MIE377 – Financial Optimization Models

Lab 2 (26–Jan–2024)

Cardinality-constrained MVO

We will formulate a cardinality constrained portfolio. This will require us to formulate a mixed-integer quadratic program (MIQP) by incorporating binary variables into mean–variance optimization (MVO). This, in turn, will serve to limit the number of assets our portfolio can hold. We will use a dataset consisting of historical prices for 50 assets and the market factor.

The instructions for this laboratory are the following. We wish to create a portfolio of only 10 assets (i.e., our cardinality will be $k = 10$). We will also impose buy-in thresholds. Any asset selected to be part of the portfolio must have a weight of at least 5% and at most 20%. Our target expected return is the mean of the market factor returns. We are implementing the following optimization model

$$
\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{y}} \quad & \boldsymbol{x}^T \boldsymbol{Q}\, \boldsymbol{x} \\
\text{s.t.} \quad & \boldsymbol{\mu}^T \boldsymbol{x} \geq R \\
& \mathbf{1}^T \boldsymbol{x} = 1 \\
& \mathbf{1}^T \boldsymbol{y} = k \\
& x_i \geq L_i \cdot y_i \quad i = 1,\ ...,\ n \\
& x_i \leq U_i \cdot y_i \quad i = 1,\ ...,\ n \\
& y_i = \{0, 1\}, \quad i = 1,\ ...,\ n
\end{aligned}
$$

MATLAB does not have a built-in MIQP solver. Instead, we will use another commercial optimization solver called 'Gurobi'. Gurobi is able to solve MIQPs and can be called from within MATLAB. Please refer to the MATLAB template provided to see the command structure on how to formulate an optimization model with Gurobi.

To call Gurobi from within MATLAB, you first need to setup Gurobi in your computer by following the instructions from the Gurobi website. Gurobi offers free academic licenses for students and it is easy to download and install the software. A detailed instruction guide can be found below.

- Gurobi quick start guide (MacOS):

  https://www.gurobi.com/documentation/9.0/quickstart_mac/index.html

- Gurobi quick start guide (Windows):

  https://www.gurobi.com/documentation/9.0/quickstart_windows/index.html

Once you have Gurobi installed in your computer, you can call it from MATLAB by following the set up instructions:
https://www.gurobi.com/documentation/9.0/quickstart_mac/matlab_setting_up_grb_for_.html

You can also find some optimization problem examples in the Gurobi website.
http://www.gurobi.com/documentation/7.5/examples/qp_m.html