In this assignment, you solve problems for Bi-Level Linear Programming (BLLP). You need to formulate the mathematical model and solve it with Python and Gurobi.

1. Submission Instructions

Submit a PDF file describing a Bi-Level Linear Programming (BLLP) model and reporting the solution to the problem instance. Also, submit a program (a Python script or a Jupyter notebook) using Gurobi to solve the problem instance. In the formulation, clearly define decision variables and state the objective function and constraints. For the problem instance, report the values of the objective and decision variables.

2. Problem

An upper-level problem of a BLLP problem is given with:

$$\max \quad x + 2\,y^*$$
$$s.t. \quad 2x - 3y^* \geq -12$$
$$x + y^* \leq 14$$

Here, x is the decision variable of the upper-level problem, and y^* is derived from the optimal solution of the following lower-level problem with given x:

$$\max \quad y$$
$$s.t. \quad 3x - y \geq 3$$
$$3x + y \leq 30$$

Question 1: use the KKT condition to reformulate the problem and then solve it using Python and Gurobi.

Question 2: try to apply an iterative approach to solve the problem. Initially, solve the upper-level problem by treating y as a decision variable, without considering the lower-level problem. Then, use the obtained x value to solve the lower-level problem for y. In subsequent iterations, iteratively update x using the upper-level problem and y using the lower-level problem based on the previous iteration.

Solve them using Python and Gurobi, describe the outcome (the objective and decision variables) of each iteration and report whether it converges to the optimal solution (as Question 1) or not.