Assignment 8

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In this assignment you solve problems regarding Mathematical Programming. For problems below,

1) Select a proper model to solve the problem. If a problem can be solved using two or more approaches, select an approach with the following priority, a) NP or DP, b) LP or IP. If a problem can be solved with either NP or DP, you can select either of both.
2) Solve Mathematical models.

Problem 1

An Organization Engineer is working on a firm's monthly production for the next 6 months. The firm can work each month using a normal shift or an extended shift. A normal shift costs $100,000 a month and can produce up to 5,000 units per month. An extended shift costs up to $180,000 a month and can produce up to 7,500 units per month. It is necessary to remember that the cost incurred for each shift type is fixed and is, therefore, independent of the quantity produced. If the firm decides to not produce in a given month, the incurred costs are zero.

It is estimated that changing from a normal shift in 1 month to an extended shift in the next month incurs an additional cost of $15,000. Additional costs are not incurred when changing from an extended shift in 1 month to a normal shift in the next month.

The cost of storing stock is estimated at $2 per unit and month (based on existing stock at the end of each month) and the initial stock is 3,000 units (produced from a normal shift). The quantity of stock at the end of month 6 should be at least 2,000 units.

The demand of the firm's product in all of the next 6 months is indicated in Table.

The production constraints are such that if the firm produces something in a particular month, it must produce at least 2,000 units.

The firm needs a production plan for the next 6 months to avoid stockouts.

Formulate a mathematical model that helps the Organisation Engineer to devise a production plan for the next 6 months that avoids stockouts.

→ SOLUTION BELOW

The firm's demand in the next 6 months

Month	1	2	3	4	5	6
Demand (in units)	6,000	6,500	7,500	7,000	6,000	6,000

Problem 2

A travel agency, TURALSA, organises 1-week journey around southern Egypt. The travel agency has a contract to provide groups of tourists with seven, four, seven and eight rented four-wheel-drive vehicles, respectively, for the next 4 weeks. The travel agency subcontracts an Egyptian car hire firm, HEZ EGYPT, which covers its car hire requirements. HEZ EGYPT charges a weekly hiring rate of $220 for each four-wheel-drive vehicles, plus a set rate of $500 for any weekly hire transaction. However, TURALSA can opt not to return hire vehicles at the end of the week, in which case the agency will be responsible only for the weekly hire ($220). What is the optimal way for TURALSA to manage the car hire situation in Egypt?

→ SOLUTION BELOW

Instructions

Submit a PDF file describing a Mathematical programming (MP) model and a solution to the problem instance. Also, submit a program (a Python script or a Jupyter notebook) using Gurobi to solve the problem instance (if needed). For the problem instance, report the values of the objective and decision variables.

Problem 1

Question Information

Normal - $100K/month 5K units/month

Extended - $180K/month 7.5K units/month

normal → extended $15K

$2/unit per month to store.

Need: 2K units in stock at end of month 6.

months.

Stages: $n \in \{1,2,3,4,5,6\}$

Decision variable: (X_n, d_n)

no production	normal production	extended production
$X_n = 0$	$2000 \le X_n \le 5000$	

Stages: $n \in \{1,2,3,4,5,6\}$

Decision variable: (X_n, d_n)

$X_n \rightarrow$ # of units produced in month n.

$h_n \rightarrow$ binary variable for production state : $[1,0,0], [0,1,0], [0,0,1]$

no production: $X_n = 0$; normal production: $2000 \le X_n \le 5000$; extended production: $5000 < X_n \le 7500$

$t_n \rightarrow$ # of units in stock at stage n.

$e_n \rightarrow$ to check if going from normal $\rightarrow$ extended $\rightarrow \begin{cases} 15000 & \text{if } h_{n+1} = [0,0,1] \\ 0 & \text{if } h = [1,0,0] \text{ or } [0,1,0] \end{cases}$

$D_n \rightarrow$ demand in stage n

State : $s_n = \{t_n, h_n\}$: how many units in stock and the production state for stage n.

Bellman Equation

$$F_n(t_n, h_n) = \min_{X_n, h_n} \{ [0, 100000, 180000]\, h_n^T + 2(X_n - D_n + t_n) + e_n + F_{n+1}(X_n - D_n + t_n, h_n) \}$$

transpose; cost to store stock; demand in stage n

$t_1 = 3000$ (start with 3000 stock)

$S_1 = (t_1, h_1)$

$S_7 = (t_7, 0)$

$\hookrightarrow$ 2000 in the stock at the end of month 6.

Dynamic Formulation Stage 6 (all states)

$\underline{n = 6}$:

$2000 = X_n - 6000 + t_n$

$F_6(t_6, h_6) = 2(2000) + 0 = 4000 \rightarrow$ pick min cost

$t_6 = 2000$, $h_6 = 0$

$\hookrightarrow$ none case

or $= 2(2000) + 100000$

$\hookrightarrow$ normal case

or $= 2(2000) + 180000 = 184000$

$\hookrightarrow$ extended case

Problem Instance states at stage 6 as said in tutorial.

$\underline{n = 5}$: possible states : $S_6 = \{8000, 7500, 7000 \ldots\ldots, 500\}$

$\hookrightarrow$ increments of 500

max possible stock + production at stage 5

$\vdots$ solve all states. (too many states to do by hand)

Final solution !!

Problem 2

Question information

\$220 per car / week

\$500 per week if new car hired.

Assumption – No need to pay \$500 if cars retained from stage n-1 are enough to meet requirement in stage n.

Stages: $n = \{1, 2, 3, 4\}$ weeks.

decision: If cars need to be returned or hired again for stage n+1.

states: S_n = how many cars carried forward from stage n-1.

Bellman Equation

$$F_n(S_n) = \min_{X_n} \{ F_{n+1}(S_{n+1}) + 220X_n + 500V_n \}$$

$D_n \le X_n$ demand for week n

X_n = # of cars hired for week n

$S_n = X_{n-1}$

$V_n = \begin{cases} 0, & \text{if } X_n < S_n \\ 1, & \text{if } X_n > S_n \end{cases}$

binary variable for if new car/s are hired in week n. 1 if yes, 0 if no.

$S_4 = \{7, 8\}$

$S_3 = \{4, 5, 6, 7, 8\}$

$S_2 = \{7, 8\}$

$S_1 = \{0\}$

states

no. of cars that can be carried over from week n-1 to week n, ensuring demand is met in week n-1.

8 is maximum needed in any week.

Dynamic Model

n = 4 (Demand = 8)

	$X_4^*(S_4)$
$F_4(8) = 220(8) = 1760$	8 ✓ pick 8
$F_4(7) = 220(7) + 220 + 500 = 2260$	7

n = 3 (Demand = 7)

	$X_3^*(S_3)$
$F_3(8) = \min_{7,8}\{220(7) + 2260,\ 220(8) + 1760\} = 3520$	8 ✓ pick 8
$F_3(7) = \min_{7,8}\{220(7) + 2260,\ 220(8) + 500 + 1760\} = 3800$	7
$F_3(6) = \min_{7,8}\{220(7) + 500 + 2260,\ 220(8) + 500 + 1760\} = 4020$	8
$F_3(5) = \min_{7,8}\{220(7) + 500 + 2260,\ 220(8) + 500 + 1760\} = 4020$	8
$F_3(4) = \min_{7,8}\{220(7) + 500 + 2260,\ 220(8) + 500 + 1760\} = 4020$	8

n = 2 (Demand = 4)

	$X_2^*(S_2)$
$F_2(8) = \min_{4,5,6,7,8}\{220(4) + 4020,\ 220(5) + 4020,\ 220(6) + 4020,\ 220(7) + 3800,\ 220(8) + 3520\} = 4900$	4
$F_2(7) = \min_{4,5,6,7,8}\{220(4) + 4020,\ 220(5) + 4020,\ 220(6) + 4020,\ 220(7) + 3800,\ 220(8) + 500 + 3520\} = 4900$	4 ✓ pick

$$F_2(7) = \min_{4,5,6,7,8} \left\{ \begin{array}{l} 220(4) + 4020,\ 220(5) + 4020,\ 220(6) + 4020, \\ 220(7) + 3800,\ 220(8) + 500 + 3520 \end{array} \right\} = 4900$$

4 ✓ pick

$n=1$ (Demand $=7$)

$X_1^*(S_1)$

$$F_1(0) = \min_{7,8} \{ 220(7) + 500 + 4900,\ 220(8) + 500 + 4900 \} = 6940$$

7 ✓ pick.

Problem Instance Solution

min cost = \$6940] minimized cost

hiring order per week ⇒

week	1 →	2 →	3 →	4
cars	7 →	4 →	8 →	8

] stagewise cars hired