In this assignment, you solve problems for duality and sensitivity analysis. You need to

(1) Formulate the dual problem of an LP model and solve it with Python and Gurobi.
(2) Perform the sensitivity analysis both manually and using Gurobi.

1. Submission Instructions

Submit a PDF file describing a linear programming (LP) model and a solution to the problem instance. Also, submit a program (a Python script or a Jupyter notebook) using Gurobi to solve the problem instance. In the LP formulation, clearly define decision variables and state the objective function and constraints. For the problem instance, report the values of the objective and decision variables.

2. Problem

For the given LP problem,

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 \\ s.t. \quad & x_1 + 2x_2 \leq 6 \\ & 2x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question 1: formulate the dual problem and then solve both the primal and dual problems using Python and Gurobi.

Question 2: we can address the primal problem using the simplex method. Its standard form is as follows:

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 \\ s.t. \quad & x_1 + 2x_2 + x_3 = 6 \quad (c_1) \\ & 2x_1 + x_2 + x_4 = 8 \quad (c_2) \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Here, x_3 and x_4 serve as slack variables. At the final iteration of the simplex, it is established that x_1 and x_2 are basis, with $x_1 = \frac{10}{3} + \frac{1}{3}x_3 - \frac{2}{3}x_4$ and $x_2 = \frac{4}{3} - \frac{2}{3}x_3 + \frac{1}{3}x_4$. Utilizing this information, along with your solutions from Q1, conduct a sensitivity analysis both manually and using Gurobi and complete the following tables.

Variables:

Variable	Objective Coefficient	Optimal Value	Reduced Cost	Allowable Decrease	Allowable Increase
x_1	2				
x_2	3				

Constraints:

Constraints	RHS	Shadow Price	Allowable Decrease	Allowable Increase
c_1	6			
c_2	8			