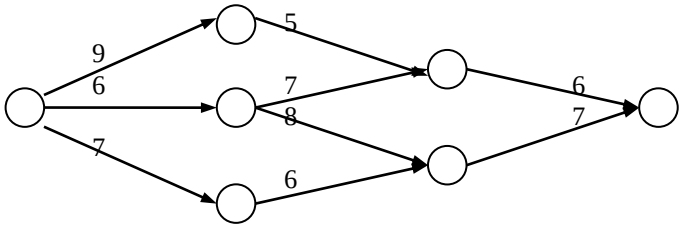
MIE376: DP Problems

1. Consider the network below, where the numbers on each arc represent the distance between the nodes. Find the shortest path in the network using dynamic programming.



2. The owner of a chain of three grocery stores has purchased five crates of fresh strawberries. The following table gives the estimated expected profit at each store when it is allocated various number of crates:

Number of crates	Store 1	Store 2	Store 3
0	0	0	0
1	5	6	4
2	9	11	9
3	14	15	13
4	17	19	18
5	21	22	20

Use dynamic programming to determine how many of the five crates should be assigned to each of the three stores to maximize the total expected profit. Note: Crates cannot be split among stores.

3. City planners are to recommend the "best" allocation of fire stations to three districts. Anywhere from zero to three stations can be located in a district. The number of stations located in a district has a bearing on the annual property damage caused by fires for that district. The table below reflects this relationship. A budget constraint restricts the total number of allocations to five stations.

Annual Property Damage (in millions of dollars)

District	Number of stations per district			
	0	1	2	3
A	2.0	0.9	0.3	0.2
B	0.5	0.3	0.2	0.1
C	1.5	1.0	0.7	0.3

a) Identify the Dynamic Programming Structure (i.e. stages, states, etc.)
b) Determine the optimal allocation by DP

4. A farmer owns K sheep. At the end of each year, a decision is made as to how many to sell or keep. The profit from selling a sheep in year n is p_n. The sheep kept in year n will double in number in year n+1. The farmer plans to sell out completely at the end of N years.

a. Formulate the dynamic programming model to maximize profits.
b. Solve the model for N=3, K=2, p_1=100, p_2=130, p_3=70.

5. The number of crimes in each of a city's three police precincts depends on the number of patrol cars assigned to each precinct (per the table below). Five patrol cars are available. The problem is to determine how many patrol cars should be assigned to each precinct to minimize the overall number of crimes. Solve the problem using dynamic programming

	No. Of Patrol Cars Assigned to Precinct					
	0	1	2	3	4	5
Precinct 1	25	17	10	4	1	0
Precinct 2	36	27	20	14	12	11
Precinct 3	30	20	12	8	6	5

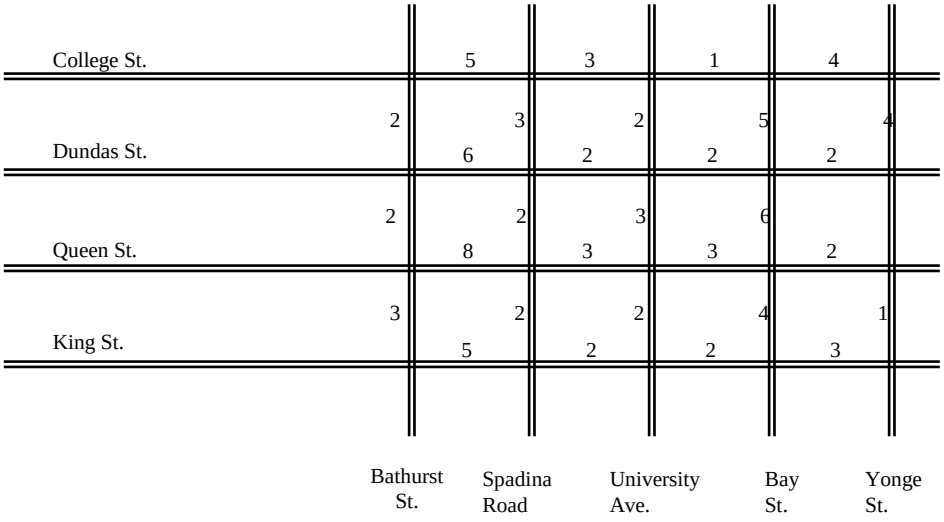
6. Howie McKim has a maximum of six hours of study time before the exam. He knows what material will be covered on the test, how well prepared he is in each area, and how much additional hour of study will help him in each area:

Hours of Study	Areas				
	LP	NP	IP	DP	
0	5	15	2	5	LP = Linear Programming
1	10	22	12	7	NP = Network Analysis
2	15	25	15	15	IP = Integer Programming
3	20	25	17	25	DP = Dynamic Programming
4	22	25	25	25	

(a) Use dynamic programming to allocate Howie's study time to maximize his point grade on the exam.
(b) How many fewer points will he get if he takes an hour off to work for the local MP candidate of his choice?

7. A job shop has 4 jobs (A, B, C & D) that must be processed on a single machine. The processing times are respectively 2, 4, 6 and 8 days. The due dates for each job are respectively 4, 14, 10 and 16 days from now. For each day a job is late the shop must compensate the customer $100. Use dynamic programming to determine how the jobs should be sequenced to minimize the total overdue penalty costs.

8. A Toronto rat crawls out of its hole at King and Bathurst Streets and decides to raid its favorite garbage can behind a store at College and Yonge Streets. Based on previous experience with cats, traffic and lighting conditions, the rat estimates travel time (in minutes) for each block as shown. Using dynamic programming find how long will the trip take by the shortest path?



9. An electronic system consists of four components: A, B, C &D. The system requires all four components to function. The system reliability can be increased by adding parallel units for one or more of the components. The component reliability and cost for different numbers of parallel components is as follows:

	Probability of functioning				Cost (in hundreds of dollars)			
# of units	A	B	C	D	A	B	C	D
1	.5	.6	.7	.5	1	2	1	2
2	.6	.7	.8	.7	2	4	3	3
3	.8	.8	.9	.9	3	5	4	4

Given a budget of $1000, use dynamic programming to determine how the money should be spent to maximize system reliability.

10. A backgammon player will be playing 3 consecutive matches. He has set aside $75 at the outset to bet any amount on each of the matches. If he wins he gains the amount bet, and if he looses he looses that amount. He assesses his odds of winning to be 50%. Use dynamic programming to determine the best betting policy for maximizing the probability that he will have exactly $100 at the end of the three matches.

11. An electric utility is considering 5 possible locations to build additional power plants over the next 20 years. The cost of building a plant at each site, the annual operating cost and the energy/year that can be provided at each site are given, as well as the total energy requirement for each year of the planning horizon. Assume that at most one plant can be placed into service in a given year, and that it can produce its full energy contribution starting the year it is placed in service. The company can currently generate 500,000 kwh/year, using its existing resources. Develop a dynamic programming model to determine the expansion plan that will minimize overall construction and operating costs over the 20 years.

12. The Mom & Pop Grocery store owners rent space in a local warehouse to which suppliers deliver their goods, and the owners arrange for a truck to pick up the goods for sale at their store. It costs them C $/day/cu.ft. for the storage space, and P$ for every truck pickup. Over the next N days they expect the flow into the warehouse on the morning of day t to be D_t cu.ft. The truck picks up the goods at the end of the day. To save in pickup costs they usually don't pick up the goods every day, but they never let the shipments remain in the warehouse more than a week. How can Mom & Pop use dynamic programming to determine an optimal schedule for picking up the goods?

13. Tanya Hyde, the local dispatcher for a moving and storage firm, must arrange to pick up x_1 different shipments of type 1 that require s_1 units of space each, and x_2 different shipments of type 2 that require s_2 units of space each. (Thus the total space requirements for the day are $s_1x_1+s_2x_2$.) She has available N different trucks. Truck i has a capacity of C_i units of space, and incurs an operating cost of E_i. (Assume that $C_1 + \ldots + C_N > s_1x_1 + s_2x_2$ and that there is a feasible solution.) Hyde wants a minimum cost selection of trucks for picking up all the loads. Formulate the decision problem as a dynamic programming problem.

14. The sales manager of a publisher of university textbooks has six traveling sales staff to assign to three different regions. She has decided that each region should be assigned one or more dedicated sales staff. Based on the estimated sales per region use dynamic programming to determine how the staff should be assigned.

No. of sales staff	Region 1	Region 2	Region 3
1	35	21	28
2	48	42	41
3	60	56	53
4	69	70	65