Aarushi Jain
102103483

Date

Maximum Likelihood Estimation

Q1 Let (X1, X2, ...) be a random sample of size n taken from a Normal Population with parameters: mean = θ1 and variance = θ2. Find the Maximum Likelihood Estimates of these two parameters.

Sol:- For Normal Distribution

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

$$L(\theta_1, \theta_2 \mid x_1, x_2, \ldots x_n) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking logarithm of the likelihood function:

$$\log L(\theta_1, \theta_2 \mid x_1, x_2, \ldots x_n) = \log \left(\frac{e^{\frac{-\Sigma(x_i-\theta_1)^2}{2\theta_2}}}{\sqrt{2\pi\theta_2}} \right)$$

$$\log L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = \log\left(e^{\frac{-\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right) - \log(2\pi\theta_2)^{1/2}$$

$$\log L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = -\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2} - \frac{n}{2}\log 2\pi\theta_2$$

differentiating with respect to θ1 and equating to 0

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial\theta_1} = \frac{+\cancel{2}\,\Sigma(x_i-\theta_1)(-1)}{\cancel{2}\theta_2} = 0$$

$$\Sigma(x_i - \theta_1) = 0$$

$$\Sigma x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\theta_1 = \bar{x} \quad \text{--- (1)}$$

#

Date

differentiating with respect to θ_2 and equate to 0.

$$\frac{\partial L(\theta_1,\theta_2)}{\partial \theta_2} = \frac{\Sigma (x_i - \theta_1)^2}{2\theta_2^2} - \frac{n}{2\theta_2} = 0$$

$$\frac{\Sigma (x_i - \theta_1)^2}{\cancel{2}\theta_2} = \frac{n}{\cancel{2}}$$

$$\theta_2 = \frac{\Sigma (x_i - \theta_1)^2}{n}$$

from eq^n ① . $\theta_1 = \bar{x}$

$$\theta_2 = \frac{\Sigma (x_i - \bar{x})}{n}$$

Therefore,

$$\hat{\mu} = \frac{\Sigma x_i}{n} = \bar{X}$$

$$\sigma^2 = \frac{\Sigma (x_i - \bar{X})^2}{n}$$

Date

Q2 Let $X_1, X_2, \dots X_n$ be a random sample from $B(m, \theta)$ distribution, where $\theta \in (0,1)$ is unknown and 'm' is a known positive integer. Compute value of θ using Maximum Likelihood Estimation.

Sol For Binomial Distribution

$$P(X=x) = \frac{\lfloor n}{\lfloor x \lfloor n-x} p^x (1-p)^{n-x} = \binom{n}{x} p^x (1-p)^{n-x}$$

Likelihood function

$$L(\theta) = \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

taking log of likelihood function

$$\log L(\theta) = \sum_{i=1}^{m} \left[\log \binom{m}{x_i} + \log \theta^{x_i} + \log (1-\theta)^{m-x_i} \right]$$

$$\log L(\theta) = \sum_{i=1}^{m} \left[\log \binom{m}{x_i} + x_i \log \theta + (m-x_i) \log (1-\theta) \right]$$

differentiating with respect to θ and equating to 0

$$\frac{\partial \log L}{\partial \theta} = \sum_{i=1}^{n} \left[0 + \frac{x_i}{\theta} + \frac{(m-x_i)(-1)}{(1-\theta)} \right] = 0$$

$$\frac{\partial \log L}{\partial \theta} = \sum_{i=1}^{n} \left[0 + \frac{x_i}{\theta} - \frac{(m-x_i)}{(1-\theta)} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m}{(1-\theta)} + \frac{x_i}{(1-\theta)} \right] = 0$$

$$\frac{\Sigma x_i}{\theta} - \frac{mn}{(1-\theta)} + \frac{\Sigma x_i}{1-\theta} = 0$$

$$\Sigma x_i \left[\frac{1}{\theta} + \frac{1}{1-\theta} \right] = \frac{mn}{(1-\theta)}$$

#

Date

$$\sum x_i \left[\frac{1 - \cancel{\theta} + \cancel{\theta}}{\theta \cancel{(1-\theta)}} \right] = \frac{mn}{\cancel{(1-\theta)}}$$

$$\frac{\sum x_i}{\theta} = mn$$

$$\theta = \frac{\sum x_i}{mn}$$

$$\theta = \frac{\bar{x}}{m}$$