ECO351 Assignment 4

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Question 1

(a)

Consider the standard *sample selection model*:

$$\begin{aligned} y_1^* &= x_1'\beta_1 + u_1, \quad \text{observe selection if } y_1^* > 0, \\ y_2^* &= x_2'\beta_2 + u_2, \quad \text{observe wage } y_2 = y_2^*, \quad \text{if} \quad y_1 = 1 \end{aligned}$$

$$\textit{where} \quad (u_1, u_2)' \sim N(0, \Sigma), \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}.$$

We are interested in the conditional mean of y_2 for the selected sample:

$$E[y_2 \mid y_1^* > 0, x] = x_2'\beta_2 + E[u_2 \mid u_1 > -x_1'\beta_1].$$

Step 1: Conditional expectation of u_2

For a bivariate normal vector, the conditional mean is (using the definition given in class) :

$$E[u_2 \mid u_1 > a] = \frac{\sigma_{12}}{\sigma_1} \cdot \frac{\varphi(t)}{1 - \Phi(t)}, \quad \textit{where} \quad t = \frac{a}{\sigma_1}$$

Setting $a = -x_1'\beta_1$ and defining $\bar{\omega}_1 = -t = \frac{x_1'\beta_1}{\sigma_1}$, we can rewrite

$$E[u_2 \mid u_1 > -x_1'\beta_1] = \frac{\sigma_{12}}{\sigma_1} \cdot \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)}.$$

$$E[y_2 \mid y_1^* > 0, x] = x_2'\beta_2 + \frac{\sigma_{12}}{\sigma_1} \cdot \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)}.$$

Step 2: Derivative with respect to $x_{1,j}$

Suppose $x_{1,j}$ appears only in the participation equation. Then

$$\frac{\partial E[y_2 \mid y_1^* > 0, x]}{\partial x_{1,j}} = -\beta_{1,j} \frac{\partial}{\partial a} E[u_2 \mid u_1 > a],$$

since $a = -x_1'\beta_1$.

We know

$$E[u_2 \mid u_1 > a] = \frac{\sigma_{12}}{\sigma_1} \cdot \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)}, \quad \text{where } \bar{\omega}_1 = -\frac{a}{\sigma_1}.$$

Differentiating with respect to a:

$$\frac{d}{da} E[u_2 \mid u_1 > a] = -\frac{\sigma_{12}}{\sigma_1^2} \frac{d}{d\bar{\omega}_1} \left(\frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} \right).$$

Using the standard derivative of the inverse Mills ratio:

$$\frac{d}{dx} \left(\frac{\varphi(x)}{\Phi(x)} \right) = -\frac{\varphi(x)}{\Phi(x)} \left(x + \frac{\varphi(x)}{\Phi(x)} \right),$$

we get

$$\frac{d}{da} E[u_2 \mid u_1 > a] = \frac{\sigma_{12}}{\sigma_1^2} \cdot \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} \left(\bar{\omega}_1 + \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} \right).$$

Finally,

$$\frac{\partial E[y_2 \mid y_1^* > 0, x]}{\partial x_{1,j}} = -\beta_{1,j} \frac{\sigma_{12}}{\sigma_1^2} \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} \left(\bar{\omega}_1 + \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} \right).$$

Step 3: Probit normalization $\sigma_1 = 1$

Now we also know that the participation equation is regressed using the probit model and hence we use the standard probit normalization of $\sigma_1^2 = 1$, the expression simplifies to:

$$\boxed{\frac{\partial E[y_2 \mid y_1^* > 0, x]}{\partial x_{1,j}} = -\beta_{1,j} \sigma_{12} \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} \left(\frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} + \bar{\omega}_1 \right).}$$

(b)

As we know the participation and wage equations are as follows:

$$\begin{aligned} y_1^* &= x_1'\beta_1 + u_1, \quad \text{observe selection if } y_1^* > 0, \\ y_2^* &= x_2'\beta_2 + u_2, \quad \text{observe wage } y_2 = y_2^*, \quad \text{if} \quad y_1 = 1 \end{aligned}$$

$$\textit{where} \quad (u_1, u_2)' \sim N(0, \Sigma), \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}.$$

Step 1: Conditional expectation of y_2 for the selected sample

The conditional expectation for the selected sample is

$$E[y_2 \mid y_1^* > 0, x] = x_2'\beta_2 + E[u_2 \mid u_1 > -x_1'\beta_1].$$

For the bivariate normal,

$$E[u_2 \mid u_1 > -x_1'\beta_1] = \frac{\sigma_{12}}{\sigma_1} \frac{\varphi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)}, \qquad \text{where } \bar{\omega}_1 \equiv \frac{x_1'\beta_1}{\sigma_1}.$$

Thus

$$\boxed{E[y_2 \mid y_1^* > 0, x] \;=\; x_2'\beta_2 \;+\; \frac{\sigma_{12}}{\sigma_1}\,\lambda(\bar{\omega}_1)}, \qquad \text{with } \lambda(z) \equiv \frac{\varphi(z)}{\Phi(z)}.$$

Step 2: Finding derivative of the conditional expectation

Assume the covariate $x_{2,j}$ enters x_2 but *not* x_1. We differentiate the conditional expectation with respect to $x_{2,j}$:

$$\frac{\partial}{\partial x_{2,j}} E[y_2 \mid y_1^* > 0, x] = \frac{\partial}{\partial x_{2,j}}\big(x_2'\beta_2\big) \;+\; \frac{\partial}{\partial x_{2,j}}\left(\frac{\sigma_{12}}{\sigma_1}\,\lambda(\bar{\omega}_1)\right).$$

Because $x_{2,j}$ does not enter the selection index $x_1'\beta_1$, we have $\partial\bar{\omega}_1/\partial x_{2,j} = 0$. Hence the second term vanishes and

$$\frac{\partial}{\partial x_{2,j}} E[y_2 \mid y_1^* > 0, x] = \frac{\partial}{\partial x_{2,j}}\big(x_2'\beta_2\big) = \beta_{2,j}.$$

$\Longrightarrow$ If a variable affects only the wage equation and not the participation decision, changing it shifts the wage directly (by $\beta_{2,j}$) but does not change who is selected into the sample - so there is no indirect selection effect to account for.

(c)

Let the model be expressed in vector form as follows:

$$\begin{aligned} y_{1i}^* &= X_{1i'}\beta_1 + \varepsilon_{1i}, \quad & y_{1i} &= \mathbf{1}\{y_{1i}^* > 0\}, \\ y_{2i}^* &= X_{2i}'\beta_2 + \varepsilon_{2i}, \quad & y_{2i} &= y_{2i}^* \text{ if } y_{1i} = 1. \end{aligned}$$

X_{1i} and X_{2i} are *vectors of regressors* (covariates) for the i^{th} individual:

$$X_{1i} = \begin{bmatrix} 1 \\ x_{1i1} \\ x_{1i2} \\ \vdots \\ x_{1im} \end{bmatrix}, \qquad X_{2i} = \begin{bmatrix} 1 \\ x_{2i1} \\ x_{2i2} \\ \vdots \\ x_{2ik} \end{bmatrix}.$$

In this notation, uppercase symbols (X_{1i}, X_{1i}) refer to the full vector of regressors for observation i, while lowercase symbols (x_{1ij}, x_{2ij}) denote the individual covariates contained within these vectors.

Rest of the conditions remain the same, now evaluating expectation and its differential we get,

$$E[y_{2i} \mid y_{1i}^* > 0, X_1, X_2] = X_{2i}\beta_2 + \sigma_{21}\lambda(\omega_i), \qquad \text{where } \omega_i = X_{1i}\beta_1. \tag{1}$$

Differentiating with respect to the covariate x_j that appears in both equations:

$$\begin{aligned} \frac{\partial}{\partial x_j} E[y_{2i} \mid y_{1i}^* > 0] &= \frac{\partial(X_{2i}\beta_2)}{\partial x_j} + \sigma_{21}\frac{d\lambda(\omega_i)}{d\omega_i}\frac{\partial\omega_i}{\partial x_j} \\ &= \beta_{2,j} + \sigma_{21}\lambda'(\omega_i)\beta_{1,j}. \end{aligned} \tag{2}$$

Using $\lambda'(\omega) = -\lambda(\omega)[\omega + \lambda(\omega)]$, we obtain

$$\frac{\partial E[y_{2i} \mid y_{1i}^* > 0]}{\partial x_j} = \beta_{2,j} - \beta_{1,j}\sigma_{21}\lambda(\omega_i)\big[\omega_i + \lambda(\omega_i)\big]. \tag{3}$$

Evaluated at the sample mean $\bar{\omega}_1$,

$$\boxed{\frac{\partial E[y_2 \mid y_1^* > 0, \bar{x}, \beta, \Sigma]}{\partial(x_{1,j}, x_{2,j})} = \beta_{2,j} - \beta_{1,j}\sigma_{21}\frac{\phi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)}\left[\frac{\phi(\bar{\omega}_1)}{\Phi(\bar{\omega}_1)} + \bar{\omega}_1\right].} \tag{4}$$

$\Longrightarrow$ The first term ($\beta_{2,j}$) captures the direct impact of the covariate on the outcome (wage), while the second term captures the indirect effect through the participation probability.

Question 2

(a)

In the fixed effects (FE) model, we make the following key assumptions regarding the error term and the relationship between the covariates and the individual-specific effects:

1. **Assumptions on the error term:**

   (a) The idiosyncratic errors have zero conditional mean:

   $$E[\varepsilon_{it} \mid x_{i1}, x_{i2}, \ldots, x_{iT}, c_i] = 0,$$

   which implies that the idiosyncratic error term is uncorrelated with the explanatory variables and the individual effect.

   (b) The idiosyncratic errors are homoskedastic:

   $$\mathrm{Var}(\varepsilon_{it} \mid x_{it}, c_i) = \sigma^2$$

2. **Assumption about the relationship between covariates and individual-specific effect:**

   $$\mathrm{Cov}(x_{it}, c_i) \neq 0.$$

   That is, in the fixed effects model, the individual-specific effects c_i are allowed to be correlated with the explanatory variables x_{it}. This distinguishes the FE model from the random effects model, which assumes $\mathrm{Cov}(x_{it}, c_i) = 0$.

(b)

Averaging equation (4) over time for each individual i gives:

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \quad \bar{x}_i = \frac{1}{T}\sum_{t=1}^{T} x_{it}, \quad \bar{\varepsilon}_i = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it}.$$

Taking the time average of equation (4):

$$y_{it} = x_{it}'\beta + c_i + \varepsilon_{it},$$

we obtain:

$$\bar{y}_i = \bar{x}_i'\beta + c_i + \bar{\varepsilon}_i.$$

This represents the individual-specific (time-averaged) version of the panel data model, where c_i remains constant across t.

(c)

Subtracting the time-averaged equation obtained in part (b)

$$\bar{y}_i = \bar{x}_i'\beta + c_i + \bar{\varepsilon}_i$$

from the original equation

$$y_{it} = x_{it}'\beta + c_i + \varepsilon_{it},$$

gives the within (deviation-from-individual-mean) form

$$\tilde{y}_{it} \equiv y_{it} - \bar{y}_i = (x_{it} - \bar{x}_i)'\beta + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

or, using the tilde notation,

$$\tilde{y}_{it} = \tilde{x}_{it}'\beta + \tilde{\varepsilon}_{it}.$$

Stacking this model over $t = 1, \ldots, T$ for a given individual i yields the vector form

$$\tilde{y}_i = \begin{pmatrix} \tilde{y}_{i1} \\ \tilde{y}_{i2} \\ \vdots \\ \tilde{y}_{iT} \end{pmatrix} = \begin{pmatrix} \tilde{x}_{i1}' \\ \tilde{x}_{i2}' \\ \vdots \\ \tilde{x}_{iT}' \end{pmatrix} \beta + \begin{pmatrix} \tilde{\varepsilon}_{i1} \\ \tilde{\varepsilon}_{i2} \\ \vdots \\ \tilde{\varepsilon}_{iT} \end{pmatrix} = \tilde{X}_i\beta + \tilde{\varepsilon}_i.$$

Here the dimensions are

$$\tilde{y}_i \in \mathbb{R}^{T\times 1}, \qquad \tilde{X}_i \in \mathbb{R}^{T\times k}, \qquad \beta \in \mathbb{R}^{k\times 1}, \qquad \tilde{\varepsilon}_i \in \mathbb{R}^{T\times 1}.$$

(d)

Stacking the within (deviation-from-individual-mean) equations over both $i = 1, \ldots, n$ and $t = 1, \ldots, T$ gives the fixed-effects model in deviation form:

$$\tilde{y} \; = \; \tilde{X}\beta + \tilde{\varepsilon}.$$

Here the components are defined as follows:

$$\tilde{y} \; = \; \begin{pmatrix} \tilde{y}_{11} \\ \tilde{y}_{12} \\ \vdots \\ \tilde{y}_{1T} \\ \tilde{y}_{21} \\ \vdots \\ \tilde{y}_{nT} \end{pmatrix}, \qquad \tilde{X} \; = \; \begin{pmatrix} \tilde{x}_{11}' \\ \tilde{x}_{12}' \\ \vdots \\ \tilde{x}_{1T}' \\ \tilde{x}_{21}' \\ \vdots \\ \tilde{x}_{nT}' \end{pmatrix}, \qquad \tilde{\varepsilon} \; = \; \begin{pmatrix} \tilde{\varepsilon}_{11} \\ \tilde{\varepsilon}_{12} \\ \vdots \\ \tilde{\varepsilon}_{1T} \\ \tilde{\varepsilon}_{21} \\ \vdots \\ \tilde{\varepsilon}_{nT} \end{pmatrix}.$$

Where for each i, t

$$\tilde{y}_{it} = y_{it} - \bar{y}_i, \qquad \tilde{x}_{it} = x_{it} - \bar{x}_i, \qquad \tilde{\varepsilon}_{it} = \varepsilon_{it} - \bar{\varepsilon}_i,$$

with $\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$, $\bar{x}_i = \frac{1}{T}\sum_{t=1}^{T} x_{it}$, and $\bar{\varepsilon}_i = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it}$.

Dimensions:

$$\tilde{y} \in \mathbb{R}^{nT\times 1}, \qquad \tilde{X} \in \mathbb{R}^{nT\times k}, \qquad \beta \in \mathbb{R}^{k\times 1}, \qquad \tilde{\varepsilon} \in \mathbb{R}^{nT\times 1}.$$

(e)

The within-group (fixed-effects) estimator is obtained by applying pooled OLS to the demeaned system from part (d):

$$\tilde{y} = \tilde{X}\beta + \tilde{\varepsilon},$$

Minimizing the sum of squared within-residuals

$$Q(\beta) = (\tilde{y} - \tilde{X}\beta)'(\tilde{y} - \tilde{X}\beta).$$

First-order condition:

$$-2\tilde{X}'(\tilde{y} - \tilde{X}\beta) = 0 \quad \Longrightarrow \quad \tilde{X}'\tilde{X}\,\hat{\beta}_{FE} = \tilde{X}'\tilde{y}.$$

Hence the fixed-effects (within) estimator is

$$\boxed{\hat{\beta}_{FE} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y} = \left(\sum_{i=1}^{n} \tilde{X}_i'\tilde{X}_i\right)^{-1}\left(\sum_{i=1}^{n} \tilde{X}_i'\tilde{y}_i\right)}$$

provided $\tilde{X}'\tilde{X}$ is nonsingular.

Dimensions: $\tilde{X}'\tilde{X}$ is $k \times k$, $\tilde{X}'\tilde{y}$ is $k \times 1$, so $\hat{\beta}_{FE} \in \mathbb{R}^{k\times 1}$.

(f)

Given $\hat{\beta}_{FE}$ from part (e), the fixed-effect for individual i is estimated by the time-average residual:

$$\boxed{\hat{c}_i \;=\; \bar{y}_i - \bar{x}_i'\,\hat{\beta}_{FE}}$$

where $\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$ and $\bar{x}_i = \frac{1}{T}\sum_{t=1}^{T} x_{it}$. This follows directly from taking the time average of the model $y_{it} = x_{it}'\beta + c_i + \varepsilon_{it}$ and solving for c_i, assuming $E[\bar{\varepsilon}_i] = 0$.

Defining the fitted residuals

$$\hat{\varepsilon}_{it} \;=\; y_{it} - x_{it}'\hat{\beta}_{FE} - \hat{c}_i \;=\; \tilde{y}_{it} - \tilde{x}_{it}'\hat{\beta}_{FE}.$$

An unbiased (degrees-of-freedom adjusted) estimator of the idiosyncratic variance σ_ε^2 is

$$\boxed{\hat{\sigma}_\varepsilon^2 \;=\; \frac{1}{nT - n - k}\sum_{i=1}^{n}\sum_{t=1}^{T}\hat{\varepsilon}_{it}^2 \;=\; \frac{1}{nT - n - k}\,\hat{\tilde{\varepsilon}}'\hat{\tilde{\varepsilon}},}$$

where $\hat{\tilde{\varepsilon}}$ is the stacked vector of within residuals and the denominator $nT - n - k$ reflects the loss of n individual means and k estimated slope parameters.

Question 3

(a)

The panel data model is as follows:

$$\text{mrdrte}_{it} = \eta_t + \beta_1 \text{exec}_{it} + \beta_2 \text{unemp}_{it} + c_i + \varepsilon_{it} \tag{5}$$

The expected signs of β_1 and β_2 are:

- **β_1: Negative**
  If past executions of convicted murderers have a deterrent effect on the intention to murder, then as the number of executions increases the murder rate should decrease.

- **β_2: Positive**
  This is expected to be positive as the unemployment rate increases, the psychological strain and frustration among people increases leading to a higher number of murders.
  More importantly, as unemployed people have no legitimate income, the opportunity cost of committing murder reduces as they do not risk losing a stable job for the same. Essentially, in general unemployment reduces the opportunity cost of committing illegal activities.

(b)Pooled OLS

Using only two time periods, let us estimate the model using Pooled OLS. We also ignore the serial correlation of the composite error $u_{it} = c_i + \varepsilon_{it}$. We ignore the fact that c_i is time invariant. The estimates for the model are described by the equation:

$$mrdrte_{it} = \beta_0 + \beta_1 exec_{it} + \beta_2 unem_{it} + \beta_{dummy} \mathbb{I}(\text{Year} = 93) + u_{it} \tag{6}$$

The different intercepts for each year denoted by η are taken care of by introducing the dummy variable to indicate if the year is 1993 and using the year 1990 to estimate the base intercept β_0. Essentially, the intercept in the case of 1993 will be: $\eta_{1993} = \beta_0 + \beta_{dummy}$
The estimates for the above model are:

Variable	Coefficients	Estimates	Standard Errors	t-statistic
Intercept(for 1990)	β_1	-5.278	4.4278	-1.192
Executions	β_2	0.12773	0.26324	0.48523
Unemployment	β_3	2.5289	0.78172	3.235
Year = 1993	β_{dummy}	-2.0674	2.1446	-0.964

According to the estimates in the pooled OLS the coefficient of *exec* is small and positive and more importantly, statistically insignificant with a standard error of 0.26324 and a t-statistic of 0.48523. This implies that the estimate of β_2 is insignificant and does not have a consistent effect across the dataset and hence **no evidence of deterrent effect.**

(c)Fixed Effects Model

Now we estimate the model via fixed effects, which eliminates the time invariant component of each individual c_i so that β's are not biased if we c_i is correlated to x_{it}. We accomplish this using first differences which gives us the following model by subtracting one from the other($t = 0$ is 1990 and $t = 1$ is 1993). We estimate the effects within-states over time whereas in part (b) we were estimating the coefficient between-states and across time both. This may lead to severe bias as some states may have a very high murder rate which may have reduced significantly while another state which doesn't have a high murder rate and subsequently did not have a drop in the murder-rate would have biased the coefficient.

$$\Delta \text{mrdrte}_i = \beta_1 \Delta \text{exec}_i + \beta_2 \Delta \text{unem}_i + \Delta \eta + \Delta \varepsilon_i$$
$$\Delta \eta = \eta_{93} - \eta_{90}$$

We estimate $\Delta \eta$ which is the change in constants. The estimates of the model are as follows:

Variable	Coefficients	Estimates	Standard Errors	t-statistic
Constant	$\Delta \eta$	0.41327	0.20938	1.9737
Executions	β_1	-0.10384	0.04341	-2.3918
Unemployment	β_2	-0.06659	0.15869	-0.4196

Each coefficient now measures the within state effect over time and ignores the time-invariants of each state.

There is **strong** evidence to account for the deterrent effect of number of executions on the murder rate as the coefficient for *exec* i.e β_1 is negative and statistically significant, which indicates it is important and consistent with what we predicted in (a). However, the strength is debatable as the t-stat value is just above 2.

However, the coefficient of unemployment is statistically insignificant, unlike the estimates in part(b).

(d) Heteroskedastic Robust Standard Errors

The Heteroskedastic Robust standard errors are calculated as follows:

$$\text{Residuals: } u = y - X\hat{\beta}$$
$$\Omega = \text{diag}(u_1^2, u_2^2, u_3^2, \ldots, u_n^2)$$
$$Var_{robust}(\beta) = (X'X)^{-1}(X'\Omega X)(X'X)^{-1}$$
$$SE_j = \sqrt{[Var(\beta)]_{jj}}$$

The estimates using the above formulas are:

Variable	Coefficients	Estimates	Robust Standard Errors	t-statistic
Intercept (for 1990)	β_1	-5.2780	5.2801	-0.9996
Executions	β_2	0.1277	0.1316	0.9707
Unemployment	β_3	2.5289	1.0856	2.3294
Year = 1993	β_{dummy}	-2.0674	1.9585	-1.0556

Comparing the non-robust and robust standard errors: The coefficient for execution remains insignificant even though the standard error for it reduces. Moreover, the standard error of unemployment increases which is evidence of heteroskedasticity, but it remains significant.

(e)

The state with the largest number for the execution variable was Texas. This value is bigger than the next highest value by 23. This can lead us to guess that Texas is an outlier in the observations since its value is very large and its difference with the next highest value is also very large.

(f) First differencing without Texas

Dropping Texas from the regression, we get the following results for the usual and robust standard errors:

Variable	Estimates	Standard Errors	t-stat	SE robust	t-stat robust
Constant	0.41327	0.20938	1.9737	0.19403	2.1299
Executions	-0.10384	0.043414	-2.3918	0.016492	-6.2964
Unemployment	-0.066591	0.15869	-0.41964	0.14254	-0.46717

Using the usual standard errors, we interpret that there is **strong evidence** to account for the deterrent effect of executions. This is because the estimates are negative and $|t - stat| > 2$ for the coefficient estimates of executions using both types of errors. Hence, the estimate values are statistically significant and we can reject the $H_0 : \beta_{\text{exec}} = 0$.
However, we see that the estimate for Unemployment also implies a negative correlation with the murder rate. This value, however, is not statistically significant according to both the usual and robust standard errors.

(g) Within group estimation

Here, we include Texas in the analysis. After performing within group estimation, we get the following results:

Variable	Coefficients	Estimates	Standard Errors	t-stat	SE robust	t-stat robust
Executions	β_1	-0.12825	0.14697	-0.87263	0.061884	-2.0725
Unemployment	β_2	0.10871	0.24244	0.44841	0.23464	0.4633
Year=1990	β_{dummy1}	0.39585	0.43535	0.90927	0.31931	1.2397
Year=1993	β_{dummy2}	0.66748	0.4184	1.5953	0.2896	2.3048

Here, we interpret that there can be some deterrent effect of executions by looking at the negative sign of the estimate. However, the normal t-stat for it is between -2 and 2, implying statistical insignificance. Hence, there is **no strong evidence** of deterrence. Unemployment has a positive relationship with murder rates as $0.10871 > 0$. It is also not statistically significant, considering the normal t-stat at a confidence level of 95%.

Compared to Pooled OLS only using 1990 and 1993, the estimate for executions is negative here while positive there. However, both are statistically insignificant when we look at the usual standard errors and t-stats. The magnitude of the estimate for unemployment has decreased for this question as compared to the Pooled OLS with only 1990 and 1993 while still remaining positive.
When we compare the results of usual SE and robust SE for this within group estimation, we see that there is **strong evidence** of deterrence due to executions while looking at the Robust standard errors. This is because the estimate for executions becomes statistically significant for robust standard errors.

Appendix

MATLAB Code

```
clear; clc;

%% Question 3: Panel data model on Murder.xlsx
T = readtable('Murder.xlsx', 'Sheet','Data');

%% Part(B) Use only 1990 and 1993

subset = (T.year == 90 | T.year == 93);
data = T(subset, :);

Y = data.mrdrte;
exec = data.exec;
unem = data.unem;
year_93 = (data.year == 93); % to create an year dummy

X = [ones(size(Y)), exec, unem, year_93];
k = size(X, 2);
n = size(X, 1);
%Estimation using OLS

beta_ols = (X' * X) \ (X' * Y);
yhat_ols = X * beta_ols;
res_ols = Y - yhat_ols;

sigma2_ols = (res_ols' * res_ols) / (n - k);
Var_ols = sigma2_ols * inv(X' * X);
se_ols = sqrt(diag(Var_ols));
tstat_ols = beta_ols ./ se_ols;

disp(table(beta_ols, se_ols, tstat_ols, 'VariableNames', {'Beta','
    StdErr','tstat'}));

%% Using Fixed effects using first differencing

ustates = unique(data.state);
dY = [];
dExec = [];
dUnem = [];

keep_idx = false(numel(ustates), 1);
for i = 1:numel(ustates)
    Si = data(strcmp(data.state, ustates{i}), :);
    if sum(Si.year == 90) == 1 && sum(Si.year == 93) == 1
        Si = sortrows(Si, 'year');
        dY(end + 1, 1) = Si.mrdrte(Si.year == 93) - Si.mrdrte(Si.
```

```
            year == 90);
        dExec(end+1,1) = Si.exec(Si.year==93)   - Si.exec(Si.year==90);
        dUnem(end+1,1) = Si.unem(Si.year==93)   - Si.unem(Si.year==90);
        keep_idx(i) = true;
    end
end

dX = [ones(size(dY)), dExec, dUnem];
beta_fe = (dX' * dX) \ (dX' * dY);
res_fe = dY - dX * beta_fe;

%homeskedastic errors
sigma2_fe = (res_fe' * res_fe) / (size(dX, 1) - size(dX, 2));
Var_fe = sigma2_fe * inv(dX' * dX);
se_fe = sqrt(diag(Var_fe));
tstat_fe = beta_fe ./ se_fe;

varnames = {'Constant', 'Executions', 'Unemployment'};
disp(table(varnames', beta_fe, se_fe, tstat_fe, 'VariableNames', {'Regressor', 'Beta','StdErr','tstat'}));

%% D) Heteroskedastic errors for the estimates in part B

meat = X' * diag(res_ols.^2) * X;
bread = inv(X' * X);
varb_robust = bread * meat * bread;
se_robust = sqrt(diag(varb_robust));
t_robust = beta_ols ./ se_robust;
varnames_robust = {'constant', 'exec', 'unem', 'dummy'};
disp(table(varnames_robust', beta_ols, se_robust, t_robust, 'VariableNames', {'Regressor', 'Beta','StdErr','tstat'}));


%Part (e)
data_93= data(data.year== 93,:);
[max_exec, idx]= max(data_93.exec);
[state_Max]= data_93.state(idx);
fprintf('The state with the highest executions in 1993 was %s', state_Max{1});
sortedExec = sort(data_93.exec, 'descend');
secondMax = sortedExec(2);
diffExec = max_exec - secondMax;
fprintf('The difference between this value and the next highest value is %d', diffExec);

```

```
%% NO texas%% Part (F) Fixed Effects (First Difference) excluding
   Texas
data_noTX = data(~strcmp(data.state,'Texas'), :);

ustates_noTX = unique(data_noTX.state);
dY_noTX = []; dExec_noTX = []; dUnem_noTX = [];

for i = 1:numel(ustates_noTX)
    Si = data_noTX(strcmp(data_noTX.state, ustates_noTX{i}), :);
    if sum(Si.year==90)==1 && sum(Si.year==93)==1
        Si = sortrows(Si,'year');
        dY_noTX(end+1,1)    = Si.mrdrte(Si.year==93) - Si.mrdrte(Si.
           year==90);
        dExec_noTX(end+1,1) = Si.exec(Si.year==93)   - Si.exec(Si.
           year==90);
        dUnem_noTX(end+1,1) = Si.unem(Si.year==93)   - Si.unem(Si.
           year==90);
    end
end

dX_noTX = [ones(size(dY_noTX)), dExec_noTX, dUnem_noTX];
beta_fe_noTX = (dX_noTX' * dX_noTX) \ (dX_noTX' * dY_noTX);
res_noTX = dY_noTX - dX_noTX * beta_fe_noTX;

sigma2_noTX = (res_noTX' * res_noTX) / (size(dX_noTX,1) - size(
   dX_noTX,2));
Var_noTX = sigma2_noTX * inv(dX_noTX' * dX_noTX);
se_noTX = sqrt(diag(Var_noTX));
t_noTX = beta_fe_noTX ./ se_noTX;

%Heteroskedastic errors
bread_f = inv(dX_noTX' * dX_noTX);
meat_f = dX_noTX' * diag(res_noTX.^2) * dX_noTX;
varb_robust_noTX = bread_f * meat_f * bread_f;
se_robust_noTX = sqrt(diag(varb_robust_noTX));
t_robust_noTX = beta_fe_noTX ./ se_robust_noTX;

varnames_noTX = {'Constant','Executions','Unemployment'};
disp('Fixed Effects (First Difference) excluding Texas:');
disp(table(varnames_noTX', beta_fe_noTX, se_noTX, t_noTX,
   se_robust_noTX, t_robust_noTX, ...
    'VariableNames', {'Regressor','Beta','StdErr','tstat', '
       StdErr_rob', 'tstat_rob'}));

%% Part (G) Fixed Effects using 1987, 1990, 1993
subset3 = (T.year == 87 | T.year == 90 | T.year == 93);
data3 = T;
```

```

Yg = data3.mrdrte;
exec3 = data3.exec;
unem3 = data3.unem;

% Create year dummies (reference year = 1987)
year90 = (data3.year == 90);
year93 = (data3.year == 93);

% Remove state means (Within / Demeaning Transformation)
ustates3 = unique(data3.state);
Yw = zeros(size(Yg));
Xw_exec = zeros(size(exec3));
Xw_unem = zeros(size(unem3));

for i = 1:numel(ustates3)
    idx = strcmp(data3.state, ustates3{i});
    Yw(idx) = Yg(idx) - mean(Yg(idx));
    Xw_exec(idx) = exec3(idx) - mean(exec3(idx));
    Xw_unem(idx) = unem3(idx) - mean(unem3(idx));
end

% Regression on demeaned data with time dummies
Xw = [Xw_exec, Xw_unem, year90, year93];
beta_g = (Xw' * Xw) \ (Xw' * Yw);
res_g = Yw - Xw * beta_g;

sigma2_g = (res_g' * res_g) / (size(Xw,1) - size(Xw,2));
Var_g = sigma2_g * inv(Xw' * Xw);
se_g = sqrt(diag(Var_g));
t_g = beta_g ./ se_g;

N      = size(Xw,1);
K      = size(Xw,2);

Bread    = inv(Xw' * Xw);
Meat_HC0 = Xw' * diag(res_g.^2) * Xw;
Meat_HC1 = (N / (N-K)) * Meat_HC0;
Var_HC1  = Bread * Meat_HC1 * Bread;
SE_HC1   = sqrt(diag(Var_HC1));
t_HC1    = beta_g ./ SE_HC1;

varnames_g = {'Executions','Unemployment','Year=1990','Year=1993'};
disp('Within (Fixed Effects) Model using 1987, 1990, and 1993:');
disp(table(varnames_g', beta_g, se_g, t_g, SE_HC1, t_HC1, ...
    'VariableNames', {'Regressor','Beta','StdErr','tstat', '
       se_robust', 'tstat_rob'}));
```