Network Programming Problems

For each of the following problems draw the appropriate network model, clearly marking the upper and lower bounds and the cost per unit flow for each directed arc, and whether it is a minimization or maximization problem.

1. Price for manufactures sedans and wagons. The number of vehicles that can be sold each of the next 3 months are shown below. Each sedan sells for $8000 and each wagon for $9000. It costs $6000 to produce a sedan and $7500 to produce a wagon. Every vehicle in inventory at the end of the month incurs an inventory charge, $150 per sedan and $200 per wagon. During each month at most 1500 vehicles can be produced. At the beginning of the first month 1200 sedans and 100 wagons are available.

Month	Sedans	Wagons
1	1100	600
2	1500	700
3	1200	50

2. You are the engineer of a Power-generation company owns 3 hydro-electric power generation stations: A, B and C. The stations are located at reservoirs with dams across the Pristine River. Station A is located 10 Km upstream from station B, and station B is located 20 Km upstream from station C. Water is measured in units of ML and electricity in units of MW. The only entry of water into the Pristine River is at station A, 100 ML in each hour. Water travels down the river at an average speed of 10 Km/hr. For each hour the station manager has to decide how much of the water arriving at the station is: a) "used" to generate electricity and then allowed to proceed downstream, b) "spilled" to proceed downstream without producing electricity, and c) "stored" in the reservoir for later use or spill. Each plant has a different generating efficiency determined by the drop in elevation at each plant; the larger the drop, the larger the efficiency. The generating efficiencies at A, B, and C, are respectively 1.5, 4.2 and 8.5 MW/ML. Each plant also has a different maximum capacity determined by the size of the generating units at each plant. The maximum capacities at A, B, and C are respectively 50, 100 and 150 MW of electricity for each hour. ABC's hourly revenue is calculated as the product of the hourly MW production at each plant times the hourly price of electricity (λ_i). As a matter of policy ABC always returns each reservoir by the end of the planning horizon to the same volume it had at the beginning of the planning horizon. Formulate the problem of determining the generation and water release policy that maximizes the revenue over an 8-hour planning horizon.

3. Hospitals in developing countries are often not kept in sanitary conditions due to their limited resources [1]. A hospital in Cameroon must meet its daily demand for clean bedsheet by either buying new bedsheet, using day-time laundry service which requires a full day turnaround, or overnight laundry service. Supplied with the demand for each of the next 5 days, and the costs for bed sheet, day-time laundry service, and overnight laundry service you have been asked to develop a model which will minimize the cost of procuring the bedsheets over the next 5 days. The hospital currently has no bedsheets in usable condition, clean or dirty, on the premises or at the laundry.

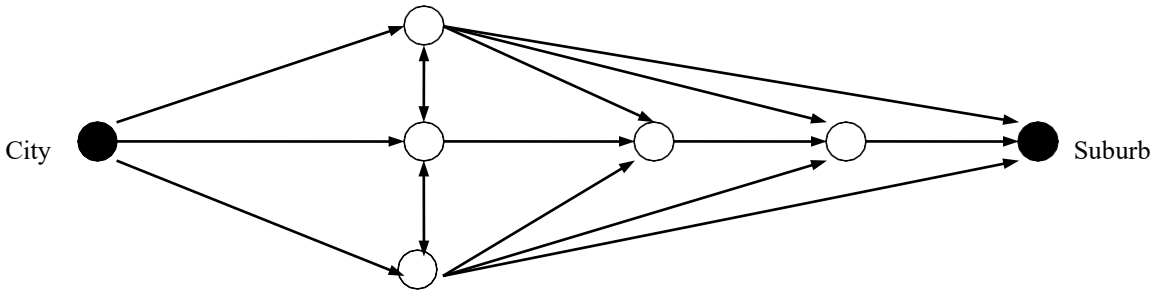
4. Micro-loans have been used to help farmers in sub-Saharan Africa to procure advanced farming equipment and improve their farming productivity [2]. You are the representatives of an NGO, responsible for distributing micro-loans. Twenty farmers are being considered for three different micro-loan packages. Each micro-loan package can only be granted to one and only one farmer. To help your assessment, each farmer has been given a rating for each micro-loan package based on their ability to improve their plantations. The micro-loan packages are to be granted based on criteria that maximize the sum of the three ratings of the placed farmers for the micro-loan package they receive.

5. You are working on the logistics for a major NGO. The NGO have leased 22 planes each of which has been assigned to a home airbase in Lusaka, the capital of Zambia. There are 5 such airbases in the surrounding regions. The planes all have the same capacity, but different operational costs. It cost c_i \$/km to fly plane i when empty, and d_i \$/km when flying the same plane loaded. Every day the dispatcher starts the day with local NGO workers' requests for pickup and delivery of medical and food supplies. Each individual request can be handled by a single plane, and the dispatcher only schedules the first 22 requests. The remaining requests remain unsatisfied. A plane is allocated to each request, which then flies empty from its base to the pickup location, loads the cargo, flies to the delivery location, unloads the cargo, and flies back empty to its home airbase before nightfall. A plane meets at most one request per day. All distances from each airbase to each pickup location, from each pickup to each delivery location, and from each delivery location back to the home airbase are readily available.

6. A distribution company produces its product at two different factories (50 units at F1 and 40 units at F2) to meet a demand for its product at its two warehouses (30 units at W1, and 60 units at W2), sometimes via a distribution center (DC). From F1 it costs 900 \$/unit to ship to W1, 400 \$/unit to ship to DC, and 200 \$/unit (up to 10 units) to ship to F2. From F2 the units can only be shipped to DC at a cost of 300 \$/unit. From DC a maximum of 80 units can be shipped to W2 at a cost of \$100/unit. In addition, units can be shipped from W1 to W2 at 300 \$/unit, and from W2 to W1 at \$200/unit. How should the goods be shipped to minimize transportation cost?

7. Internet has allowed rice farmers in China to circumvent local distribution chain and sell their produce directly to global buyers [3]. Each autumn the village co-op draws up a territorial plan for harvesting several hundred thousand hectares of rice plantation that are shipped to several market locations across the globe. Rice can be harvested from any one of 5 rice fields and shipped to any one of 3 markets in different parts of the world. The cost of rice harvesting varies from fields to fields, the revenue varies from market to market, and the transportation cost from each field to each market can be determined. This year the co-op plans to harvest a total of 50,000 hectares. Due to the limited resource in your co-op, there are upper and lower limits on the number of hectares that can be harvested from each patty, and upper and lower limits to the amount of rice to be shipped to each market. An optimal plan is required to determine how many hectares of rice to be harvested from each field for each market.

8. In 1955, Lagos is a small port city with a population less than half million peoples. Today it is one of the fastest growing cities in Africa with a population of over 14 million peoples [4]. Transportation congestion is now a major problem in Lagos. You are the city planner of Lagos and wants to plan a new suburb. The alternative routes from the city to the suburb are depicted by the following diagram, where the clear nodes are existing suburbs.



The mayor wants to determine the

a. the shortest route from the city to the new suburb, based on inter-suburb distances.
b. the fastest route from the city to the new suburb, based on distances and speed limits
c. maximum capacity, i.e. number of vehicle/hr that can travel from the city to the new suburb, based on inter-suburb capacities.

9. Most of China's electricity are generated from coal that can harm human health and the environment. Most of the electricity are used for the manufacturing sector to meet globalized demand. You are the utility planner in a major manufacturing city in China, and the city needs to meet an hourly requirement for electricity (in MWh) over each day. It can generate power from oil, coal, nuclear and hydro-electric sources. Cost per MWh for oil, coal and nuclear are respectively 80, 40 and 10. Hydro-electric energy is free but there is only sufficient daily MWh energy available to meet 30% of the daily MWh demand. Given the hourly electricity price of λ_i, what is the optimal mix of sources for meeting the daily demand?

10. Three professors must be assigned to teach two out of a total of six sections of finance. Each professor has assigned a rank (from one to ten, ten being the preferred choice) for each of the sections. The sections are to be assigned to maximize the overall sum of rankings.

11. You are the logistics person of a major NGO that seeks to distribute vaccines across different countries that is prone to infectious diseases. You have six distribution staff to assign to three different disease prone countries. you have decided that each country should be assigned one or more dedicated sales staff. Based on the reported cases of different disease per region how should the staff be assigned.

No. of distribution staff	country 1	country 2	country 3
1	35	21	28
2	48	42	41
3	60	56	53
4	69	70	65

12. Globalization has allowed goods to be transported between different countries. It has also increased the pace of urbanization for many parts of the developing world. More people are moving to the cities. Many developing countries are struggling to accommodates to the changes brought by globalization. One track of the Railroad system runs from the Rural town of Ato the major port city of B. This track is heavily used by both express passengers and freight trains. The passenger trains are carefully scheduled and have priority over the slower freight trains. The freight trains must pull over to one of S siding in order to keep the passenger trains on schedule. Each siding can hold n_i trains i=1..., S, and it takes t_i minutes for a freight train to move from siding i to siding i+1. A freight train may pass or leave siding i, at a particular time j, only if D_{ij}=1. The setting of the D_{ij} flag has been determined based on the passenger schedule, the distance between siding i and i+1, the relative speeds of the trains, and appropriate safety margins. A freight train is also required to stop at a siding if there will not be sufficient room for it at all subsequent sidings that it would reach before being overtaken by a passenger train. It has become necessary to maximize the capacity of the freight train service without interfering with the passenger service.

13. You are currently in Goma and you need to take a trip by car to Nairobi. You are studying a map to determine the shortest route to Nairobi. Depending on which route you choose, there are five other towns (A, B, C, D, E) that you might pass through on the way. The map shows a mileage chart as shown. A dash indicates that there is no direct route between the towns.

	Kms between adjacent towns					
Towns	A	B	C	D	E	Nairobi
Goma	400	600	500	-	-	-
A		100	-	700	-	-
B			200	550	400	-
C				-	500	-
D					100	600
E						800

14. Joe Cheapskate is a frequent traveler who always finds a way to take best advantage of air travel discounts. He is scheduled for 8 flights between Toronto and New York during the month shown below. As you can see, he always flies from Toronto on a weekday and returns back from New York on a weekday later in the same week.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2 Leave	3	4	5	6 Return	7
8	9	10 Leave	11	12 Retur	13	14
15	16 Leave	17	18	19	20 Return	21
22	23	24	25 Leave	26 Return	27	28
29	30	31				

Return tickets between the two cities come in a variety of discount options:

- Full fare of $500 – no restrictions
- 20% discount if the period between the ticket departure date and the ticket return date includes a weekend
- 30% discount if the period is at least 10 days
- 35% discount if the period is at least 21 days

Discounts cannot be combined, but the same prices apply if the ticket shows Toronto as the starting point, or New York as the starting point. For example, he could buy a Toronto-New York return ticket to cover his flights on the 2nd and 12th and a New York-Toronto return ticket to cover flights on the 6th and 10th.

Draw the network model that will determine the 4 tickets that cover the 8 flights at least cost.

15. An NGO in West Africa manages regional blood banks to meet the demand for platelets in each of the five provincial hospitals. It now needs to determine where these units should be collected, where they should be processed, and which blood bank should supply which hospitals, directly or trans-shipped via one or more other hospitals. To determine the optimal plan, the next day's demand for platelets by hospital is given, as well as the number of platelet units that can be collected by each regional blood bank and processed by each regional bank. In addition, both the collection cost and the processing cost at each regional blood bank are given in $/unit, as well as the transportation cost (per unit of platelet) and transportation link capacities (in terms of total number of platelet units) among all the blood banks and hospitals. Draw a network optimization model that will collect, process and transport the platelets at minimum cost.

16. Globalization has made goods available across the world. This also means that local producers can be affected by the volatility of the world markets. A co-op has packed 5five shipping containers worth of berries and ready to ship across the world. The estimated probability distribution of potential sales of the berries before spoilage differs among the three countries. Therefore, the co-op wants to know how he should allocate the five containers to the three countries to maximize expected profit. For administrative reasons, the co-op does not wish to split containers between countries. However, the co-op is willing to distribute zero crates to any of the countries. The following table gives the estimated expected profit at each store when it is allocated various number of containers:

Number of containers	Country 1	Country 2	Country 3
0	0	0	0
1	5	6	4
2	9	11	9
3	14	15	13
4	17	19	18
5	21	22	20

17. As manager of a western wheat cooperative you have one week to arrange delivery of an order for 6000 tons of wheat, stored among 5 silo locations, to arrive at the port of Rotterdam no later than 3 weeks hence. You will be ready to transport the wheat from the silos starting the morning of day 7, first by rail to one of 4 ports on the Great Lakes, and from there by ship. There is only one ship sailing for Rotterdam within the next three weeks at each port. The amount of wheat at each silo is provided, as well as the rail cost/ton and transport time from each silo to each port. Also, each ship departure date, expected arrival date in Rotterdam, ship capacity and shipping cost per ton to Rotterdam are all known. To simplify describing the mathematical model, you obtained the following specific travel times:

	Port A	Port B	Port C	Port D
Days From Silo 1 to	5	2	1	4
Days From Silo 2 to	2	1	6	2
Days From Silo 3 to	1	4	2	3
Days From Silo 4 to	6	3	5	1
Days From Silo 5 to	4	1	3	4
Ship Departure Day	10	8	11	9
Ship Arrival Day	19	17	21	17

The trains deliver the goods at each port early in the morning and the ships depart late in the day, providing plenty of time to load the ships. There is an incentive for early delivery in $/ton/day, and an overnight storage costs at each port in $/ton/day. Provide an LP model deriving the optimal shipping plan.

18. When items are being donated to the developing world, there is often a mismatch between the local needs and what the donors choose to donate [6]. A benefactor has donated to your school valuable books of four different heights: 6, 8, 10 and 12 inches. You receive a quote from your school repairman for four different shelf heights, which includes both a fixed cost and a variable cost. Shelves to accommodate 6 or 8 inch books cost $22 to set-up, plus a per foot-length cost of $2.50 for the 6 inch books and $3.50 for the 8 inch books. Shelves to accommodate 10 or 12 inch books cost $25 to set-up, plus a per foot-length of $4.50 for the 10 inch books and $5.50 for the 12 inch books. You measure the total amount of shelf length you require for each book size: 12 feet for the 12 inch books, 18 feet for the 10 inch books, 9 feet for the 8 inch books and 10 feet for the 6 inch books. Given that smaller books can be stored on larger shelves, the problem is to determine what combination of shelf lengths minimizes the cost.

[1] https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3225304/
[2] http://www.shareagfoundation.org/projects/micro-enterprise.html
[3] http://www.chinadaily.com.cn/china/2017-06/29/content_29927746.htm
[4] http://worldpopulationreview.com/world-cities/lagos-population/
[5] https://journals-scholarsportal-info.myaccess.library.utoronto.ca/pdf/09213449/v129icomplete/290_tfocic.xml
[6]http://apps.who.int/iris/bitstream/handle/10665/44568/9789241501408_eng.pdf;jsessionid=6F87E1430A0C70841B1B1EEFC1EB58E0?sequence=1