Investment Portfolio Optimization Using Markowitz' Modern Portfolio Theory

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Abstract

This report investigates portfolio optimization using Modern Portfolio Theory (MPT). The goal of this project is to create an optimized investment portfolio of selected stocks, aiming to maximize the risk-adjusted return (Sharpe Ratio) by combining assets based on their expected returns and volatilities. Python was used for simulations and optimizations, utilizing real stock market data to generate efficient frontiers and Capital Allocation Lines (CAL). The final portfolio composition is discussed based on the results of this optimization.

1 Introduction

Portfolio optimization is a key problem in finance that seeks to allocate wealth across a range of assets to maximize expected returns for a given level of risk, or minimize risk for a given level of expected return. This project focuses on optimizing a portfolio of five technology stocks (Apple, Microsoft, Amazon, Google, and Tesla) using Markowitz's Modern Portfolio Theory (MPT). The theory emphasizes diversification and provides a framework for constructing portfolios that offer the best risk-return tradeoff.

These stocks are highly liquid and are part of large-cap, technology-oriented portfolios. They tend to exhibit growth characteristics but also have varying risk-return profiles, which makes them suitable for portfolio optimization using Modern Portfolio Theory.

In this project, we apply MPT to real historical stock data, simulate random portfolios, and generate the efficient frontier. The optimal portfolio is identified using the Sharpe Ratio, which is maximized to find the best possible combination of assets.

2 Methodology

The methodology involves several key steps, from data collection to optimization and visualization. Below are the detailed steps:

2.1 Data Collection

We begin by gathering historical stock data for the following five technology stocks:

- Apple Inc. (AAPL)
- Microsoft Corporation (MSFT)
- Amazon.com Inc. (AMZN)
- Alphabet Inc. (GOOGL)
- Tesla Inc. (TSLA)

Using the `yfinance` Python library, we downloaded daily adjusted closing prices for these stocks over a 5-year period (2018–2023). The data was then processed to calculate the daily returns for each stock.

2.2 Portfolio Simulation

The returns and covariance matrix for the stocks were computed using historical daily return data. To explore possible portfolios, 10,000 random portfolios were simulated by assigning random weights to each stock. Each portfolio's expected return, volatility (risk), and Sharpe Ratio were calculated. The key metrics used were:

$$\text{Expected Portfolio Return} = \sum_i w_i \mu_i$$

$$\text{Portfolio Risk (Std. Dev.)} = \sqrt{w^\top \Sigma w}$$

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Return} - \text{Risk-Free Rate}}{\text{Portfolio Risk}}$$

where w_i represents the weight of stock i, μ_i is the expected return of stock i, and Σ is the covariance matrix of stock returns.

2.3 Optimization

To identify the optimal portfolio, we used the `SciPy` optimization library to maximize the Sharpe Ratio. The constraints included ensuring the sum of the portfolio weights equals 1 (i.e., fully invested) and that no short-selling was allowed (weights between 0 and 1).

The optimization problem was defined as:

$$\text{Maximize } \frac{E(R_p) - R_f}{\sigma_p} \text{ subject to } \sum w_i = 1$$

where R_p is the portfolio return, R_f is the risk-free rate, and σ_p is the portfolio risk. The optimization provided the optimal weights for the tangency portfolio on the efficient frontier.

2.4 Capital Allocation Line (CAL)

The risk-free rate was assumed to be 2% annually, representing the yield on short-term government bonds. The Capital Allocation Line (CAL) was plotted to show the trade-off between risk and return when combining the optimal risky portfolio with the risk-free asset. The tangent point of CAL with the efficient frontier corresponds to the portfolio with the maximum Sharpe Ratio.

3 Results

3.1 Efficient Frontier and Optimal Portfolio

The figure below shows the efficient frontier generated from the simulated portfolios and the Capital Allocation Line (CAL) representing the optimal risk-return tradeoff.

The optimal portfolio composition that maximizes the Sharpe Ratio is as follows:

- **Apple (AAPL)**: 13.77%
- **Microsoft (MSFT)**: 0%
- **Amazon (AMZN)**: 0%
- **Google (GOOGL)**: 62.47%
- **Tesla (TSLA)**: 23.75%

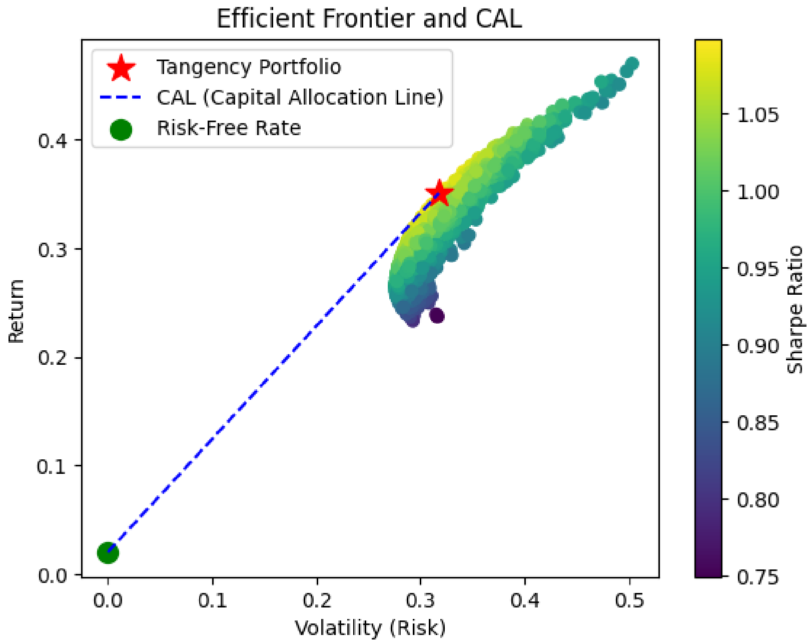

Figure 1: Efficient Frontier and Capital Allocation Line

4 Discussion

This portfolio achieves a high Sharpe Ratio, indicating the best risk-adjusted return. Google (GOOGL) has the highest allocation at 62.47%, which means it has the most favorable risk-return profile among the chosen stocks for maximizing the Sharpe Ratio. Tesla (TSLA) is allocated 23.75%, indicating that it's the second most significant stock in the optimized portfolio. Apple (AAPL) is allocated 13.77%. Microsoft (MSFT) and Amazon (AMZN) have near-zero allocations, meaning that these stocks don't contribute to the optimal portfolio, given the risk-return dynamics.

The Capital Allocation Line illustrates the tradeoff between the risk-free rate and the tangency portfolio, offering investors a choice of risk profiles. By leveraging the risk-free rate, investors can adjust their exposure to risk while maintaining an optimal return.

5 Conclusion

This project successfully demonstrates the application of Modern Portfolio Theory to construct an optimized investment portfolio using real-world stock data. By simulating portfolios and identifying the efficient frontier, we derived the optimal portfolio that maximizes the Sharpe Ratio. The methodology can be expanded to include other asset classes or constraints, such as short-selling or sector-specific portfolios.

Overall, this analysis highlights the value of diversification, risk management, and quantitative analysis in investment strategies, making it highly relevant to both the finance and consulting industries.

References

- Markowitz, H. (1952). Portfolio Selection. *Journal of Finance.*
- Sharpe, W. F. (1966). Mutual Fund Performance. *Journal of Business.*