Assignment 1

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13. An oil company must decide upon the blends to be used for this week's gasoline production. Two types of gasoline must be blended. Their characteristics are:

GASOLINE	VAPOUR PRESSURE	OCTANE NUMBER	SELLING PRICE (in $/barrel)
Lo-lead	≤ 7	≥ 80	$19.80
Premium	≤ 6	≥ 100	$22.00

The characteristics of the components from which the gasoline can be blended are shown.

COMPONENT	VAPOUR PRESSURE	OCTANE NUMBER	AVAILABLE THIS WEEK (in barrels)
Cat-Cracked Gas	8	83	2700
Isopentane	20	109	1350
Straight Gas	4	74	4100

The vapour pressure of a blend is simply the weighted average of the vapour pressure of its components. Similarly the octane number of a blend is simply the weighted average of the octane number of its components. Components not used can be sold to "independents" for $19 per barrel. Formulate an appropriate LP model.

Question 13

Variable Definitions

1) Let X_1 be the number of barrels of cat-cracked gas used to produce Lo-lead
2) Let X_2 be the number of barrels of isopentane used to produce Lo-lead
3) Let X_3 be the number of barrels of straight gas used to produce Lo-lead

4) Let Y_1 be the number of barrels of cat-cracked gas used to produce Premium
5) Let Y_2 be the number of barrels of isopentane used to produce premium
6) Let Y_3 be the number of barrels of straight gas used to produce premium

7) Let Z_1 be the number of barrels of cat-cracked gas not used in production
8) Let Z_2 be the number of barrels of isopentane not used in production
9) Let Z_3 be the number of barrels of straight gas not used in production

Assumptions

1) Assuming that part of a barrel can be used in production and the barrel usage does not have to be a whole number, instead belonging to $\mathbb{R}_+$.

Constraints

1) vapour pressure for lo-lead blend $\Rightarrow$ $$\frac{8X_1 + 20X_2 + 4X_3}{X_1+X_2+X_3} \leq 7$$

2) vapour pressure for premium blend $\Rightarrow$ $$\frac{8Y_1 + 20Y_2 + 4Y_3}{Y_1+Y_2+Y_3} \leq 6$$

constraints to maintain the max weighted average of vapour pressure of components under 7 and 6 for lo-lead and premium blends.

3) octane no. for lo-lead blend $\Rightarrow$ $$\frac{83X_1 + 109X_2 + 74X_3}{X_1+X_2+X_3} \geq 80$$

4) octane no. for premium blend $\Rightarrow$ $$\frac{83Y_1 + 109Y_2 + 74Y_3}{Y_1+Y_2+Y_3} \geq 100$$

constraints to maintain the max weighted average of octane number of components over 80 and 100 for lo-lead and premium blends.

5) cat-cracked gas barrel availability : $X_1 + Y_1 \leq 2700$

6) isopentane barrel availability : $X_2 + Y_2 \leq 1350$

7) straight gas barrel availability : $X_3 + Y_3 \leq 4100$

constraints to maintain the number of barrels used for each type of component to be less than the number of barrels available for each.

8) cat-cracked gas unused barrels : $Z_1 = 2700 - X_1 - Y_1$

9) isopentane unused barrels : $Z_2 = 1350 - X_2 - Y_2$

10) straight-gas unused barrels : $Z_3 = 4100 - X_3 - Y_3$

constraints to maintain the number of barrels used and unused for each type of component to be less than the number of barrels available for each.

11) Lower bound on barrel usage of all components : $X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3 \geq 0$.

not letting any component barrel usage be less than 0.

Objective Function - to maximize profit in $.

$$\text{max Profit } (\$) = 19.8(X_1+X_2+X_3) + 22(Y_1+Y_2+Y_3) + 19(Z_1+Z_2+Z_3)$$

Here 19.8 and 22 are the selling price is $/barrel for lo-lead and premium gasoline blends, whereas 19 is the selling price is $/barrel for each unused component during the week

16. A transport company owns a cargo plane with one front, one center and one back storage compartment, with the following weight and space capacities. Customers have offered four different cargo shipments to be loaded on an upcoming flight, and the company needs to determine the mix of cargoes that it will accept to maximize its profit.

Compartment	Weight Capacity (tons)	Space Capacity (cu. ft.)
Front	12	7,000
Center	18	9,000
Back	10	5,000

Cargo	Weight (tons)	Volume (cu. ft.)	Profit ($/ton)
A	20	500	220
B	16	700	280
C	25	600	250
D	13	400	200

Any portion of these cargoes can be accepted. In making this decision, the company must also ensure the plane's load is properly balanced, i.e. the weight of the cargo in each of the compartments must be in the same ratio as the maximum weight capacity among the compartments. Formulate this problem as a Linear Programming model.

Question 16

Variable Definitions

1) Let X_i be the units of cargo in front compartment
2) Let Y_i be the units of cargo in center compartment
3) Let Z_i be the units of cargo in back compartment

where i is the type of cargo and $i \in \{A, B, C, D\}$

Constraints

- weight capacity constraints
  1) Front compartment $\Rightarrow$ $20X_A + 16X_B + 25X_C + 13X_D \leq 120$
  2) Center compartment $\Rightarrow$ $20Y_A + 16Y_B + 25Y_C + 13Y_D \leq 180$
  3) Back compartment $\Rightarrow$ $20Z_A + 16Z_B + 25Z_C + 13Z_D \leq 100$

  constraint for weight capacity of each compartment of the plane for all units of cargo in it.

- space capacity constraints
  1) Front compartment $\Rightarrow$ $500X_A + 700X_B + 600X_C + 400X_D \leq 7000$
  2) Center compartment $\Rightarrow$ $500X_A + 700X_B + 600X_C + 400X_D \leq 9000$
  3) Back compartment $\Rightarrow$ $500X_A + 700X_B + 600X_C + 400X_D \leq 5000$

  constraint for space capacity of each compartment of the plane for all units of cargo in it.

- compartment weight capacity balancing constraints

1) Front to center compartment weight ratio balance $\Rightarrow$ $$\frac{20X_A + 16X_B + 25X_C + 13X_D}{120} = \frac{20Y_A + 16Y_B + 25Y_C + 13Y_D}{180}$$

constraint to ensure the weight balance b/w all compartments of the

1) Front to center compartment weight ratio balance $\Rightarrow$ $\dfrac{20X_A + 16X_B + 25X_C + 13X_D}{120} = \dfrac{20Y_A + 16Y_B + 25Y_C + 13Y_D}{180}$

2) Center to back compartment weight ratio balance $\Rightarrow$ $\dfrac{20Y_A + 16Y_B + 25Y_C + 13Y_D}{180} = \dfrac{20Z_A + 16Z_B + 25Z_C + 13Z_D}{100}$

constraint to ensure the weight balance b/w all compartments of the plane based on weight limit.

We do not need a 3rd constraint to balance the weight ratio's for the front and back compartments as the 2 constraints above already satisfy that requirement. ie. F = C , C = B , then F = B by transitivity.

- Lower bound constraint on cargo units of each type

1) $X_i, Y_i, Z_i \geq 0$, where $i \in \{A, B, C, D\}$ to keep the number of units to be 0 or greater.

Objective Function - to maximize the profit in $

Max Profit ($) $= 220 \times 20 \times (X_A + Y_A + Z_A) + 280 \times 16 \times (X_B + Y_B + Z_B) + 250 \times 25 \times (X_C + Y_C + Z_C) + 200 \times 13 \times (X_D + Y_D + Z_D)$

Here 220, 280, 250 and 200 are profit in $/ton for cargos A, B, C, D types and 20, 16, 25 and 13 are weight in tons for cargos A, B, C, D