Heston Model: Theory and Applications

Quantitative Research

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1 Key Fundamentals

The Heston model (1993) is a **stochastic volatility model** that extends the Black-Scholes framework by allowing volatility itself to be random rather than constant. It's one of the most widely used models in quantitative finance for pricing derivatives and managing risk.

1.1 Core Mathematical Structure

The model is defined by two coupled stochastic differential equations (SDEs):

Asset Price Process:

$$dS_t = \mu S_t\, dt + \sqrt{v_t}\, S_t\, dW_t^S \tag{1}$$

Variance Process (CIR process):

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}\, dW_t^v \tag{2}$$

Where:

- S_t = asset price at time t
- v_t = instantaneous variance (volatility squared) at time t
- μ = drift rate (expected return)
- κ = mean reversion speed (how fast variance reverts to long-term mean)
- θ = long-term mean variance level
- σ = volatility of volatility (vol-of-vol)
- W^S and W^v = Wiener processes with correlation ρ (typically negative)

1.2 The Feller Condition

A critical constraint is the **Feller condition**:

$$2\kappa\theta \geq \sigma^2 \tag{3}$$

This ensures variance remains strictly positive (never goes negative), which is essential for mathematical consistency and realistic modeling.

2 What It Models

2.1 Stochastic Volatility

Unlike Black-Scholes (constant volatility), Heston captures the empirical observation that volatility changes randomly over time. Periods of market calm alternate with periods of high turbulence.

2.2 Volatility Clustering

The mean-reverting variance process (CIR) naturally produces **volatility clustering** – the tendency for high-volatility periods to cluster together, and likewise for low-volatility periods. This matches real market behavior.

2.3 Leverage Effect (Asymmetric Volatility)

The correlation parameter ρ (typically -0.5 to -0.8) captures the **leverage effect**: when stock prices fall, volatility tends to rise. This asymmetry is crucial for modeling equity markets and produces realistic volatility smiles.

2.4 Volatility Smile/Skew

The Heston model produces the characteristic **volatility smile** and **skew** observed in options markets – out-of-the-money puts are more expensive than Black-Scholes predicts, while deep out-of-the-money calls are cheaper.

2.5 Semi-Closed Form Solutions

One of Heston's key advantages is that European option prices can be computed via **Fourier inversion** and the characteristic function:

$$C(S_0, v_0, K, T) = S_0 P_1 - K e^{-rT} P_2 \tag{4}$$

where P_1 and P_2 are probabilities computed using characteristic functions, providing fast and accurate pricing without Monte Carlo simulation.

3 What It Ignores

3.1 Jumps in Asset Prices

The Heston model assumes continuous price paths. It doesn't capture sudden **discrete jumps** (like earnings announcements, geopolitical shocks, or market crashes). Extensions like Bates model add jump components.

3.2 Time-Varying Parameters

All parameters $(\kappa, \theta, \sigma, \rho)$ are assumed constant over time. In reality, market dynamics evolve – mean reversion strength and long-term volatility levels change with market regimes.

3.3 Multiple Volatility Factors

Heston uses a single stochastic volatility factor. Empirical research suggests multiple volatility components (short-term vs. long-term) may be needed for accurate modeling, especially for long-dated options.

3.4 Interest Rate Risk

The model assumes a constant risk-free rate. For long-dated derivatives or in volatile rate environments, stochastic interest rates (hybrid models) may be necessary.

3.5 Microstructure Effects

Transaction costs, bid-ask spreads, liquidity constraints, and market impact are ignored. The model assumes frictionless markets with continuous trading.

3.6 Non-Normal Return Distributions

While better than Black-Scholes, the model still doesn't fully capture extreme tail events (fat tails, extreme kurtosis) observed during market crises.

4 Why People Use It

4.1 Realistic Volatility Dynamics

The stochastic volatility feature captures real market behavior better than constant-volatility models, producing more accurate option prices across strikes and maturities.

4.2 Volatility Smile Consistency

Heston naturally generates the volatility smile/skew without ad-hoc adjustments. This makes it ideal for pricing exotic derivatives and managing volatility risk.

4.3 Computational Efficiency

The semi-closed form solution via characteristic functions makes Heston **much faster** than pure Monte Carlo methods while maintaining accuracy. Option prices can be computed in milliseconds.

4.4 Calibration Stability

With only 5 parameters $(\kappa, \theta, \sigma, \rho, v_0)$, the model strikes a good balance between flexibility and parsimony. It can be calibrated to market data reasonably well without overfitting.

4.5 Industry Standard

Heston is a **benchmark model** in quantitative finance. It's widely implemented in trading systems, risk management platforms, and academic research. This standardization facilitates communication and comparison.

4.6 Path-Dependent Products

For path-dependent derivatives (Asian options, barrier options, variance swaps), Heston provides a tractable framework that can be simulated efficiently or sometimes solved semi-analytically.

4.7 Volatility Trading

The model is essential for **volatility arbitrage** and **variance swap** trading. The variance process directly relates to realized and implied volatility, allowing traders to take positions on volatility itself.

4.8 Risk Management

Financial institutions use Heston for:

- **VaR (Value at Risk)** calculations with dynamic volatility
- **Stress testing** under various volatility scenarios
- **Hedging strategies** that account for changing volatility (vega risk)

4.9 Model Extensions

Heston serves as a foundation for more sophisticated models:

- **Bates model**: Heston + jumps
- **Double Heston**: Two variance factors
- **Heston-Hull-White**: Stochastic volatility + stochastic interest rates

5 Practical Considerations

5.1 When to Use Heston

- Pricing equity options (especially short-to-medium term)
- Volatility surface modeling and arbitrage
- When volatility risk is significant
- When you need balance between realism and computational speed

5.2 When to Consider Alternatives

- **Local volatility models** (Dupire): Better calibration to vanilla options but poor for exotics
- **Jump-diffusion models**: When discontinuous price movements matter (earnings, crises)
- **SABR model**: Popular for interest rate derivatives
- **Rough volatility models**: Cutting-edge research for very high-frequency data

6 Conclusion

The Heston model represents a sweet spot in quantitative finance: sophisticated enough to capture key market realities (stochastic volatility, leverage effect, volatility smile), yet tractable enough for practical implementation. While it makes simplifying assumptions and ignores certain market features, its widespread adoption reflects its robust performance across a wide range of derivatives pricing and risk management applications.

Understanding the Heston model is essential for any quantitative analyst working in derivatives markets. Its mathematical elegance combined with practical utility has made it the gold standard for stochastic volatility modeling over the past three decades.

References

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