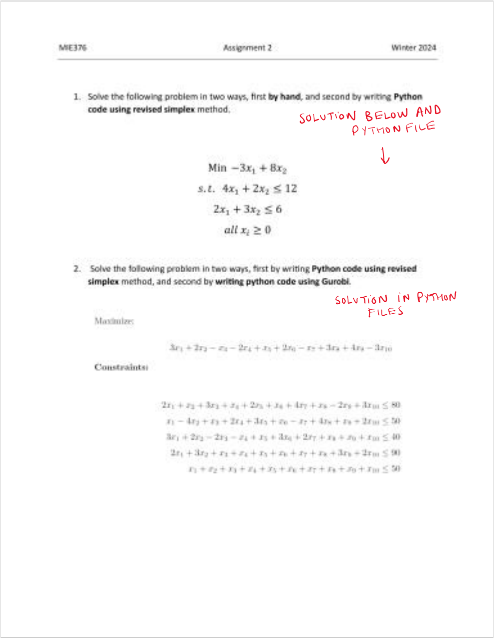
Assignment 2

January 22, 2024 1:15 PM



MIE376 Assignment 2 Winter 2024

1. Solve the following problem in two ways, first **by hand**, and second by writing **Python code using revised simplex** method.

SOLUTION BELOW AND PYTHON FILE ↓

$$\text{Min } -3x_1 + 8x_2$$
$$s.t.\ \ 4x_1 + 2x_2 \le 12$$
$$2x_1 + 3x_2 \le 6$$
$$all\ x_i \ge 0$$

2. Solve the following problem in two ways, first by writing **Python code using revised simplex** method, and second by **writing python code using Gurobi.**

SOLUTION IN PYTHON FILES

Maximize:

$$3x_1 + 2x_2 - x_3 - 2x_4 + x_5 + 2x_6 - x_7 + 3x_8 + 4x_9 - 3x_{10}$$

Constraints:

$$2x_1 + x_2 + 3x_3 + x_4 + 2x_5 + x_6 + 4x_7 + x_8 - 2x_9 + 3x_{10} \le 80$$
$$x_1 - 4x_2 + x_3 + 2x_4 + 3x_5 + x_6 - x_7 + 4x_8 + x_9 + 2x_{10} \le 50$$
$$3x_1 + 2x_2 - 2x_3 - x_4 + x_5 + 3x_6 + 2x_7 + x_8 + x_9 + x_{10} \le 40$$
$$2x_1 + 3x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + 3x_9 + 2x_{10} \le 90$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \le 50$$

Question 1

Simplex Method

$z = -3x_1 + 8x_2$] → objective function

$4x_1 + 2x_2 + x_3 = 12$
$2x_1 + 3x_2 + x_4 = 6$] → constraints x_3, x_4 = slack variables

$x_1, x_2, x_3, x_4 \geq 0$] → lower bound.

Iteration 1:

$B = \{x_3, x_4\}$
$N = \{x_1, x_2\}$] basic, non-basic variables

$z = -3x_1 + 8x_2 = 0$] non-basic variables are 0.

$4\underset{0}{\cancel{x}}_1 + 2\underset{0}{\cancel{x}}_2 + x_3 = 12 \Rightarrow x_3 = 12$
$2\underset{0}{\cancel{x}}_1 + 3\underset{0}{\cancel{x}}_2 + x_4 = 6 \Rightarrow x_4 = 6$] basic feasible solution.

→ (-3 for x_1)

To minimize : lowest z-coefficient : x_1 enters] entering variable

Now, writing basic variables in terms of non-basic variables check maximum value for entering variable (x_1) when other variables are 0.

(pick maximum)

$\overset{0}{\cancel{x}}_3 = 12 - 4x_1 - 2\overset{0}{\cancel{x}}_2 \Rightarrow 12 = 4x_1 \Rightarrow x_1 = \frac{12}{4} \Rightarrow x_1 = 3$
$\underset{0}{\cancel{x}}_4 = 6 - 2x_1 - 3\underset{0}{\cancel{x}}_2 \Rightarrow 6 = 2x_1 \Rightarrow x_1 = \frac{6}{2} \Rightarrow x_1 = 3$ } → picking x_3 as departing variable

Can also be x_4] departing variable required to maintain feasibility while minimizing the most

Iteration 2:

update basic and non-basic variables

$B = \{x_1, x_4\}$
$N = \{x_2, x_3\}$

Rewriting basic variables in terms of non-basic variables

$x_1 = 3 - \frac{1}{4}x_3 - \frac{1}{2}x_2$] algebraic manipulation of equation for x_3 from iteration 1

$x_4 = 6 - 2(3 - \frac{1}{4}x_3 - \frac{1}{2}x_2) - 3x_2$] using x_1 from above to plug into equation for x_4 from iteration 1.

$\Rightarrow 6 - 6 + \frac{1}{2}x_3 + x_2 - 3x_2 \Rightarrow -2x_2 + \frac{1}{2}x_3$

$z = -3(3 - \frac{1}{4}x_3 - \frac{1}{2}x_2) + 8x_2$] rewriting the objective function using x_1, x_4 found in iteration 2

$\Rightarrow -9 + \frac{3}{4}x_3 + \frac{3}{2}x_2 + 8x_2 \Rightarrow -9 + \frac{19}{2}x_2 + \frac{3}{4}x_3$ STOP!

] we stop because the coefficients on both the non-basic variables x_2, x_3 are positive and giving them any value ≥ 0 as set by constraints will not minimize

$\Rightarrow -9 + \frac{3}{4}x_3 + \frac{3}{2}x_2 + 8x_2 \Rightarrow -9 + \frac{19}{2}x_2 + \frac{3}{4}x_3$ STOP!

+ve coefficients.

we stop because the coefficients on both the non-basic variables x_2, x_3 are positive and giving them any value ≥ 0 as set by constraints will not minimize the Function any further than it already is.

$\therefore$ set x_2, x_3 to 0 $\Rightarrow z = -9 + \frac{19}{2}(0) + \frac{3}{4}(0) = -9$

$\therefore$ variable values to mimimize : $x_2 = 0$ (non-basic variable in 2nd iteration)

$x_1 = 3 - \frac{1}{4}x_3 - \frac{1}{2}x_2$ (From earlier in iteration 2)

since x_3, x_2 = non-basic variables $= 0$ as established

$x_1 = 3 - \frac{1}{4}(0) - \frac{1}{2}(0) = 3$

$\therefore$ -9 is the minimized the objective → minimized objective function value.

and $x_1 = 3$, $x_2 = 0$ are the variable values for optimal minimization → variable values for minimization.