CO2020 HW5

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Q.2

Solution:

a. 4th order centred-difference scheme for spatial discretization is:
For i > 1 and i < nx-2 :

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{(dx)^2}\left(\frac{4}{3}(T_{i+1} + T_{i-1}) - \frac{1}{12}(T_{i+2} + T_{i-2}) - \frac{5}{2}T_i\right)$$

For i == 1:

$$\frac{\partial^2 T}{\partial x^2} = (10T_{i-1} - 4T_{i+1} + 14T_{i+2} - 6T_{i+3} + T_{i+4} - 15T_i)/12$$

For i == nx-2:

$$\frac{\partial^2 T}{\partial x^2} = (10T_{i+1} - 4T_{i-1} + 14T_{i-2} - 6T_{i-3} + T_{i-4} - 15T_i)/12$$

b. When we perform stability analysis on 4th order scheme, we get:

$$\epsilon_j^{n+1} = \epsilon_j^{n}(1 - 5\lambda/2) + \frac{4\lambda}{3}(\epsilon_{j-1}^{(n)} + \epsilon_{j+1}^{(n)}) - \frac{\lambda}{12}(\epsilon_{j+2}^{(n)} + \epsilon_{j-2}^{(n)})$$

Therefore, we get

$$|1 - \frac{5\lambda}{2} + \frac{4\lambda}{3}(2cos(kh)) - \frac{\lambda}{12}(2cos(2kh))| < 1$$

Solving this we get

$$\lambda < \frac{3}{8}$$

To find maximum value of nx for a given time step

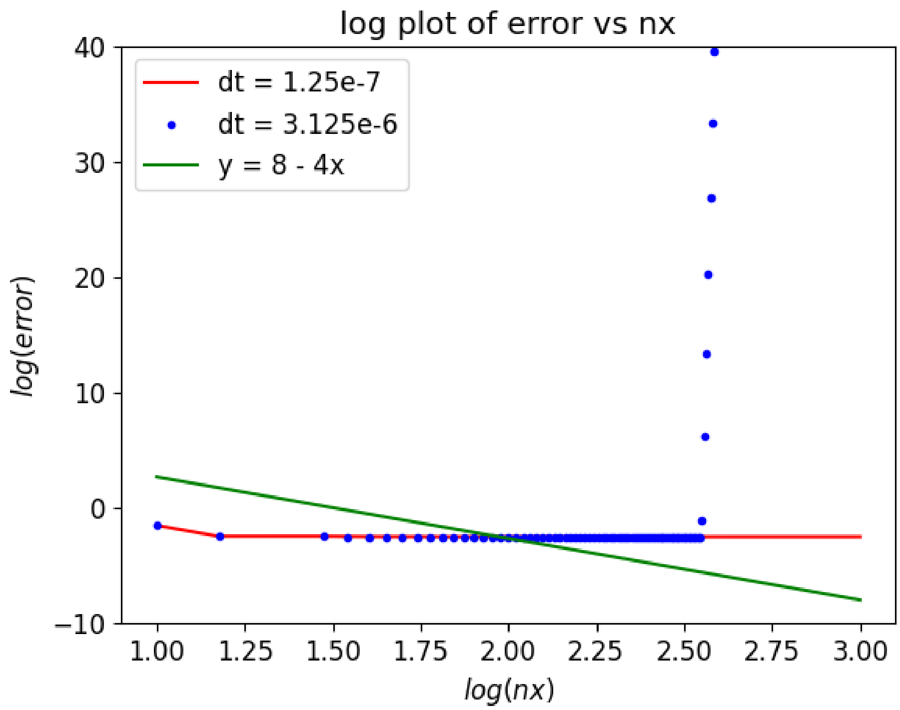
$$\frac{k\Delta t}{(\Delta x)^2} < \frac{3}{8}$$

$$\Delta x = \frac{L}{nx-1}$$

$$\frac{k\Delta t(nx-1)^2}{L^2} < \frac{3}{8}$$

$$nx < 1 + \sqrt{\frac{3L^2}{8k\Delta t}}$$

c. The log plot of error vs nx is following



We can see that error is decreasing with a slope of approximately -4, then the error becomes almost constant w.r.t. nx.

In case of $dt = 3.125e - 6$, since value of dt is more, the solution blows up after a value of $nx \cong 350$.

Whereas for a lower value of $dt = 1.25e - 7$, the solution is stable till nx = 1000.