MIE376: LP Formulation Problems

1. A drill and a lathe are used exclusively in the manufacture of two products P1 and P2. The time units necessary for each step (drilling and turning) in the manufacture of both products are shown in the table below.

	P1	P2
Lathe	1 hr	3 hr
Drill	1 hr	1 hr
Sales profit per unit	$1	$2

If the lathe and drill are available for a maximum of 16 hrs and 7 hrs respectively, find the maximum weekly profit possible in the manufacturing of these items.

2. U.S. labs manufactures mechanical heart valves from the heart valves of pigs. Different heart operations require valves of different sizes. U.S. labs purchases pig valves from three different suppliers. The cost and size mix of the valves purchased from each supplier are given as follows:

Supplier	Cost/Valve	% large	% med.	% small
1	512	42	45	13
2	451	32	37	31
3	373	24	22	54

Each month U.S. labs places one order with each supplier. At least 500 large, 300 medium and 300 small valves must be purchased each month. Because of limited availability of pig valves, at most 500 valves per month can be purchased from each supplier. How would you go about finding the best combination?

3. Two investments with varying cash flows are available as follows:

Investment	Oct. 1, 2012	Oct. 1, 2013	Oct. 1, 2014	Oct. 1, 2015
A	($6,000)	($5,000)	$7,000	$9,000
B	($8,000)	($3,000)	$9,000	$7,000

Any fraction of an investment can be purchased. By Oct. 1, 2012 you will have $10,000 available for investment, and a year later you will have $7,000 available. Assuming that the prevailing interest rate is 4% how should the money be used to maximize its Net Present Value.

4. A post office has the following workload:

Day of the week	S	M	T	W	T	F	S
Full time equivalent (FTE) required	11	17	13	15	19	14	16

An FTE represents 8 hrs/day. The post office may meet its daily labour requirements with a combination of full time and part time employees. Both work 5 consecutive days per week. Full-time employees work 8 hrs/day at a rate of $15/hr, and part time employees work 4 hrs/day at $10/hr. Agreements with the union limit the composition of part-time labour to 25% of the labour requirements.

5. A company faces the following varying demand and unit production cost for its product over the next three months:

Month	Oct.	Nov.	Dec.
Demand	20	10	15
Unit Production cost	13	14	15

Because the product is sold throughout each month, assume that only half the goods produced during a month can be sold within that month, the other half is used to meet the demand for the following month. For goods stored in inventory a holding cost of $2 per unit is assessed against each months ending inventory. At the end of September the company has 5 units on hand. What production schedule meets the demand at minimum overall cost?

6. Priceler manufactures sedans and wagons. The number of vehicles that can be sold each of the next 3 months are shown below. Each sedan sells for $8000 and each wagon for $9000. It costs $6000 to produce a sedan and $7500 to produce a wagon. Every vehicle in inventory at the end of the month incurs an inventory charge, $150 per sedan and $200 per wagon. During each month at most 1500 vehicles can be produced. Production line restrictions dictate that during month 1 at least two-thirds of all cars produced must be sedans. At the beginning of the first month 1200 sedans and 100 wagons are available.

Month	Sedans	Wagons
1	1100	600
2	1500	700
3	1200	50

7. An airline flies planes on the following route: Vancouver – Calgary – Toronto – New York – Vancouver. The distances are as follows: Vancouver to Calgary, 687 km, Calgary to Toronto, 2690 km, Toronto to New York 571 km, and New York to Vancouver 3910 km. At each stop the plane may purchase up to 10,000 litres (l) of fuel. The price of fuel varies widely from city to city. In Vancouver it is 0.88 $/l, in Calgary 0.15 $/l, in Toronto 0.95 $/l, and in New York 1.05 $/l. The plane's fuel tank holds 12000 l, and for safety, the plane must carry at least 600 l. upon landing. The amount of fuel used per km on each leg of the flight is [1 + (average amount of fuel in tank on leg of flight/2000)]. Assume that this average amount of fuel is [(1/2) (amount of fuel in tank at start of leg + amount of fuel in tank at end of leg)]. Formulate an LP that can be used to minimize the fuel cost incurred in completing the schedule.

8. Alexis Cornby makes her living buying and selling corn. On January 1st, she has 50 tons of corn and $1,000. On the first day of each month Alexis can buy corn at the following prices per ton: January, $300; February, $350; March, $400; April, $500. On the last day of each month, Alexis can sell corn at the following prices per ton: January, $250; February, $400; March, $350; April, $550. Alexis stores her corn in a warehouse that can hold at most 100 tons of corn. She must be able to pay cash for all corn at the time of purchase. Formulate the linear program which will determine how Alexis can maximize her cash on hand at the end of April.

9. An electric company operates 3 generating stations - Al, Bruce and Curt - which together produce sufficient power to meet the demand from 2 towns: Daleview and Elsville. Each station produces a minimum output of 50 MW and a maximum capacity of 250 MW. The demand for power is 375 MW from Daleview and 335 MW from Elsville. The production cost in $/MW generally increases as the station increases its output, and can be approximated as follows:

MW Output	Cost in $/MW		
	Al	Bruce	Curt
50 to 100	1.5	0.5	1.4
100+ to 200	3.1	1.0	3.5
200+ to 250	5.7	1.5	6.2

Suppose that the power generated at each station reaches the customers in Daleview and Elseville through an extensive transmission grid. Based on the various resistances of the lines, the power is distributed from the stations to the towns according to the following percentages:

	To Daleview	To Elsville
From Al	30%	70%
From Bruce	80%	20%
From Curt	40%	60%

Thus for example, 30% of the power generated at Al flows to Daleview, and 70% flows to Elsville.

10. CBR is informed that one of its 1000 units of blood is contaminated with Factor X. To test if a unit is contaminated they take a small sample from the unit, add to the sample a certain amount of enzyme E12, and place the sample in a centrifuge. If the sample turns green within a certain waiting time period, then the unit is contaminated and all testing can stop, otherwise the unit is clear and more sample tests are required. The details of the test vary by blood type; type i requires a_i grams of enzyme E12 to be added, and t_i hours of required waiting time, and p_i is the % of samples of blood type i. Unfortunately there is only one centrifuge, and not enough time nor enzyme to isolate the contaminated unit for sure. Formulate a linear programming model to help CBR
    a. maximize the number of tested units within the available time and enzyme E12,
    b. test a certain number of units, with a given amount of enzyme in the fastest possible time.
    c. test a certain number of units, within a given time, with the least amount of enzyme.

11. A particular project consists of nine tasks. The minimum task times, normal times, costs to reduce normal task times, and precedence relationships are given below.

Task	Min Task Time (days)	Normal Task Time (days)	Cost incurred for each one-day reduction in normal task time	Immediate Predecessors
A	1	3	4	none
B	2	4	1	none
C	0.5	2	1	A
D	2	5	1	A
E	1	6	3	B, C
F	1	2	7	D, E
G	3	4	9	D, E
H	2	3	5	F
I	4	5	8	G

The project manager needs to complete the project in 15 days, and quickly realizes that it is impossible to do so using the normal task times. He will need to carefully select which normal task times should be reduced and by how much, in order to minimize the overall cost of meeting the deadline. Specify the appropriate LP model to accomplish this task.

12. As manager of a western wheat cooperative you have one week to arrange delivery of an order for 6000 tons of wheat, stored among 5 silo locations, to arrive at the port of Rotterdam no later than 3 weeks hence. You will be ready to transport the wheat from the silos starting the morning of day 7, first by rail to one of 4 ports on the Great Lakes, and from there by ship. There is only one ship sailing for Rotterdam within the next three weeks at each port. The amount of wheat at each silo is provided, as well as the rail cost/ton and transport time from each silo to each port. Also each ship departure date, expected arrival date in Rotterdam, ship capacity and shipping cost per ton to Rotterdam are all known. To simplify describing the mathematical model, you obtained the following specific travel times.

	Port A	Port B	Port C	Port D
Days From Silo 1 to	5	2	1	4
Days From Silo 2 to	2	1	6	2
Days From Silo 3 to	1	4	2	3
Days From Silo 4 to	6	3	5	1
Days From Silo 5 to	4	1	3	4
Ship Departure Day	10	8	11	9
Ship Arrival Day	19	17	21	17

The trains deliver the goods at each port early in the morning and the ships depart late in the day, providing plenty of time to load the ships. There is an incentive for early delivery in $/ton/day, and an overnight storage costs at each port in $/ton/day. Provide an LP model deriving the optimal shipping plan.

13. An oil company must decide upon the blends to be used for this week's gasoline production. Two types of gasoline must be blended. Their characteristics are:

GASOLINE	VAPOUR PRESSURE	OCTANE NUMBER	SELLING PRICE (in $/barrel)
Lo-lead	≤ 7	≥ 80	$19.80
Premium	≤ 6	≥ 100	$22.00

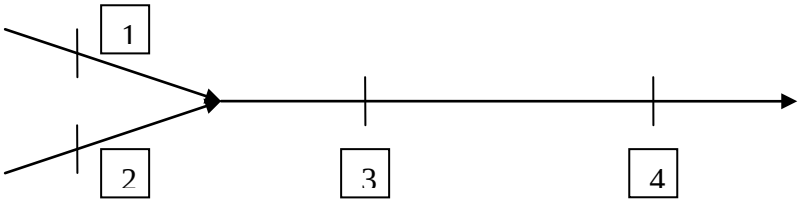
The characteristics of the components from which the gasoline can be blended are shown.

COMPONENT	VAPOUR PRESSURE	OCTANE NUMBER	AVAILABLE. THIS WEEK (in barrels)
Cat-Cracked Gas	8	83	2700
Isopentane	20	109	1350
Straight Gas	4	74	4100

The vapour pressure of a blend is simply the weighted average of the vapour pressure of its components. Similarly the octane number of a blend is simply the weighted average of the octane number of its components. Components not used can be sold to "independents" for $19 per barrel. Formulate an appropriate LP model.

14. The ABC electric company owns 3 hydro-electric power generation stations: A, B and C. The stations are located at reservoirs with dams across the Pristine River. Station A is located 10 Km upstream from station B, and station B is located 20 Km upstream from station C. Water is measured in units of ML and electricity in units of MW. The only entry of water into the Pristine River is at station A, 100 ML in each hour. Water travels down the river at an average speed of 10 Km/hr. For each hour the station manager has to decide how much of the water arriving at the station is: a) "used" to generate electricity and then allowed to proceed downstream, b) "spilled" to proceed downstream without producing electricity, and c) "stored" in the reservoir for later use or spill. Each plant has a different generating efficiency determined by the drop in elevation at each plant; the larger the drop, the larger the efficiency. The generating efficiencies at A, B, and C, are respectively 1.5, 4.2 and 8.5 MW/ML. Each plant also has a different maximum capacity determined by the size of the generating units at each plant. The maximum capacities at A, B, and C are respectively 50, 100 and 150 MW of electricity for each hour. ABC's hourly revenue is calculated as the product of the hourly MW production at each plant times the hourly price of electricity (λ_i). As a matter of policy ABC always returns each reservoir by the end of the planning horizon to the same volume it had at the beginning of the planning horizon. Formulate the problem of determining the generation and water release policy that maximizes the revenue over an 8 hour planning horizon.

15. One measure of the quality of water in a river is its dissolved oxygen (DO) concentration. Certain minimum concentration levels of DO are necessary to permit fish and other aquatic animals to survive. A large portion of the waste released into streams is organic material, which is a source of nutrients for many organisms found in streams. In the process of utilizing the organic material, the organisms withdraw DO from the stream. The larger the amount of these wastes the larger the biochemical oxygen demand (BOD). Consider the river system depicted below consisting of two tributaries leading into the main stream. The daily flow rate of water at cities 1 to 4 is respectively known to be f1, f2, f1+f2, f1+f2. Water at 1 or 2 reaches 3 in one day, and water at 3 reaches 4 in one day. Let Di be the known DO concentration of the water just above city i (i = 1 or 2). Similarly, let Bi be the known waste concentration of the water, measured by its BOD concentration, just above city i. (i=1 or 2). Wastes are discharged from cities 1 to 3 into the stream in known amounts w1, w2 and w3 per day. (These are negligible in comparison to f1 and f2.)If the waste discharged from city i (i=1, 2 or 3) is untreated, its BOD concentration would be Ui. By using appropriate treatment processes the BOD concentration can be lowered to any level between Li and Ui. The cost of reducing the BOD concentration from Ui is ci per unit reduction. However, some treatment is necessary at some or all of the cities to at least achieve a minimum standard S for the DO concentration at cities 3 and 4. The problem is to choose the BOD concentration of wastes discharged from cities 1 to 3 that minimizes the cost of meeting this standard .The following biochemical model has been developed to help solve some problems of this type. Suppose the river water has a daily flow rate of f and a BOD concentration of b, and then has waste discharged at a rate of w per day with a BOD concentration of x. The effect immediately downstream is to raise the BOD concentration of the water to



$$v_0 = \frac{bf + xw}{f + w} \approx b + \left(\frac{w}{f}\right)x$$

There is no immediate effect on the DO concentration. However, both the DO and BOD concentration would change gradually downstream. If u0 and v0 are respectively the DO and BOD concentrations of the water at a particular location on a river, and no waste is added to this water, then the respective concentrations u1, v1 of DO and BOD in this same water 1 day downstream become u1 = $\alpha+\beta u0-\gamma v0$ and v1= $\delta+\varepsilon v0$, whereα,β, γ, δ and ε are positive constants reflecting the various underlying physical and biochemical processes. Formulate the linear programming model for this problem.

16. A transport company owns a cargo plane with one front, one center and one back storage compartment, with the following weight and space capacities. Customers have offered four different cargo shipments to be loaded on an upcoming flight, and the company needs to determine the mix of cargoes that it will accept to maximize its profit:

Compartment	Weight Capacity (tons)	Space Capacity (cu. ft.)
Front	12	7,000
Center	18	9,000
Back	10	5,000

Cargo	Weight (tons)	Volume (cu. ft.)	Profit ($/ton)
A	20	500	220
B	16	700	280
C	25	600	250
D	13	400	200

Any portion of these cargoes can be accepted. In making this decision, the company must also ensure the plane's load is properly balanced, i.e. the weight of the cargo in each of the compartments must be in the same ratio as the maximum weight capacity among the compartments. Formulate this problem as a Linear Programming model.

17. A company makes two products: A and B. Below is a schedule of the demand for the next four weeks, and the resource requirements (labour hours, material and storage space) to produce each unit of A and B. The company has a labour force which can contribute 100 man-hrs per week at a regular labour rate of 12 $ per hr, plus another 40 man-hrs per week at the overtime labour rate of 18 $ per hr. The company has 800 Kg of material on hand and due to lead time constraints cannot obtain any more during the next 4 weeks. Up to 100 mt^3 can be rented in a local warehouse at a rate of 2 $ per mt^3 per week.

	Demand in number of units				Per Unit Resource Requirements		
Product	Week 1	Week 2	Week 3	Week 4	Labour (hrs)	Material (Kg)	Storage (mt3)
A	20	20	30	20	3	2	3
B	10	30	10	20	4	1	5

18. Air Cargo is an airline company that provides cargo pickup and delivery services to local customers. It owns 22 planes each of which has been assigned to a home airbase. There are 5 such airbases. The planes all have the same capacity, but different operational costs. It cost ci $/km to fly plane i when empty, and di $/km when flying the same plane loaded. Every day the dispatcher starts the day with a given list of customer requests for pickup and delivery. Each individual request can be handled by a single plane, and the dispatcher only schedules the first 22 requests. The remaining requests remain unsatisfied. A plane is allocated to each request, which then flies empty from its base to the pickup location, loads the cargo, flies to the delivery location, unloads the cargo, and flies back empty to its home airbase before nightfall. A plane meets at most one customer request per day. All distances from each airbase to each pickup location, from each pickup to each delivery location, and from each delivery location back to the home airbase are readily available. Formulate a LP model that will minimize the operational costs for one day.

Winston LP problems with a financial flavour:

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