

Jupyter Notebooks

Madhavan Mukund

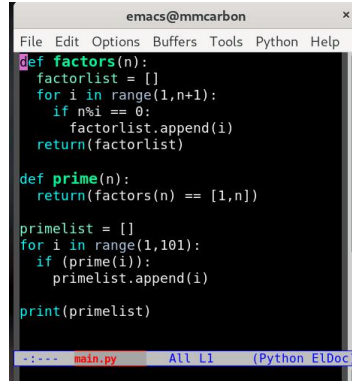
<https://www.cmi.ac.in/~madhavan>

Programming, Data Structures and Algorithms using Python
Week 1

Writing and running code

■ Manual

- Text editor to write code
- Run at the command line



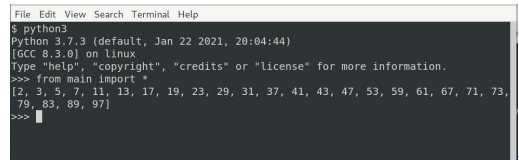
```
File Edit Options Buffers Tools Python Help
def factors(n):
    factorlist = []
    for i in range(1,n+1):
        if n%i == 0:
            factorlist.append(i)
    return(factorlist)

def prime(n):
    return(factors(n) == [1,n])

primelist = []
for i in range(1,101):
    if (prime(i)):
        primelist.append(i)

print(primelist)

-:--- main.py All L1 (Python ElDoc)
```



```
File Edit View Search Terminal Help
$ python3
Python 3.7.3 (default, Jan 22 2021, 20:04:44)
[GCC 8.3.0] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> from main import *
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73,
79, 83, 89, 97]
>>>
```

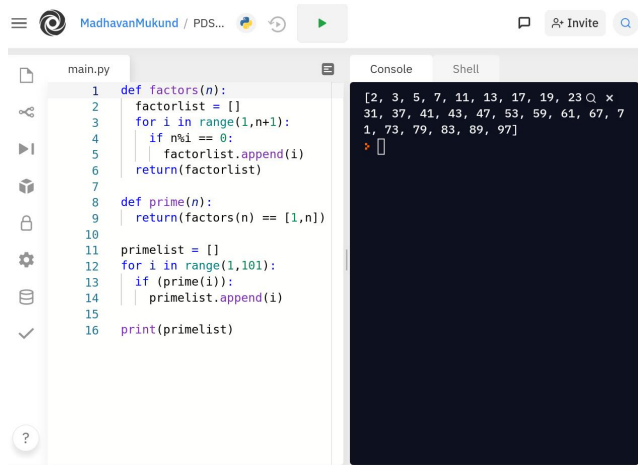
Writing and running code

■ Manual

- Text editor to write code
- Run at the command line

■ Integrated Development Environment (IDE)

- Single application to write and run code
- On desktop or online, [replit](#)
- Quick update-run cycle
- Debugging, testing, ...



The screenshot shows a Jupyter Notebook interface with a file named 'main.py'. The code defines two functions: 'factors(n)' which returns a list of factors for a given number 'n', and 'prime(n)' which returns a list of prime factors for a given number 'n'. The code then uses these functions to find the prime factors of all numbers from 1 to 101 and prints the resulting list.

```
1 def factors(n):
2     factorlist = []
3     for i in range(1,n+1):
4         if n%i == 0:
5             factorlist.append(i)
6     return(factorlist)
7
8 def prime(n):
9     return(factors(n) == [1,n])
10
11 primelist = []
12 for i in range(1,101):
13     if (prime(i)):
14         primelist.append(i)
15
16 print(primelist)
```

The console output shows the list of prime factors for all numbers from 1 to 101, including 2, 3, 5, 7, 11, 13, 17, 19, 23, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Writing and running code

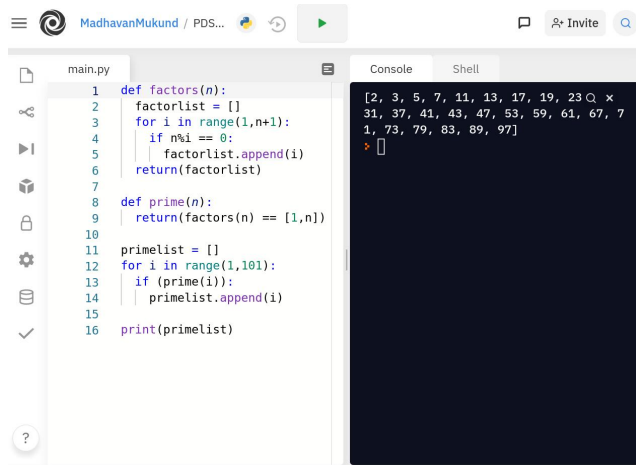
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- Debugging, testing, ...

■ What more could one want?



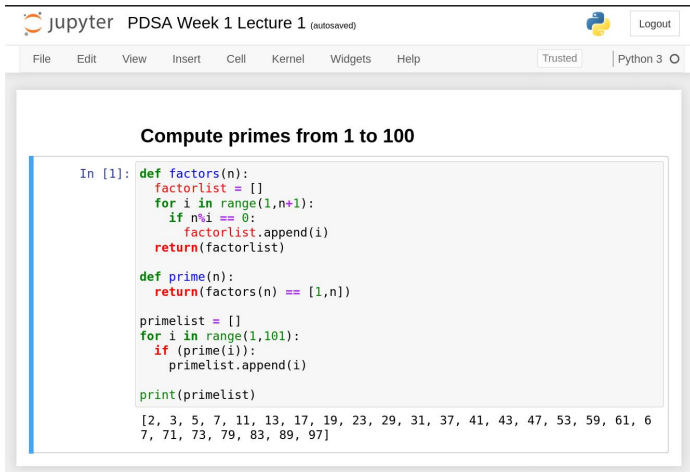
The screenshot shows a Jupyter Notebook interface with a file named `main.py`. The code defines two functions: `factors(n)` which returns a list of factors of `n`, and `prime(n)` which returns a list of prime factors of `n`. The code then finds the prime factors of 101 and prints them.

```
1 def factors(n):
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3     for i in range(1,n+1):
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8 def prime(n):
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15
16 print(primelist)
```

The console output shows the list of prime factors of 101: `[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97]`.

Collaboration

- Share your code
 - Collaborative development
 - Report your results



The screenshot shows a Jupyter Notebook window titled "jupyter PDSA Week 1 Lecture 1 (autosaved)". The interface includes a top menu bar with options: File, Edit, View, Insert, Cell, Kernel, Widgets, Help. On the right, there is a "Logout" button and a "Python 3" indicator. The notebook content is titled "Compute primes from 1 to 100". It contains a code cell with the following Python code:

```
In [1]: def factors(n):
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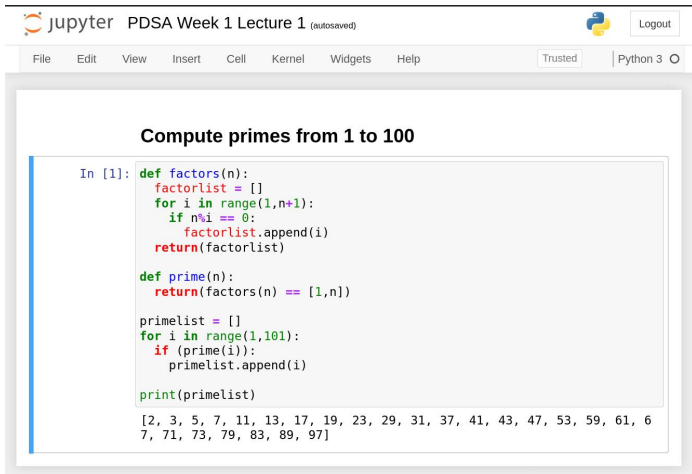
        print(primelist)
```

The output of the code is displayed below the code cell:

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[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97]
```

Collaboration

- Share your code
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 - Report your results
- Documentation
 - Interleave with the code



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In [1]: def factors(n):
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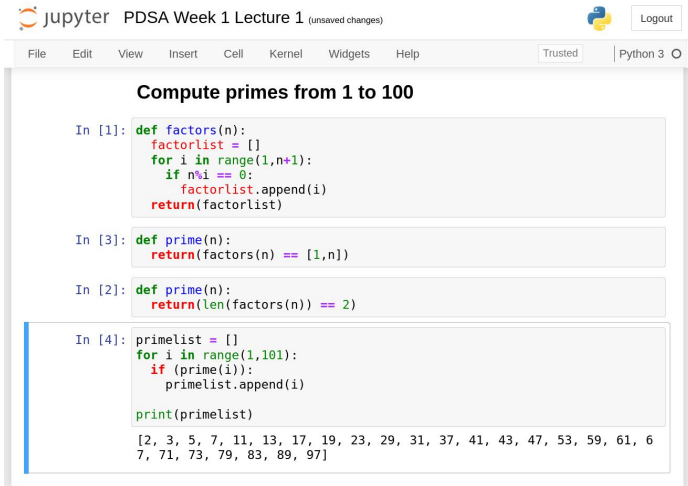
        print(primelist)
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The output of the code is displayed below the cell:

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[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97]
```

Collaboration

- Share your code
 - Collaborative development
 - Report your results
- Documentation
 - Interleave with the code
- Switch between different versions of code

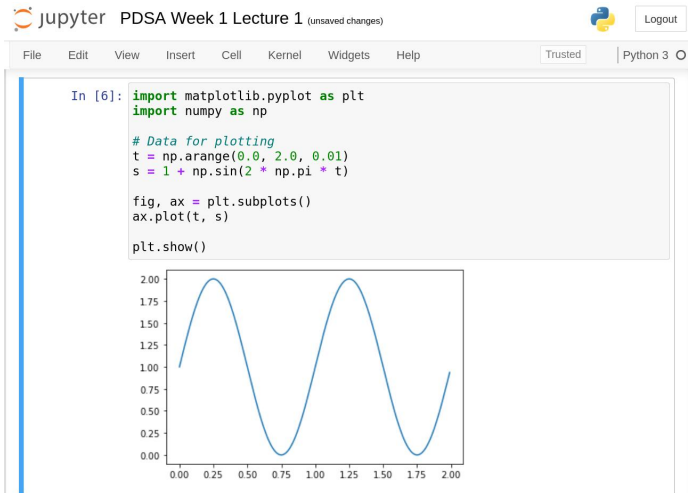


The screenshot shows a Jupyter Notebook window titled "PDSA Week 1 Lecture 1 (unsaved changes)". The interface includes a top bar with the Jupyter logo, a menu bar (File, Edit, View, Insert, Cell, Kernel, Widgets, Help), and a status bar (Trusted, Python 3). The notebook content is titled "Compute primes from 1 to 100" and contains four code cells:

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In [1]: def factors(n):  
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        return(factorlist)  
  
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```

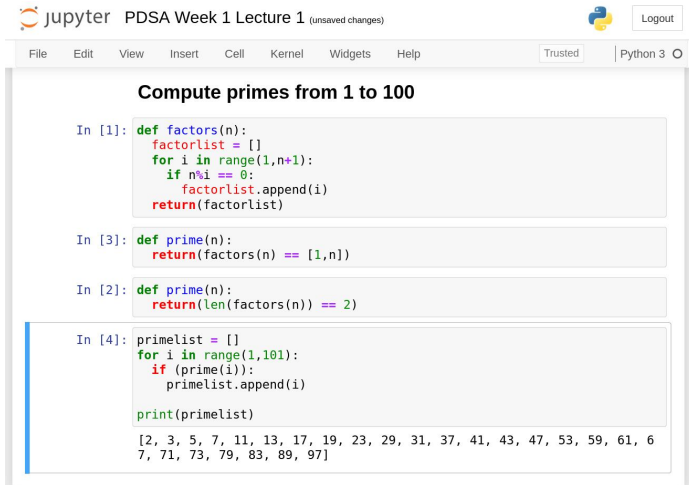
Collaboration

- Share your code
 - Collaborative development
 - Report your results
- Documentation
 - Interleave with the code
- Switch between different versions of code
- Export and import your project
- Preserve your output



Jupyter notebook

- A sequence of cells
 - Like a one dimensional spreadsheet



Jupyter PDSA Week 1 Lecture 1 (unsaved changes) Logout

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3

Compute primes from 1 to 100

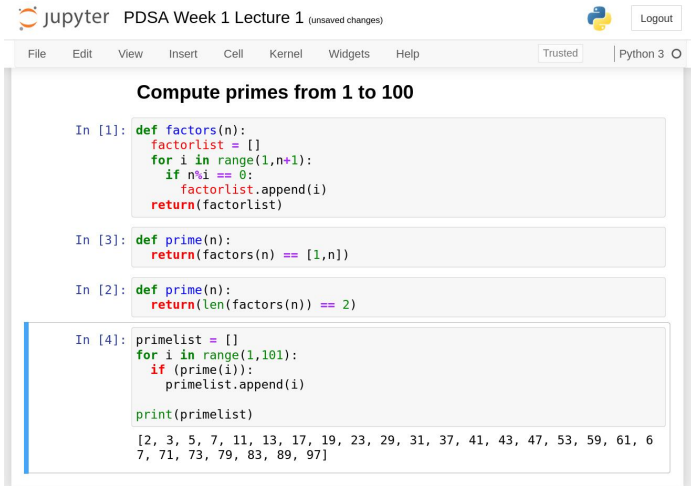
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Jupyter notebook

- A sequence of cells
 - Like a one dimensional spreadsheet
- Cells hold code or text
 - Markdown notation for formatting
 - <https://www.markdownguide.org/>

Jupyter notebook

- A sequence of cells
 - Like a one dimensional spreadsheet
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 - <https://www.markdownguide.org/>
- Edit and re-run individual cells to update environment



The screenshot shows a Jupyter Notebook window titled "PDSA Week 1 Lecture 1 (unsaved changes)". The interface includes a top bar with the Jupyter logo, a menu bar (File, Edit, View, Insert, Cell, Kernel, Widgets, Help), and a status bar (Trusted, Python 3). The notebook contains four code cells:

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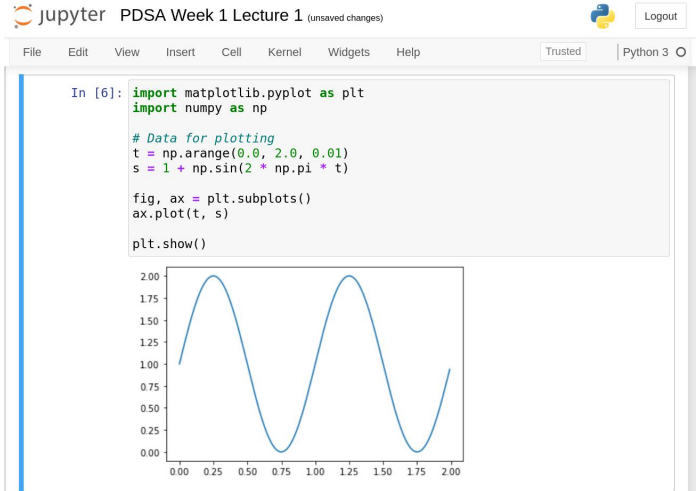
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In [4]: primelist = []  
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The output of the final cell is displayed below the code:

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[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97]
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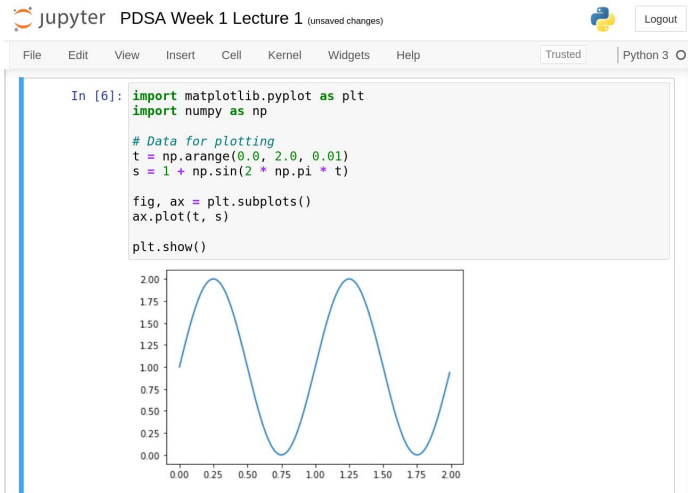
Jupyter notebook ...

- Supports different kernels
 - Julia, Python, R
- We will use it only for Python



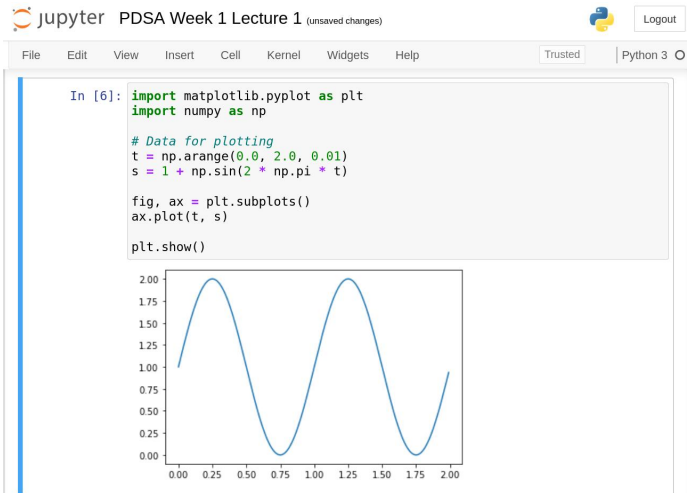
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- Widely used to document and disseminate ML projects
 - Solutions to problems posed on platforms like Kaggle <https://www.kaggle.org>

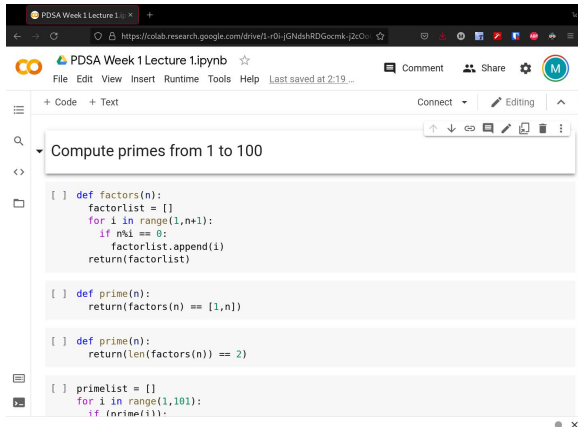


Jupyter notebook ...

- Supports different **kernels**
 - **Julia, Python, R**
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- ACM Software Systems Award 2017



- Google Colaboratory (Colab)
 - `colab.research.google.com`
 - Free to use



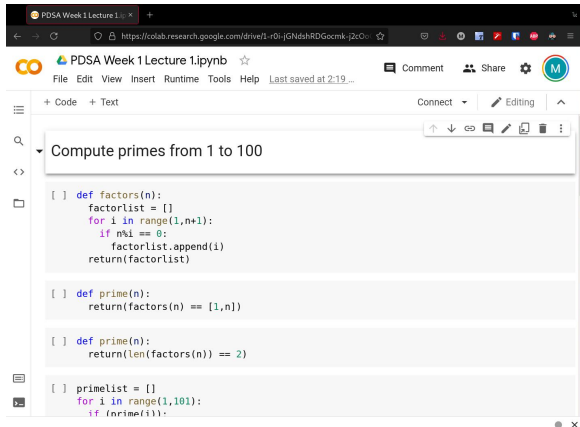
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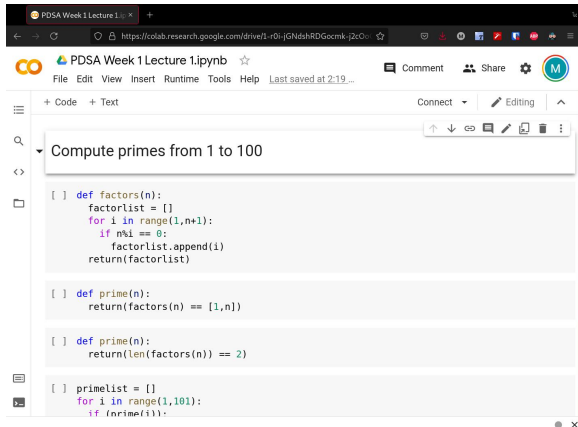
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- Google Colaboratory (Colab)
 - `colab.research.google.com`
 - Free to use
- Customized Jupyter notebook
- All standard packages required for ML are preloaded
 - `scikit-learn`, `tensorflow`
 - Access to GPU hardware



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- Google Colab — free to use version configured for ML

Python Recap – I

Madhavan Mukund

<https://www.cmi.ac.in/~madhavan>

Programming, Data Structures and Algorithms using Python
Week 1

Computing gcd

- $\text{gcd}(m, n)$ — greatest common divisor
 - Largest k that divides both m and n
 - $\text{gcd}(8, 12) = 4$
 - $\text{gcd}(18, 25) = 1$
 - Also **hcf** — highest common factor

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- $\text{gcd}(m, n)$ always exists
 - 1 divides both m and n
- Computing $\text{gcd}(m, n)$
 - $\text{gcd}(m, n) \leq \min(m, n)$
 - Compute list of common factors from 1 to $\min(m, n)$
 - Return the last such common factor

```
def gcd(m,n):  
    cf = []    # List of common factors  
    for i in range(1,min(m,n)+1):  
        if (m%i) == 0 and (n%i) == 0:  
            cf.append(i)  
    return(cf[-1])
```

Computing gcd

Points to note

- Need to initialize `cf` for `cf.append()` to work
 - Variables (names) derive their type from the value they hold

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def gcd(m,n):  
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 - Variables (names) derive their type from the value they hold
- Control flow
 - Conditionals (`if`)
 - Loops (`for`)
- `range(i,j)` runs from `i` to `j-1`
- List indices run from `0` to `len(l) - 1` and backwards from `-1` to `-len(l)`

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def gcd(m,n):  
    cf = []    # List of common factors  
    for i in range(1,min(m,n)+1):  
        if (m%i) == 0 and (n%i) == 0:  
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Computing gcd

Eliminate the list

- Only the last value of `cf` is important

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Computing gcd

Eliminate the list

- Only the last value of `cf` is important
- Keep track of most recent common factor (`mrcf`)

```
def gcd(m,n):  
    for i in range(1,min(m,n)+1):  
        if (m%i) == 0 and (n%i) == 0:  
            mrcf = i  
    return(mrcf)
```

Computing gcd

Eliminate the list

- Only the last value of `cf` is important
- Keep track of most recent common factor (`mrcf`)
- Recall that `1` is always a common factor
 - No need to initialize `mrcf`

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def gcd(m,n):  
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Efficiency

- Both versions of `gcd` take time proportional to $\min(m, n)$

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Computing gcd

Eliminate the list

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- Recall that 1 is always a common factor
 - No need to initialize `mrcf`

Efficiency

- Both versions of `gcd` take time proportional to $\min(m, n)$
- Can we do better?

```
def gcd(m,n):  
    for i in range(1,min(m,n)+1):  
        if (m%i) == 0 and (n%i) == 0:  
            mrcf = i  
    return(mrcf)
```

```
def gcd(m,n):  
    cf = []    # List of common factors  
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        if (m%i) == 0 and (n%i) == 0:  
            cf.append(i)  
    return(cf[-1])
```

Python Recap – II

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Programming, Data Structures and Algorithms using Python
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Checking primality

- A prime number n has exactly two factors, 1 and n
 - Note that 1 is not a prime

Checking primality

- A prime number n has exactly two factors, 1 and n
 - Note that 1 is **not** a prime
- Compute the list of factors of n

```
def factors(n):  
    fl = []    # factor list  
    for i in range(1,n+1):  
        if (n%i) == 0:  
            fl.append(i)  
    return(fl)
```

Checking primality

- A prime number n has exactly two factors, 1 and n
 - Note that 1 is **not** a prime
- Compute the list of factors of n
- n is a prime if the list of factors is precisely $[1, n]$

```
def factors(n):  
    fl = []    # factor list  
    for i in range(1, n+1):  
        if (n%i) == 0:  
            fl.append(i)  
    return(fl)
```

```
def prime(n):  
    return(factors(n) == [1, n])
```


Counting primes

- List all primes upto m

```
def primesupto(m):  
    pl = []    # prime list  
    for i in range(1,m+1):  
        if prime(i):  
            pl.append(i)  
    return(pl)
```

Counting primes

- List all primes upto m
- List the first m primes
 - Multiple simultaneous assignment

```
def primesupto(m):  
    pl = []    # prime list  
    for i in range(1,m+1):  
        if prime(i):  
            pl.append(i)  
    return(pl)
```

```
def firstprimes(m):  
    (count,i,pl) = (0,1,[])  
    while (count < m):  
        if prime(i):  
            (count,pl) = (count+1,pl+[i])  
        i = i+1  
    return(pl)
```

Counting primes

- List all primes upto m
- List the first m primes
 - Multiple simultaneous assignment
- `for` vs `while`
 - Is the number of iterations known in advance?
 - Ensure progress to guarantee termination of `while`

```
def primesupto(m):  
    pl = []    # prime list  
    for i in range(1,m+1):  
        if prime(i):  
            pl.append(i)  
    return(pl)
```

```
def firstprimes(m):  
    (count,i,pl) = (0,1,[])  
    while (count < m):  
        if prime(i):  
            (count,pl) = (count+1,pl+[i])  
        i = i+1  
    return(pl)
```

Computing primes

- Directly check if n has a factor between 2 and $n - 1$

```
def prime(n):  
    result = True  
    for i in range(2,n):  
        if (n%i) == 0:  
            result = False  
    return(result)
```

Computing primes

- Directly check if n has a factor between 2 and $n - 1$
- Terminate check after we find first factor
 - Breaking out of a loop

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            break    # Abort loop  
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```

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- Directly check if n has a factor between 2 and $n - 1$
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- Alternatively, use `while`

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            result = False  
            break    # Abort loop  
    return(result)
```

```
def prime(n):  
    (result,i) = (True,2)  
    while (result and (i < n)):  
        if (n%i) == 0:  
            result = False  
            i = i+1  
    return(result)
```

Computing primes

- Directly check if n has a factor between 2 and $n - 1$
- Terminate check after we find first factor
 - Breaking out of a loop
- Alternatively, use `while`
- Speeding things up slightly
 - Factors occur in pairs
 - Sufficient to check factors upto \sqrt{n}
 - If n is prime, scan $2, \dots, \sqrt{n}$ instead of $2, \dots, n - 1$

```
import math
def prime(n):
    (result,i) = (True,2)
    while (result and (i <= math.sqrt(n))):
        if (n%i) == 0:
            result = False
        i = i+1
    return(result)
```

Properties of primes

- There are infinitely many primes

Properties of primes

- There are infinitely many primes
- How are they distributed?

Properties of primes

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- Use a dictionary
- Start checking from 3, since 2 is the smallest prime

Properties of primes

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- Twin primes: $p, p + 2$
- **Twin prime conjecture**
There are infinitely many twin primes
- Compute the differences between primes
- Use a dictionary
- Start checking from 3, since 2 is the smallest prime

```
def primediffs(n):  
    lastprime = 2  
    pd = {} # Dictionary for  
            # prime differences  
    for i in range(3,n+1):  
        if prime(i):  
            d = i - lastprime  
            lastprime = i  
            if d in pd.keys():  
                pd[d] = pd[d] + 1  
            else:  
                pd[d] = 1  
    return(pd)
```

Python Recap – III

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Programming, Data Structures and Algorithms using Python
Week 1

Computing gcd

- Both versions of `gcd` take time proportional to `min(m, n)`
- Can we do better?

```
def gcd(m,n):  
    cf = []    # List of common factors  
    for i in range(1,min(m,n)+1):  
        if (m%i) == 0 and (n%i) == 0:  
            cf.append(i)  
    return(cf[-1])
```

```
def gcd(m,n):  
    for i in range(1,min(m,n)+1):  
        if (m%i) == 0 and (n%i) == 0:  
            mrcf = i  
    return(mrcf)
```


Computing gcd

- Both versions of `gcd` take time proportional to $\min(m, n)$
- Can we do better?
- Suppose d divides m and n
 - $m = ad, n = bd$
 - $m - n = (a - b)d$
 - d also divides $m - n$

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 - $m = ad, n = bd$
 - $m - n = (a - b)d$
 - d also divides $m - n$
- Recursively defined function
 - Base case: n divides m , answer is n
 - Otherwise, reduce `gcd(m, n)` to `gcd(n, m - n)`

```
def gcd(m,n):  
    (a,b) = (max(m,n), min(m,n))  
    if a%b == 0:  
        return(b)  
    else  
        return(gcd(b,a-b))
```

Computing gcd

- Unfortunately, this takes time proportional to $\max(m, n)$

```
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Computing gcd

- Unfortunately, this takes time proportional to $\max(m, n)$
- Consider $\text{gcd}(2, 9999)$
 - $\rightarrow \text{gcd}(2, 9997)$
 - $\rightarrow \text{gcd}(2, 9995)$
 - ...
 - $\rightarrow \text{gcd}(2, 3)$
 - $\rightarrow \text{gcd}(2, 1)$
 - $\rightarrow 1$

```
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Euclid's algorithm

- Suppose n does not divide m

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- Suppose n does not divide m
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 - If n divides m , $\text{gcd}(m, n) = n$
 - Otherwise, compute $\text{gcd}(n, m \bmod n)$

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- Can show that this takes time proportional to number of digits in $\max(m, n)$

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```

- Can show that this takes time proportional to number of digits in $\max(m, n)$
- One of the first non-trivial algorithms

Exception handling

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Programming, Data Structures and Algorithms using Python
Week 1

When things go wrong

- Our code could generate many types of errors
 - `y = x/z`, but `z` has value 0
 - `y = int(s)`, but string `s` does not represent a valid integer
 - `y = 5*x`, but `x` does not have a value
 - `y = l[i]`, but `i` is not a valid index for list `l`
 - Try to read from a file, but the file does not exist
 - Try to write to a file, but the disk is full

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 - Try to read from a file, but the file does not exist
 - Try to write to a file, but the disk is full
- Recovering gracefully
 - Try to anticipate errors
 - Provide a contingency plan
 - Exception handling

Types of errors

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 - Not much you can do!

Types of errors

- Python flags the type of each error
- Most common error is a syntax error
 - `SyntaxError: invalid syntax`
 - Not much you can do!
- We are interested in errors when the code is running
 - Name used before value is defined
`NameError: name 'x' is not defined`
 - Division by zero in arithmetic expression
`ZeroDivisionError: division by zero`
 - Invalid list index
`IndexError: list assignment index out of range`

Terminology

- Raise an exception
 - Run time error → signal error type, with diagnostic information
`NameError: name 'x' is not defined`
- Handle an exception
 - Anticipate and take corrective action based on error type
- Unhandled exception aborts execution

Terminology

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 - Run time error → signal error type, with diagnostic information
`NameError: name 'x' is not defined`
- Handle an exception
 - Anticipate and take corrective action based on error type
- Unhandled exception aborts execution

Handling exceptions

```
try:  
    ... ← Code where error may occur  
    ...  
except IndexError:  
    ... ← Handle IndexError  
except (NameError, KeyError):  
    ... ← Handle multiple exception types  
except:  
    ... ← Handle all other exceptions  
else:  
    ... ← Execute if try runs without errors
```

Using exceptions “positively”

- Collect scores in dictionary

```
scores = {"Shefali": [3, 22],  
          "Harmanpreet": [200, 3]}
```

- Update the dictionary
- Batter `b` already exists, append to list

```
scores[b].append(s)
```

- New batter, create a fresh entry

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scores[b] = [s]
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Traditional approach

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```

Using exceptions

```
try:  
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except KeyError:  
    scores[b] = [s]
```

Flow of control

```
...  
x = f(y,z)
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`IndexError` not
handled in `h()`

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`IndexError`
inherited from `h()`

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Flow of control

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```
def f(a,b):
```

```
...
```

```
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```

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```
def g(m):
```

```
...
```

```
h(m)
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Not handled?

```
def h(s):
```

```
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```

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Flow of control

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Flow of control

```
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`IndexError`

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Not handled?

Abort!

```
def f(a,b):
```

```
...
```

```
g(a)
```

`IndexError`

inherited from `g()`

Not handled?

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def g(m):
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Classes and objects

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Programming, Data Structures and Algorithms using Python
Week 1

Classes and objects

- Abstract datatype

- Stores some information
- Designated functions to manipulate the information
- For instance, stack: last-in, first-out, `push()`, `pop()`

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- Template for a data type
- How data is stored
- How public functions manipulate data

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■ Class

- Template for a data type
- How data is stored
- How public functions manipulate data

■ Object

- Concrete instance of template

Example: 2D points

- A point has coordinates (x, y)
 - `__init__()` initializes internal values `x, y`
 - First parameter is always `self`
 - Here, by default a point is at $(0, 0)$

```
class Point:  
    def __init__(self, a=0, b=0):  
        self.x = a  
        self.y = b
```

Example: 2D points

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 - `__init__()` initializes internal values `x, y`
 - First parameter is always `self`
 - Here, by default a point is at $(0, 0)$
- Translation: shift a point by $(\Delta x, \Delta y)$
 - $(x, y) \mapsto (x + \Delta x, y + \Delta y)$

```
class Point:
    def __init__(self, a=0, b=0):
        self.x = a
        self.y = b

    def translate(self, deltax, deltay):
        self.x += deltax
        self.y += deltay
```

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- Distance from the origin
 - $d = \sqrt{x^2 + y^2}$

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class Point:
    def __init__(self, a=0, b=0):
        self.x = a
        self.y = b

    def translate(self, deltax, deltay):
        self.x += deltax
        self.y += deltay

    def odistance(self):
        import math
        d = math.sqrt(self.x*self.x +
                       self.y*self.y)
        return(d)
```

Polar coordinates

- (r, θ) instead of (x, y)
 - $r = \sqrt{x^2 + y^2}$
 - $\theta = \tan^{-1}(y/x)$

```
import math
class Point:
    def __init__(self, a=0, b=0):
        self.r = math.sqrt(a*a + b*b)
        if a == 0:
            self.theta = math.pi/2
        else:
            self.theta = math.atan(b/a)
```

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        return(self.r)
```

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 - $r = \sqrt{x^2 + y^2}$
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- Distance from origin is just r
- Translation
 - Convert (r, θ) to (x, y)
 - $x = r \cos \theta, y = r \sin \theta$
 - Recompute r, θ from $(x + \Delta x, y + \Delta y)$

```
def translate(self, deltax, deltay):  
    x = self.r*math.cos(self.theta)  
    y = self.r*math.sin(self.theta)  
    x += deltax  
    y += deltay  
    self.r = math.sqrt(x*x + y*y)  
    if x == 0:  
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 - Convert (r, θ) to (x, y)
 - $x = r \cos \theta, y = r \sin \theta$
 - Recompute r, θ from $(x + \Delta x, y + \Delta y)$
- Interface has not changed
 - User need not be aware whether representation is (x, y) or (r, θ)

```
def translate(self, deltax, deltay):  
    x = self.r*math.cos(self.theta)  
    y = self.r*math.sin(self.theta)  
    x += deltax  
    y += deltay  
    self.r = math.sqrt(x*x + y*y)  
    if x == 0:  
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Special functions

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- `__str__()` — convert object to string
 - `str(o) == o.__str__()`
 - Implicitly invoked by `print()`

```
class Point:  
  
    ...  
  
    def __str__(self):  
        return(  
            '('+str(self.x)+', '  
            +str(self.y)+')'  
        )
```

Special functions

- `__init__()` — constructor
- `__str__()` — convert object to string
 - `str(o) == o.__str__()`
 - Implicitly invoked by `print()`
- `__add__()`
 - Implicitly invoked by `+`

```
class Point:
    ...

    def __str__(self):
        return(
            '('+str(self.x)+', '
            +str(self.y)+')'
        )

    def __add__(self,p):
        return(Point(self.x + p.x,
                      self.y + p.y))
```

Special functions

- `__init__()` — constructor
- `__str__()` — convert object to string
 - `str(o) == o.__str__()`
 - Implicitly invoked by `print()`
- `__add__()`
 - Implicitly invoked by `+`
- Similarly
 - `__mult__()` invoked by `*`
 - `__lt__()` invoked by `<`
 - `__ge__()` invoked by `>=`
 - ...

```
class Point:
    ...

    def __str__(self):
        return(
            '('+str(self.x)+', '
            +str(self.y)+')'
        )

    def __add__(self,p):
        return(Point(self.x + p.x,
                      self.y + p.y))
```

Timing our code

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Programming, Data Structures and Algorithms using Python
Week 1

Timing our code

- How long does our code take to execute?
 - Depends from one language to another

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- `perf_time()` is a performance counter
 - Absolute value of `perf_time()` is not meaningful
 - Compare two consecutive readings to get an interval
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- Python has a library `time` with various useful functions
- `perf_time()` is a performance counter
 - Absolute value of `perf_time()` is not meaningful
 - Compare two consecutive readings to get an interval
 - Default unit is seconds

```
import time

start = time.perf_counter()

...
# Execute some code
...
end = time.perf_counter()

elapsed = end - start
```


A timer object

- Create a timer class

```
import time  
class Timer:
```

A timer object

- Create a timer class
- Two internal values
 - `_start_time`
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```
import time
class Timer:
    def __init__(self):
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```

A timer object

- Create a timer class
- Two internal values
 - `_start_time`
 - `_elapsed_time`
- `start` starts the timer

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import time
class Timer:
    def __init__(self):
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    def start(self):
        self._start_time = time.perf_counter()
```

A timer object

- Create a timer class
- Two internal values
 - `_start_time`
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- `start` starts the timer
- `stop` records the elapsed time

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import time
class Timer:
    def __init__(self):
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    def start(self):
        self._start_time = time.perf_counter()

    def stop(self):
        self._elapsed_time =
            time.perf_counter() - self._start_time

    def elapsed(self):
        return(self._elapsed_time)
```

A timer object

- Create a timer class
- Two internal values
 - `_start_time`
 - `_elapsed_time`
- `start` starts the timer
- `stop` records the elapsed time
- More sophisticated version in the actual code

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A timer object

- Create a timer class
- Two internal values
 - `_start_time`
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- `start` starts the timer
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- More sophisticated version in the actual code
- Python executes 10^7 operations per second

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Why Efficiency Matters

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Week 1

A real world problem

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- Simple nested loop

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for each SIM card S:  
    for each Aadhaar number A:  
        check if Aadhaar details of S  
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- Every SIM card needs to be linked to an Aadhaar card
- Validate the Aadhaar details for each SIM card
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- How long will this take?
 - M SIM cards, N Aadhaar cards
 - Nested loops iterate $M \cdot N$ times

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- Validate the Aadhaar details for each SIM card
- Simple nested loop
- How long will this take?
 - M SIM cards, N Aadhaar cards
 - Nested loops iterate $M \cdot N$ times
- What are M and N
 - Almost everyone in India has an Aadhaar card: $N > 10^9$
 - Number of SIM cards registered is similar: $M > 10^9$

for each SIM card S:

for each Aadhaar number A:

check if Aadhaar details of S
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- Assume $M = N = 10^9$

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 - $10^{11}/60 \approx 1.67 \times 10^9$ minutes

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 - $(1.17 \times 10^6)/365 \approx 3200$ years!

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 - $(1.17 \times 10^6)/365 \approx 3200$ years!
- How can we fix this?

```
for each SIM card S:  
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```

Guess my birthday

- You propose a date

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- I answer, *Yes*, *Earlier*, *Later*

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- Suppose my birthday is 12 April

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 - September 12? *Earlier*

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 - February 23? *Later*

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- Suppose my birthday is 12 April
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 - September 12? *Earlier*
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 - July 2? *Earlier*

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- I answer, *Yes*, *Earlier*, *Later*
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 - September 12? *Earlier*
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- What is the best strategy?

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- Suppose my birthday is 12 April
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 - ...
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- Interval of possibilities

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- I answer, *Yes*, *Earlier*, *Later*
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 - September 12? *Earlier*
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 - ...
- What is the best strategy?
- Interval of possibilities
- Query midpoint — halves the interval

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 - September 12? *Earlier*
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 - June 30? *Earlier*

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 - September 12? *Earlier*
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- Interval shrinks from $365 \rightarrow 182 \rightarrow 91 \rightarrow 45 \rightarrow 22 \rightarrow 11 \rightarrow 5 \rightarrow 2 \rightarrow 1$

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- Under 10 questions

A real world problem

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for each SIM card S:  
    probe sorted Aadhaar list to  
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- Assume Aadhaar details are sorted by Aadhaar number
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- Halving 10 times reduces the interval by a factor of 1000, because $2^{10} = 1024$

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- Total time $\approx 10^9 \times 30$

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- 3000 seconds, or 50 minutes

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- From 3200 years to 50 minutes!

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- Arranging the data results in a much more efficient solution

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- Arranging the data results in a much more efficient solution
- Both algorithms and data structures matter