MIE377 – Financial Optimization Models
Portfolio Resampling

We will construct a robust MVO portfolio using the Michaud resampling method. Portfolio resampling is an alternative 'robust' MVO technique developed with a practitioner's needs in mind by Michaud and Michaud (2007). The concept deviates away from traditional robust optimization techniques, but works well in practice.

We will use the same dataset as Laboratory 2, loading historical prices for 50 assets. For this laboratory, we are not required to use factor models to estimate our asset expected returns or covariance matrix. Instead, we can find the geometric mean and the sample covariance of the raw returns.

The conditions for this laboratory are the following. We will compare the efficient frontier of our nominal MVO problem against the efficient frontier of the resampled MVO problem. We are implementing the following optimization model

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \boldsymbol{x}^T \boldsymbol{Q}\ \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{\mu}^T \boldsymbol{x} = R \\ & \mathbf{1}^T \boldsymbol{x} = 1 \end{aligned}$$

The reason we use an equality constraint on our target return is to ensure we have a smooth efficient frontier. To find the nominal efficient frontier, we will solve this problem 100 times with increasing values of our target return. First, calculate the average return of the market, $\mu_m \in \mathbb{R}$. Use this average return to set our target return during optimization. Start with a target return $R = c_i \times \mu_m$. Set $c_1 = 0.05$ and linearly increase this value by 0.05 for a total of 100 steps, where $c_{100} = 5$.

To create the resampled efficient frontier, we must take the following steps:

1. Collect a sample of T observations by drawing randomly generated values from your distribution $r_t \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{Q})$ for $t = 1, ..., T$, with each $\boldsymbol{r}_t \in \mathbb{R}^n$ and with $T = 100$. Use the function 'mvnrnd()' from MATLAB to do this.

2. Use your T randomly generated observations to estimate a new vector of expected returns $\mu^l \in \mathbb{R}^n$ and covariance matrix $\boldsymbol{Q}^l \in \mathbb{R}^{n \times n}$.

3. Use $\boldsymbol{\mu}^l$ and $\boldsymbol{Q}^l$ to construct a new efficient frontier in the same fashion as before, starting with a target return $R = c_i \times \mu_m$. We have $c_1 = 0.05$ and $c_i = c_{i-1} + 0.05$ for $i = 2, ..., 300$. Note that we have increased the step-wise increase in the target from 100 steps to 300 steps. Save your portfolios $\boldsymbol{x}_i^l \in \mathbb{R}^n$ for each step within this efficient frontier.

4. Repeat steps 1 to 3 for $l = 1, ..., 30$. Thus, you will end up with 30 efficient frontiers, each with 300 different portfolios $\boldsymbol{x}_i^l$ for each step of the frontier.

5. Next, average your portfolios for each step of the 30 efficient frontiers. For example, start with the first portfolio in each frontier, $\boldsymbol{x}_1^l$ for $l = 1, ..., 30$. We should average the 30 sample weights of asset 1 to find the weight of asset 1 for our resampled portfolio. We must do this for all 50 assets. For example, $\boldsymbol{x}_1 \in \mathbb{R}^n$ is the first of the resampled portfolios, which we use to produce the first point in our 'resampled efficient frontier'.

6. Finally, compute the resampled efficient frontier by computing $\boldsymbol{\mu}^T \boldsymbol{x}_i$ and $\sqrt{\boldsymbol{x}_i^T \boldsymbol{Q} \boldsymbol{x}_i}$ for $i = 1, ..., 300$. Plot both the nominal and the resampled frontier.