**Starting Idea:** Leverage honest majority to remove OT.

# [Garg-Srinivasan17]

Use of OT in [GS17]

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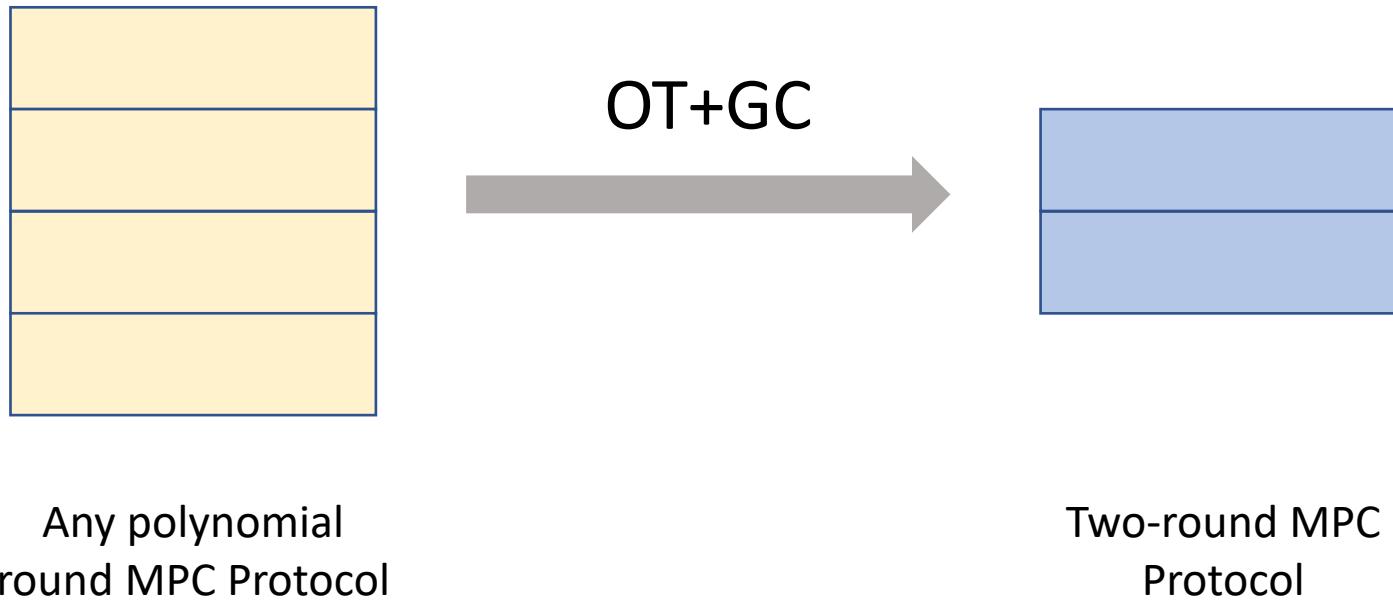


Any polynomial  
round MPC Protocol

## Use of OT in [GS17]

Start with any dishonest majority protocol based on OT over broadcast channels

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Compile it into a 2 round protocol using OT and Garbled circuits

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	Use of OT in [GS17]	Our approach
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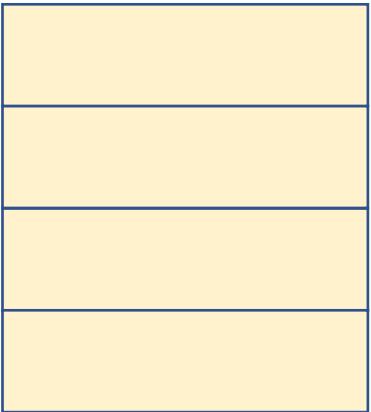
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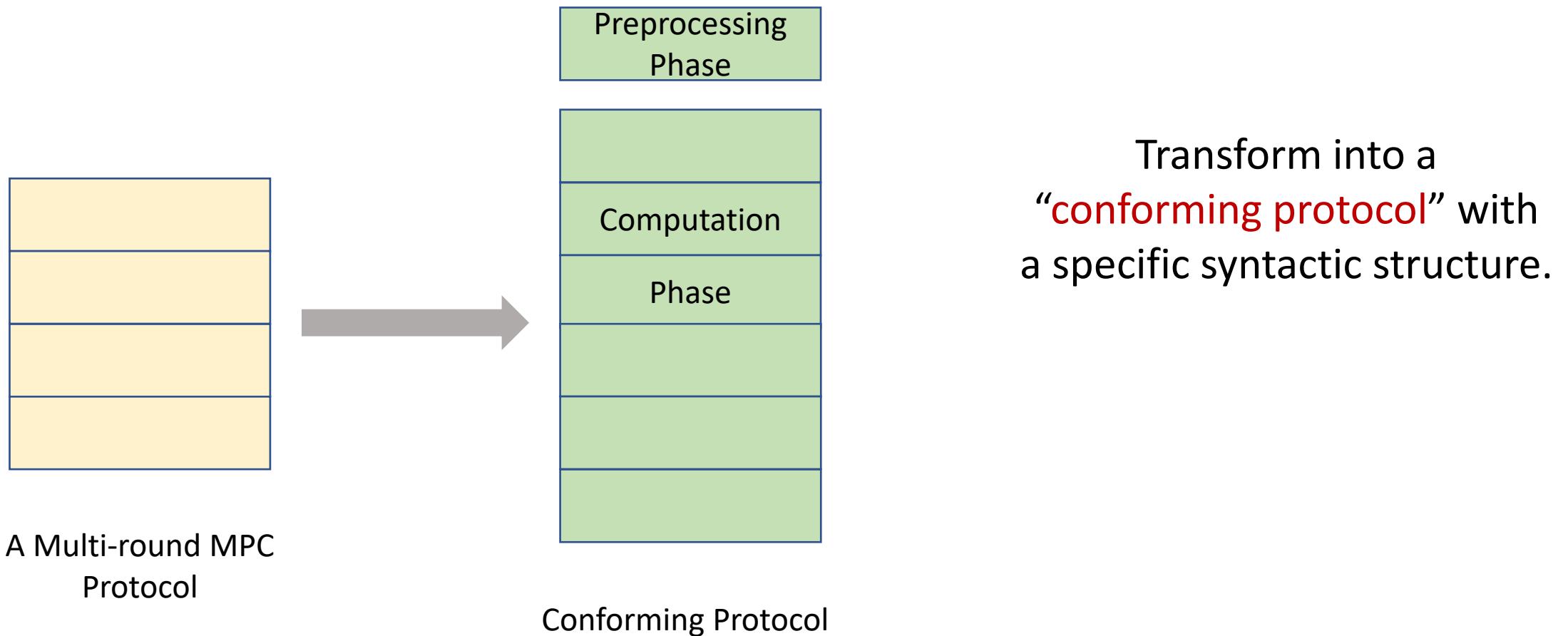
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# Recap of [Garg-Srinivasan17]

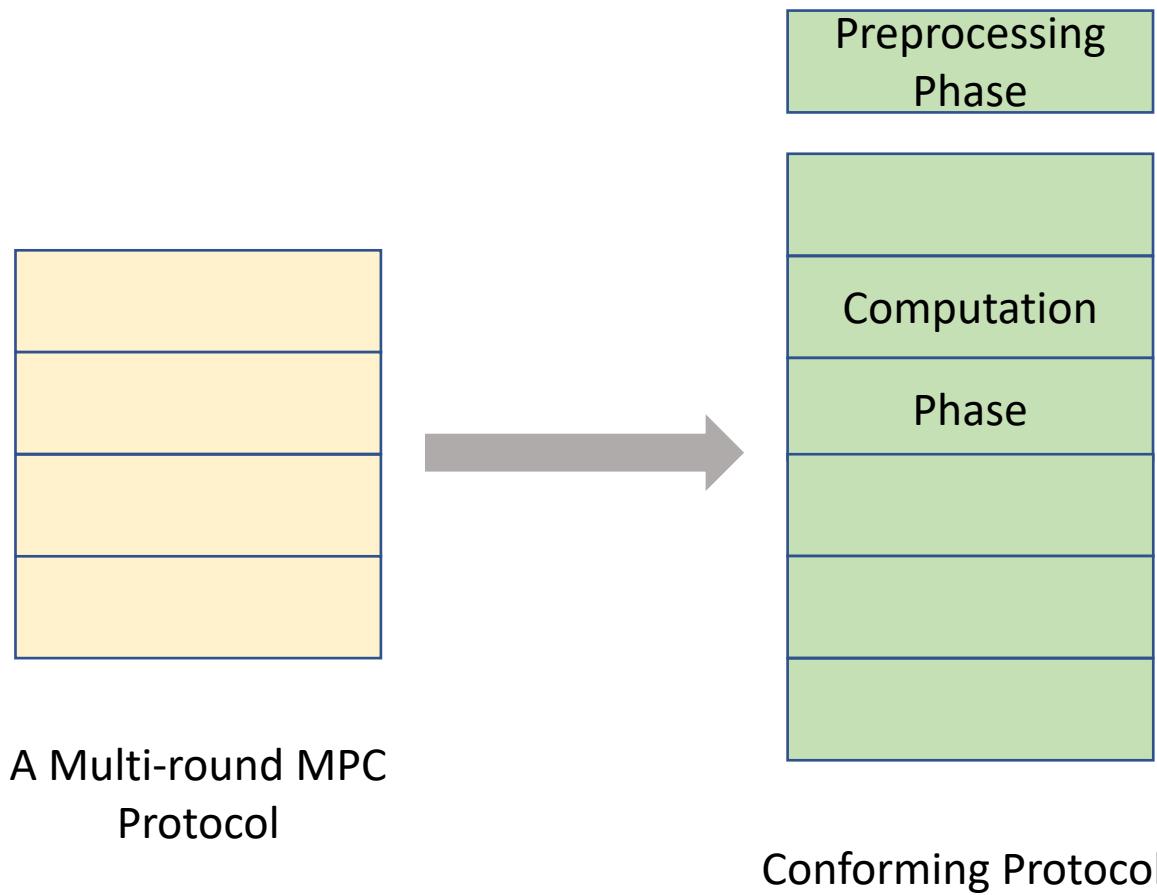


A Multi-round MPC  
Protocol

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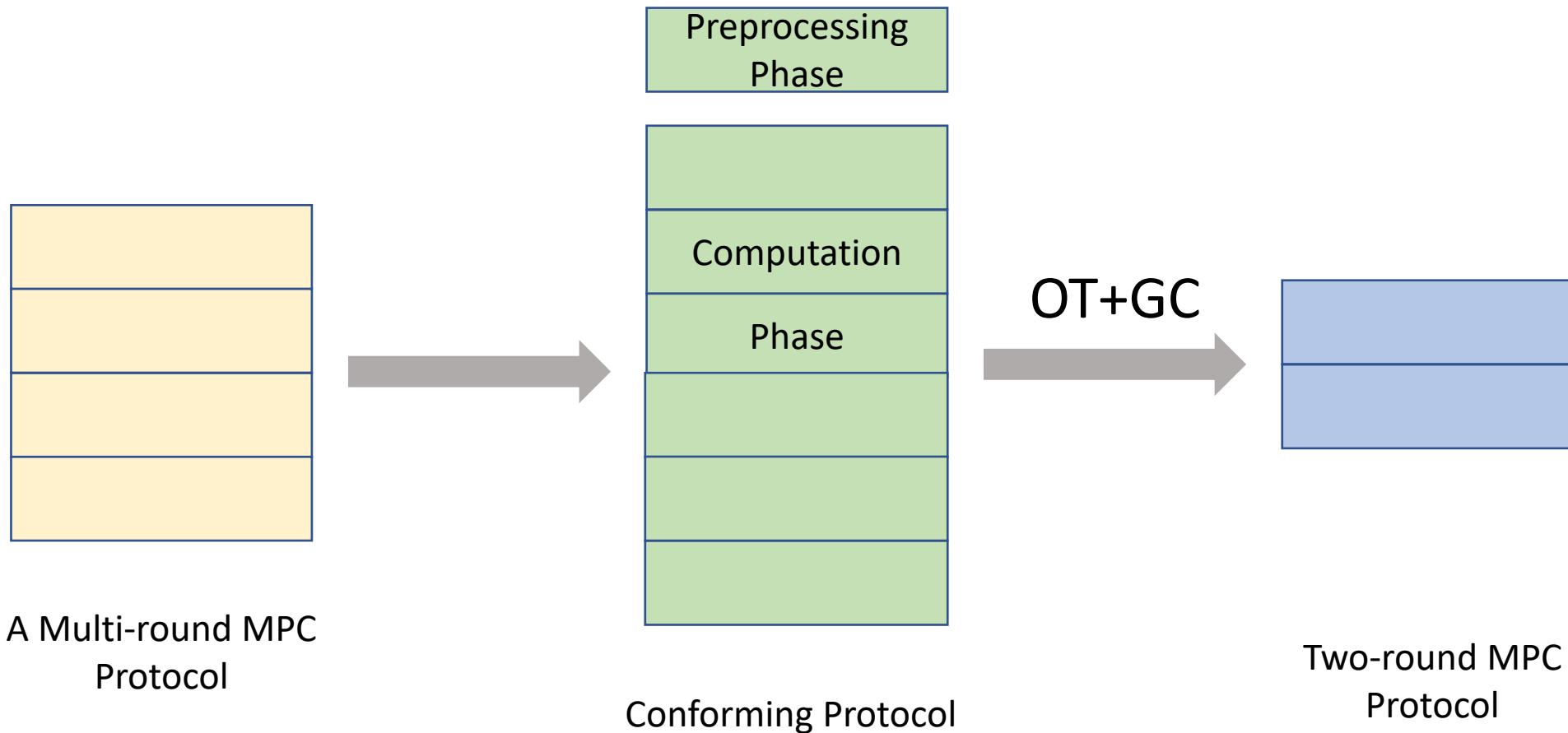


Computation Phase:

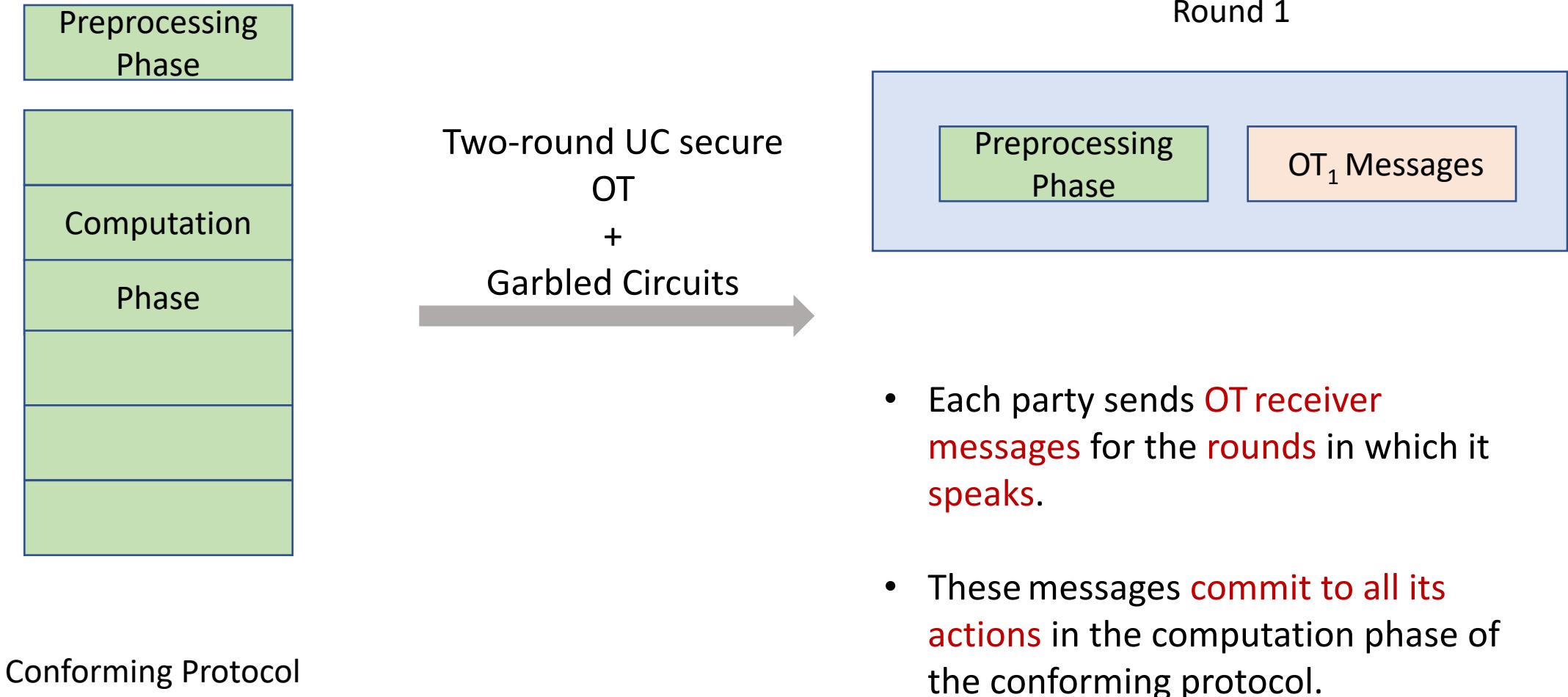
Only a **single bit** is broadcasted by a single party (**speaker**) in each round.

All other parties are **listeners** for that round.

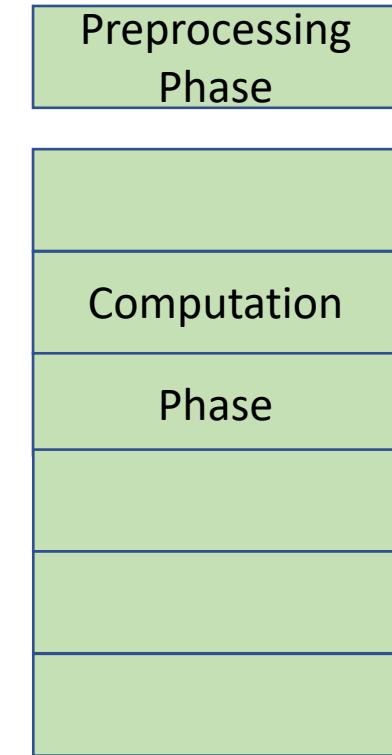
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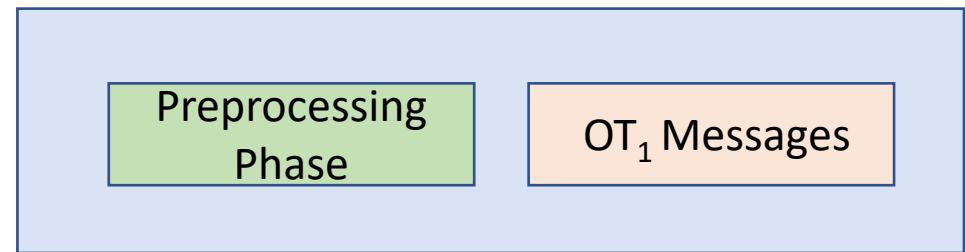


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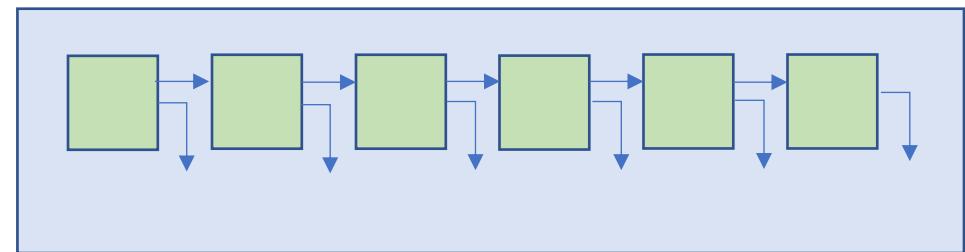


Conforming Protocol

Round 1

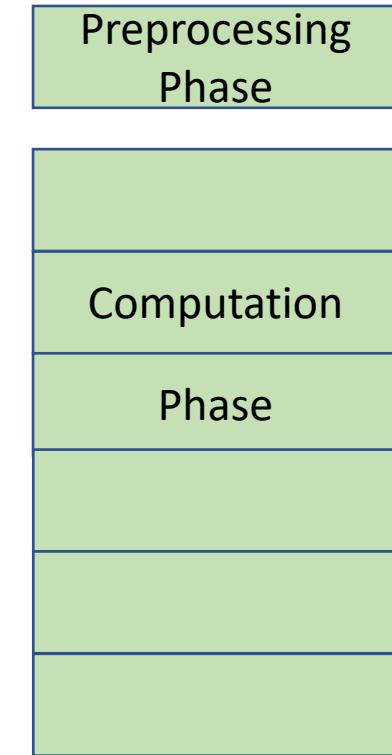


Round 2



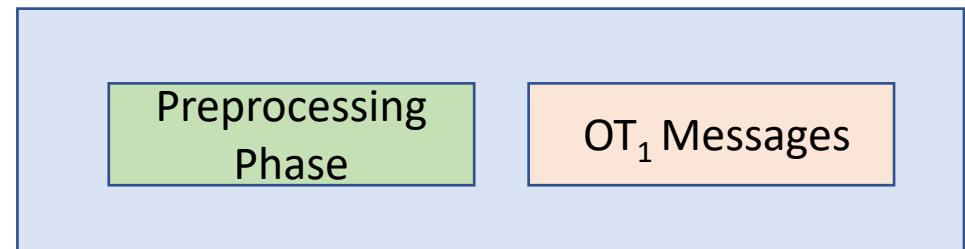
Each party sends garbled circuits corresponding to each round in the computation phase.

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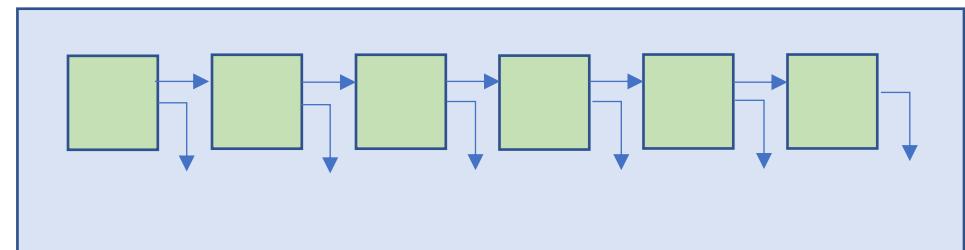


Two-round UC secure  
OT  
+  
Garbled Circuits

Round 1



Round 2



GCs output the OT sender messages.

Goal of these OTs is to deliver wire labels of GC.

# Our Strategy: Challenge 2

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OT functionality for sender inputs  $(m_0, m_1)$  and receiver input  $(b)$  can be represented as a **degree 2 polynomial** in  $\mathbb{F}_2$ .

$$m_b = m_0(1 + b) + m_1(b)$$

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Later: How to implement

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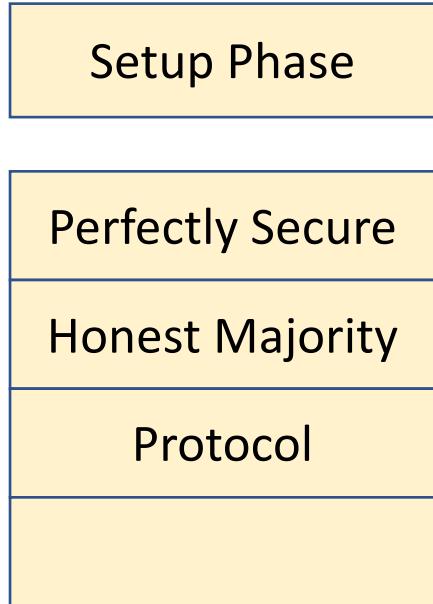
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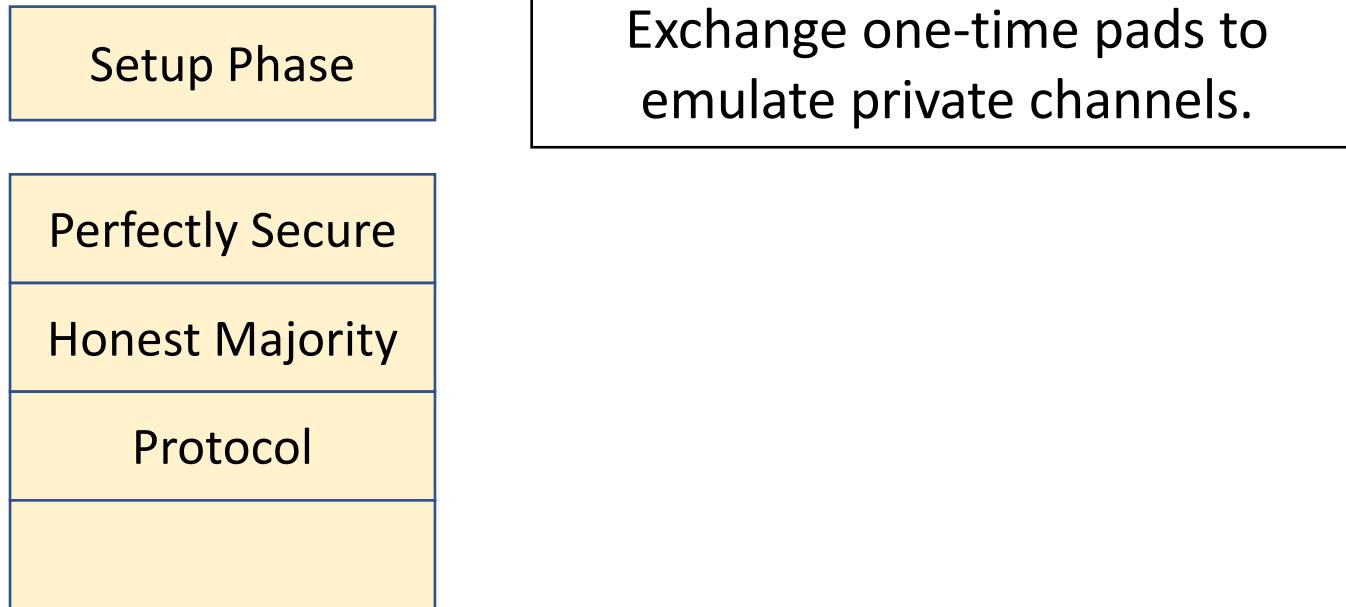


Uses both broadcast and  
private channels.

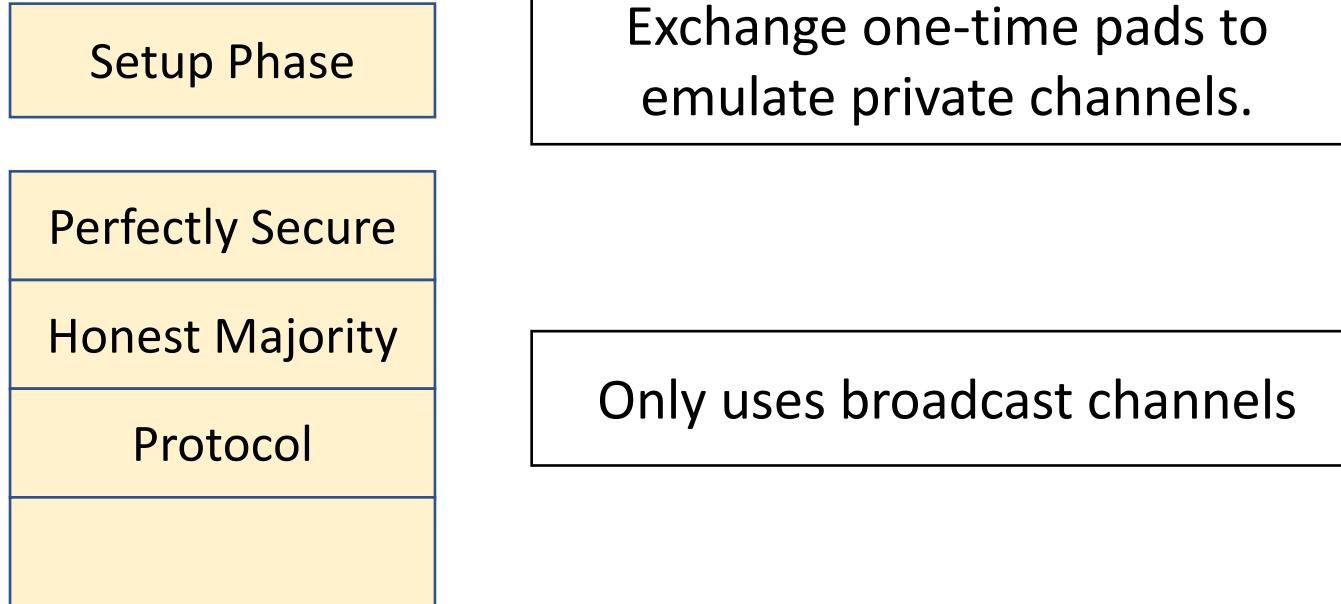
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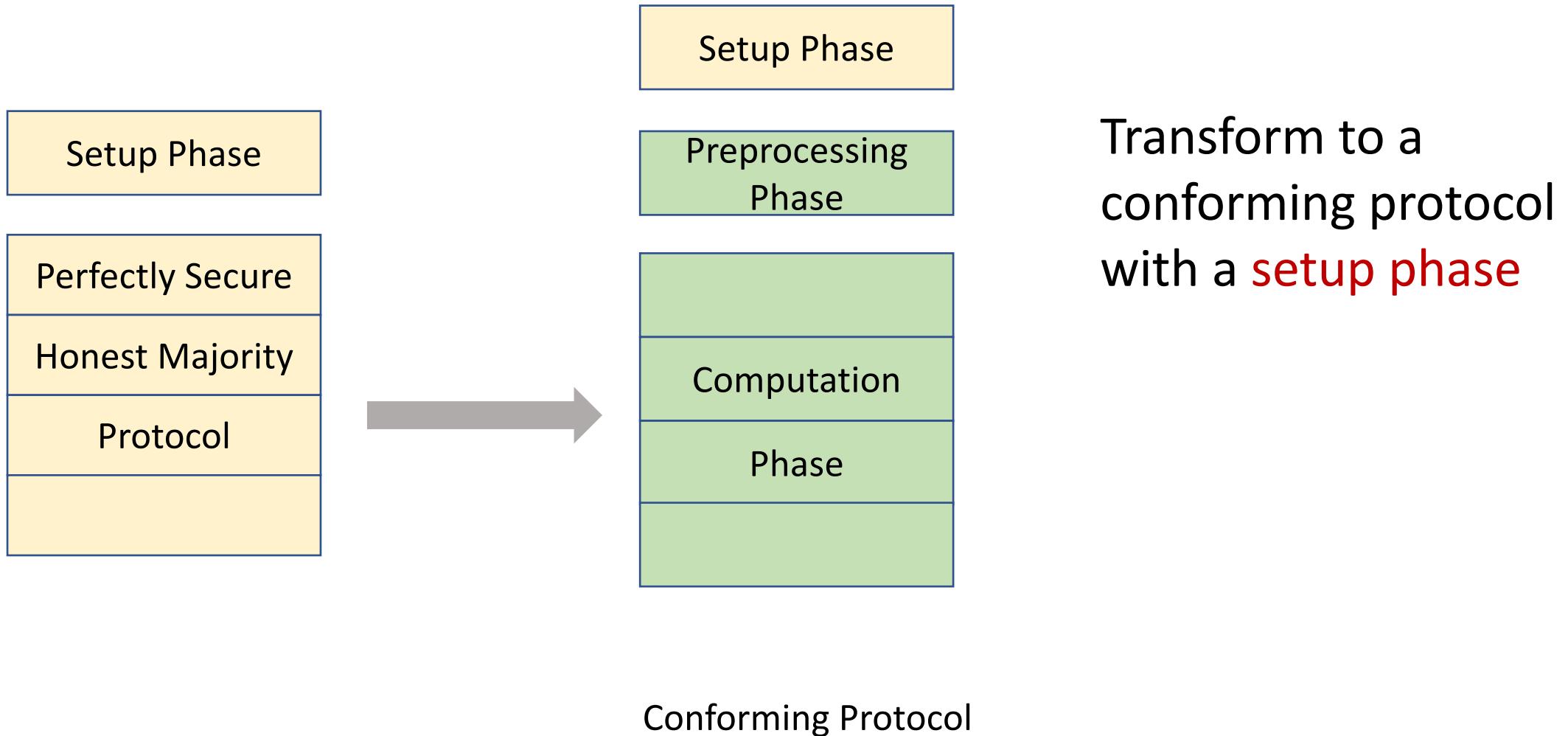
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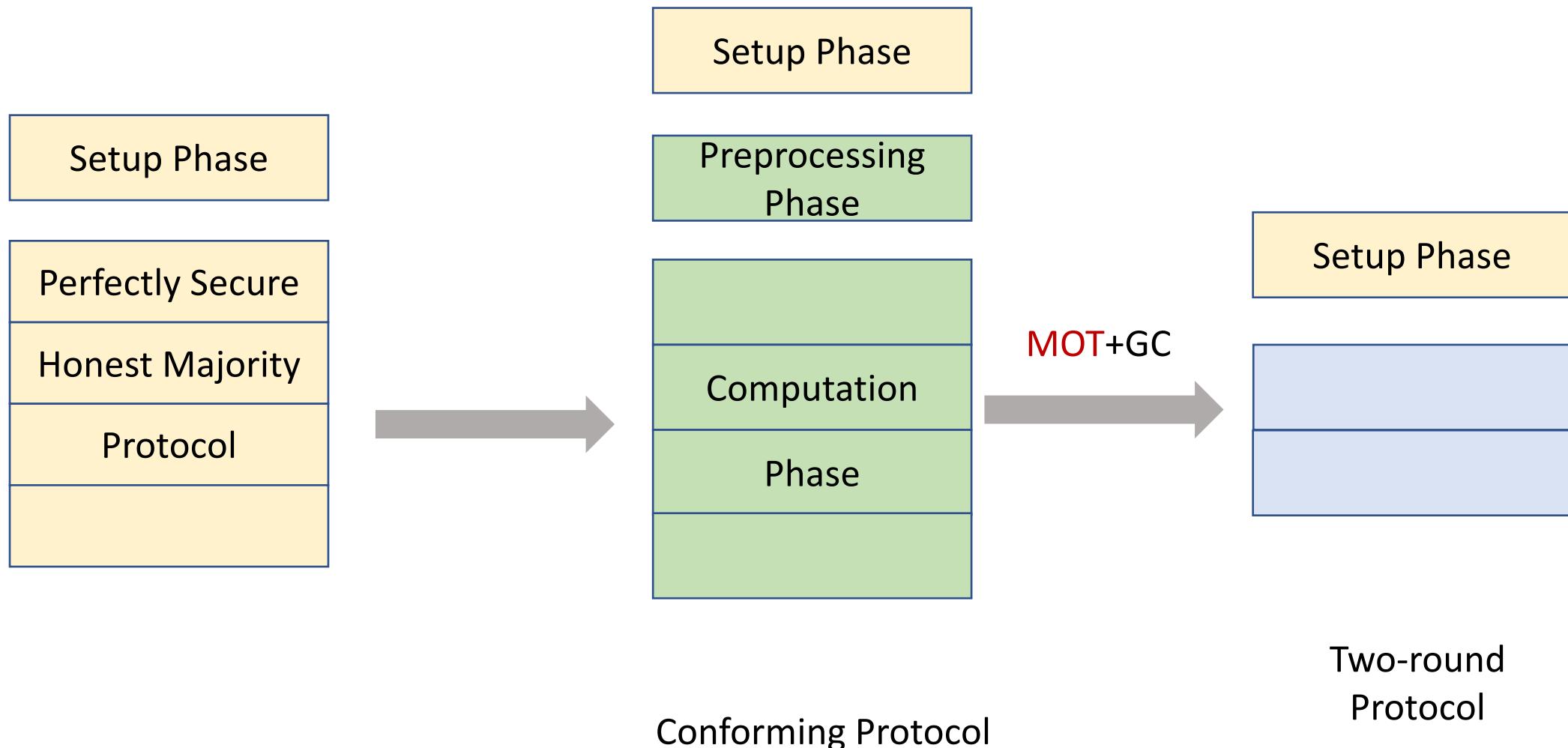
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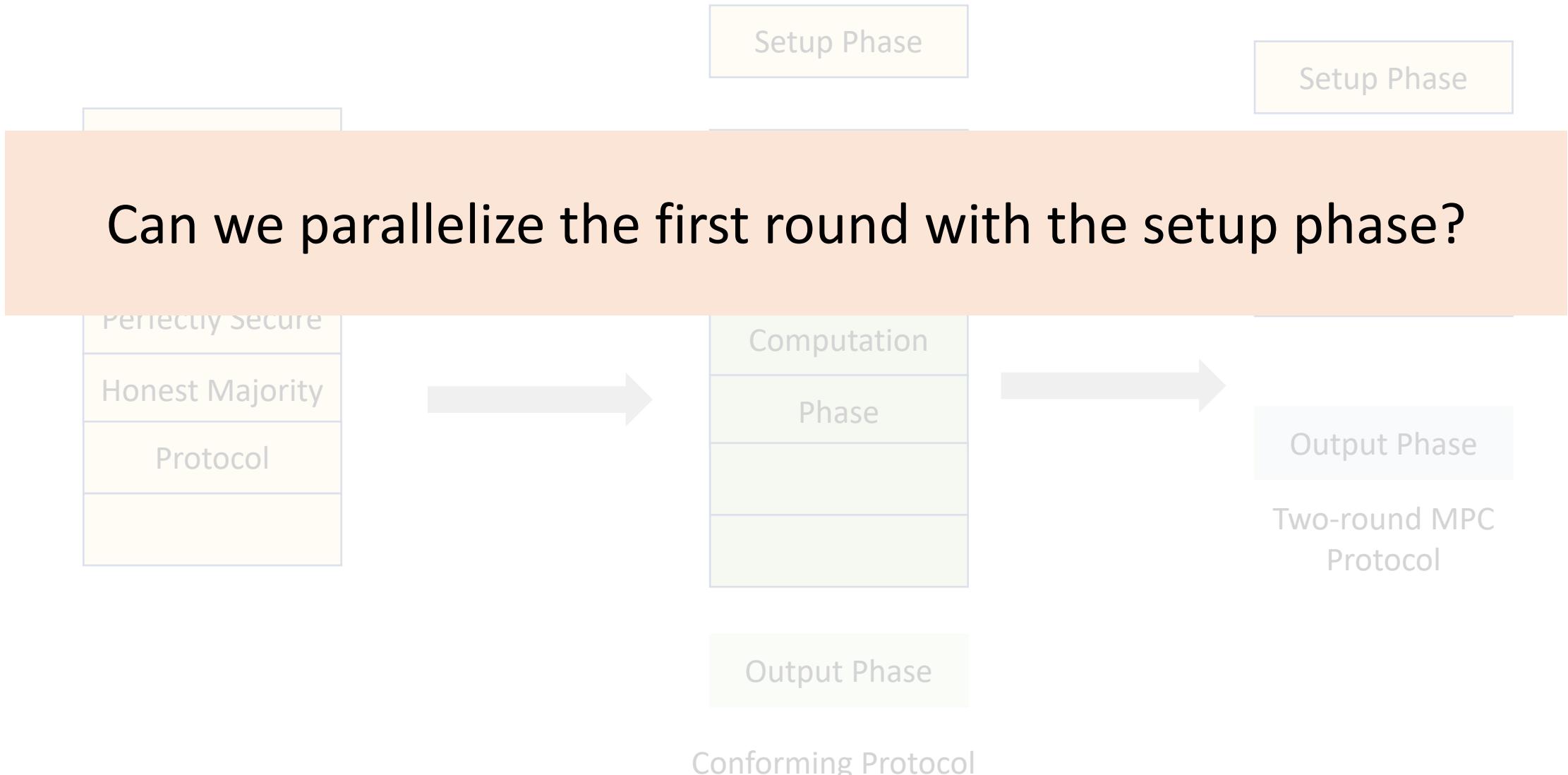
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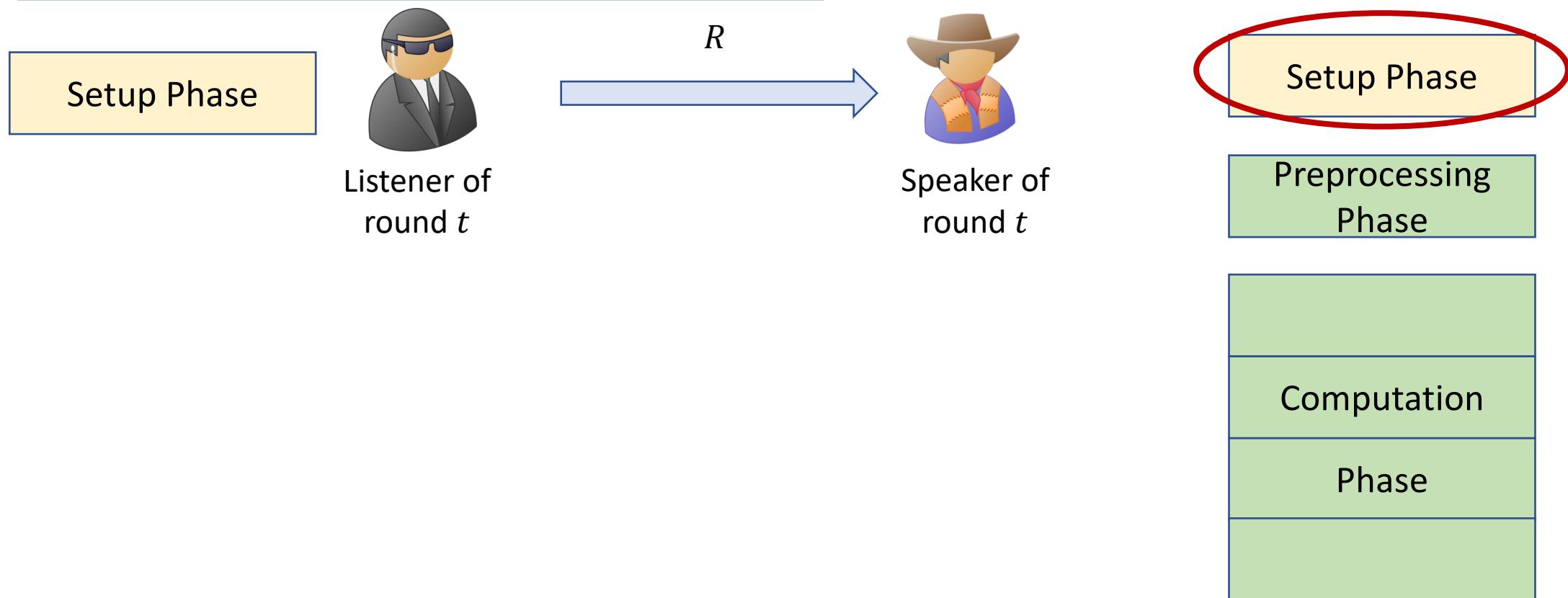


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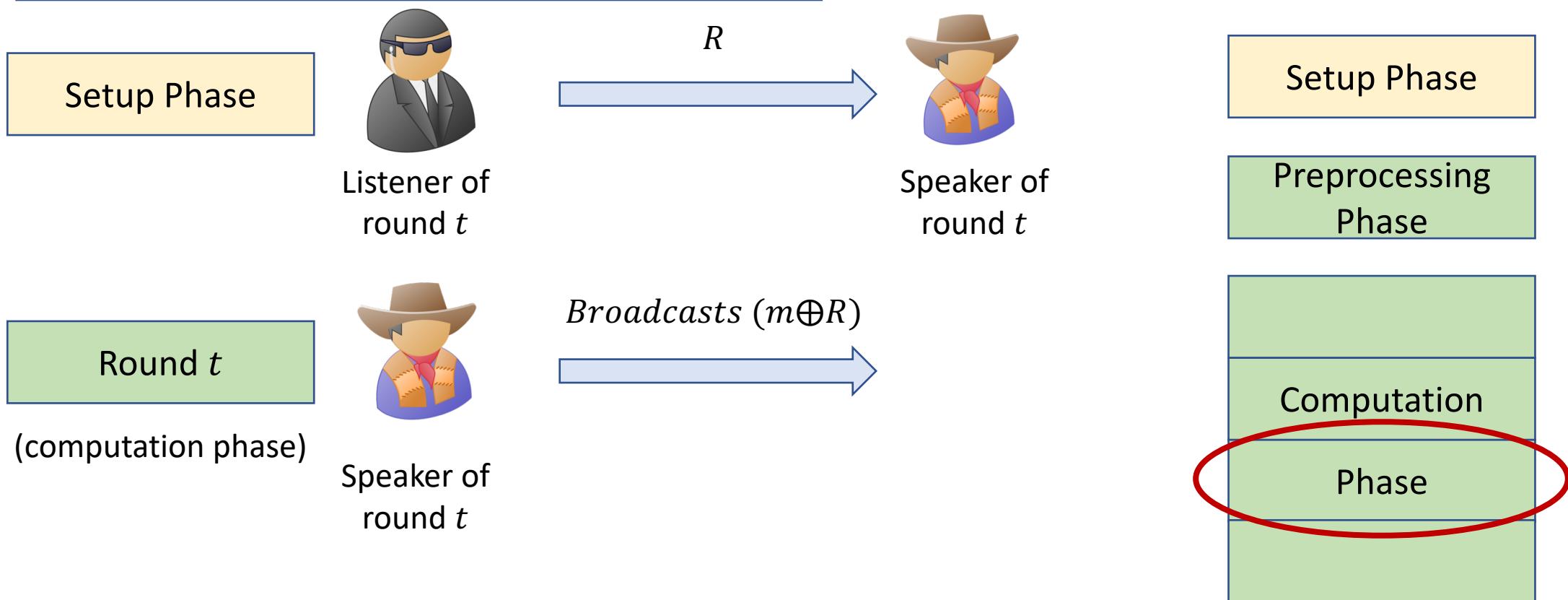
# Can we parallelize the first round with the setup phase?

## Conforming Protocol with setup



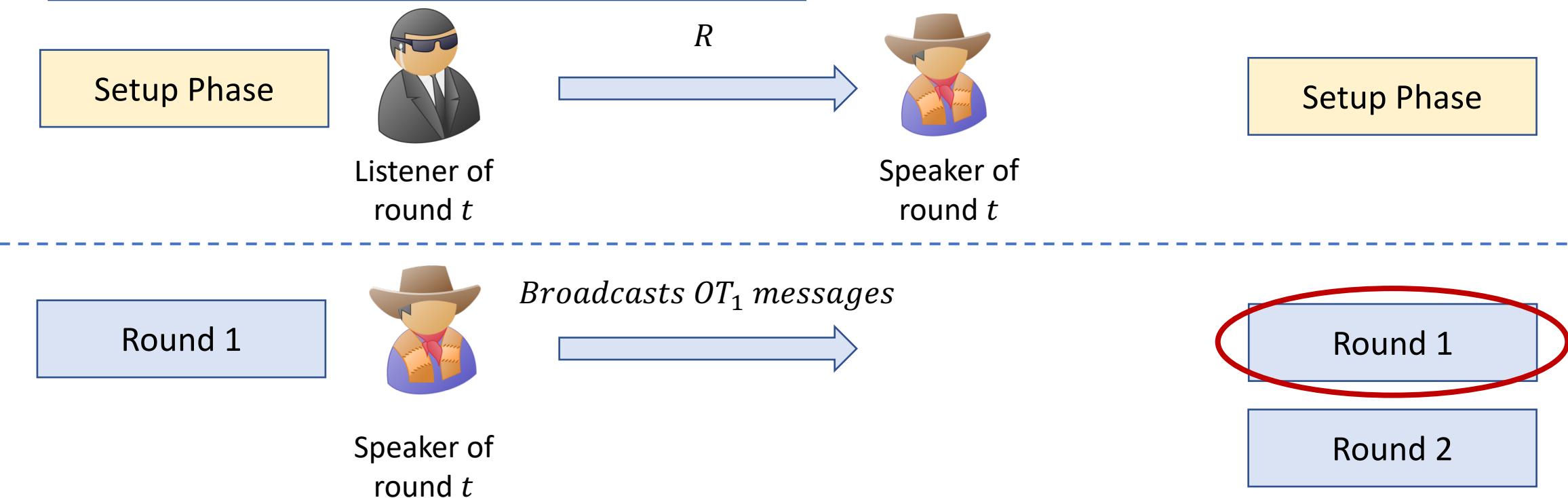
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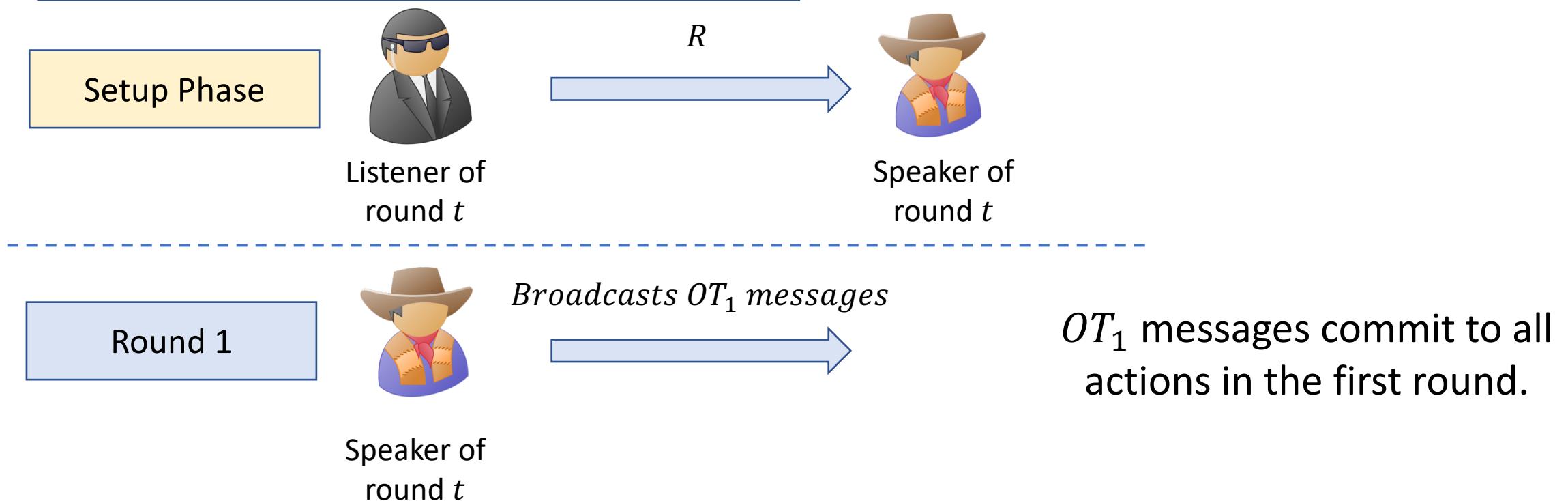
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## 2 Round Protocol with setup



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## 2 Round Protocol with setup

Setup Phase



Listener of  
round  $t$

$R$



Speaker of  
round  $t$

Round 1



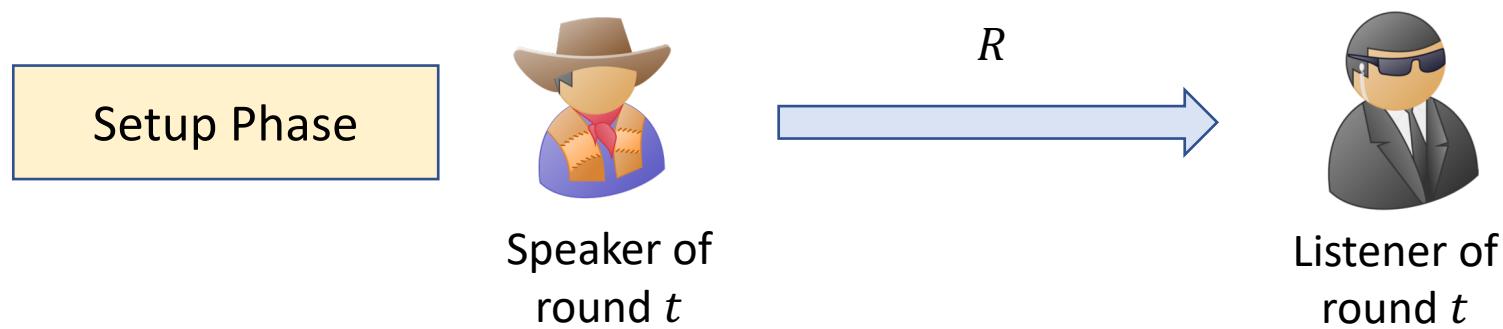
Speaker of  
round  $t$

Broadcasts  $OT_1$  messages

$OT_1$  messages depend on  $R$   
which is not known before  
setup.

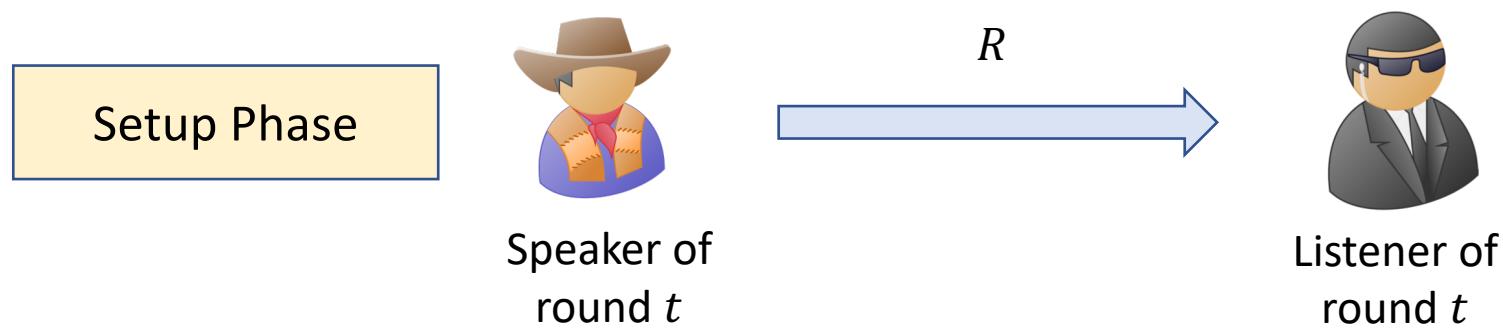
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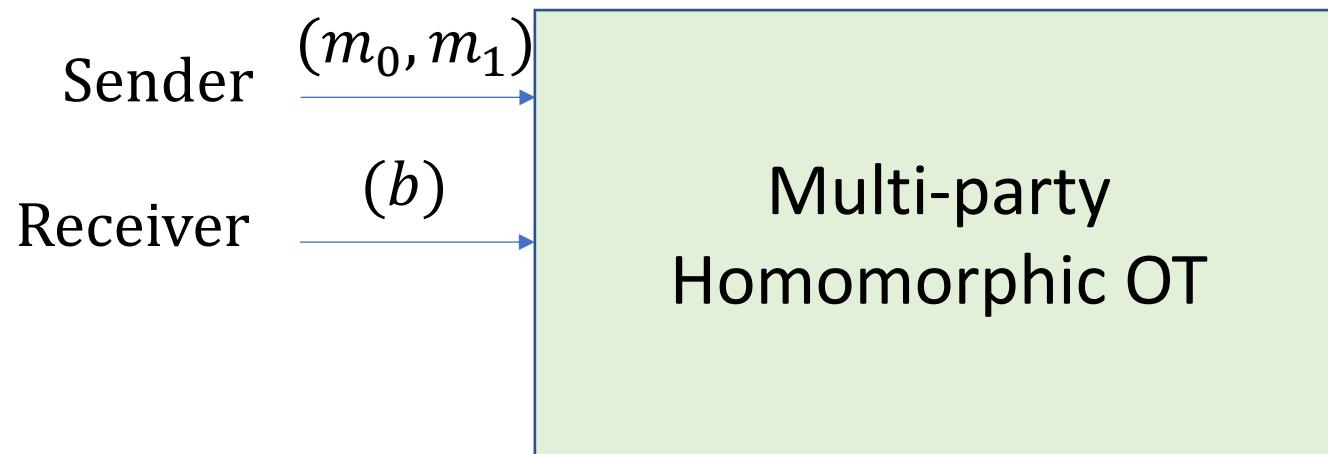
This approach doesn't seem to work!

- Similar problem arises.
- Transfers the problem to another round.

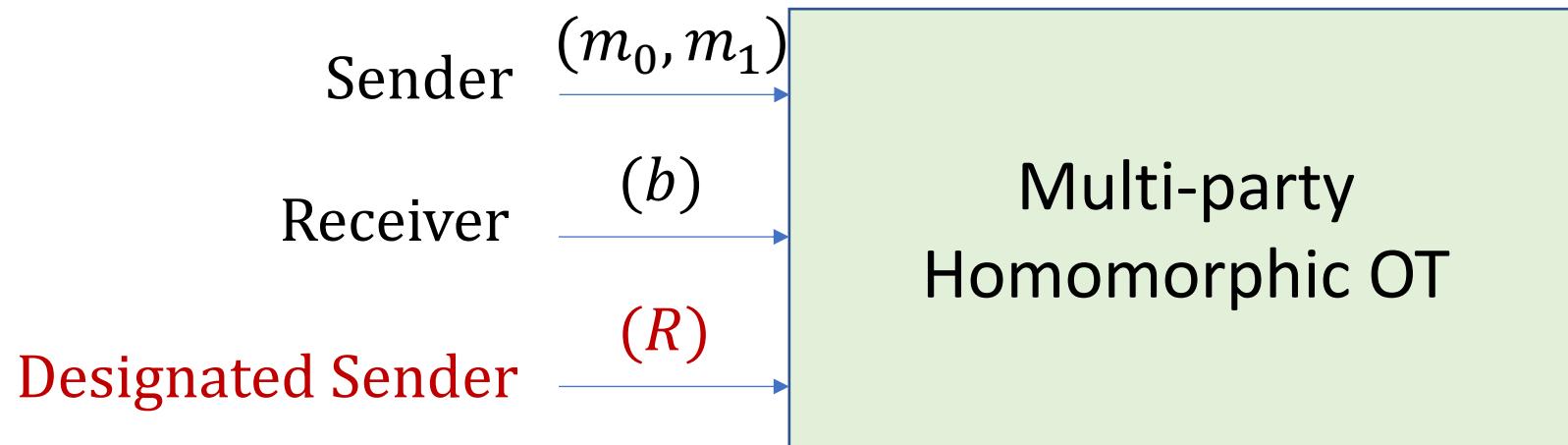
# Multi-party Homomorphic OT

- Multi-party protocol.
- Only 3 parties have inputs, others have no input.
- Every party receives the output.

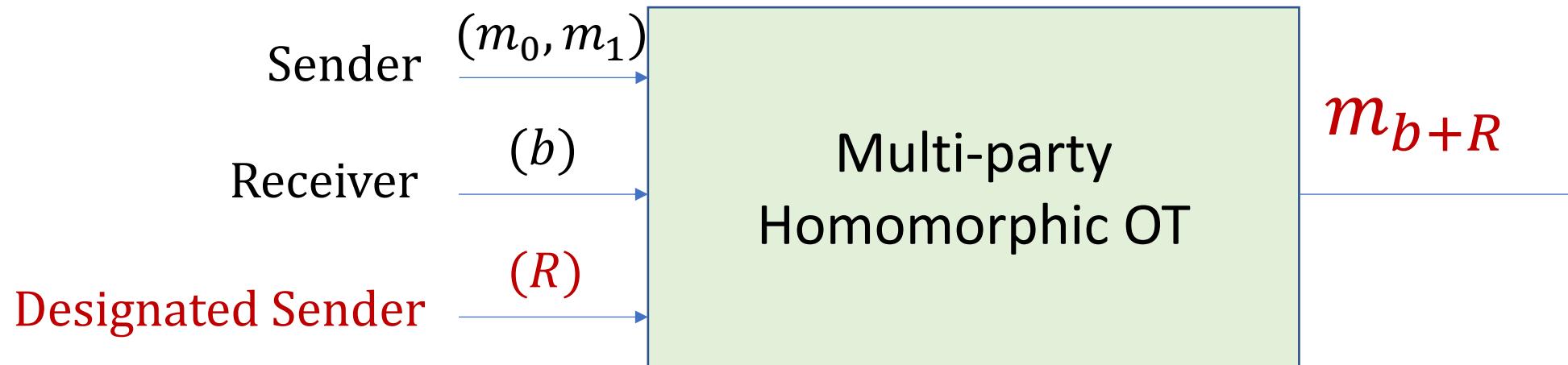
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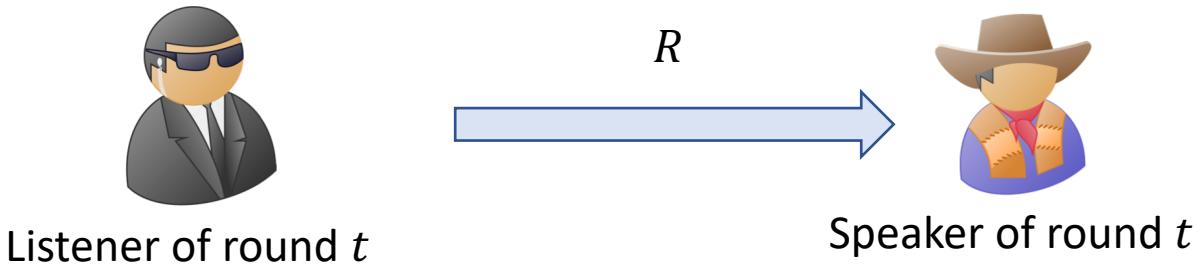
# Multi-party Homomorphic OT

- The homomorphic OT functionality with sender inputs  $(m_0, m_1)$ , receiver input  $(b)$  and designated sender input  $(R)$  can be represented as degree 2 polynomial in  $\mathbb{F}_2$ .

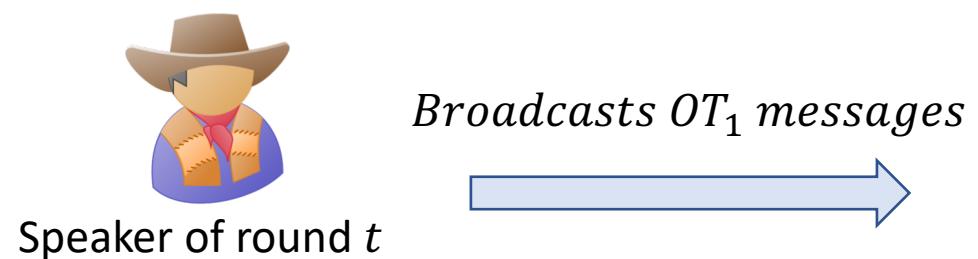
$$m_{b+R} = m_0(1 + b + R) + m_1(b + R)$$

# Parallelizing using MHOT

## 2 Round Protocol with setup



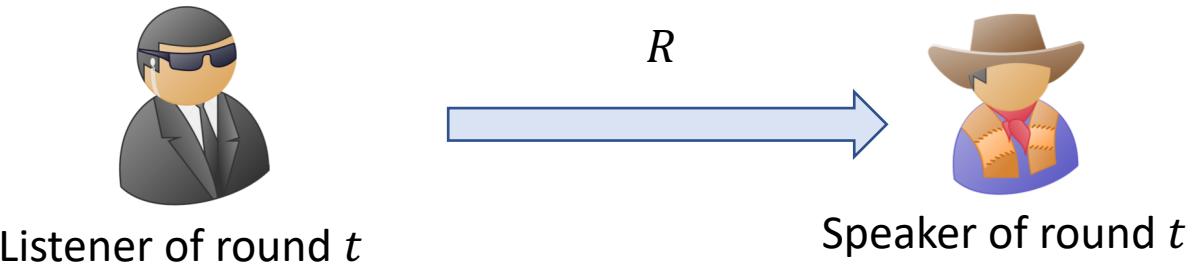
Setup Phase



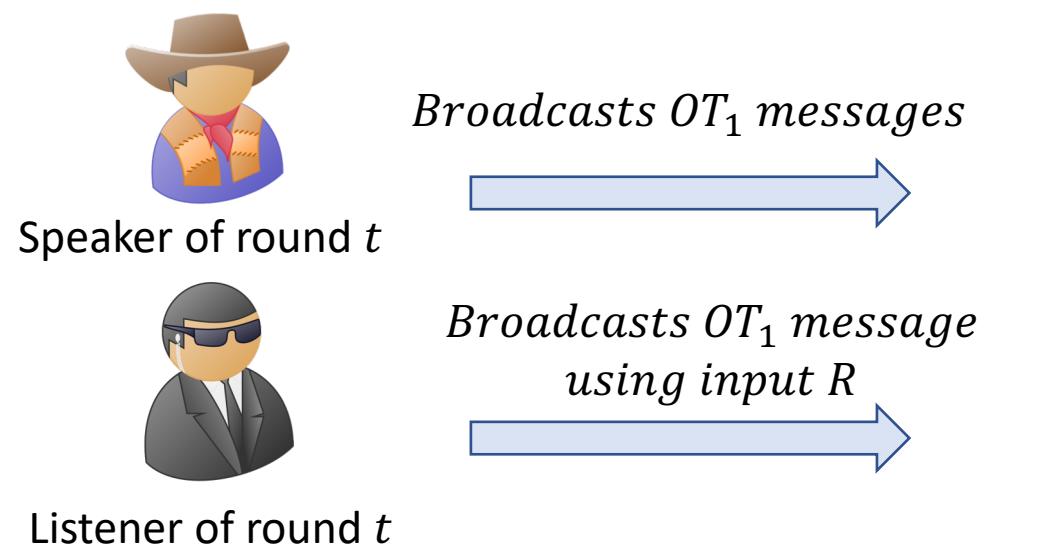
Round 1

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## 2 Round Protocol with setup



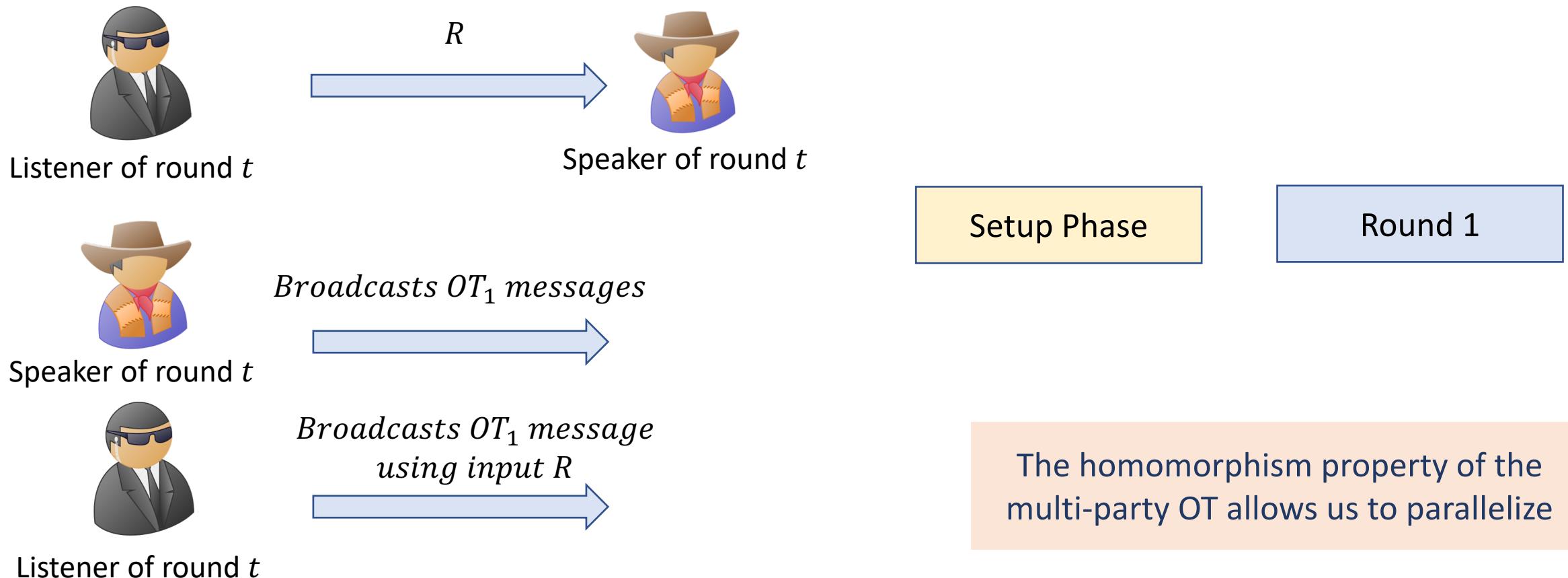
Setup Phase



Round 1

# Parallelizing using MHOT

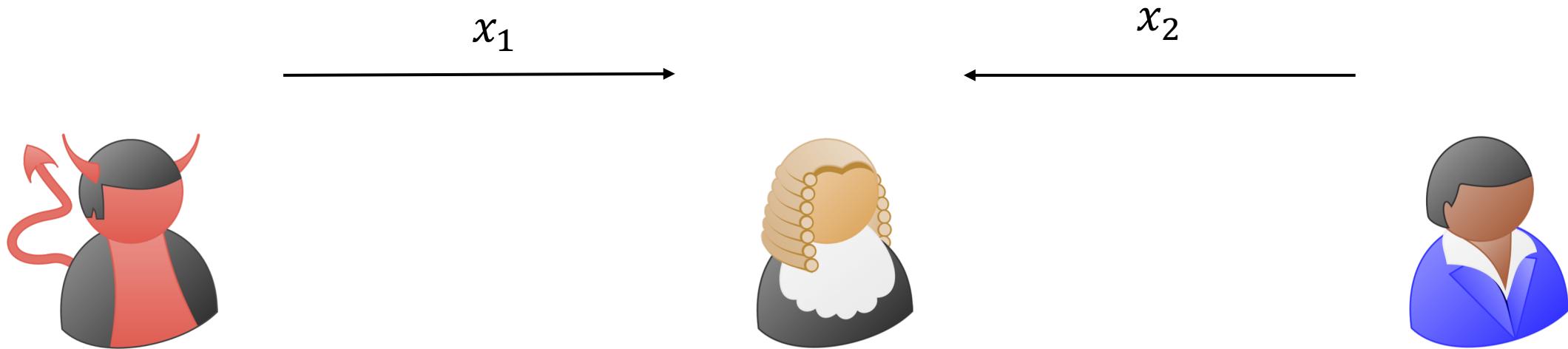
## 2 Round Protocol with setup parallelized



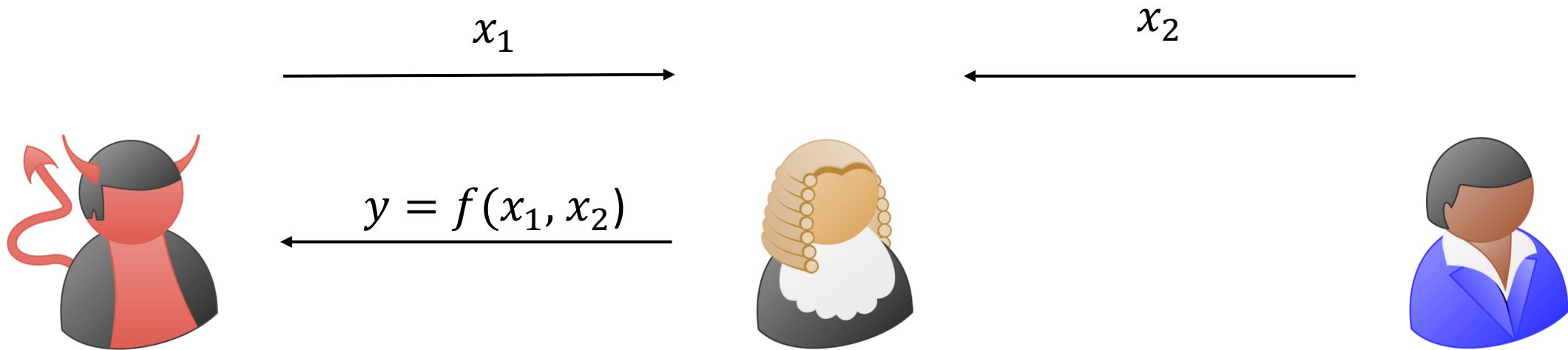
# Instantiating Multi-party Homomorphic OT

- [Ishai-Kushilevitz-Paskin10] give a construction for such a degree 2 polynomial computation protocol that satisfies statistical **t-privacy with knowledge of outputs**.

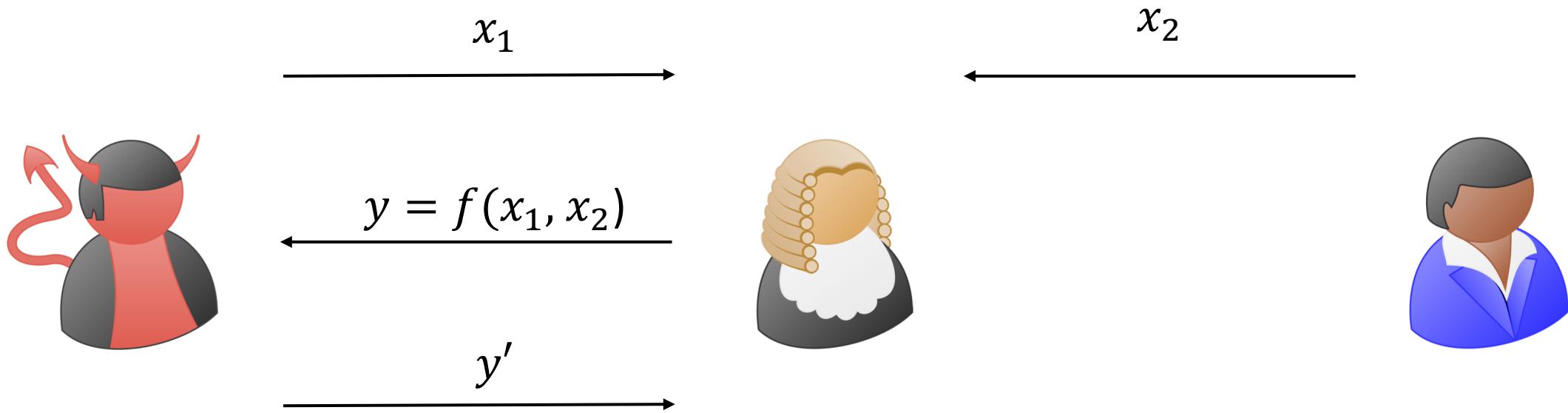
# Ideal World: Privacy with Knowledge of Outputs



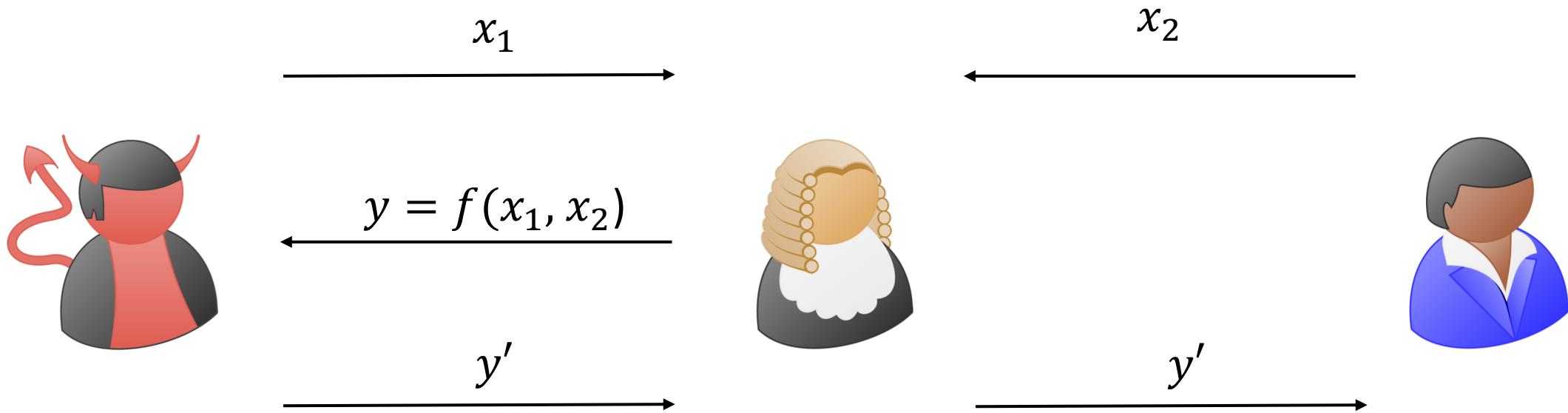
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**Privacy with knowledge of outputs:** A weaker notion than security with abort that does not guarantee correctness of output of the honest parties.

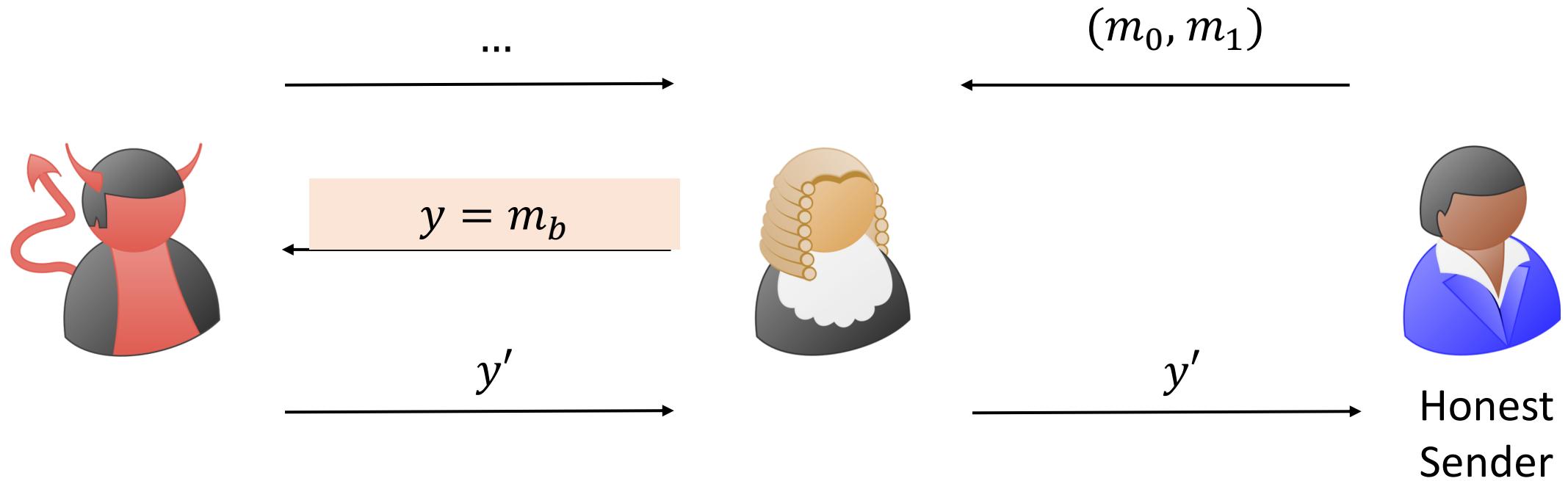
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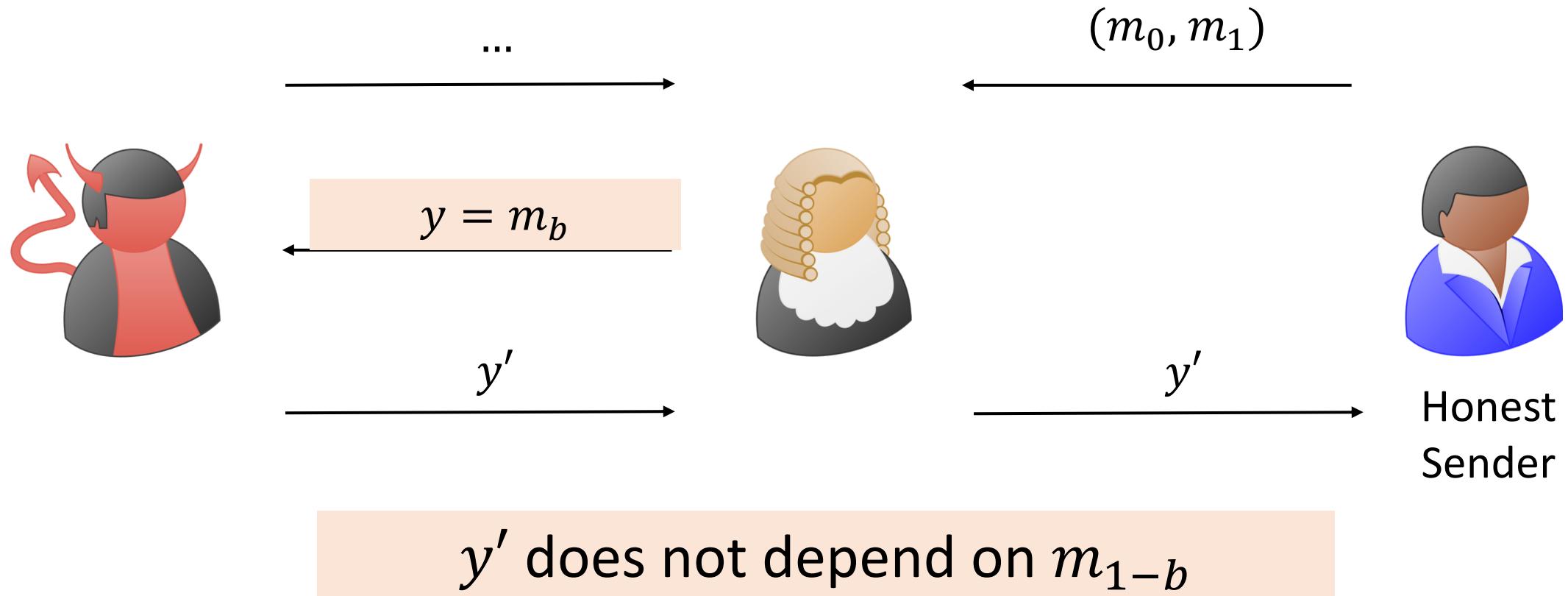
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**Challenge:** How to ensure correctness of honest party outputs?

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- OT functionality transmits wire labels for GC.
- Unless valid labels are transmitted, **GC remains private**.

<https://eprint.iacr.org/2018/572>

Thank You.

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