Homework 4

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Q.1

Solution:

a. $T_{ex} = x(1 - x)cos\pi y$
   $\partial T/\partial x = (1 - 2x)cos\pi y$
   $\partial^2 T/\partial x^2 =- 2cos\pi y$
   $\partial T/\partial y =- \pi x(1 - x)sin\pi y$
   $\partial^2 T/\partial y^2 =- \pi^2 x(1 - x)cos\pi y$
   $- 2cos\pi y - \pi^2 x(1 - x)cos\pi y + \dot{q} = 0$
   $=> \dot{q} = 2cos\pi y + \pi^2 x(1 - x)cos\pi y$

b. To solve the equation numerically, the coefficients aP, aN, aS, aE, aW are calculated by discretizing the equation by using a second-order finite difference scheme and substituted in the `get_coeffs` function.

Contour plot of exact solution and numerical solution for grids of different sizes are following:

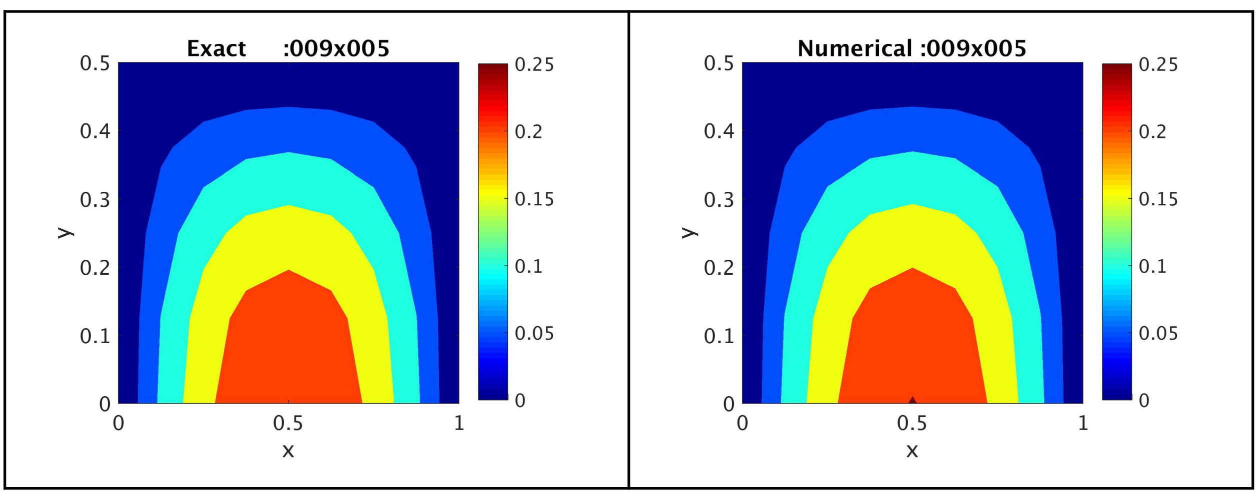

Fig.1 Exact and numerical solutions for 9x5 grid

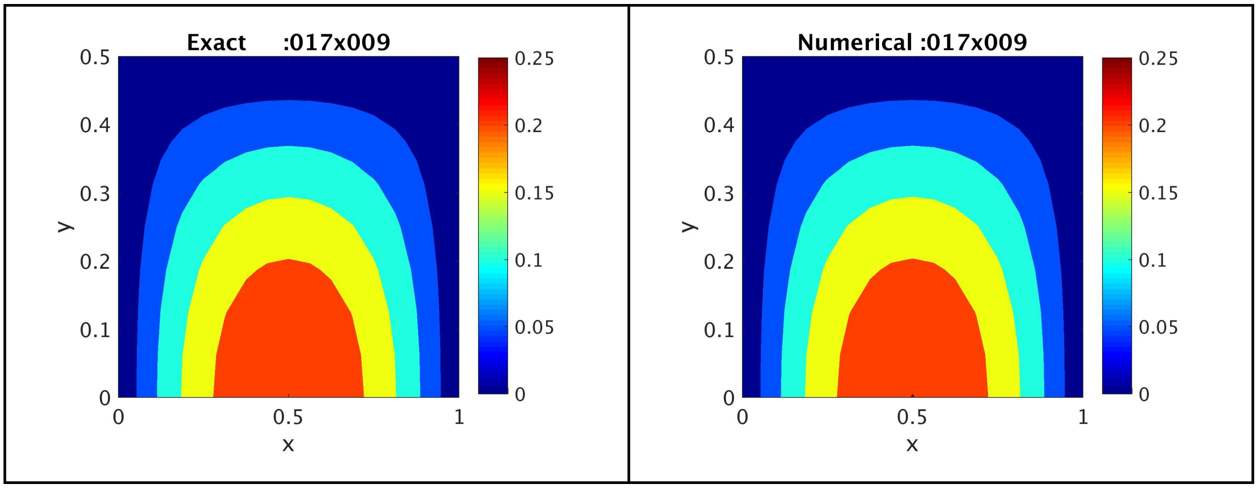

Fig.2 Exact and numerical solutions for 17x9 grid

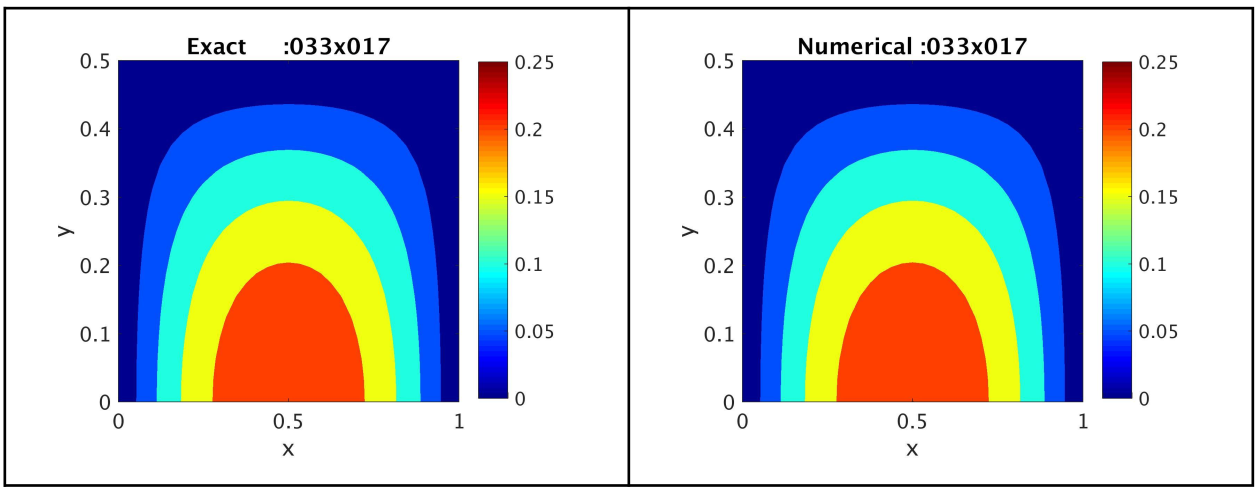

Fig.3 Exact and numerical solutions for 33x17 grid

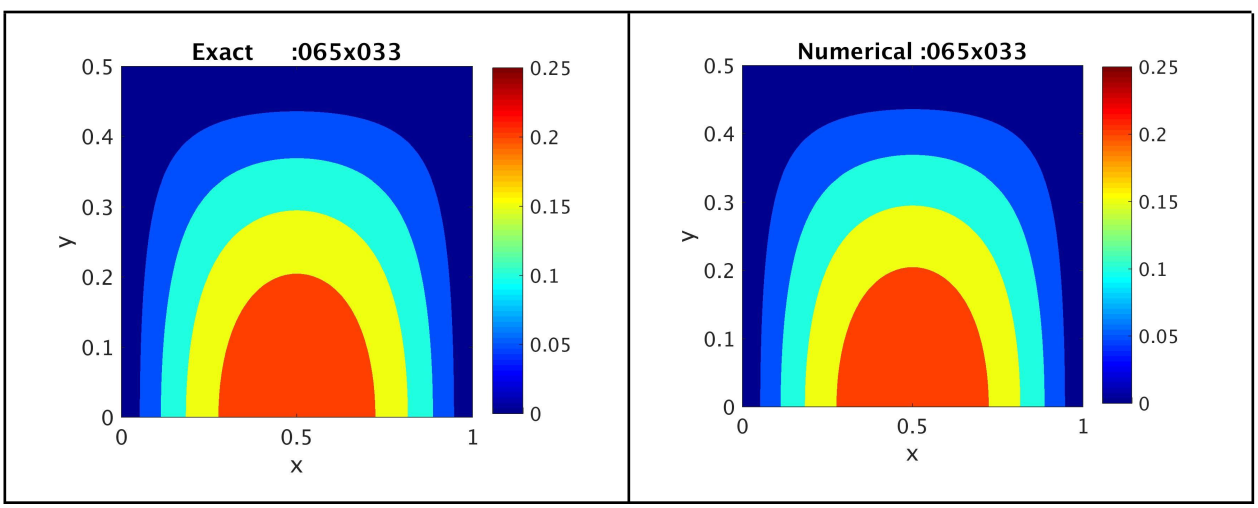

Fig.4 Exact and numerical solutions for 65x33 grid

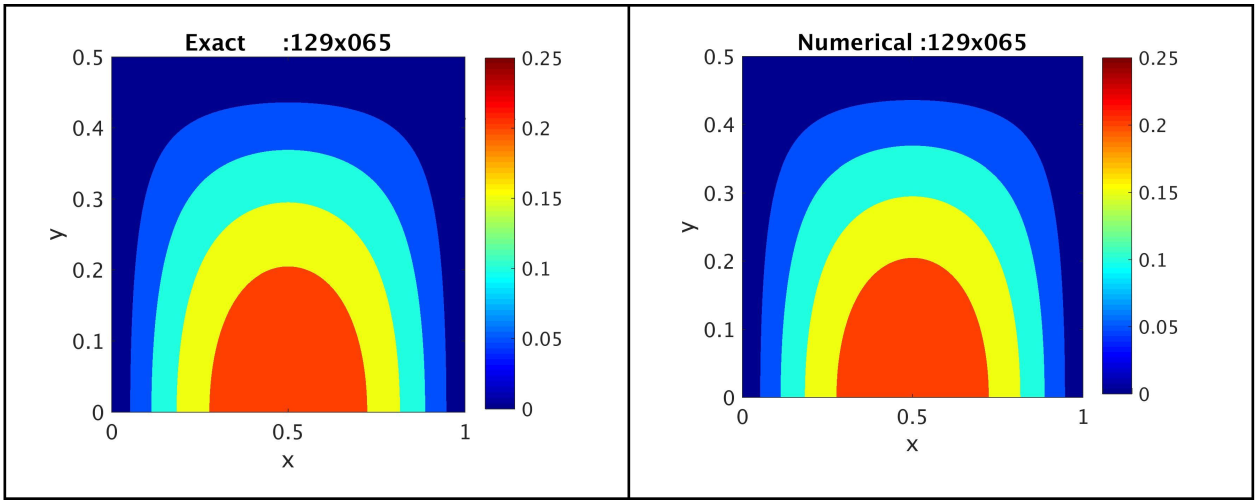

Fig.5 Exact and numerical solutions for 129x65 grid

Plot of error norm vs number of grid points in x direction is following:

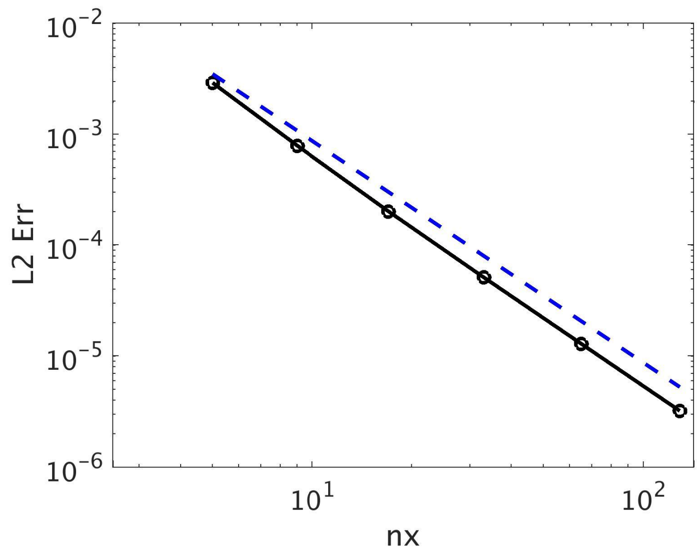

Fig.6 Plot of L2 Error vs number of grid points

This plot shows that the solution is second order accurate.

Q.2

Solution:

a. $\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \dot{q} = 0, \ k = A + BT$

$=> \ \frac{\partial k}{\partial x}\frac{\partial T}{\partial x} + k\frac{\partial^2 T}{\partial x^2} + \frac{\partial k}{\partial y}\frac{\partial T}{\partial y} + k\frac{\partial^2 T}{\partial y^2} + \dot{q} = 0$

$$=> \ B(\frac{\partial T}{\partial x})^2 + k\frac{\partial^2 T}{\partial x^2} + B(\frac{\partial T}{\partial y})^2 + k\frac{\partial^2 T}{\partial y^2} + \dot{q} = 0$$

Substituting $T = x(1 - x)cos\pi y$

We get

$$\dot{q} = 2kcos\pi y + k\pi^2 x(1 - x)cos\pi y - B((1 - 2x)cos\pi y)^2 + (\pi x(1 - x)sin\pi y)^2$$

This is a nonlinear problem, to solve this we need to discretize the first order derivative as well as second order derivative.

However since the square of first order derivative appears in the equation, we can split it in following way:

$$(\frac{\partial T}{\partial x})^2 = \frac{\partial T}{\partial x} * \frac{\partial T}{\partial x}$$

While iterating we can calculate the first order derivative using the previous guess and treat it as a constant and use that to calculate the coefficients aP, aN, aS, aE, aW.

$$(\frac{\partial T}{\partial x})^2 = (\frac{T_{i+1,j}^{new} - T_{i-1,j}^{new}}{2h_x}) * (\frac{T_{i+1,j}^{old} - T_{i-1,j}^{old}}{2h_x})$$

Similarly we can discretize the first order derivative w.r.t y.

Since the coefficients aP, aN, aS, aE, aW and matrix b are dependent on temperature, we have to update the coefficients and b matrix on every iteration in the `solve_gssor` function.

Contour plot of exact solution and numerical solution for grid of size 129x65 is following:

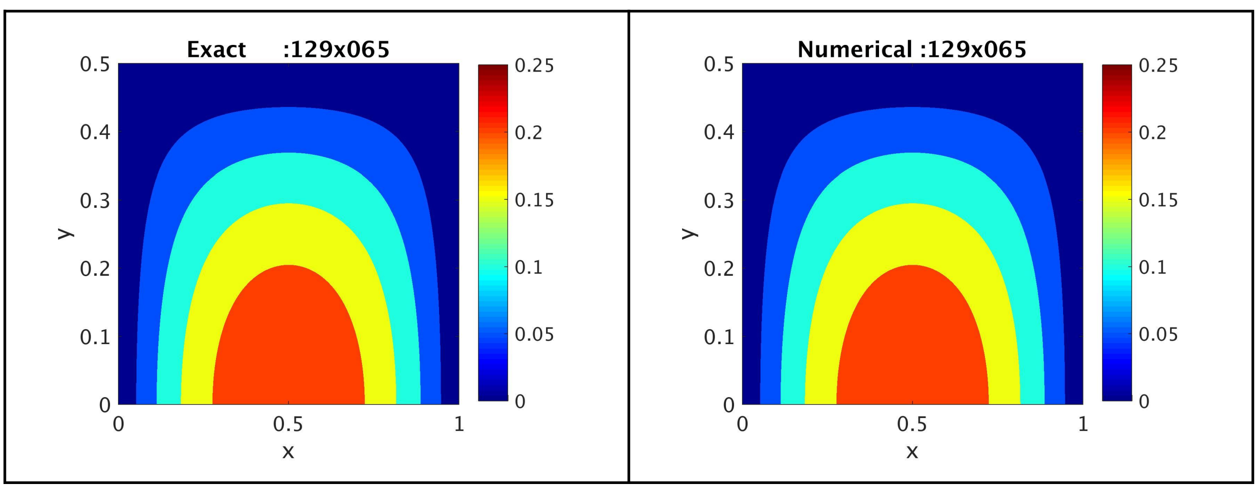

Fig.7 Exact and numerical solutions for 129x65 grid

Plot of error norm vs number of grid points in x direction is following:

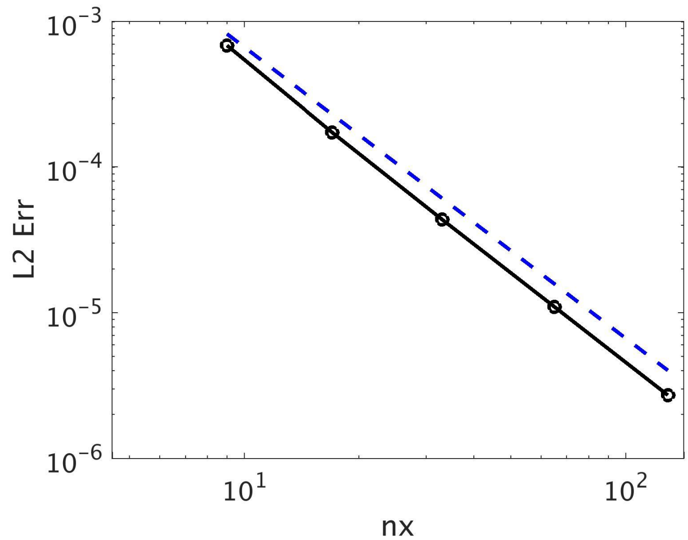

Fig.8 Plot of L2 Error vs number of grid points

This plot shows that the solution is second order accurate.

Since the second problem is a nonlinear problem, it is evident that it will converge in more iterations as compared to the first problem which was linear.
To show the same, the plot for number of iterations vs the number of grid points for both problems is following:

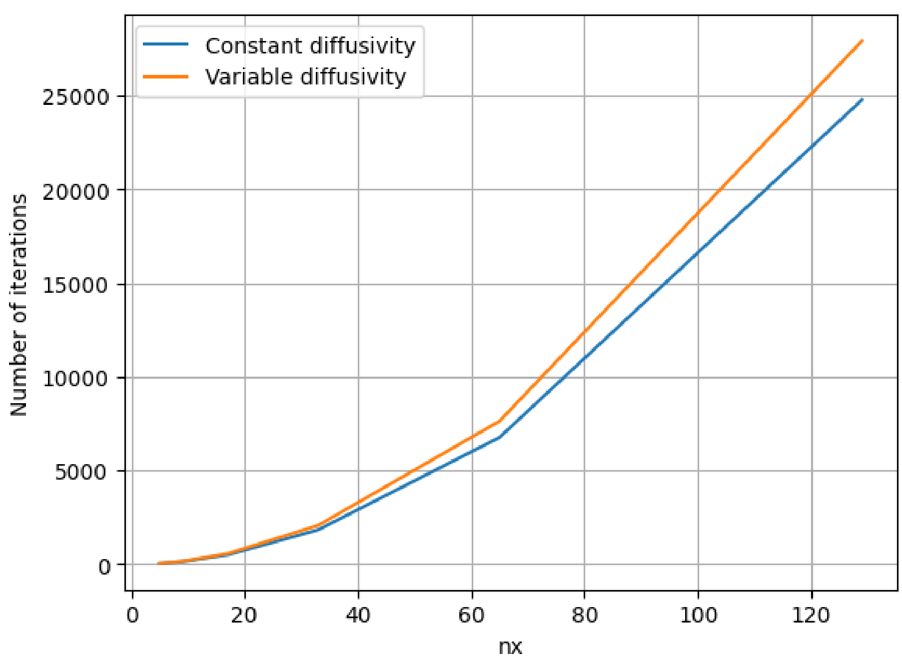

Fig.9 Plot of number of iterations vs number of grid points for both problems