MIE377 – Financial Optimization Models

Lab 5 (05–April–2025)

CVaR Optimization

In this lab we will construct a portfolio by minimizing its Conditional Value-at-Risk (CVaR). In MVO, we use the portfolio variance as the measure of risk. This implies we are averse to both positive and negative deviations. However, as investors, we are only concerned with reducing our losses, but we welcome positive deviations. CVaR is a tail-based (or downside) risk measure, meaning we are only concerned with the loss at the tail of a Profit and Loss (PnL) distribution. Figure 1 presents a visual example of VaR and CVaR for some arbitrary set of PnL scenarios (this plot does not correspond to the dataset of this laboratory).

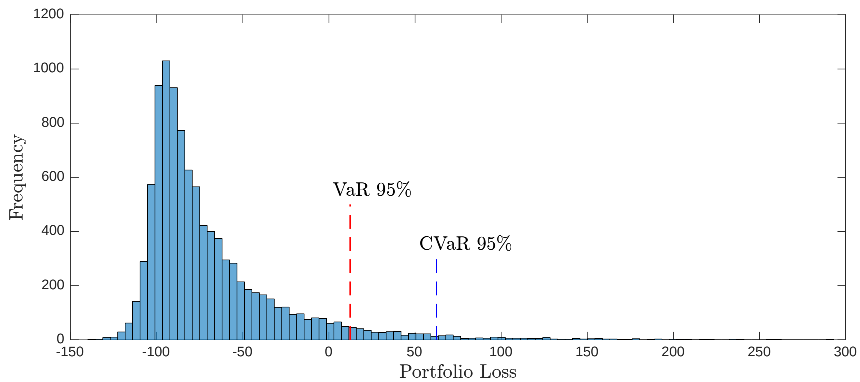

Figure 1: Example of scenario-based VaR and CVaR

We will use the same dataset as Laboratory 2, loading historical prices for 50 assets. For this laboratory, we are not required to use a factor model to estimate our asset expected returns. Instead, we can take the geometric mean of our historical returns as the asset expected returns, $\boldsymbol{\mu} \in \mathbb{R}^n$. We will set our target expected return, $R \in \mathbb{R}$, by taking the geometric mean of the market, i.e., $R = \mu_m$. We do not need to calculate the asset covariance matrix.

The objective of this laboratory is the following. We will use all the historical asset returns available ($S = 210$ observations) as our scenarios, $\hat{\boldsymbol{r}}_s \in \mathbb{R}^n$ for $s = 1, ..., S$. Since we are using daily historical returns as our scenarios, the time horizon of our CVaR is daily. We are implementing the following linear program

$$
\begin{aligned}
\min_{\boldsymbol{x}, \boldsymbol{z}, \gamma} \quad & \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s \\
\text{s.t.} \quad & z_s \geq 0, && s = 1, ..., S, \\
& z_s \geq -\hat{\boldsymbol{r}}_s^T \boldsymbol{x} - \gamma, && s = 1, ..., S, \\
& \boldsymbol{\mu}^T \boldsymbol{x} \geq R, \\
& \boldsymbol{1}^T \boldsymbol{x} = 1,
\end{aligned}
$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is our vector of asset weights, $\gamma \in \mathbb{R}$ is a variable that serves as a placeholder for the portfolio Value-at-Risk (VaR), and $0 < \alpha < 1$ is our confidence level. We wish to optimize CVaR with a 95% confidence level, i.e., $\alpha = 0.95$. The auxiliary variables $\boldsymbol{z} \in \mathbb{R}^S$ represent the losses in excess of our VaR. We have one z_s per historical scenario, where $z_s = \max(0, \ -\hat{\boldsymbol{r}}_s^T \boldsymbol{x} - \gamma)$.

Note that our loss function is $f(\boldsymbol{x}, \hat{\boldsymbol{r}}_s) = -\hat{\boldsymbol{r}}_s^T \boldsymbol{x}$, where we take the negative of the portfolio return per historical scenario $s = 1, ..., S$. Since we are trying to minimize our CVaR, we consider the losses to be a positive number, while profits are a negative number.