MIE377 – Financial Optimization Models

Lab 4

Robust optimization

We will construct a robust MVO portfolio. This will require us to incorporate an ellipsoidal uncertainty set into our optimization problem. This, in turn, will serve to mitigate the effects of estimation error surrounding our asset expected returns.

We will use the same data set as Laboratory 2, loading the historical price data for 50 assets. This laboratory will focus on formulating a robust portfolio optimization model. Thus, we are not required to use factor models to estimate our asset expected returns and covariance matrix. Instead, we can find the geometric mean and the sample covariance of the raw returns.

The conditions for this laboratory are the following. We wish to incorporate an ellipsoidal uncertainty set into our optimization model. The uncertainty set is

$$\boldsymbol{\mu}^{\text{true}} \in \mathcal{U}_{\boldsymbol{\mu}} = \{\boldsymbol{\mu}^{\text{true}} \in \mathbb{R}^n : (\boldsymbol{\mu}^{\text{true}} - \boldsymbol{\mu})^T \boldsymbol{\Theta}^{-1} (\boldsymbol{\mu}^{\text{true}} - \boldsymbol{\mu}) \leq \varepsilon^2\}$$

where $\boldsymbol{\Theta} \in \mathbb{R}^{n \times n}$ is our matrix of squared standard errors arising from the estimation of the asset expected returns and $\varepsilon \in \mathbb{R}_+$ is a scaling parameter that determines the size of our uncertainty set. We can calculate $\boldsymbol{\Theta}$ and ε as follows

$$\begin{aligned} \boldsymbol{\Theta} &= \frac{1}{T}\text{diag}(\boldsymbol{Q}) \quad \rightarrow \quad (\Theta^{1/2})_{ii} = \frac{\sigma_i}{\sqrt{T}}, \quad \text{and} \quad (\Theta^{1/2})_{ij} = 0 \text{ for } i \neq j \\ \varepsilon &= \sqrt{\chi_n^2(\alpha)} \end{aligned}$$

where T is the number of observations used to estimate our parameters, $\text{diag}(\cdot)$ is an operator that creates a diagonal matrix by setting all off-diagonal elements equal to zero, $\alpha \in \mathbb{R}_+$ is our confidence level, and $\chi_n^2(\cdot)$ is the chi-square distribution with n degrees of freedom.

We are implementing the following optimization model

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \lambda \boldsymbol{x}^T \boldsymbol{Q}\, \boldsymbol{x} - (\boldsymbol{\mu}^T \boldsymbol{x} - \varepsilon \|\boldsymbol{\Theta}^{1/2} \boldsymbol{x}\|_2) \\ \text{s.t.} \quad & \boldsymbol{1}^T \boldsymbol{x} = 1 \\ & \boldsymbol{x} \geq 0 \end{aligned}$$

where $\lambda \in \mathbb{R}_+$ is a risk aversion parameter. We will use the following parameters: $\lambda = 20$ and $\alpha = 0.9$.

We will solve this problem using the MATLAB function 'fmincon', which is MATLAB's built-in general non-linear optimization solver. This function is able to handle the ℓ_2 norm term in our objective function, which we can write in MATLAB using the 'norm()' function. For reference, please visit MATLAB's website to learn more about the fmincon function.