In this assignment, you solve problems for Markov Decision Processes (MDP) - Discounted. You need to formulate the mathematical model and solve it by hand and with Python and Gurobi.

## 1. **Submission Instructions**

Submit a PDF file describing a Markov Decision Processes (MDP) model and reporting the solution to the problem instance. Also, submit a program (a Python script or a Jupyter notebook) using Gurobi to solve the problem instance. In the MDP formulation, clearly define the stages, states, and actions and formulate the Bellman equations. In the Linear Programming (LP) formulation, clearly define decision variables and state the objective function and constraints. For the problem instance, report the values of the objective and soluions.

## 2. **Problem**

**Question 1**: At the beginning of each year, an aircraft engine is in good, fair, or poor condition. It costs \$500,000 to run a good engine for a year, \$1 million to run a fair engine for a year, and \$2 million to run a poor engine for a year. A fair engine can be overhauled for \$2 million, and it immediately becomes a good engine. A poor engine can be replaced for \$3 million, and it immediately becomes a good engine. The transition probability matrix for an engine is as follows:

| | **Good** | **Fair** | **Poor** |
|---|---|---|---|
| **Good** | 0.7 | 0.2 | 0.1 |
| **Fair** | 0.0 | 0.6 | 0.4 |
| **Poor** | 0.0 | 0.0 | 1.0 |

Suppose the discount rate $\beta = 0.9$. and our goal is to minimize expected discounted cost.

a) Use the policy iteration method to determine an optimal policy.

b) Use linear programming to determine an optimal policy.

c) Perform two iterations of value iteration.

d) Find a policy that minimizes average cost.

**Question 2**: At any time, the size of tree is 0, 1, 2, or 3. We must decide when to harvest the tree. Each year, it costs \$1 to maintain the tree. It costs \$5 to harvest a tree. The sales price for a tree of each size is as follows:

| **Tree Size** | **Sale Price** |
|---|---|
| 0 | \$20 |
| 1 | \$30 |
| 2 | \$45 |
| 3 | \$49 |

The transition probability matrix for the size of the tree is as follows:

| | **0** | **1** | **2** | **3** |
|---|---|---|---|---|
| **0** | 0.8 | 0.2 | 0.0 | 0.0 |
| **1** | 0.0 | 0.0 | 0.9 | 0.1 |
| **2** | 0.0 | 0.0 | 0.7 | 0.3 |
| **3** | 0.0 | 0.0 | 0.0 | 1.0 |

For example, 80% of all size 0 trees begin the next year as size 0 trees, and 20% of all size 0 trees begin the next year as size 1 trees. Assuming the discount factor for cash flows is 0.9 per year, determine an optimal harvesting strategy.

a) Use the policy iteration method to determine an optimal policy.

b) Use linear programming to determine an optimal policy.

c) Perform two iterations of value iteration.

d) Find a policy that minimizes average cost.