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CURRICULAR MATERIAL
FOR
DIPLOMA IN ELEMENTARY EDUCATION (D.El.Ed) COURSE
IN DIETs OF
ARUNACHAL PRADESH

Course Code: 19

PEDAGOGY OF MATHEMATICS
AT
UPPER PRIMARY LEVEL

STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING
GOVT. OF ARUNACHAL PRADESH
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The curricular material has been developed keeping in view the learning needs of the D.El.Ed Course trainees as per the current PSTE curriculum. While developing the material authentic textual/reference materials from various sources have been referred. As far as possible the content of the materials have been presented in an objective manner. The ideas and opinions as presented in the content of the materials are entirely of the developer of the material.

FOREWORD

The Diploma in Elementary Education (D.El.Ed.) curriculum for 2 year PSTE course of the DIETs in Arunachal Pradesh was revised and updated as an exercise deemed necessary in the context of National Curriculum framework-2005 and enforcement of Right to Education (RTE) Act-2009. The curriculum was revised on the basis of recommendations of the National Council for Teacher Education, National Curriculum Framework for Teacher Education (NCFTE) and the guidelines of Bordia Committee Report entitled "Implementation of RTE, Act and Resultant Revamp of SSA" (2010). Since 2013-14 the revised D.El.Ed Curriculum is being implemented in all the eleven DIETs of the state. However, in view of change in the structure and content of the revised curriculum, there has arisen a pressing need for content specific and contextualized curricular materials which could be handy for both teacher educators and student teachers of the DIETs in the state. Further Justice Verma Commission Report on Teacher Education-2012, constituted by the Hon'ble Supreme Court of India observed," our prospective teachers are educated through substandard readymade materials available in the form of 'guides' which are conceptually confusing and regressive in perspectives". Hence, the commission strongly recommended for development of learner friendly curricular materials for different types of teacher education courses.

The D.El.Ed curricular material has been developed in workshop situation with participation of Resource Persons from Department of Education, Rajiv Gandhi University, Itanagar and faculty members of SCERT and DIETs of the state.

I am immensely grateful to the Joint Director, SCERT, Mr. Gania Leij for his guidance, Professor Jaydev Sahu, Dept. Of Education, Rajiv Gandhi University, Itanagar for his academic support, members of SCERT Academic Team, Assistant Directors, Shri G.C.Baral, Sri S.Pradhan and Sri V.R.Sharma for supervision and finalisation of curricular materials. I am specifically thankful to the author on **Pedagogy of Mathematics at Upper Primary Level Course Code-19, Sri J.P.Sharma, Assistant Director SCERT, Itanagar** for his efforts in writing the texts of the course materials as per the need of the syllabus.

Lastly, it is hoped that the curricular materials will be highly useful as reference materials for the teacher educators and student teachers of the DIETs of the Arunachal Pradesh.

Moto Nyori, Joint Director SCERT, Itanagar

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I also take this opportunity to express my gratitude to all the learned faculty of SCERT for their suggestions and moral support in the completion of this work.

I shall be failing in my duty if I do not convey my indebtedness to all the fellow participants of the workshop and the members of the review team for their valued suggestions for bringing the material to its present shape.

J.P.Sharma

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UNIT-01

TEACHING AIDS IN MATHEMATICS

"It is easier to believe what you see than what you hear, but if you both see and hear, then you can understand readily and retain more lastingly"

----- Alberecht Durer

Introduction:

The main goal of mathematics education in schools is the mathematics of the child's thinking. There are many ways of thinking and kind of thinking. An important area of concern is the teacher's own perception of what mathematics education is and what constitute the goal of mathematics education. There are many mechanising that needs to be ensured to offer better teaching support and professional development. But the essential and central requirement is that of a large treasury of teaching aids and resource materials which teachers can access freely as well as contribute to and teacher's pedagogic competence will be strengthened immensely.

The use of sensory aids in the teaching of mathematics is of recent origin. In fact, all teaching has always evolved the communication of ideas through the senses either orally through the medium of speech or visually by the use of written or printed material.

Teaching aids in mathematics are essential for effected teaching. Teaching aids make the lesion more interesting, meaningful useful. It stimulates the interest of the learner and the facilitate learning. Teaching aids are specially used for elementary classes because at this level student not have high perfection and maturity. They cannot think abstractly like us.

Piaget, a famous educationist, experimentally proved that student must be active in the class and activeness can be created with the help of tools or teaching aids. Mathematics is essentially a subject where doing is more prominent then reading that is why a certain amount of equipment is indispensably in order to make even a start in this subject. The successes of teaching methods also depend upon the use of teaching aids. The difficult principle or concept of lesion that are new can be made clear with the use of concrete objects.

1.1 Meaning of teaching Aids:-

Any device, object or machine used by a teacher to clarify or enliven a subject, to supplement classroom or to stimulate the interest of the students is known as teaching aids. Teaching aids help teachers to deliver curriculum more effectively and assist students in accessing a topic, improving a skill and lessening anxiety.

1. According to Berton - Audio- visual aids are those sensory objects or images which initiate or stimulate and reinforce learning.
2. According to Edgar Dale: - Audio – visual aids are those devise by the use of which communication of ideas between persons and groups in various teaching and learning situations is helped. These are also termed as multi sensory materials.
3. Good's Dictionary of Education: - Audio – visual aids are anything by means of which learning process may be encouraged or carried on through the sense of hearing or sense of sight.

1.2 Role and Importance of Teaching Aids:

i) *Role of teaching aids in learning* :Teaching aids are useful to

- Reduce dullness
- Clear the confusion
- Save time and money
- Give direct Experience
- Increase vocabulary
- Reinforce what you are saying
- Ensure that your point is understand
- Enable students to visualise or experience something that is important to see or do in real life.
- Engage students other senses in the learning process
- Facilitate different learning style.

ii) Important of Teaching Aids:

The process of teaching and learning largely depends on the available teaching materials. Today there are a wide range of teaching aids which can be audio, video, books, DVDs etc. ***What we hear we know but what we see we remember***.

Individuals are prone to forgetfulness and hold on more to memorizes of things they were seen than heard. When on audio visual aid is correctly used it will help in retention of the acquired knowledge.

i. Motivation

Motivation is almost everywhere in teaching and learning process , every individual whether teen or adult is more attentive to practical knowledge , the use of teaching aids can motivate and capture the student's interest in what you are teaching .

ii. Imagery

Images last longer in the mind , when a learner correctly see, hears and touches a knowledge he can himself put the pieces together and get the picture even without the teacher speaking.

iii. <u>Prompts reasoning</u>

Just like the previous teaching aids prompts the students reasoning and is a good example or conceptual thinking.

iv. <u>Interest</u>

Its spark of interest in schools where student skip classes, and then one day they see a teacher going to the class with a projector, DBD or large paper you will get a full class not because of your sternness but the aids had sparked their interest.

v. <u>EXPERIANCE</u>

It reduces the work load of explanation on the teacher and give a more permanent knowledge. Provide a more direct experience for the students. A student may have seen an object without knowing its name or use ,the teacher will teach on that same subject but they just won't get it, that's where the aid come into remind them and you will have things like ' OKEY' ! "We know it! "

***Type of teaching aids**:* Teaching aids are classified in to three-groups.

1. Visual aids: - these are aids that use the actual vision like- maps, charts, objects, picture, models, flash card, chalkboard, projectors, slides, bulletin board.
2. Audio aids: - these are aids uses only the students sense of hearing. Example is films, television, DVDs, filmstrips, radio strips, radio etc.
3. Audio- visual aids- these aids combine both vision and hearing sense and always give a more vivid picture, example; film. The use of audio – visual teaching aids is known to the more effective and comprehension in learners. The type of teaching aid used will also depend largely on the level and age of students.

Precautions in uses of teaching Aids :

Improper presentation of teaching aids can confuse as well as irritate students causing them to loss interest. As a teacher, ensure that both subject matter and teaching aids are properly coordinated and related so that all teaching objective may be achieved at the end of the class.

1.3 Mathematical kit and other useful teaching aids models, charts, geometry box, garboard and abacus etc.

i) ***Mathematical kits***: A child is motivated to learn mathematics by getting involved in handling various concrete manipulates in various activities. In addition to activities, games in mathematics above help the child's involvement in learning by strategizing and reasoning for learning mathematical concepts through the above mentioned approach , a child centred mathematical kit has been developed for the students of upper primary stage by NCERT and other agencies are available in opera market. The kit includes various kit Items along with a manual for performing activities and playing games. The kit broadly covers the activities in the areas of number system, geometry and menstruation.

ii) The kit has the following advantages:-

- Availability of necessary materials at one place
- Multipurpose use of Items
- Economy of time in doing the activities
- Portability one place to other
- Provision for teacher's innovation , and
- Low-cost, no – cost material and use of indigenous resources

iii) Special feature of the kit items:

Plastic strips with slots and markings are provided. This help creating angles, triangles and quadrilaterals. The slots facilitate the adjustment of the strips over one another so that triangles quadrilaterals of different dimensions can be made. Markings are provided on the strips which help in measuring the lengths, whenever required. The 360* protractors can be fixed on the strips which forming angles, triangles and quadrilaterals and are used for measuring angles.

Nets of different solids are provided for formation of solids by folding.

iv) **Geoboard**: It is a board of dimensions 19 cm x 19 cm x 1 cm. Holes have been drilled on the board on one side at equal distance of 1 cm each. Small pins (called Dowels) can be filled in the holes and with the help of rubber bands different geometrical shapes can be formed.

Cut-outs of plastic corrugated sheets in the form of parallelogram, triangle, trapezium and circle help in learning concepts related to areas. For an activity regarding different views of solids from various prospective, plastic cubes are provided in the kit. Each cube has a notch on one if their face which helps in fixing it to other cubes to form different shapes like cubes or cuboids. These plastic cubes are also helpful in learning regarding surface area and volume of a solid and fractions.

An abacus is also provided to inculcate understanding of place values of numbers. In the base dowels have to be fixed in them indicating different place value. Beads are provided in the kit to be put in these dowels. This abacus is useful in creating an understanding of addition and subtraction of the decimal numbers. Counters of different colours are provided in the kit for the activities related to integers. These can also be used for playing one of the games using integers. A number board marked with numbers from + 104 to – 104 is also provided for this game.

v) **Models:** Model is an imitation of the real thing. It should always be noted that model gives the real image in the mind of students. A teacher can use lever screw, coins watch, balance etc. According to the requirement, Models of solid figure are necessary as they are three dimensional and it is difficult to make students understand them with imagination only. Hence models of cube, cuboids, cone, cylinder, sphere, hemisphere are shown in the class, students can easily and readily acquire the knowledge about them.

Models of circle, angle at a point, right angle, different types of angles, sector. etc. can be made from cardboard to understand the concepts. Cardboard can also be used as a model to find the area of square, rectangle, four walls etc.

Model of fractions, different solid figures, model of Pythagoras theorem, model of different triangle etc. can also be made and they have a great importance in teaching of mathematics.

vi) **Chart:** Chart is an effective medium in teaching in mathematics'. They save a lot of time of the teacher as in place of draining figures in the class. These charts can create a healthy environment in mathematics' class room. Charts can be made on different topics of mathematics' like coin, weight and measurement school or class pens percentage, different geometrical figures and their properties etc. Teacher can use figures already made on the chart. It is not always possible that teacher may immediately draw accurate and beautiful diagrams on the blackboard. In starting if new topic, its relevant charts should be used.

Important and necessary things related to the topic should be shown in the chart. It should be colourful and attractive but not too much. Figures and writing on the chart should be clear and correct without spelling mistake. If possible it should be made by students.

vii) Geometry Box.

It is an important instrument for teaching and learning of mathematics. Geometry Box contains scale (6 inches long), compass, two set squares, protectors, pencil, sharpener, eraser etc. With the help of compass, different constructions can be done and circles of various radius and diameters can be made. The protector is used to measure the degrees of an angle. One set square has angles 45^0, 90^0 and 45^0 and another has angles 60^0, 90^0 and 30^0. It can be used for making angle 30^0, 60^0, 45^0 and 90^0 angles and to draw parallel or perpendicular lines. Geometry box also contains a divider. It has two equal arms having sharp pointed ends and are attached to the circular pulley and is used for comparing two line segments.

1.4 Preparation of lowest cost / no cost teaching aids from local resources

There is a district trend and determined effort in the country to utilize low cost, simple and indigenous materials which include handmade educational charts, maps, models, mathematical kits, educational toys numbers games and number pattern for children. Attention has been focused on systematic approach to reviewing the current efforts and products and exploring new strategies and directions in designing low cost/ no cost simple educational materials, using the environment and mobilizing unused and under-utilized local resources.

In the preparation of low cost/no cost:

- Use of local materials makes teaches and learners aware of the resources to be found in the environment and stimulates creativity to use them
- The experiments and models can be constructed in a very short time, with a few tools, with locally available materials even by unskilled persons as part of pre-service and in-service teachers training
- The self- construction develops a sense of proud ownership and promotes a more frequent use
- Repair and replacement of broken parts are possible locally without technical or administrative problems
- For the storage no special storage facility is needed. A lockable cupboard is enough. Security is no problem because of the lower no material value.
- For the implementation no special infrastructure is needed. The innovations go straight to the school, what has been learnt today in a training workshop can be applied tomorrow in the class room.

Chapter-- 02

NUMBERS SYSTEM

Counting numbers 1,2,3,4,…were all that were required in a primitive way of life .When we enter our schools we begin our number lessons. At the first stage our development; it is only the number which we require. In order to become proficient in counting we have to undergo many a monotonous repetitions of 1,2, and 3, …etc. It is the usual practice with us to represent our numbers as the points on a straight line. That makes some basic ideas and notions perhaps more clear as also facilitate the need of the extension of numbers.

Let us begin with a picture of counting numbers marked off unit by unit upon a straight line extending infinitely to our right.

|0 |1 |2 |3 |4 |5 |6 |7

We note a curious thing about this picture. While 1 marks the distance of 1 unit from the beginning of the line , 2 the distance of 2 units from the beginning, 3 ,3 units from the beginning, 4 , 4 units from the beginning and so on, there is no number among the counting numbers which can make the beginning of the line .And if we try to take away 3 units from the point marked 3 this beginning point is exactly where we obtain our answer. What is the answer to the question 'how many is3 minus 3 ? ' The answer is none at all or '0'. So, let us call '0' a number because it answers the question ***how many just as the other*** counting numbers do and so we mark the beginning of the number line with 0

|0 |1 |2 |3 |4 |5 |6 |7

Although ,'0' was not invented in this way, the invention of 'zero' was significant , for it made possible the subtraction of a number from itself . The domain of natural numbers is closed with respect to addition and multiplication.

Every natural number lies between two others—its immediate successor and its immediate predecessor and between any two numbers immediately following one another no further number can be inserted in them.

There is also one exception that is of '0' which has no predecessor and on the other hand there is no natural number (however large) which has no successor.

2.1 WHOLE NUMBER (W)

Teachers should tell students that counting numbers begin from one and goes infinitely. Natural numbers are a group of counting numbers i.e. 1, 2, 3, 4, 5, 6, 7, 8, 9…. If zero is added to this group then the group is called whole number .i.e. whole numbers is the extension of natural number.

Group of natural numbers: N = {1, 2, 3, 4, 5, 6,…}

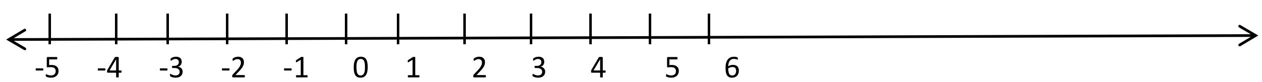
Similarly the group of whole numbers: W ={ 0,1,2,3,4,5,6,7…}.

From the above fact it is said that all natural numbers are whole numbers but convers is not true. Whole numbers are closed with respect to the operations of addition and multiplication only. The smallest whole number is ‘0’ and greatest whole number is not defined (not known). ‘0’ is the additive identity for whole numbers e.g. 0+a = a+ 0 = a for whole numbers ‘a’. ’1’ is the multiplicative identity for whole numbers as a x 1 = 1 x a = a for all whole number ‘a’.

2.2 INTEGER (I or Z)

The negative number can be thought of as form by reversing the formative rule of natural number series. This instead of successfully adding the numbers to the right we descend from numbers 3 to 2, from 2 to 1 , from 1 to 0 from 0 to -1 ,-2,-3,etc.etc.

We let this equally spaced point to the left of ‘0’ represents the number -1,-2,-3, and so on giving from right to left as shown below :



Thus the negative numbers make subtraction always possible. Discovery of zero made only the subtraction of a number from itself possible—but that did not make subtraction Omni possible.

This enlarge system {….,-3,-2,-1,0,1,2,3,…..} is known as the system of integers. It is denoted by ‘I’ or ‘Z’

Numbers 1, 2, 3, 4,… etc. which are counting numbers are also called positive numbers. Likewise -1,-2,-3,…. etc. are called negative numbers.’0’ is neither positive nor negative number.

Integers are also an ordered system as natural numbers and whole numbers. A significant difference is that the sequence of integers has neither a beginning nor an end. There is no number like ‘0’ in natural number sequence that precedes all of these. It is extended infinitely both ways – right and left.

All whole numbers are integers. Of the four fundamental operations, system of integers is closed under **addition, multiplication and subtraction but** not under division (for the quotient of two integers is not always an integer). O we must make an attempt to make this fundamental operation of ‘division ‘also omni possible.

2.3 RATIONAL NUMBER (Q)

In order to make division of integers possible, let us introduce a new number rational number. Let us introduce fraction which leads to the other group of numbers i.e. Rational Number. On the number line we have represented so far -2,-1,0,1,2,3,,4etc but we have kept silent about the points falling in between any two integers. Let us now try to divide each unit intervals into two equal parts. That introduces a new point in between any two integers. This point represents

½ between integers 0 and 1

1½ between integers 1 and 2

-½ between integers -1 and 0

-1½ between integers -2 and -1 and so on.

If we divide each interval into three parts we introduce points

$\frac{1}{3}$, $\frac{2}{3}$, $1\frac{1}{3}$, $1\frac{2}{3}$, $2\frac{1}{3}$, $2\frac{2}{3}$ etc. on the right of the '0' and - $\frac{1}{3}$,- $\frac{2}{3}$, - $\frac{4}{3}$, $-\frac{5}{3}$, -$2\frac{1}{3}$, - $2\frac{2}{3}$ etc. on the left of zero. Thus we get a point to represent every fractional number both positive and negative where we further consider each unit interval divided into one –fourth, one-fifth, one –sixth, one seventh etc. . etc.

If for a moment we now consider positive rationals in their ascending or descending order omitting those points which correspond to fraction without common factors (since they are already represented) we find that placing them into one-one correspondence with the rational numbers is impossible . Not only there is no smallest fraction but also there is no largest fraction in any of the unit interval. Between any of the two fraction $\frac{a}{b}$ and $\frac{c}{d}$, there is always a fraction which is larger than $\frac{a}{b}$ and smaller than $\frac{c}{d}$. And can be infinity of fractions between any two -- this fact is very confusing and defies actual representation on the number line. In any interval in between $\frac{a}{b}$ and $\frac{c}{d}$ there is always a fraction $\frac{a+c}{b+d}$ and this process can be repeated without end.

$$\frac{1}{3} \quad \frac{1}{2}$$

$$\frac{1}{3} \quad \frac{2}{5} \quad \frac{1}{2}$$

$$\frac{1}{3} \quad \frac{3}{8} \quad \frac{2}{5} \quad \frac{3}{7} \quad \frac{1}{2}$$

$$\frac{1}{3} \quad \frac{4}{11} \quad \frac{8}{3} \quad \frac{5}{13} \quad \frac{2}{5} \quad \frac{5}{12} \quad \frac{4}{9} \quad \frac{1}{2}$$

Between a large set of fractions, we soon see the same fact. We see between

$\frac{1}{43651}$ *and* $\frac{1}{23518}$ We have $\frac{2}{67169}$.

This third system (containing natural numbers, whole numbers and integers) is useful for measurement and known as 'Rational Number System'. This system is denoted by 'Q' .It has made division always possible except in the case of division by zero.

(i) $\frac{any\ number}{0}$= not defined

(ii) $\frac{0}{any\ number}$= 0

(iii)$\frac{0}{0}$ = not defined

It is mentioned above that between any two elements in any unit interval we can introduce many more elements corresponding to fractions lying in between the two elements. This specific characteristic is connected with the concept of denseness and we define an ordered system of elements as dense if between any two elements there always lies another elements of the system. The natural numbers and integers do not have this property.

2.4 Irrational Number ($Q^/$)

The idea of denseness creates difficulty for proper grasp of the rational numbers. Since we can insert infinity of points between any two integers we find it hard to obtain an intuitive picture of their distribution over the number line. In any interval however small (between any two fractions) we can insert infinity of other rational numbers. Any interval thus at best can be treated as an infinitely fine dust. From the structure of rational numbers we realise that between any interval there is neither any smallest number nor any greatest number. This fact can tempt many of us to believe that v rational points can fill up the number line completely and there is no 'hole' left in it or a point to which a number does not correspond. But that is a mistake because even though these rational numbers are infinitely dense; yet do not cover the entire number axis. This great discovery is due to Pythagoras.

He was the first to recognise that these are numbers completely different from rational numbers. We know that Pythagoras and his followers know the well-known property of right - angled triangle that the square of its hypotenuse is equal to the sum of the squares on the other two sides ;i.e. with sides 3,4 the hypotenuse of the right-angled triangle is √25 = 5 units.

But when we apply the same principle on unit square , we get its diagonal as $\sqrt{1^2 + 1^2} = \sqrt{2}$

Obviously $\sqrt{2}$ can not be a whole number for $1^2 = 1$, $2^2 = 4$ but a feel intuitively that the diagonal of the square must have definite length and it must have a corresponding point on that dense line of rationals. Lying the diagonal out on the number line is merely a figurative expression, for our eyes could detect the exact point on the line which marked its end and therefore would be an infinity of numbers which would appear to us to measure it exactly. In order to measure the diagonal of the unit square we have to use our minds and not merely eyes to find out a rational number which when multiplied itself gives 2 . Pythagoras gave a wonderful proof for the fact that there is no such number.

Let there be a number $\frac{a}{b}$ such that $(\frac{a}{b})^2 = 2$ (where a & b are co prime numbers i.e. no common factors)

i.e. $a^2 = 2b^2$

Here , a^2 is even because $2b^2$ is divisible by 2.

Now if a^2 is even , 'a' must also be even.

Say, a = 2k

$(2k)^2 = 2b^2$

$4k^2 = 2b^2$

$2k^2 = b^2$

b^2 is also even i.e. b is even.

So, a is even, b is also even and the rational number $\frac{a}{b}$ contrary to our assumptions that a and b has no common factors. Here a and b have the common factor 2.Since our assumption has been contradicted, therefore it follows that there is no rational number which represent $\sqrt{2}$ .The number we seek is irrational number and there is no such number found amongst the rational numbers.

Where does then $\sqrt{2}$ lies in the number axis? We have seen $\sqrt{2}$cannot berepresented as a fraction .We can say that it lies between1 and 2; we say that

1 is too small for it and 2 is too large

$\frac{4}{3}$ is again too small for it and $\frac{3}{2}$ is too large

$\frac{7}{5}$ is again too small for it and is $\frac{10}{7}$ too large

$\frac{24}{17}$ is again too small for it and $\frac{17}{12}$ is too large

Thus we see that rational points though cover the number axis as an infinitely fine dust they still do not completely fill it up. They at best leave many holes and thus we infer that many cracks and crevices are left out to be filled up by another type of numbers known as **Irrationals**. It is represented by Q'.

In this way we see that this number system also become dense and all the irrational numbers somehow or other are squeezed in between the rational numbers But since these irrational numbers can be generated in many ways , for instance not only by square roots , but also by cube roots, fourth roots fifth roots etc,etc. We may conclude that there are many points on the number axis corresponding to the irrational numbers and there different fractional multiples. All this leads us to surmise that the irrational numbers make up the principle part of the structure of the number axis instead of being exceptional points.

2.5 REAL NUMBER (R)

The system of rational and irrational numbers is called the set of real numbers. It is also defined as a number whose square is non-negative, is called a real number. It is denoted by 'R'. All the former system of numbers (N,W,Z, Q & Q') are contained in it and the discovery of possible irrational numbers made the extraction of roots of positive numbers always possible.

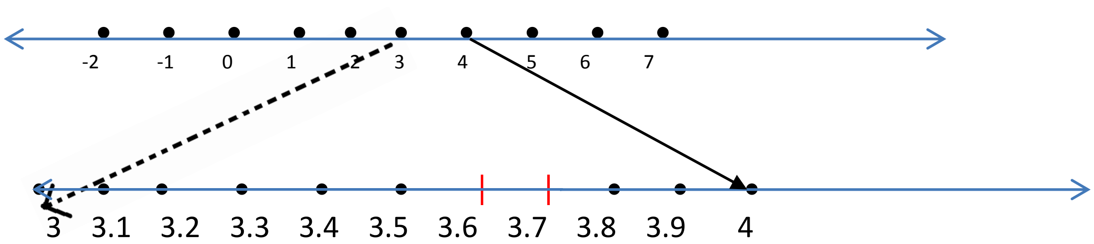
NOTE: (i) If it is integer or it has a terminating or repeating decimal representation then it is rational

(ii) If it has a non terminating and non repeating decimal representation, then it is irrational. The totality of rational and irrationals forms the collection of all real numbers. In the 1870s two German mathematicians Cantor and Dedekind showed that corresponding to every real number there is a point on the number line and corresponding to every point on the, there exists a unique real number. This is why we call the number line, the real number line.

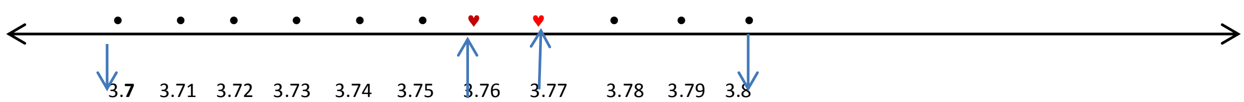
Density Property: Between any two real numbers, there exist infinitely many real numbers.

<u>Representing Real Numbers on The Number Line :</u>

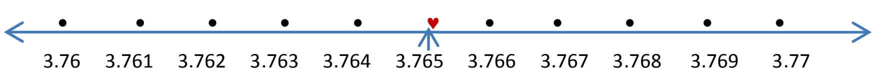
Any real number has a decimal expression. This helps us to represent it on the number line. Suppose we want to locate 3.765 on the number line. We know that this lies between 3 and 4. Let us look closely at the portion of the number line between 3 and 4 .



Suppose we divide this into 10 equal parts and each point of division as 3.1, 3.2, 3.3, ---- -, 3.9 as in above figure. Since 3.765 lies between 3.7 and 3.8 and divide again it into 10 equal parts as shown below. Mark each point of division as 3.71, 3.72, 3.73 and so on Again we observe that 3.765 lies between 3.76 and 3.77 .



Let us focus on this portion of the number line and imagine dividing it again into 10 equal parts.



The first mark represents 3.761, the next 3.762 and so on.

3.765 is the fifth mark in these subdivisions. We call this process of visualization of representation of real numbers on the t\number line as the process of successive magnification. It is possible by sufficient successive magnifications to visualize the position (or representation) of a real number with a terminating decimal expansion on the number line. In similar manner we can visualise the position (or representation) of a real number with a non-terminating recurring decimal expansion on the number line. e.g.

We can proceed endlessly in this manner successively viewing through magnifying glass and simultaneously imagining the decrease in the length of the portion on the number line in which 3.47 is located .The size of the portion of the line we specify depends on the degree of accuracy we would like for the visualization of the position of the number line.

$3.4\overline{7}$: You might have realised by now that the same procedure can be used to visualize a real number with non-terminating non-recurring decimal expansion on the number line .

Unit-3

Teaching of Algebra

3.1 What is algebra?

The word algebra is from Arabic in origin, it is part of the title of the book written by mohammeibn –musa al- Khwarizmi. The book title was 'Hisab al-jabrw' al-nuqabalah'. Al-jabra means something like 'completion' or 'restoration' and refers to the transposition of subtracted terms to the other side o an equation. Nuqabalah refers to r' reduction' or 'balancing' that is cancelling out like terms on opposite sides of the equal sign in equations .

Algebra is also a science of numerals like Arithmetic .It has originated and developed from arithmetic. The only difference is that in algebra letters are used instead of numerals or digits . symbols of these letters are called ' Algebraic terms ' and mathematics of these letters is called as " Algebra" .

Algebra is a detailed and simplified form of Arithmetic only. If gives a wider and general form to the facts of arithmetic by using terms and letters .it is also called as " generalized Arithmetic"

3.2 Aim of teaching Algebra:

With the introduction of algebra, the student learns to extend the number system of arithmetic. So that four fundamental operations can be performed in all cases. It is considered to be the most difficult and abstract of all the branches of mathematic. Solution of problems by equations, a power of generalization, use of formulae and idea of functionality. Various facts of arithmetic can be generalized and presented assign symbols and terms by various formulae of algebra for examples.

$(a + b)^2 = a^2 + 2ab + b^2$ (Square of sum of two numbers)

$(a - b)^2 = a^2 - 2ab + b^2$ (Square of difference of two numbers)

$a^2 - b^2 = (a+b)(a-b)$ (Difference of Square of two numbers)

As algebra is most important and effective branch of mathematics . it has a lot of practical applicability and is related with the daily life of students . it also help in achieving the objectives &of mathematics, for example it develops various & kills in students like problem solving , analytical thinking etc. so, it must be taught at upper primary level.

3.3 When and why we use variables

Algebra is great fun. It is just like a puzzle. What is the missing number?

$$\boxed{\quad\quad} - 2 = 4$$

OK, the answer is 6, right? Because 6 -2 = 4 easy staff.

Well, in algebra we don't use blank boxes; we use a letter (usually an x or y, but any letter is fine). So we write x – 2 = 4

It is really that simple .The letter (in this case an x) just means " we don't know this yet" and is often called unknown or the variable . And with we solve it we write.

Why use a letter:

We use a letter because it is easier to write "X" than drawing empty boxes . if there are several boxes (several variable) we can use a different letters for each one.

The main difficulty in algebra is the concept of variable. The idea that a letter is used to represent any number taken from a artier set like N. Z. Q. or R, is hard for pupils to understand. Letters in algebraic expressions are frequently viewed in a different way by pupils. They might see the letters to present an object. It in a square the measure of the length of the side is indicated with 'n" pupils see this denoting the side rather than the measure of the side. This error of labelling is extremely common. For example pupils will write " let the side be x," " let the pencil be P " " let the exercise book be m " etc. – giving labels to objects , instead of introducing a variable.

This misconception is reinforced by several teachers who say, in order to explain 2a+3a, 2 apple plus 3 apples (so called fruit salad' algebra). But letters are not shorthand for objects! 4 y does not mean that you have four y's (which is not the same as four times the numbers y). Using a letter as an object, which amounts to reducing the letter's meaning from the abstract variable to something for more concrete and 'real' allows many pupils to arrive at correct answer.

3.4 Forming and solving simple linear equations up to two variables algebraically and graphically.

Equation - an equation is a statement of equality of two algebraic expressions involving one or more variable.

Linear equation: The equation in which the variable i.e. x, y or z etc. are in first order is called linear or simple equations e.g.

$6x = 16 - 2x$

is a linear equation in one variable x because higher power of x is of order one .

$ax + by + c = 0$

is a linear equation in two variables x and y and higher power of x & y is of order one where a, b, c are real numbers .

Simultaneous Equations- two or more equation which contain two various are called simultaneous equations.

e.g. $ax+by=c$ and $dx+ ey= f$

where a , b, c, d, e, f, are real numbers .

3.41 Formation of Linear Equations in two Variables

You must have come across situations like given below; Gopa went to a fair in her locality. She wanted to enjoy rides on the Giant wheel and play Hoopa. The number of times she played Hoopa is half the number of rides she had on the Giant wheel . it each ride costs Rs. 4, How would you find out the number of rides she had and how many times she played Hoopa, provided she spent Rs. 20.

Hence first we have to form the equations and then to solve it . know the situation can be represented by the two equations .

$$Y = \tfrac{1}{2} X \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

$$3x + 2y = 20 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (ii)$$

3.42 Solutions of linear equations up to two variables:

There are several ways of finding the solution of these equations. Commonly used methods are (A) algebraic method:

i) The method of elimination by comparison

ii) The method of elimination by addition and subtraction

iii) The method of elimination by substitution

iv) The method of determinations (i.e. cross-multiplication method) and

(B) The graphical method

The graphical method can be used, of course, only with those students who have previously studied the meaning and construction of graphs of linear equations . its principal advantages lies in the fact that if is interesting and that is illustrated things in a very convincing manner.

Its main disadvantages are that it provides only approximate solutions instead of exact ones, which often introduce apparent discrepancies and gives the student a consequent feeling of dissatisfaction. Secondly, it is comparatively slow, tedious and inefficient method for solving simple linear equations. For this reason, after students have learnt more exact and efficient algebraic method, they are likely to prefer them to the graphical method.

The solving of two linear equations will involve the drawing of the straight lines representing the two separately their points intersection gives the solution of the two equations ; since lies on both the lines and satisfy both .

3.5 Nature of straight lines-parallel, intersecting and coincident lines (using graph) :

If the graphs of two lines drawn intersect each other at one point only, then they have unique solution and are called consistent. Also, if the two lines drawn coincide i.e. overlap, then they have infinite solutions and are called consistent. In this case every point (solution) of one line (equation) is also the point (solution) of the other line .

But it the two lines drawn do not meet each other i.e. they are parallel to each other, then the lines are said to have no solution such system of equations is called inconsistent.

Let us consider a general system of linear equations $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

on solving it we set.

$X = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

(i) $a_1b_2 - a_2b_1 \neq 0$

i.e, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

then we say the given system of equations has unique solution i.e lines are intersection lines

ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{C_1}{C_2}$

then system of equations has no solution i.e lines are parallel.

You may cheek the nature of pair of lines (i) the intersecting lines (ii) the coincident lines or (iii) the parallel lines of the following system of equations. Also draw their graphs.

1) $3x + 6y = 3$ 　 2) $2x + 3y = 12$ 　 3) $3x – 6y = 15$

$5x + 10y = 20$ 　 $x – y=1$ 　 $x – 2y = 5$

3.6 CONCEPT OF SQUARE, SQUAREROOTS , EXPONENTS AND RADICALS

3.61 Square: A square root is defined as a number which h when multiplied by itself gives a real non – negative number called a square . Numbers like 1, 4, 9, 16, 25, 36 … are square numbers or perfect squares .

3.62 Square Root: A square root is best defined using geometry where, considering a square (which is a four sided polygon whose sides are all equal) , a square not is defined as the length of the diagonal of this square (a diagonal is a line drawn from one vertex / corner to the opposite vertex of the square) .

A **radical** is a root of a number A square root is a radical . Root can be square not , cube root, fourth root and so on.

A square root is commonly shown as

$\sqrt{a^2}$ = a , Where, $\sqrt{\ }$ is known as the radical sign (or positive Square root) and a^2as the radicand.

A square root of a number can above represented as

$(x)^{1/2}$ in place of $\sqrt{x}$

Every square has two square root, one is positive and the other negative. This is shown as

$(x)\frac{1}{2} = +\sqrt{x}\ and\ -\sqrt{x}$

Properties of square roots and radicals

i) $\sqrt{x}$ x$\sqrt{y}$= $\sqrt{xy}$, when x & y are positive.

ii) $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$ 　 (iii) $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{x/y}$ 　 iv) a $\sqrt{x} = \sqrt{a^2\text{x}}$

v) A perfect Square can always be expressed as the product of pair of prime factors .

vi) The unit digit of a perfect .square can be only.

0, 1, 4, 5, 6, or 9

vii) the square of number having

- 1 or 9 at the unit place ends in 1
- 2 or 8 at the unit place ends in 4
- 3 or 7 at the unit place ends in 9
- 4 or 6 at the unit place in 6
- 5 at the unit place ends in 5 .

viii) There are 2 n natural (numbers between the squares of numbers n and n+1.

ix) The sum of first n odd natural numbers is given by n^2

e.g, $1 + 3 + 5 + 7 + 9 + 11 = 6^2$

$1 + 3 + 5 = 3^2$ and so on .

x) If a perfect square is of n digits then its square root will have $\frac{n}{2}$ digit if in even , or ($\frac{n+1}{2}$ digit if n is odd.

Note : pair the numbers starting from decimal in both the direction (i.e. left and right). In decimal part put Zero to make the pair but no in the whole part .

3.63 Exponents :-

Exponents are another way of writing multiplication you use exponents when you want to write that a number is being multiplied by itself many times .

The base 'a" is raised to the power of n is equal to the multiplication of a n times.

a^n= a x a x a x …. ………... x a n times

a is called the base and n is the exponent .

eg. $3^1 = 3$

$3^2 = 3 \times 3 = 9$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$ etc

We can call a^n as "a raised to the power of n" or "a to the power of n" or simply "a to the on"

From this definition, we can deduce some basic rules that that exponentiation must follow as add as some hand special case that follow from the rules. We can summary the rule and special cases as given below.

Rule or special case	(Formula or (Laws & exponents)	Example
Product	$X^m \times x^n = x^{m+n}$	$2^2 \times 2^3 = 2^{2+3} = 2^5 = 32$
Quotient	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{2^3}{2^2} = 2^1 = 2$
Power of power	$(x^m)^n = x^{mn}$	$(2^3)^2 = 2^6 = 64$
Power of product	$(xy)^m = x^m . y^m$	$(2 \times 3)^2 = 2^2 \times 3^2 = 4 \times 9 = 36$
Power of one	$X^1 = x$	$2^1 = 2$
Power of Ziro	$X^0 = 1$	$2^0 = 1$
Power of negative one		
Change Sign of Exponents	$X^{-1} = \frac{1}{x}$	$2^{-1} = \frac{1}{2}$
Fractional exponents	$X^{-m} = \frac{1}{xm}$	$2^{-3} = \frac{1}{8}$

Special cases:

The expression 0^0 is indeterminate. You can see that it must be indeterminate because you can come up with good reasons for it to be two different values . first, from the above, if $x \neq 0$ then $x^0 = 1$, no matter how small x is . if we just let x go all the way to zero then it seems that 0^0 should be 1.

On the other hand ,$O^a = O$ as loggias $a \neq 0$. Repeat multiplication of O still gives Zero; and we can use the above rules to show O^a still is Zero, no matter how small a is as long as it is non Zero . if just let a so all the way to Zero .

Then it seems like O^o should be O "

($x^o = 1$ so, $O^o = 1$, $O^n = O$ so $O^o = O$, when in doubt .., O^o = indeterminate ")

So, $O^n =$) if n . 0, $O^o =$ undefined if n < O

3.54 Radical:

$(X)^{\frac{1}{n}} = \sqrt[n]{x}$ = n th root of x

$\sqrt[m]{(x)n} = x^{n/m}$

$(x)^{\frac{m}{n}} = \sqrt[n]{(x)m}$, do the m-th power , then take the n- th root

or, $(\sqrt[n]{x})^m$, take the n th root and then do the m-th power.

$(X^m)^n = (x)^{mn}$

$(-1)^n = 1$ if n is even

=-1 if n is odd.

$X^{1/2} = \sqrt{x}$

A negative exponent means divide because the opposite of multiplying is dividing.

3.7Playing with Common number pattern as triangular numbers, Fibonacci numbers.

3.71Square number: Numbers can have interesting patterns .heremwel is the most common patterns and how they are mad. Square numbers :

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, …..

They are the squares of whole numbers

O (= 0 x0)

1 (= 1x 1)

4 (=2 x2)

9=(3 x 3) etc.

How to square a number . To square a number, Just multiply it by itself .

Example .what is 3 squared ?

3

3 Squared 3

1	2	3
4	5	6
7	8	9

= 3 x 3 = 9

"Squared "is often written as a little 2 like this:

2 ⟵ this means square of 4 = 16

This is says " 4 squared equals 16 " (the little 2 years the number appears twice in multiplying)

The Squares are also on the multiplication table

Negative Number : we can also square negative numbers .

Example : what happens when we square (-5) ?

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Answer : (-5) x (-5) = 25 (because negative times a negative gives a positive)

When we square a negative number we get a positive result

3.72 Triangular number

1, 3, 6, 10, 15, 21, 28, 36, 45…

A rule : we can make a " Rule " so we can calculate any triangular number . First , rearrange the dots (and give each pattern a number n) , like this

N = 1	2	3	4
o	o	o	o
	o o	o o	o o
		o o o	o o o
			o o o o
1 dot	3 dots	6 dots	10 dots

This triangular number is generated from a pattern of dots which form a triangle.
Then double the number of dots , and form them into a rectangle

N = 1	2	3	4
oo	ooo	o o o o	o o o o o
	ooo	o o o o	o o o o o
		o o o o	o o o o o
			o o o o o

By a adding another row of dots and counting all the dots we can find the next number of the square.

N = 1	2	3	4
o	o o	o o o	o o o o
	o o	o o o	o o o o
		o o o	o o o o
			o o o o

The rectangles are n high and n+1 wide and x_n is how many dots in the triangle (the value of the triangular number n) and we get (remembering we doubled the dots).

Dots in rectangle= $x_n = n(n+1)$

Dots in trainagle = $X_n = \frac{n(n+1)}{2}$

(Rule : $x_n = \frac{n(n+1)}{2}$: The 5^{th} triangular number is $x_5 = \frac{5(5+1)}{2} = 15$

The Rule is $\boxed{X_n = x_{n-1} + x_{n-2}}$

3.73 Powers of two

In mathematics, a power of two a number of the form 2^n where n is an integer , i.e the result of exponentiation with number two as the base and integer n as the exponent .

In a context where only integers are considered n is restricted to non- negative valudes, so we have 1, 2 and 2 multiplied by itself a certain number of times.

What is the power of 2 ?

Power it 2 enables any one to make progress with their matters it teaches all the mental matter required by the national numeracy strategy, and through its use enables pupils to access the full maths curriculum . Power if 2 is essentially about putting in place the building blocks of number and developing skills with mental calculations.

Power of 2 is ideal for anyone who benefits from repeated maths practice. It has been with students aged & and upwards, right up to adult basic skills.

Power it 2 begins with teaching number bonds to 10, and then moves on to introducing doubling , halving , addition and subtraction I rounding numbers, multiplying and dividing . it then introduces fraction and looks at worded problems and time. it covers all the aspects of mental maths needed for the national Numeracy Strategy .

3.74 Fibonacci Number : the febonecci Sequence is found by adding the two numbers before it together

e.g. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ….

The 2 is found by adding the two numbers before it (1+1) THE 21 5 FOUND BY ADDING THE TWO NUMBERS BEFORE IT (8 + 13) and so on. Can you figure out the next few numbers?

The Rule

N = 0 1 2 3 4 5 6 7 9 10 11 12 13 14 ….

Xn = 0 1 1 2 3 5 8 13 21 34 55 89 55 89 144 233 337 …

$X_8 = x_9 + x_6$

UNIT 04

PRACTICAL ARITHMETIC

4.1Ratio and Proportion:

i) **RATIO:** In every walk of life, whether it is education, agriculture and business are all compare things, objects at many instances. Even children compare their marks among themselves. Using the concept of mathematics, often are compare two quantities of the same type. There are different methods of comparison of one quantity with another quality of same type. Let us discuss here the different methods of comparison of two qualities to develop the concept of ratio in the mind of the children.

Make students to sit in group's ad ask them to compare the length of the playground with its breadth in different ways. Let us not the varieties of answers submitted by the group on the blackboard possible answers.

i. Length is 80 m more than the width
ii. Breadth is 80 m less than the lengths
iii. Length is 5 times that of width
iv. Width is $\frac{1}{5}$th of length
v. 5 times the width gives us the length

The first two comparisons are made on the basis of difference between length and breadth. The last three comparisons made on the basis of division of length with breadth.

The comparison of one quantity with another quantity by division is called ratio i.e the ratio of two quantities of the same kind and in the same unit is the fraction that one quantity is the other.

From the above activity compare breadth of the playground with that of its length as;

$$\frac{Breadth}{length} = \frac{20m}{100m} = \frac{1}{5} = \quad \mathbf{1:5}$$

Thus, Breadth: Length = 1:5

Or we say, the ratio of breadth to length is 1:5.

Note that 1:5 is not equal to 5:1 of the 2 number in a ratio, a:b, 'a' is called the first term or antecedent and 'b' is called the second term or consequent.

Note: (1) The value of a ratio remains the same if both its terms are multiplied or divided by the same non-zero number; e.g $\frac{a}{b} = \frac{ma}{mb}$, m # 0

ii. The ratio a **:** b is always written in its lowest terms i.e. 25:: 35 is same as 5**:7**

iii. a:b > c:d = $\frac{a}{b} > \frac{c}{d}$ = ad > bc

iv. a:b = c:d = $\frac{a}{b} = \frac{c}{d}$ = ad = bc

v. a:b < c:d = $\frac{a}{b} = \frac{c}{d}$ = ad = bc

ii) **Proportion:**

The statement showing the equality of two ratios is said to be a proportion. e.i, if a: b = c: d are in proportion and it can be written as then a, b read as "a is to b is as c is to d" where 'a' and 'd' are called extremes,

B and C are known as means who D is called the fourth proportional of a,b,c.

Different form of proportion:

a: b = c:d is also written as

(i) a, b, c, d are proportional (Statement form)

(ii) a: b = c: d (Ratio form)

(iii) $\frac{a}{b} = \frac{c}{d}$ (fraction form)

or ad: = bc

i.e. product of extremes = product of means

(iv) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then each ratio equal to $\frac{a+c+e}{b+d+f} = \frac{Sum\ of\ Antecedents}{Sum\ of\ Consequents}$

iii) Direct Proportion:

When the relation between two amounts or quantities is such that when one of them increases, the other a b o increases in the same ratio and when one decreases, the other also decreases in the same ratio they are said to be in Direct Proportion. If two amounts x and y increases or decrease together so that the ratio between them remains constant, they are in direct proportion for example:

A car runs 400 Kms in 40 litres of petrol. How far will it run in 15 litres.

Petrol	40	15
Distance	400	x

$$\frac{40}{100} = \frac{15}{x}$$

$\therefore$ x = 150km

iv) Inverse Proportion:

When the relation between two amounts or quantities is such that when one of the increase, the other decreases in inverse proportion and when one decreases the other increases in inverse ratio, they are said to be in inverse proportion.

i.e. $x \infty \frac{1}{y}$ (or $x = \frac{k}{y}$ or $y = \frac{k}{x}$ $\therefore$ $xy = k$, constant)

For example: Ram and Shyam can together plough a field in 4 days Shyam can alone plough if in 6 days. How much time till Ram takes to plough the field done?

At least four methods are available for the effective teaching of proportion. Taught any of these four methods children soon attain considerable skill and accuracy with straight forward examples.

1) **<u>Tradition Method depending upon extremes and means:</u>**
   If 5 books cost Rs. 765, what would 11 cost ?
   The putting of dots can straightway lead to its solution - 5:11 = 7.85: x
2) **The method of Units:** The advocates of this method claim that it is reasonable based on common sense, easily thought and capable of simple explanation. But is is hardly a suitable method, since to proceed always thought the 'unit' and to write out the three lines of statement in often cumbersome and even unnecessary.

3) **<u>The Fractional Method:</u>** Here the unit line of the previous method is discarded and the final statement of the answer in correct form is obtained in one step, calculation only being employed to simplify the answer. For example - if 12m of caret cost Rs. 56.40, what would 18m cost ?

$\frac{18}{12}$ or $\frac{3}{2}$ of Rs. 56.40

Thus students must be taught to reason along the same simple line, viz -

a) Is my answer a number or money or a quantity, etc?
b) Is it more or less?
c) In what proportion is it more or less?

Thus, it would appear to combine the method of unity and the fractional method, and mot to require the students to adhere slavishly or mechanically to either of the two methods. The merits of the problem should determine the method. The unit line should never be used when it can obviously be avoided.

To attempt to teach the one or the other as the rule to be always flowed, is me chemical and cramps thinking and reasoning.

4) **The Educational Method**: This method of setting down the result of reasoning in comparative arithmetic in simple and useful.

Take the example; 66 A map scale was 12 km to the inch. What length of the line represented a distance of 42 km?

It might be set out as $\frac{x}{1} = \frac{42}{12}$

It is a good method. It is readily taught and mathematically sounds. The process of solution is sensible, and at the same time specify and accurate.

To sum up, teach had to avoid any method that is ill-understand by the average child even though that may be his choicest method. He should rather give a free scope to every child's mathematical common sense. If he can "Jump" to way and at his own pace. This attitude will be more profitable in the end. People who can think accurately in comparisons are of more value to the world at large than those who can patiently and obediently work in a mathematical manner.

<u>4.2 PERCENTAGES AS A FORM OF FRACTIONAL NUMBER</u>

Percentage is another way to compare quantities of same kind. As we already know that percentage' means out of hundred e.g. 5% means 5 out of hundred.

Thus, if the consequent (2nd term) in a ratio is 100 then the antecedent expresses the equivalent percentage.

The teacher may introduce the idea of percentage by taking it with fractions as;
$\frac{40}{100}$, $\frac{2}{100}$, $\frac{19}{100}$, $\frac{125}{100}$ etc are all percents.

i.e. a fraction with its denominator as 100 is called percentage.

CONVERTING FRACTIONS OR DECIMALS INTO PERCENTS

Since a percent is a fraction with denominator 100 and a fraction or decimal can always be expressed in the form having denominator 100,

Therefore, a fraction or decimal can always be converted into the form of a percent.

e.g: $\frac{2}{5} = \frac{2x20}{5x20} = \frac{40}{100} =$ 40% (symbol % means per hundred)

$\frac{2}{100}$ in written as 2%

$\frac{2}{3} = \frac{2}{3}$ x $\frac{100}{100} = \frac{200}{3}$ x $\frac{1}{100} = \frac{200}{3}$ %

$\frac{26}{25} = \frac{26x4}{25x4} = \frac{104}{100}$ = 104%

3:4 = $\frac{3}{4} = \frac{3x25}{4x25} = \frac{75}{100}$ = 75%

0.52% = $\frac{52}{100}$ = 52%

1.234 = $\frac{1234}{1000} = \frac{1234}{10}$ x $\frac{10}{100} = \frac{1234}{10}$ % = 123.4 %

Conversely, 135% = $\frac{135}{100}$ = 1.35 (decimal)

25% = $\frac{25}{100} = \frac{1}{4}$ (fraction)

Teacher may now help the students to do these types of conversions.

Example: Out of 16,000 voters in a constituency, 67% voted. How many voters voted?

4.3 Profit and Loss

What is meant by profit and loss? It may be safety assumed that the student of today must have learnt the significance of profit and loss from his environment. Every bargain usually involves either of the two.

The knowledge of percentage is the prerequisite of the study of this topic.

The topic may be introduced effectively with the help of a few oral problems.

Given an example like this: A man buys a cow for Rs. 1800/- after a year when it gets dry he sells it for Rs. 1200. A student with common sense can conclude in no time that the man suffers a loss of Rs. 600. Similarly from the bargain that man buys a buffalo for Rs. 1600 and sells it for Rs 2000, the students can easily conclude that the man gets a profit or gain of Rs. 400.

i) **Profit:** If the selling price (S.P) is greater than the cost price (C.P), the shopkeeper will gain profit. In this ways, the difference of selling price and the cost price is equal to profit.

It means, profit = selling price - cost price (S.P > C.P)

S.P = C.P + Profit

C.P = S.P - Profit

Profit % = $\frac{\Pr ofit}{C..P}$ x 100

Sometimes, shopkeeper has to spend some extra money in transportation, repair etc. of the article, called over head expenses. These are added to the cost price.

ii) **Loss:** If selling price of any object is less than the cost price, the shopkeeper will have to face a loss.

$\therefore Loss = C.p - S.p(S.p < C.p)$

$$Loss\% = \frac{Loss}{C.P} x100$$

Now teacher may explain the process of finding profit / loss through some examples as shown below:

Example: The cost price of a toy is Rs. 80, if it is sold at 15% profit; find the selling price of the toys.

Example: The selling price of a flower vase is Rs. 220. If it was sold at a loss of 8%, find the cost price of the flower vase.

While calculating Gain or Loss%, it should be made clear to the students that profit or loss always reckoned on cost price. Cost price is the investment of the person who gains or losses. Nothing but investment can be the basis for calculating profit or loss. Finding out profit or loss% is a simple case of application of the unitary method. Total gain or loss is on total investment. The question is what would the person gain or loss by investing Rs. 100 only?

4.4 Simple and Compound Interest

Beginning from the close confinement of our day to day life to the business in the international markets, money has been seen as the central driving force at every step of an individual, a family, an institution or a government a big business house, earning, spending, investing, taking loans and repaying those have become inseparable processes. In order to function as an effective individual one has to be familiar with the matters of financial bans action.

Besides, the persons concerned with business, every one of us need to use our income in an optimal manner to get maximum benefits out of it at present and future. All of us save a portion of our earning and deposit in a bank, invest in some business or lend out to somebody. After a period we earn interest on the amount saved, invested or given as loan.

To know the nature and ways to calculate interest on the deposits or loans has become a basic necessary for living. Therefore, the topic on 'interest calculation' finds place in elementary school mathematics curriculum. In this section, the concepts of simple and compound interest and their application have been discussed to enable the teachers to deal effectively in the classroom transactions

If children understand that 5% of a thing is $\frac{5}{100}$ of it (and they are expected to have learnt it from other work on percentages and know the unitary method) he should have no difficulty with simple interest.

Its knowledge can be easily introduced and effectively developed from the previous knowledge of percentage and unitary method. Some general information about interest can be collected by the students themselves from the banks, post officers, small saving authorities and business agencies.

i) Simple Interest (SI):

The rate at which SI is to be calculated is decided by the individual or the organization which lends the money in case of nationalized banks, the rate of interest is decided by the central government (**i.e**. **RBI**).

If R = 10%, it means that on the principal of Rs. 100, the interest for 1 year is Rs. 10. 66 rate of interest is the interest to be charged on a principal of Rs. 100/- for 1 year. It is expressed as a percentage and denoted by 'r'.

***Example**:* Find the simple interest on Rs. 500 for 3 years at 5% per annum?

Interest on Rs. 100 for one year = Rs 5

Interest on Re. 1 for one year = Rs $\frac{5}{100}$

Interest on Re. 500 for one year = Rs $\frac{5}{100}$ x 500

$\therefore$ Interest on Re. 500 for three years = Rs $\frac{5}{100}$ x 500 x 3

So in generalized form, the formula can be put as

S.I. = $\frac{Principal\ x\ Rate\ x\ Time}{100}$

or S.I. = $\frac{PRT}{100}$

ii) Compound Interest

What is meant by compound interest? There should be no difficultly about explaining its significance. Money is said to be borrowed on compound interest when at the end of a year, the due interest is mot paid by the borrower, but gets added to the sum borrowed and the amount thus obtained becomes principal for the next year. The same process i.e. addition of the yearly

Interest to the previous amount, is repeated until the amount for the last year has been obtained the difference between the original sum borrowed and the final amount is the required compound interest.

The teacher will enable the learner to realize how amount accumulate sat a surprising and unimagined rate, when interest on interest is charged year after year.

It is obvious that the knowledge of compound interest will have to be developed on the basis of that of simple interest. The teacher will draw the attention of the learners to the formula for calculating amount, which they must have learned during work on simple interest.

Amount = A = P + I $= P + \frac{PRT}{100} = P\left(1 + \frac{RT}{100}\right)$

This formula will enable the learner to find out the value of amount after one, two three, four or any number of years. It will be possible for him to find that…

At the end of 1st year, amount $A_1 = P\left(1 + \frac{r.t}{100}\right)$ **i.e.** $A_1 = 1 + \frac{r.t}{100})^1$

At the end of 2nd year, amount $A_2 = P\left(1 + \frac{r.t}{100}\right)^2$

At the end of 3rd year, amount $A_3 = P\left(1 + \frac{r.t}{100}\right)^3$

Similarly, in general, amount after n years $= A_n = P\left(1 + \frac{r.t}{100}\right)^n$

Note: (i) For the money that we deposit in the post office or in the banks, under saving bank account scheme, we get compound interest.

(ii) The compounding of interest could be annual or half yearly or quarterly.

In case of half yearly compounding the interest becomes more as compared to annual compounding.

After the stage the finding of the compound interest required only the subtraction of original sum or principal from the final amount. i.e.

Compound interest $= p\left(1 + \frac{r.t}{100}\right) - p$ i.e., $C.I = P\left[\left(1 + \frac{r.t}{100}\right)^n - 1\right]$

The remaining work is only the efficient application of this formula.

Example: Determine the compound interest on Rs. 1000 for 1 year at 8% per annum compounded half yearly.

Here, interest is compounded half yearly,

$\therefore r = \frac{1}{2}$ x rate % per annum = $\frac{1}{2}$ x8% per/ half yearly

= 4% half yearly

n = 1 year = 2 x no. of year = 2 x 1 = 2 half year

P = Rs. 1000.

Using the formula calculate the C.I

as C.I = P [(1 + $\frac{r.t}{100}$) ^{n} - 1]

4.5 Discount:

Sometimes to attract the customers to buy goods or o promote sales of good, the shopkeeper generally make various plans. Very often we come across the sale advertisements in newspapers, magazines or handbills. In sale the shopkeeper gives some percent rebate on the goods price. This type of incentives to the3 customers in one form or the other called discount on the marked price of the article. Shopkeeper marks the price on good according to themselves.

Marked price of good is also called list price shopkeeper gets after giving discount on the marked price (means selling price) generally more than the cost price. Hence, even after giving discount he gets profit.

Shopkeepers, generally declare, high cash discount, 'discount sale' on various festivals to attract customers. Generally, this discount is given in the form of cash or percentage of marked price of goods or in the form of tree gift. This may be explained through some examples.

Example*:* A item marked at Rs. 800 is sold it Rs. 750.Find the discount and discount Percentage.

M.P = Rs.800
S.P = Rs. 750
$\therefore Discount = Rs(800 - 750) = Rs.50$
Hence, discount percent = $\frac{Discount}{m.p} x100 = \frac{50}{800} x100 = 6.25\%$

UNIT – V
GEOMETRY WAYS OF LOOKING AT SPACE AND SHAPE

Geometry is the science of space and extent. It deals with the position, shape and size of bodies but has nothing to do with their material or physical properties. It is the key to mathematical thinking and demonstrates the nature and power of pure reason.

The teacher of mathematics and curriculum framers have now began to realize that nature and practical arts are the primary and permanent sources of geometric learning. The function of the middle school geometry is to systematize the information received by the pupils at the pre-school and primary school stage from nature and practical arts. The emphasis will be on the understanding of fundamental concepts and techniques such the meaning drawing and use of lines, angles, triangles and polygons. The primary object is not to teach the pupils to know geometry, but rather to lead them to think geometry.

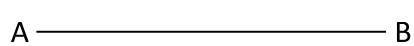
A point, line and plane are undefined terms. If we say a point that which has no length, no breadth and no thickness then it does not exist in the universe. But very close to that we define a point is that which has position but length breadth and thickness are negligible.

Line: it is the boundary of the surface. It has only one dimension-length. A lie is that which has length but negligible breadth and thickness. The best method to explain it is to draw a line before the students.

Straight Line: It is a line which has the same direction from point to point throughout its whole length. It is the shortest distance between any two points. Only one straight line can be drawn between two points.

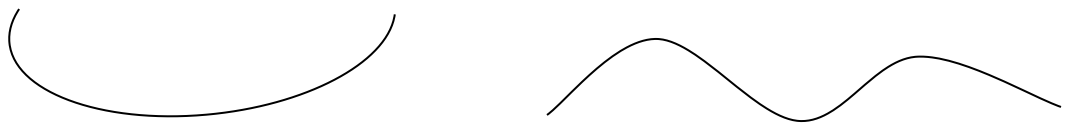
Place a foot rule and draw a line with it to show the shape and nature of straight line;

The name of a straight line can also be indicated by writing two letters, one at each end e.g;

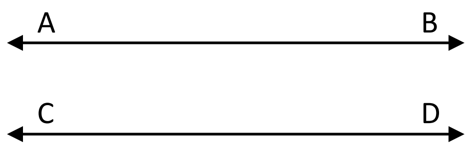


Curved Line: A line which doesn't have the same direction from point to point throughout its whole length is called a curved line.

This concept should be explained by drawing an arc or a line with bends e.g;



Parallel Line: Let us draw two lines AB and CD such that distance between the two lines remains same in every direction as shown below;



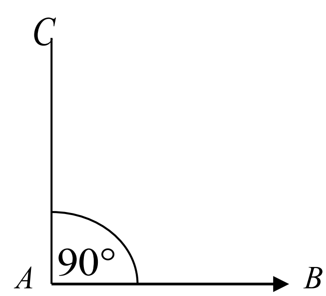
In the similar manner, when in a plane two or more lines never meet each other whether they are extended in any direction, then these lines are known as parallel lines. e.g; Edges of a book, edges of blackboard, railway track etc.

Note: If two lines are parallel then they are at same distances throughout their length. The symbol of representing two parallel lines is two vertical lines between then as

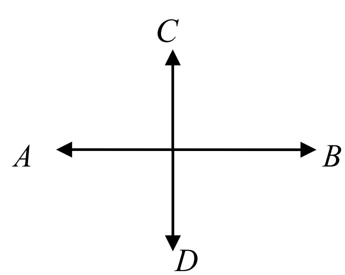
$$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$$

Perpendicular line:

By rotating a ray AB, bring it in the position of ray AC and it makes the angle of 90°, the ray AC is perpendicular on the ray AB as shown in the figure.



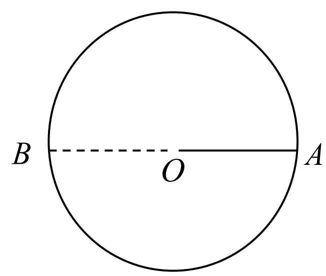
Thus two intersecting lines AB and CD are said to be perpendicular to each other if angle between them is a right angle (90°)



Circle: One of the most common simple closed curves is the circle. In this part we see some of the properties of a circle as a mathematical figure.

A circle is the locus (or path) of a point which moves in such a way that its distance from a fixed point in always a constant.

The fixed point is called the centre the constant distance is called the radius. i.e; a line segment whose one end meet the centre while the other end touches the boundary line is called radius.



In fig. OA is the radius.

<u>Diameter of Circle:</u> A line Segment which passes through the centre and whose both end points touches the circle (i.e boundary line) . In fig; BOA is a diameter.

<u>Circumterencefo a circle</u>: The measure of boundary of a circle is called its perimeter or circumstances of the circle.

$$\frac{Circumterence}{Diameter} = \text{a constant } (= \pi)$$

$\frac{C}{D} = \pi$ C = Circumstance

D = Diameter = 2 x radius (r)

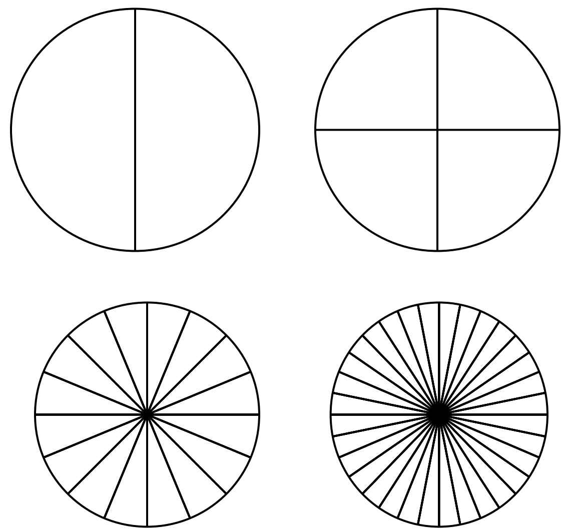
$\frac{C}{D} = \pi d$

or $C = 2\pi r$

Arc of a Circle: Any part of the circumference of a circle is called an arc of the circle.

Chord: A straight line which joins any two points on the circumference of a circle is called a chord of circle.

Area of a Circle:



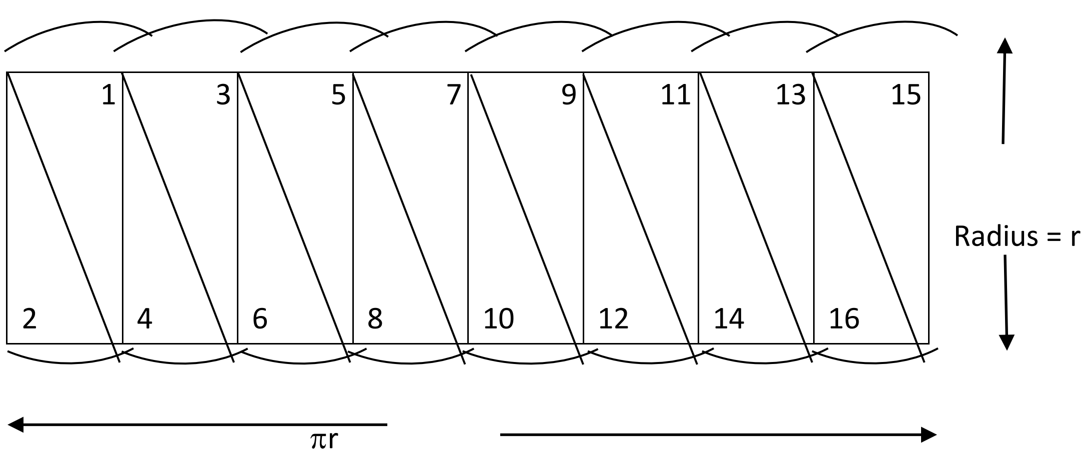
Draw a circle on a plane paper of any radius and fold it along any diameter. Again fold it along radius and continue two times more.

Lastly open the paper after pressing it. Now cut; along the folded lines, mark the paper strip 1 to 16 and paste them alternatively as 1,3,5…….15 as shown in figure below; watching the figure carefully, you will notice that it looks like a rectangle. The area of this figure will be the area of theπ rectangle.

$\therefore$ Area of Circle = area of the rectangle

= length ($\frac{1}{2}$ x circumference) x breadth (radius) = $\frac{1}{2}$ x 2 πr = πr^2

$\frac{1}{2}$ **Circumference = πr**



Therefore, area of circle = πr^2

= $\pi r x (radius)^2 \pi$

Radius(r) = $\sqrt{area\ of\ circle/\pi}$

Application: The Students apply the formula in doing problem.

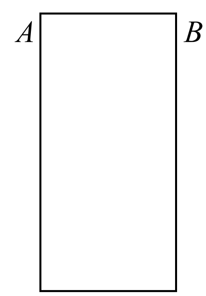
i. Students analyze the problem
ii. Students select the relevant data
iii. Students generalize the rules.
iv. Students develop the skill in drawing figures and doing problems.

<u>Study of Solids</u>: Cylinder, cone, sphere and hemisphere: surface area and volume.

***<u>Cylinder</u>*:**

In figure (i) AOCB is a rectangle where AO=BC

(i)



If BC rotates around AO then you get a cylindrical surface of radius AB and height OA as shown in -

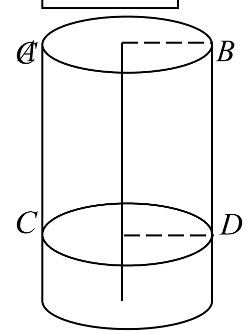


fig (ii) it is a hollow cylinder open from both ends 'OA' is called perpendicular axis. If cylinder is in vertical position, then lower circular area is called base and its height is called the height of cylinder. Radius of circular ends are called radius of cylinder.

<u>Curved Surface Area of Cylinder</u>:

When we cut and open the cylinder through perpendicular, we get a rectangle in the form B B^1C^1B

where, $BB^1 = CC^1$ = Perimeter of top and bottom surface = 2 πr

$\therefore$ **Curved surface area of cylinder**

= area of rectangle B B^1 C^1 C

$= l x b$

$= h x 2 \pi r$ Sq. unit

$= 2 \pi r h$ Sq. unit

Total surface area of cylinder

= curved surface area + area of circular ends

$= 2 \pi r h + 2 \pi r h 2$

$= 2 \pi r h \ (h + r)$

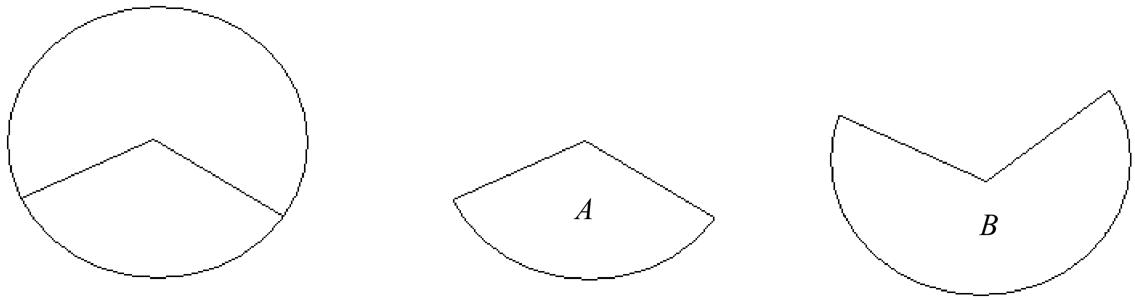
<u>Volume of cylinder</u>: The space occupied by the cylinder is known as volume of cylinder. Volume of cylinder of radius(r) and height (h)

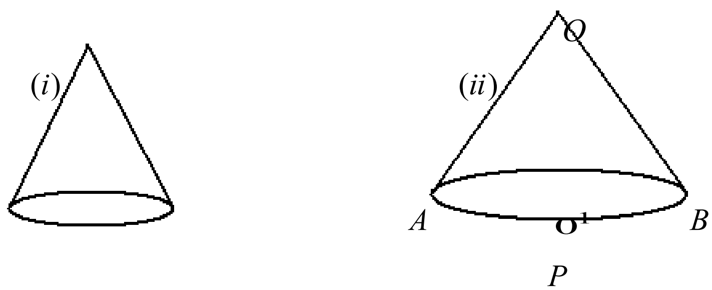
= Area of base x height

$= \pi r^2$ x h

$= \pi r^2$ h cubic unit

<u>Cone</u>: A circular sheet of cardboard is cut along two of its radius. Thus the circular sheet is divided into two sheets A and B shown below. Each part could be turned in the manner as shown so that both the straight edges coincide. Then with the help of cello tapes the two edges are kept fixed. Thus first sheet give rise to a cone of smaller base circle and the larger height as shown in (i) and the sheet B gives rise to a cone of bigger base-circle and of smaller height as shown in (ii).





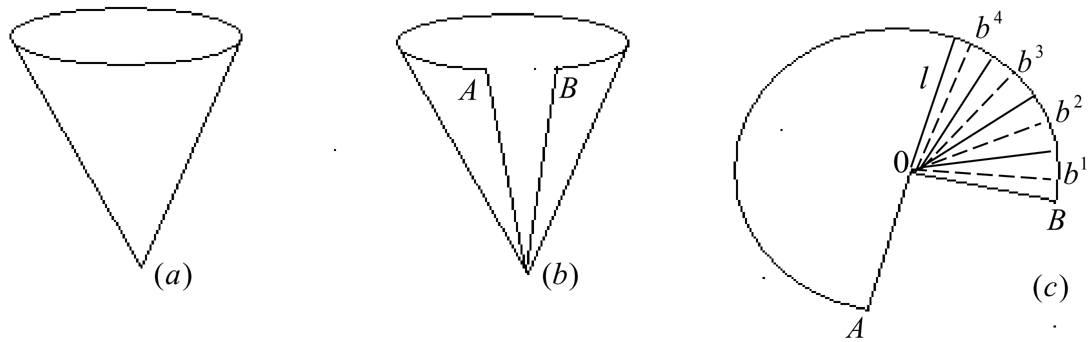
A right circular cone shown above in fig (ii) APB is a plane circular end, called base of the cone, O is the vertex, O’ is the centre of circular base., O^0 is the height of the cone and OA is called the slant height of the cone.

Volume of perpendicular cone is equal to one-third volume of the same height and radius of a cylinder.

$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} x$ (area of base) x height

<u>Curved surface Area of cone</u>: Cut out a neatly made paper cone that doesn’t have any overlapped paper, straight along its side, and opening it out, to see the shape of the paper that forms the surface of the cone (the line along which you cut the cone is the stand height of the cone which is represented by l). it looks like a part of a round cake.

If you bring the sides make a and B at the tips together, you can see that the curved portion of fig (c) will form the circular base of the cone.



If the paper like the one in fig (c) is now cut into hundreds of little pieces, along the line drawn from the point 0, each cut portion is almost a small triangle; whose height is the slant height ***l*** of the cone.

Now area of each triangle = $\frac{1}{2}$ x base of each triangle x l

So the area of entire piece of paper

= sum of the areas of all the triangles

$= \frac{1}{2} b_1 l + \frac{1}{2} b_2 l + \frac{1}{2} b_3 l + \ldots$

$= \frac{1}{2} l (b_1 \times b_2 \times b_3 + \ldots$

= $\frac{1}{2}$ *l x height of entire curved boundary of fig (c)*

(*as* b_1 x b_2 x b_3+ … makes up curved portion of the figure)

But the curved portion of the figure makes up the perimeter of the base of the cone and the circumference of the base of the cone = 2 πr , where r is the base radius of the cone.

Where r is its base radius and *l* its slant height and $b^2 = h^2 + r^2$

$\therefore l = \sqrt{h^2 + r^2}$

Now if the base of the cone is to be closed, a circular piece of paper of radius r is also required where area is πr^2

So, totals surface area of a cone = $\pi rl = \pi r^2 = \pi r(h + r)$

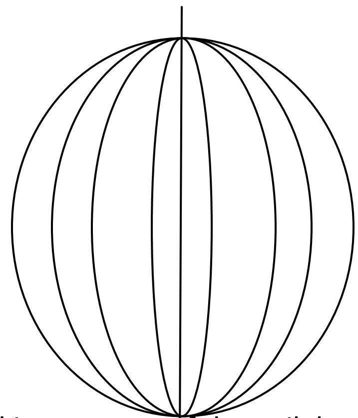
Example: Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

<u>Surface Area of a Sphere</u>:

What is Sphere?

Let us cut-out a circle from a cardboard. Now if you past a string along a diameter of a circular disc and rotate it, you see a new solid as shown in the figure. It resembles as a ball, which is called a sphere.

So, a sphere is a three dimensional figure (solid figure), which is made up of all points in the space, which lies at a constant distance called the radius, from a fixed point called the centre of the sphere.



Now let us take a rubber ball and drive a nail into it. Taking support of the nail, let us wind a string around athe ball. When you have reached the 'farthest' part of the ball, use pins to keep the string in place and continue to wind the string around the remaining part of the ball, till you have completely covered that ball as shown in fig (i) mark the starting and finishing points on the string, and slowly unwind the string from the surface of the ball. Now measure the diameter of the ball with help of teacher and hence the radius. Then on the sheet of paper, draw four circles with radius equal to the radius of the ball. Start filling the circles one by one, with the string you had wounded around the ball as shown in fig (ii)

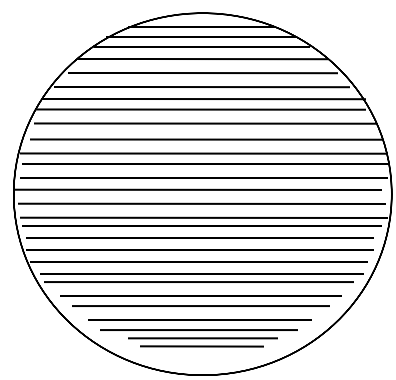


Fig (i)

What you achieve in all this? The string which had complete covered the surface area of the sphere, has been used completely fill the regions of four circles, all of the same radius as of the sphere.

So, what does that mean?

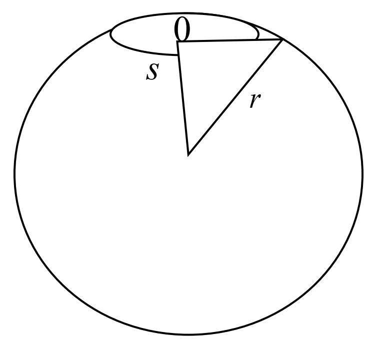
This suggests that the surface area of the sphere of radius (r) = 4 times the area of circle of radius ®

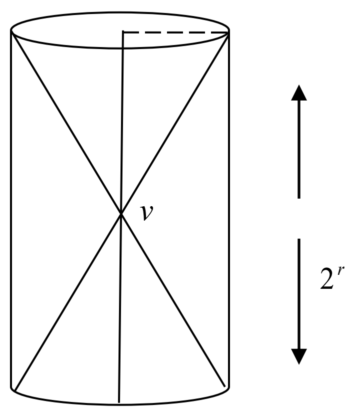
= 4 x (πr^2)

So, $\boxed{\text{surface Area of Sphere} = 4\pi r^2}$

Volume of Sphere:

The space occupied by a sphere is called volume of sphere for finding the column of sphere, take a sphere of radius r and also a cylinder with height 2r shown in the fig. now imagine two solid cones each of radius r and height also r as shown in the above figure. Both cones have vertex V. now this cylinder has two empty cones





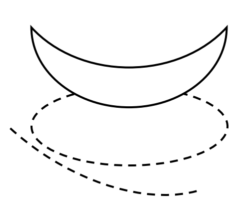
$\therefore$ Volume of solid part of cylinder

= volume of cylinder -2 volume of cone (height of cylinder = height of cone=r_1)

= πr^2 x 2r – 2 x $\frac{1}{3}\pi r^2$ xr = $\frac{4}{3}\pi r^3$

Let us compare this with a sphere of radius r, place the sphere and cylinder on a common plain surface such that centre 'O' of sphere and vertex V of cylinder are at equal level and both are placed at a distance 2r.

Hemisphere:



Volume of Sphere;

= $\frac{1}{2}$ x volume of sphere

= $\frac{1}{2}$ x $\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$

as hemisphere is obtained by dividing the sphere by a plane into two halves.

Surface Area of Hemisphere:

I. Curve surface Area of Hemisphere

= $\frac{1}{2}$ x surface Area

= $\frac{1}{2}$ x $\frac{4}{3}\pi r^2 = \frac{2}{3}\pi r^3$

II. Total surface Area of Hemisphere.

= curved surface area + area of cross section circle

= $2\pi r^2 + \pi r^2$

= $3\pi r^2$

Geometrical constructions

The geometry the word "to construct" means to draw accurately. The only instruments permissible in these constructions are the straight edge for drawing straight lines and the compasses for drawing arcs and measuring lengths of lines. Efficiency in construction requires skilful manipulation of these instruments. Actual doing is very necessary. A student may verbally know how to construct a certain geometrical figure, and yet may fail to draw it accurately when actually required to do so., lack of practical experience is generally responsible for this failure.

Accuracy and neatness require good accurate geometrical instruments. Further constant practice is the only way to acquire self-confidence in construction work.

Construction is an art. Mainly two types of geometrical construction is theirs. One is that in which figure are constructed form the given facts and other is that in which the length is to be found out in addition to construction

Parts of a construction problem: -

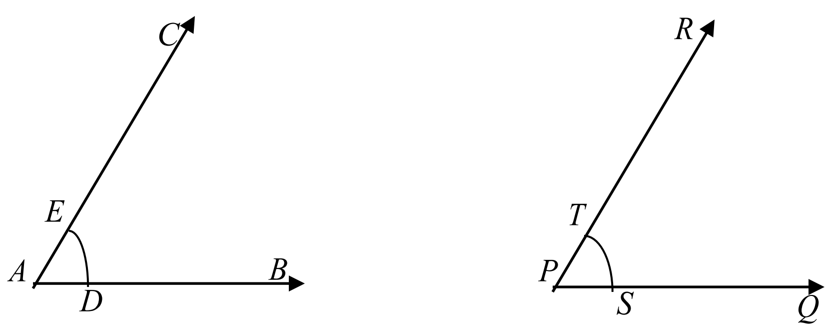
i. A statement of a problem; to acquaint with it;
ii. A representation of given part, describe it orally.
iii. A representation of the given parts in terms of drawing figure.
iv. A statement of what is to be constructed i.e classification of objective.
v. The actual construction, with a description of and relevant authorities for each step.
vi. A statement that the required construction has been made.
vii. A proof of the construction, writing reasons for each step.

For every new construction a thought must be given and deep analysis must be done utilizing previous knowledge.

To construct an angle equal to given angle;

Given: $\angle CAB$

Required: To draw another angle equal to $\angle CAB$



Steps of Constructions:

1. Draw a line PQ
2. Taking a as centre and radius of any measurement draw an arc cutting AB in D and AC in D.
3. Now take P as centre and radius same as in step2, draw an arc (as shown above) intersecting PQ in S.
4. Consider S as centre and DE as radius draw another arc which cut arc of step 3 in T
5. Join PT and extend line to PR. Now $\angle RPS$ is the required angle equal to $\angle CAB$.

   Verification: $\angle CAB$ = $\angle RPS$

<u>To bisect the Line Segment</u>:

Given: Line Segment AB = 8 cm

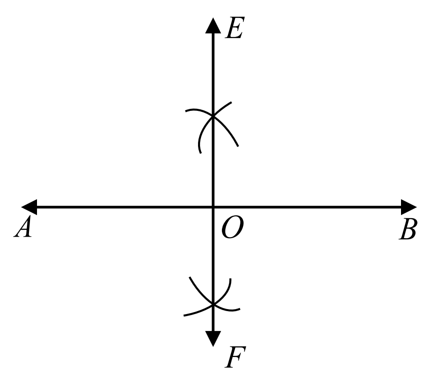
Required: To bisect the line segment.

Steps of Construction: -

1. Consider A as centre and take radius more than half of AB, draw two arcs on both (above & below) side of the line AB.
2. Taking B as centre and same radius as in step 1, draw other two arcs intersecting the previous two arcs (in step 1) at E and F.
3. Join EF, which bisect AB at O, now point ‘O bisects line segment AB.

Verification: - measuring, OA = OB = 4 cm

∴ EF is the bisection.



<u>To construct a perpendicular from a given point to a given line</u>

Given: A given point P and a given line AB.

Required: to draw PQ perpendicular on AB.

<u>Steps of construction:</u>

1. Taking ‘P’ as centres draw an arc of any radius which intersect line AB in two distinct points E and F.
2. Consider E as centre draw an arc with radius same as in step 1 below the line AB(opposite of point P) as shown in fig.
3. Taking F as centre and with same radius as in step 2, draw another arc to intersect the previous arc drawn in step 2 at R.

4. Join PR as shown in fig (above the line full line and below the line AB dotted line). PQ is required perpendicular from P to AB.

   Verification: $m\angle PQA = m\angle PQB : PQ \perp AB$

<u>To construct Angle of 30^0, 45^0,60^0 and 90^0 using compasses as sell as set square</u>.

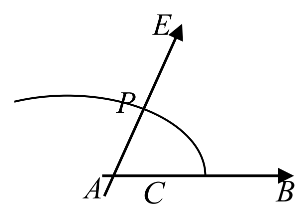
In geometry box there are two set squares having angles 30^0,$60^{0,}$ 90^0 and $45^{0,}$ 45^0, 90^0. On its one edge there is a mark of cm and other edge has mark of inch.

To make angle of 30^0, 45^0, 60^0 and 90^0. Take suitable set square and hold the set squares firmly on a paper and draw two rays from its pint where $30^0$is marked. Similar procedure may be used for constructing other angles.

Verification: Now take a protractor and measure the angles and see whether the degree measures are of required angles.

<u>To construct 30^0, 45^0,$60^{0\ and}$ 90^0 by using</u> <u>compasses</u> Construction **of 60^0**

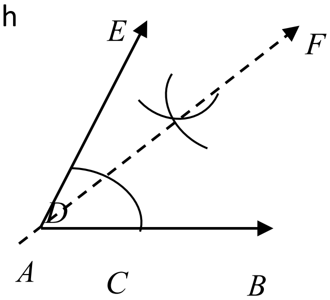
Steps of construction: -



1. Draw a line AB
2. Taking A as centre and radius of any measurement draw an arc intersecting AB in C.
3. Consider C as centre and with the same radius, draw another arc to cut the arc in step 2 at D
4. Join AD and proceed to E. $\angle EAB = 60^0$

   Verification: Verify the measure of $\angle EAB$ with protractor = 60^0

<u>Construction of 60^0</u>.



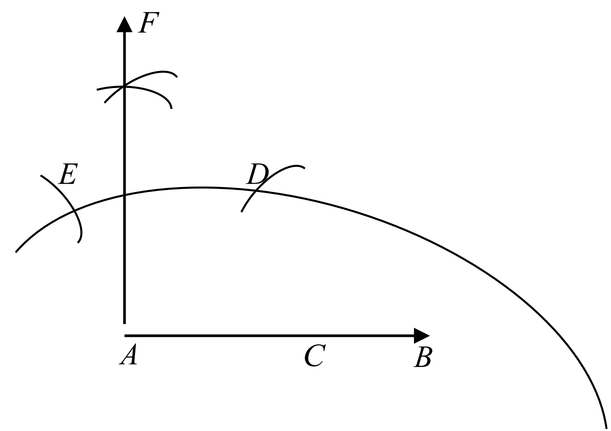
Steps of construction:

1. Draw angle $\angle EAB$ = 60^0
2. Bisect $\angle EAB$ as show in fig.
3. $\angle FAB$ = 30^0

   Verification: verify the measure of $\angle FAB$ with protractor =30^0

Construction of 90^0

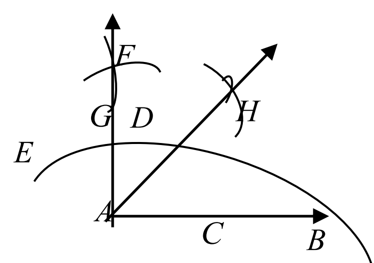
Steps of construction:



1. Draw a line AB
2. Draw an arc with centre A and of any of any radius intersecting AB in C.
3. Taking C as centre and with same radius (in step1) draw an arc to intersect previous arc at D.
4. Again taking D as centre with same radius draw another arc to intersect the previous arc (in step 1) at E.
5. With centre E and D and with same radius (in previous) draw two arc intersecting each other at E.
6. Join AF, $\angle FAB = 90^0$

   Verification: verify the measure of $\angle FAB = 90^0$

Construction of 45^0



1. Bisect $\angle FAB = 90^0$ as shown in fig.
2. $\angle HAB = 45^0$ is the required angle

Verification: verify the measure of $\angle HAB = 45^0$

Construction of congruency and similarity of two triangles:

Concept of Congruency of two triangles

Two geometrical figures having exactly the same shape and size are known as congruent figure.

When the student will understand the shape and dimension of every object then only they should be made aware of congruency. If students understand congruency they will come to the conclusion that the all sum rays are congruent two line segments of same size are congruent, two line segments of same size are congruent, two equal circles of same radii are congruent, two squares are congruent if their sides are equal, if corresponding sides and corresponding angles are equal, then triangles are congruent.

The sides that are in front of equal angles are corresponding sides and the angles that are in front of equal sides are corresponding angles. The symbol $\cong$ is used between two congruent triangles.

Theorem of Congruent Triangles:

1. In two triangles, if two sides and one angle between these two sides are equal

respectively, then the triangles are congruent.

2. Two triangles are congruent if any two angles and the included side of one triangle are equal to any two angles and the included side of the other triangle.
3. Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.
4. Two right triangles are congruent if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.
5. If two figures are congruent to their figure, are all of them congruent.

***Similarity of two triangles*: -**

It is a very important principle of elementary mathematics, but it is not yet achieved its rightful position. It has great mathematical simplicity and beauty about it and has an extraordinarily wide range of application. Similar figure are related to one another in the same way as an object and its image or as a lantern slide and its projection on a screen or two maps of the same country drawn on different scales. These are figures of the same appearance, irrespective of their size.

There are two necessary conditions for similarity, namely:

(i) The same shape (equal corresponding angles)

(ii) Proportional dimensions.

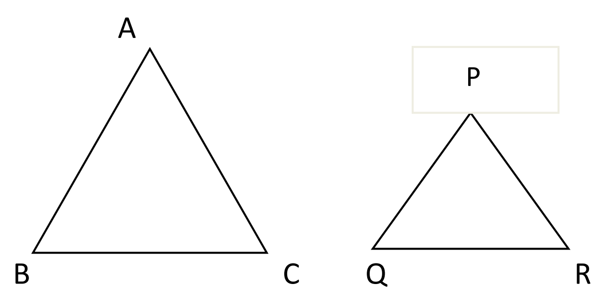
1. If two triangles are equiangular, there corresponding sides are proportional and hence, the two triangles are similar; it is denoted y the symbol ‘ ‘ $\sim$

In fig. ABC $\triangle \sim$ PQR $\triangle$

$$\angle A = \angle P, \angle B = \angle Q$$

$$\because and \angle C = \angle R$$

$$\therefore \frac{AB}{PG} = \frac{BC}{QR} = \frac{AC}{PR}$$



2. If two angles of one triangle are respectively equal to two angles of another triangle, then the two Triangles are similar.
3. If in two triangles, sides of one triangle are proportional (i.e in the same ratio). The sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.
4. If one angle of a triangle is equal to one angle to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

The principle of similarity helps in measuring the height of poles, building, width of rivers and in enlarging and reducing maps

UNIT- 6

HANDLING OF DATA

Collecting Information or Data, arranging or processing those data and drawing interferences from them to solve the problem confronted have been components of the domain of knowledge called statistics.

Think for a while about the number of information you need when planning for effective measurement of different school activities.

- ❖ The number of students [boys & girls] attending school every day.
- ❖ The number of students [boys & girls] who remain absent more than two days.
- ❖ The number of students who dropped out of the school during last five years (year wise and grade wise)
- ❖ The number of students attending unit tests and annual examinations (grade wise & subject wise).
- ❖ Promotion of students of different classes and in different subjects. The number of teachers actually present every working day during the last month.
- ❖ Cleanliness habits of children.
- ❖ Number of working days per month during the year

The list may be quill big.

Often there is a need to collect various information which is required to solve the problems we confront in our daily life. The information that we want to know are drawn from some numerical (description. In each of the above mentioned problems numerical descriptions like number of absentees, number of drop out etc. are to be collected for eliciting information from them. Such numerical descriptions are called “data” in other words; a data is a collection of numbers gathered to give some information.

<u>6.1 Collection of data</u>: -

First we decide the sources of collection of data. The data may be collected directly from the source, it may collected from some documents which as already been prepared for some other purpose like census report and other report of the kind, from various news items published in newspapers etc.

Before the data is collected, the purpose of which it is being collected should be known to the person who collects it.

Example: - to know the achievement levels of the students of a school at the end of class-v in maths.

This is an instance of raw data collected from the 'Primary source'

Example: - to know the age-group of patients who are affected by heart disease in an industrial area.

Source of information about the incidence of heart disease is the record available in the local hospital. This is not a direct source. We have to go through the records of all patients attended in the hospital. Such type of data collected from other sources is called secondary data.

The data collected from the field are not well organized and does not help us in drawing up any inference then it is called 'raw data'.

6.2 Classification and interpretation of Data:

Collected data are always ungrouped. If the data includes a few scores, we simply arrange them in magnitude order (increasing or decreasing order). Then it is known as arranged data.

e.g.: *Raw data:* ---	12,	11,	19,	17,	10,	15,	19,	12,	8
Arranged data: --	8,	10,	11,	12,	12,	15,	17,	19,	19

As soon as the work related to collection of data is over, the investigator has to find out ways to present them in a form which is meaning full, easily understood and gives its main features at a glance. Let us now recall the various ways of presenting the data through some examples.

First method:

Example: consider the marks obtained (out of 100 marks) by 30 students of class vii of a school:

10,	20,	36,	92,	95,	40,	50,	56,	60,	70
92,	88,	80,	70,	72,	70,	36,	40,	36,	40
92,	40,	50,	50,	56,	60,	70,	60,	60,	8

Recall that the number of students when have obtained a certain number of marks is called the frequency of those marks. For instance, 4 students got 70 marks, so the frequency of 70 marks is 4. To make the data more easily understandable, we write it in a table as given below.

Marks	Tally Marks	Number of students (*i.e. frequency*)
10	I	I
20	I	I
36	III	3
40	IIII	4
50	III	3
56	II	2
60	IIII	4
70	IIII	4
72	I	1
80	I	1
88	II	2
92	III	3
95	I	1
Total		30

This table is called an ungrouped frequency distribution table or simply a frequency distribution table.

<u>Second Method</u>: The collected data can be classified and arranged in the following manner as group frequency distribution: ---

(i) First of all the range of data is found out.
Range of data: = highest value lowest value
= difference between highest and lowest observations (or scores).

(ii) After finding out the range, data are classified. There is not any hard and fast rule for classifying it. Gargets talked about forming the category from 5 to 15 and gill ford about 10 to 100.

(iii) The difference of the last and first number is called class-interval. The rule of class internal is very simple divide the total numbers by range and add one to the quotient then the class interval is found out.

(iv) After making class interval (C-i), the data is tabulated.

Example: 50 persons reserved their tickets in the morning of a particular day at a railway reservation counter-Their ages in years recorded as follow: -

22, 35, 28, 40, 38, 6, 35, 20, 15, 55, 45, 27, 39, 40, 25, 30, 59

48, 21, 44, 29, 32, 40, 10, 44, 11, 46, 32, 56, 9, 11, 40, 36, 20

51, 41, 38, 39, 21, 29, 7, 28, 27, 57, 57, 21, 26, 33, 37, 20

Classify this data

Age group	**Tally Marks**	**Frequency**
0 – 10	II	3
10 – 20	~~IIII~~	4
20 – 30	~~IIII~~ ~~IIII~~ ~~IIII~~	15
30 – 40	~~IIII~~ ~~IIII~~ II	12
40 – 50	~~IIII~~ ~~IIII~~	10
50 – 60	~~IIII~~ I	6
	Total	**50**

Range = 59 – 6 = 53

Class is to be formed for 10

Class size = 10, No. of C.I = $\frac{53}{10} + 1$

$$= 5 + 1 = 6$$

Graphical Representation of Data:

The representation of data by tables has already been discussed. Now term our attention to another representation of data i.e. the graphical representation. It is well said that one picture is better than a thousand words usually comparisons among the individual items are best shown by means of graphs. The representation the become easier to understand and looks beautiful and impressive than actual data. The data are presented in colourful pictures and colourful graphs so that at a glance observer may get clear idea about the data.

A Bar Graph: -

A bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis (say the X-Axis), depicting the variable. The values of the variable are shown on the other axis (say, the Y-Axis) and height of the bars depends on the value of the variable.

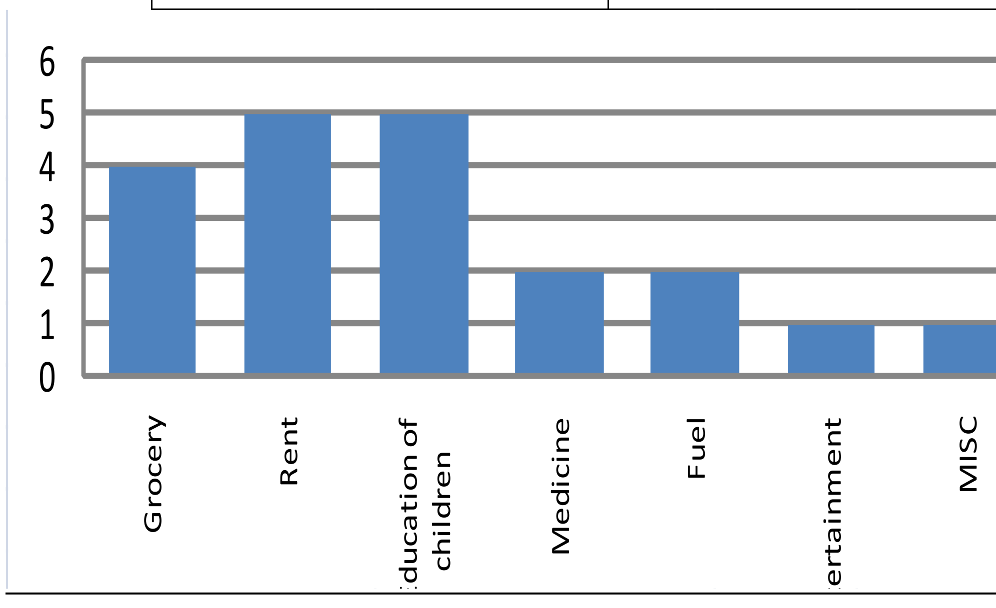
Bar diagram are of different types:

(i) Simple Bar diagram, (II) Component Bar diagram, (iii) Multiple Bar diagram, (iv) Percentage Bar diagram etc.

i) **Simple Bar Graph/diagram**

A family with a monthly income of Rs. 20, 000 had planned the following expenditures per month under various head.

Heads	Expenditure (in thousand rupees)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
MISC	1



ii) **<u>Histogram</u>**

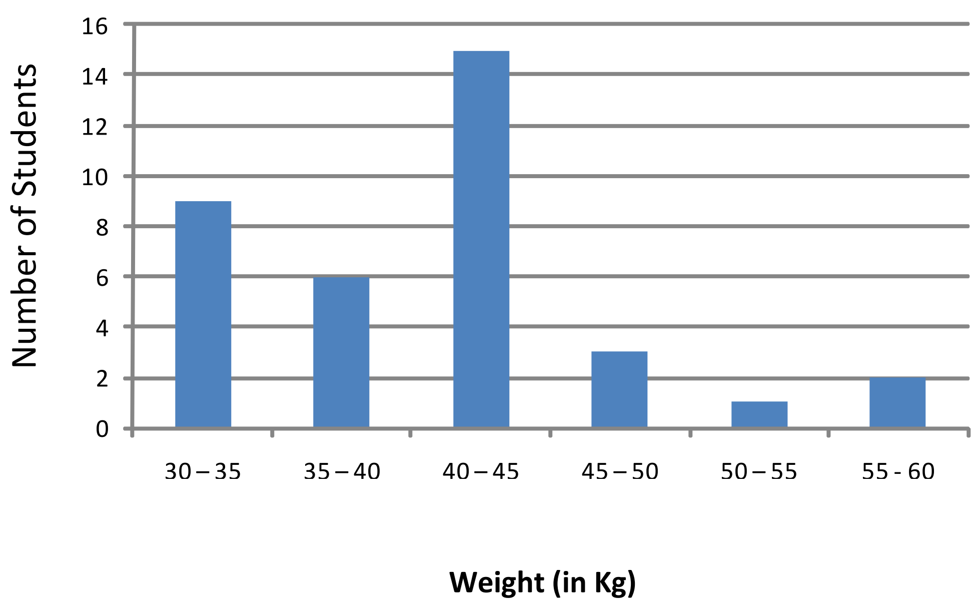
In this the frequency number are represented as rectangle. Its basis is the length of C.I the specialty of this figure is that the frequency is evaluated on the whole class interval.

Example: The weights of 36 students of a class are given below in the table. Represent the data graphically.

Weight in kg	Number of Students
30 – 35	9
35 – 40	6
40 – 45	15
45 – 50	3
50 – 55	1
55 - 60	2
Total	36

The numbers of the above table are put into the histogram by the following method:

i. Base line /horizontal axis OX is drawn
ii. A vertical axis/line OY is drawn
iii. The ratio of these lines is according to the number of frequencies.
iv. The C.I on X-axis and frequencies on OY are to be put and therefore a division is done.
v. To divide OY, about $\frac{3}{4}$ the length of OX is found out and OX is divided by the most/largest frequency this divides OY.
vi. Scale should be written on every axis.
vii. In class intervals, the part is left as class and (   ) or N is put on its part, if the class interval starts from a number other than zero.



6.3 **<u>Measures of Central Tendency:</u>**

Often in our daily life we make statements of the following kinds: -

(i) Average monthly attendance of a student is 95 percent.
(ii) Average temperature in a week is 35 degree Celsius
(iii) Average height of the students of class viii is 130 cm
(iv) Average daily income of a labourer is Rs. 95.00

What does such a statement mean? Let us consider the first statement. Suppose in the month of January in which there were 20 working days in a certain school and s student remained present in the school for 18 days, then his percentage of attendance for that month is $\frac{18}{20}x100 = 90\%$. There will be such 10 different percentages obtained for 10 months (summer holiday for 2 months).

We feel it necessary to determine a single percentage to represent all the 10 percentages. Such a number is known as a representative score or central tendency for all the 10 percentages.

Central Tendency is a statistical measure that identifies a single score as a representative of an entire distribution. The goal of Central Tendency is to find the single score that is most typical or most Representatives of Central Tendency.

They are: -

(i) Arithmetic mean or mean
(ii) Median and
(iii) Mode

i) Arithmetic Mean: -

Arithmetic man of a set of scores, commonly known as average, is calculated by adding all the scores in the distribution and dividing the sum by the number of scores. The mean $\overline{X}$ of a number of scores (x) is given by;

$$\overline{x} = \frac{\sum_{i=1}^{n} x2}{n}$$

If there are four scores say 2, 3,5,10 then the mean is

$$\overline{x} = \frac{2+3+5+10}{4} = \frac{20}{4} = 5$$

Mean for grouped data is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} fiXi}{\sum_{i=1}^{n} fi}$$

Where, fi = frequency of x_i

X_i = observation/class mark of c.i.

Example: In an ungrouped frequency distribution mean is determined as follows:

Scores x	Frequency (fi)	fxx
5	8	40
6	12	72
7	17	119
8	9	72
9	4	36
Total	$\sum f = 50 = N$	$\sum fx = 339$

∴ Mean = $\frac{\sum fX}{\sum f} = \frac{339}{50} = 6.78$

ii) **Median:**

The median of a set of numbers arranged in order of magnitude (ascending or descending) is the middle most value of the array. It divides the observations into exactly two parts.

(i) When the number of observations (n) is odd. The median is the value of the $\frac{n+1}{2}^{th}$ observation.

(ii) When the number of observation (n) is even, then the median is the mean of the $(\frac{n}{2})^{th}$ and the $(\frac{n}{2}+1)^{th}$ observations.

Example: Median of *3, 5, 7,* (13)*, 21, 29, 30 is 13*

and median of *2, 3, 4, 8, 9, 11, 12, 15 is*

$\frac{8+9}{2} = \frac{17}{2}$

$= 8.5$

iii) **Mode: -**The mode is that value of observation which occurs most frequently i.e. an observation with the maximum frequency is called the mode.

Example: - The mode of following data:

4, 6, 5, 9, 3, 2, 7, 6, 5, 6 is *6*

The readymade garment and shoe industries make great use of this measure of Central Tendency. Using the knowledge of mode, these industries decide which size of the product should be produced in large number.

6.4 Time tabling including Railway Time Table

The time table is a condensed statement which indicates the division of work and time in the school or college. It is a chart which indicates all the activities of an institution. It shows hours of work, kind of subjects, the allotment of rooms for various periods, the teachers at work and at rest, time of roll call, recreational time, time of interval, time of drill and time of games and other co-curricular activities. It has rightly been said 66 the schedule is the plug of the school which sets into motion its various activities and programmes.

i) Values of Time Table

There are different but important values that are served by time-table.
They are as follows; -

1. Utility of time and energy,
2. Develops moral values,
3. Ensures the attention to every subject and activity
4. Brings system of school life,
5. Ensure regular progress and increases efficiency and
6. Psychological value.

It removes the fatigue of teachers and taught which they may acquire during the long hours of teaching and learning-

In making the time-table there are several things which should be kept in mind. The preparation of the some depends much upon the ingenuity and skill of its maker. The framing of time-table is difficult and time absorbing job. Some of the important factors which should be kept in mind in framing the time table are given below

1. Types of school
2. Independence of fatigue
3. Time factor

4. Relative importance of the subject
5. Free period for teachers
6. Equitable distribution
7. Variety of class room etc.

It should be remembered that the time-table is a tool and not the master and as such it should never be allowed to dictate procedures and methods.

Implementation of time-table is not only for school only. It can be seen in different sectors also, like but stand and Railway Stations etc.

ii) **The Time-Table of APST Bus service** (***or your reference is given below)***

APST Bus Service Ph.:0360 2212338 0360 2351490				
Night Service				
Sl. No	**Name of Routes**	**Departure**	**Distance (KM)**	**Bus Fare (in Rs)**
1.	Itanagar–shilling (Daily Service)	5.00 pm	510	460/-
2.	Itanagar– Nyapin (Mon, Thu, Sat)	2.00 pm	325	400/-
3.	Itanagar– Aalo (Daily Service)	2.30 pm	325	350/-
4.	Itanagar– Daporijo (Daily Service from Nlg)	12 noon	432	470/-
5.	Itanagar - Yingkiong (Alternate day)	01.00 pm	436	550/-
6.	Itanagar– Seppa (Alternate day)	12.00 noon	385	420/-
7.	Itanagar–Khonsa (sun, Mon, Wed &thu)	1.00 pm	677	620/-
8.	Itanagar–Namsai (Daily Service)	1.00 pm	655	580/-
9.	Itanagar–Koloriang (Alternate day)	2.30 pm	334	400/-
DAY SERVICES				
10.	Itanagar–Guwahati (Daily Service)	6.00 am	395	350/-
11.	Itanagar–Pasighat (Daily Service)	6.00 am	270	250/-
12.	Itanagar–Tribin (Mon, Thu & Sat)	5.30 am	328	330/-
13.	Itanagar–Bomdila (Daily except Friday)	6.00 am	346	320/-
14.	Itanagar - Aalo (Daily Service)	5.30 am	325	350/-
15.	Itanagar–Sagalee (Sun, Tue, & Thu)	7.00 am	121	135/-
16.	Itanagar–Pakke Kessang (Mon, Wed & Fri)	7.00 am	160	200/-
17.	Itanagar– Mengio (Sun, Tue & Thu)	6.00 am	198	230/-
18.	Itanagar–Kimin/H-Camp (Daily except Fri)	12.00 noon	94	80/-
19.	Itanagar–Gerukhamukh(Daily except Sun)	12.00 noon	134	120/-
20.	Itanagar–Balijan (Daily except Sun)	12.30 pm	105	105/-
21.	Itanagar – Sonajuli (Daily except Sun)	01.00 pm	70	80/-
22.	Itanagar – Jote (Daily except Sun)	07.00 am	22	30/-

iii) Importance of Railway Time-Table

In our country more than 2.5 crores people travel by train every day. There are 8000 railway stations and 12617 trains running per day as per the record of 2015. The web of Indian railways stands at 2nd place in world. Railway is a complicated institution. Passengers and tourists must have some pre-information for planning their journey from where they can derive various information such as train running between two stations of their travel, time of arrival and departure of various trains all these detailed information can be obtained from railway time- table.

Railway time table contains various information about arrival and departure of train from particular stations, distance between two station, fare of various distances, availability of reservations, etc.

For reference, see the Railway Time table given below and see the information it has shown: -

1. Each table is given a serial number on the basis of which we recognize it.
2. Each time table is associated with the routes which are indicated on the headlines.
3. There are total thirteen columns in this time table giving information of 13 trains.
4. Time shown in the upper part shows arrival time and time shown in the lower part of box shows departure time i.e. ‘a’ demotes arrival and ‘d’ denotes departure.
5. If in a box, time is enclosed by a rectangle then it shows that train starts from that station at that time

&&&

Unit -7

Recreational Mathematics and Mathematical Reasoning

In fact, the teaching of mathematics should be carried out not only in the class room but it should also be carried out in the form of co-curricular activities. When the students are out of the class room, they feel themselves free and so, while playing, they learn many things. This thing may be done with the help of the mathematics library, mathematics laboratory, mathematics clubs, mathematics class room and such other associations or organisations that provide forum for the co-curricular activities of mathematics.

7.1 Setting of mathematics lab/Resource Room:-

Mathematics is a technical subject in which the objective of education in not only to gain knowledge but the important point is how to use that knowledge. And for that knowledge it is important that it should be gained through activities from daily life probably using local mathematics. Therefore a separate mathematics laboratory must be established as there is in science. A mathematics laboratory can faster mathematical awareness, skill, building, positive attitudes and learning by doing experiences in different branches of mathematical such as Algebra, Geometry, menstruation ,trigonometry ,co-ordinate Geometry, statistic ,probability ,etc. It is place where student can learn certain concept using concrete objects and verify many mathematical facts and properties using models, measurements and other activities. It will also provide the opportunity to the students to do certain calculations using tables, calculators, etc, and also listen or view certain audio –video cassettes, remedial instruments, and enrichments etc. of his / her choice on a computer. Thus mathematical will act as an individualised learning centre for the students. It provides opportunities for discovering, remedial instruction, reinforcement and enrichment. Mathematics laboratory will also provide an opportunity for the teacher to explain and demonstrate many mathematical concept facts and properties using concrete materials, models, chart etc. The teacher may also encourage students to prepare similar models and charts using materials like thermocol, cardboard etc. in the laboratory. The laboratory will also act as a forum for the teachers to discuses and deliberate on some important mathematical issue and problem of the day. It may also act as a place for teachers and the students to perform a number of mathematical celebrations and recreational activities.

The main goal of the mathematical education is to ‘Develop the Child’s resources to think and reason mathematically, to peruse assumptions to their logical conclusion and bindle abstractions (NCF-2005) .

To achieve this, a variety of methods and skill have to be adopted in the teaching learning situation.

One of the biggest challenges of mathematics teacher is to create and sustain interest in his students. This is a general feeling that mathematics is all about formulas and mechanical procedures. Under this circumstance, a mathematics laboratory will help teacher to reorient their strategies and make mathematics also an activity oriented programme in schools.

7.2 Equipments for mathematics laboratory

As the students will be involved in a lot of model making a activities under the guidance of the teacher, the smooth running of the mathematics laboratory will depend upon the oddments such as strings and threads, which cardboard, hard-cardboard, needle and pins, abacus and beads, number lines presented on rods, tape and rope , number flash cards, wall clock, squares, cube , cuboids , cone pyramid , cylinder etc , tin and plastic sheets, glazed paper , books, journals, computers reference books, mathematics dictionary , etc. in the library. CDs and film regarding teaching of mathematics especially on some selected topics must be there. Mathematics instruments set some geometry boxes, metre scales of 100cm and 50cm, measuring tape, diagonal scale, clinometers, and calculators are also the necessary equipments of mathematics laboratory.

7.3 Mathematics clubs

Like the other subject club and societies, there is a genuine place for a mathematics club in the school of today. Broadly speaking, it will be organised in the same way or on the same pattern as other subject club or even co- curricular societies. The mathematics club is no longer a vague idea; it has become a reality in many progressive schools. It gives right direction to the students. Establishment of the mathematics club is very useful for the mathematics students and also for those who have interest in mathematics.

Its purpose

1. Through mathematics club students come near teacher which helps in understanding each other and thus relationship between them becomes stronger.
2. Mathematics club attracts talented students towards mathematics.
3. Problem solving and searching attitude is developing in students.
4. It can arrange mathematical film strip shows and film shows for its member.
5. It can look after the mathematics section of the school magazine.
6. It helps presenting theatrical knowledge of mathematics by activity method.
7. It can provide a forum to those interested in mathematics for mathematical activities.
8. It will be provide to the student an opportunity of listening to experts and teachers from outside. They can be invited through this agency to address the students on different topics.
9. It invites and develops mathematical expression among the students.

10. The club will be a medium of developing student's interest in the subject.

Organisation of the club.

Head of the institute will be the patron and senior teacher of the subject will be in charge. Office bearer of the club should be selected from students to discharge various specific duties yearly or monthly donation should be taken from the members of the club. Membership of club should be open for all students because mathematics is a compulsory subject up to 10th class. If the number of member is very large, there can be too clubs are for the senior students and the other for junior students.

*1.chairmen/president

2. Vice-president
3. Secretary
4. Joint secretary
5. Cashier

<u>7.4 Progress of Generation, pattern recognition and inductive reasoning, process enable formation of hypothesis.</u>

"A lesson without the opportunity for learning to generalise is not a mathematics lesson."

J. mason

Process of Generalisation:-

Making Generalisation is fundamental to mathematics. Developing the skill of making generalisation and making it part of the student's mental disposition or habits of mind in learning and dealing with mathematics is one of the important goals of mathematics education. Making generalisation is a skill, vital in the functioning of society. It is one of the reasons why mathematics is in the curriculum. Learning mathematics (if though properly) is the best context for developing the skill of making generalisations

There are three meanings attached to generalisation from the literature. The first is as synonyms for abstraction. That is the process of generation is the process of finding and sending out (of properties) in a whole class of similar objects. His sense it is a synonym of abstraction. What is abstraction in mathematics? Abstraction is inherent to mathematics. It is a must for mathematics teachers to know and understand what this process is and what its producers are knowledge of it can enrich our relation of our own practice as well as guidance us and make us conscious of the type of learning activities we provide our students. The second meaning includes extensions (empirical or mathematical) of existing concept or a mathematical invention. Perhaps the most famous example of the better is the invention of non-Euclidean geometry. The third meaning define generalisation in terms of its product if the product of abstraction is a concept, the product of generalisation is a statement relating the concept that is a theorem.

Mathematical investigation and open-ended problem solving tasks are ways of engaging

students in making generalisation ----shorting number expression,
---teaching subtraction of integers
----polygons and algebraic expression.

Generalisation can be extended to symbols, something that elementary students can do and middle school students must do. Let us take an example of multiplication.

32	45	24	57
X 45	x 32	x 57	x 24

When students solve these question then focus your attention on the factors, asking the following question "what..

7.5 PATTERN INDUCTIVE REASONING

7.3 STRUCTURE OF MATHEMATICS: AXIOMS, DEFINITIONS AND THEOREMS.

During Euclid's time the Greek mathematicians thought of geometry as an abstract model of the world in which they lived. The notion of point, line, plane/surfaces and so on were derived from what was seen around them. An abstract geometrical notion of a solid object was developed after studies solid and space. A solid has shape, size and position and can be moved from an place to another. Its boundaries called surfaces, they separate one point of the space from another and said to have no thickness.

I) Definition:

Euclid summarised some statements a definitions as:

1. A point in that which has no part.
2. A line is breadth-less length.
3. The ends of a line are points.
4. The straight line is a line which lies evenly with points on itself.
5. Surface is that which has length and breadth only.
6. Edges of a surface are lines.

If you carefully study these definitions, you find that some of the terms like part, length, breadth, evenly etc need to be further explained clearly. For example, consider his definition of point. In this definition, 'a part' needs to be defined. Suppose if you define 'a part' to be that which occupies "area", a gain 'an area' needs to be defined. So, to define one thing, you need to define many other things and you may get a long chain of definition without end. For such reason mathematics agree to leave some geometric terms undefined like point, line, planes etc. A definition is a statement of the meaning of a term (a word, phrase or other set of symbols). It is sometimes challenging, common dictionaries contain lexical, descriptive definitions, all with different purposes and focus.

A term may have many different senses and multiple meanings, and thus require multiple definition.

In mathematics, a definition is used to give a precise meaning to a new term, instead of describing a pre-existing term. Definitions and axioms are the basis on which all mathematics is constructed.

iii) Axioms and Postulates:

Starting with his definition, Euclid assumed certain properties, which were not to be proved.

These assumptions are actually 'obvious universal truth' He divided them into two types: AXIOMS and POSTULATES. He used the term "postulates" for the assumptions that were specific to geometry common motion often called axioms, on the other hand, were assumptions used throughout mathematics and not specifically linked to geometry.

In modern mathematics there is no longer an assumption that axioms are 'obviously true' Axioms are merely 'background' assumptions we make. The best analogy i know is that axioms are the "Rule of the game"

In Euclid's geometry the main axioms are:

1. Things which are equal to the same thing are equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.
6. Things which are double to the same things are equal to one another.
7. Things which are halves of the same things are equal to one another.

These common notions refer to magnitudes of some kind.

Five Postulates of Euclid's are as follows:

1. A straight line may be drawn from any one point to any other point.

2. A terminated line can be produced indefinitely.

3. A circle can be drawn with any centre and any radius.

4. All right angles are equal to one another.

5. If a straight line falling on the two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced infinitely, meet on that side on which the sum it angles is less than two right angles.

iii) Theorem:

What is theorem? A theorem is a statement that can be demonstrated to be true by accepted mathematical operations and arguments. In general a theorem is an embodiment of same general principle that makes part of a larger theory. The process of showing a theorem to be correct is called a proof.

A theorem is also a logical consequences of the axioms in geometry, the 'propositions' are all theorems. They are derived using the axioms and the valid rules. A 'corollary' is a theorem that is usually considered as "easy consequences" of another theorem. What is or is not corollary is entirely subjective. Sometimes what an author thinks is a 'corollary' is deemed more important than the corresponding theorem.

7.4 Problem solving:

Process and its application in real life situations:

Problem solving method is an important and very popular method in teaching and learning of mathematics. In this method, the facilitator presents the problems to the students and they solve those problems in the classroom itself by using previous learned knowledge facts, laws, principles and formulae. The life is full of problems and the successful man in life is he, who is fully equipped with adequate knowledge and reasoning power to tackle these problems. We must teach the students how to think, to reason logically to solve a problem. The solution of these problems enables him to have a mastery over his environment of mathematical situations. The problems should be related to daily life so that the studies should be motivated to solve them. This method helps us by solving problems on each and every path of the progress of learning. Free environment should be provided in teaching learning process to solve the problems.

Process in problem solving:-

1. To understand the facts and their relations to the given problems.

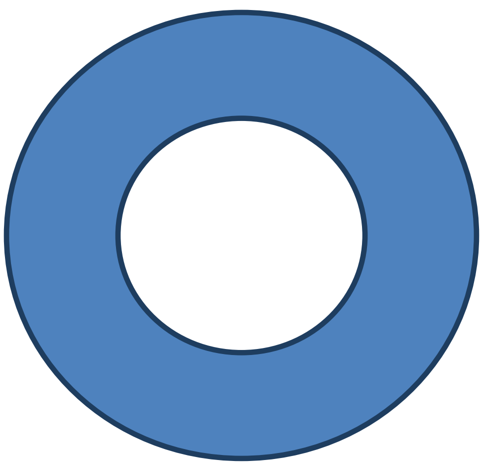
2. Analyse the problem.

3. To search the expected answer (formation of hypothesis).
4. Calculation to find the solution.
5. To find the solution and check the answer.

Example: place an X on the number line about 11/8 would be. Explain why you put your X where you did.

Notice that the task can be solved in a variety of ways- - for example with ruler or by folding a strip of paper. Studies will have to justify where they placed their mark. In the follow-up discussion, the teacher will be able to help the class refine ideas about fractions greater than 1 (for example, that 11 eights are equivalent to whole and 3 more eights)

Example: give values for X and Y.



Answer: X =

Mathematical puzzles:

Mathematical puzzles require mathematics in order to solve them. They have scientific rules, as do multiplayer games; mathematical puzzles do not usually involve competition between two or more players. Instead in order to solve such a puzzle, the solver must find a solution that satisfies the given conditions.

Logical puzzles:

Logical puzzles are a common type of mathematical puzzle conway's game of life and fractals are also considered mathematical puzzles, even though the solver only interacts with them by providing a set of initial conditions.

As they often include or require game like features or thinking, mathematical puzzles are sometimes also called mathematical games.

Example: put the numbers 1, 2, 3,4,5,6 and 7 in the circle so that each straight line of three numbers adds up to the same total.

UNIT –08

EVALUATION

Evaluation means modification of behaviour in a socially desirable way. It is the work of educator to define the goal, in which direction the behaviour is to be modified. This goal is commonly known as educational objectives. Before teaching, the teacher decides the educational objectives. Before teaching, the teacher decides the educational objectives depending on the subject, ability of the students, age of the students etc and social conditions in which the child will be taught.

After providing learning experiences, it is the duty of the educator to assess to what extent the objectives has been achieved and what desired changes have been taken place in the child's behaviour. The process by which we assess the effectiveness of learning experience and to what degree the educational objectives have been achieved is known as evaluation.

When one objective is achieved a new objective is fixed and suitable learning experience is provided to the child and evaluation is made to know the effectiveness of this learning experience in achieving the objectives.

After objectives have been identified and clarified for each area of evaluation, the next work is to select appropriate available tests or measures that will provide evidence about the growth and development of each objective in the child.

8.1 Various Kind of Test Items:

Test is a systematic procedure for observing persons and describing them with either a numerical scale or a category system. Thus test may give either qualitative or quantitative information.

Test commonly refers to a set of items or questions under specific conditions. Testing is a broad term that indicates exposing students to questions, quiz, puzzles, problems or situations that will make students to 'bring out' certain behaviour (Knowledge, skill, competencies) for observation. A test creates opportunity to confirm what transpire between teachers and learners.

<u>Type of Test</u>

(i) Essay Type

(ii) Objective Type

<u>i)</u> **<u>Essay type Test</u>** : It is an item format that requires the student to structure a rather long written response up to several paragraphs.

<u>Characteristics of Essay Test</u>

- ❖ Generally Essay test contain more than one question in the test.
- ❖ Essay tests are to be answered in written only.
- ❖ Essay test require completely long answers.
- ❖ Essay tests are attempted on the basis of recalling the memory.
- ⇨ Note: The short answer type tests in which short answers items requires the examinee to respond to the item with a word, short phrase, number or a symbol.

<u>ii)</u> **<u>Objective Type Test</u>**: Any test having clear and unambiguous scoring criteria is called objective type test, i.e., test that can be objectively scored.

Behaviours focused by test items are

- ➢ Remember : Factual, conceptual, procedural, meta cognitive knowledge
- ➢ Understanding : Translation, interpretation and Extrapolation
- ➢ Apply: Using knowledge to solve practical problems and something hypothetical examples may help.
- ➢ Analyze: Cause and effect relationships involving facts, laws, principles and providing except able explanations for these.
- ➢ Evaluate: Make decisions and draw inferences and conclusions.
- ➢ Create: Bringing out new idea, method, product in the form of writing, drawing and patterns.

<u>Characteristics of Objective Test items</u>:

- ❖ There are usually two parts : Stem and options
- ❖ They give room to both the tester and the testee to be free from influencing the test score: Objective scoring.
- ❖ There is a large number of items on the test.
- ❖ The type of response expected from the testees can be easily scored as right or wrong.
- ❖ Problems involving reasoning and judgement are presented.
- ❖ It could be used for checking memory of facts.
- ❖ Check recognition but not recall.
- ❖ Free from subjectivity.

<u>Structure of Objective Test Items</u>

(i) True or False / Yes or No

(ii) Matching Type

(iii) Fill in the Gaps / Completion item type

(iv) Multiple Choice Test

(v) Interpretive Exercise

The nature of the stem may require two forms of responses:

(a) Supply type response format requires that the testees either supply short answers or complete a sentence.

(b) Selection type provides the options from which tested pick the one they consider most appropriate.

The natures of the options are of two types:

(a) Correct answer type are applicable to simple questions in the nature of what, who, where or when based on facts, laws and principles.

(b) Best answer type is used for more complex questions which present options that are correct to various degrees and so there may not be an absolutely correct option. Questions here are of the why and how nature and only one of them is most appropriate.

8.2 **Criteria of good test** paper:

1) **Objective Base**: Questions of test paper should be based on pre-determined objectives and should be framed in such a way that it tests the objective effectively.
2) **Instruction:** It should specify a particular task through the instructions. For this appropriate directional words should be used and structures situation should be given.
3) **Scope**: It should indicate the limit and the scope of the answer (length of the answer) in accordance with the estimated time and make allotted to it.
4) **Content**: The question should test the same area of content which it intends to test.
5) **Form of the Question**: The form of questions depends on the objective and the content area to be tested. Some forms are better than the others for testing certain abilities.
6) **Language**: A good test paper is formed in a clear, precise and unambiguous, well within the comprehensive of the students.
7) **Difficulty Level**: A question should be written keeping in view the level of the students or for whom it is meant. The difficulty of the question depends upon the ability to be tested the content area to be tested and the time available to answer it.
8) **Discriminating Power**: A good test paper discriminate between bright students and others.

8.3 Construction of Test Paper

Construction of test paper is an art. There are many aspects keeping in mind when a test paper is constructed.

A. General Guidelines

1. Make sure you have the latest version of the syllabus and you are familiar with the assessment criteria.
2. Before and after setting the paper, check that all the test items are based on the respective syllabus and that the items are graded in difficulty.
3. Do not use material reflecting race, ethnic or sex bias.
4. Check that the duration of the examination is entered correctly on the paper and that the time allotted is sufficient to enable the students complete the paper and revise their work.

5. Pass on the finalized draft of the paper to an external reviser who has to proof read the text again, ensure that no test item is out of syllabus. Check that all set tasks are workable and that the paper can be completed in set time.
6. Make necessary changes in the test paper and the marking scheme as advised by the reviser.
7. Hand in the marking scheme together with the test paper for printing.
8. Examine printed papers for printing defects and for any error corrige that may be required.

B. <u>Layout</u>

1. The layout of the paper should be as clear as possible to make it as student friendly as possible. For write on papers enough space for working or writing must be provided.
2. Instructions to candidates should be clear and unambiguous. They should be presented in bold type.
3. Whenever possible, use a straight forward and consistent format with regular line length.
4. Use typesetting features such as bold, italics, indentation or boxes effectively to help candidates focus their attention on the task.
5. Long complex questions are best split up by the use of subsidiary numbering systems.
6. Structured questions should follow a graded and logical sequence.
7. The information contained on a page should be well structured through the appropriate use of headings and sub-headings. This would help candidates organize text in advance of reading.
8. Check that the diagrams, pictures or photographs used are necessary, helpful and of high quality.
9. Place the text close to the relevant diagrams or pictures to enable the candidates relate the two effectively. Comprehension text and questions should be set on the same page or on adjacent pages.
10. Ensure that marks assigned for each item / exercise / section are clearly indicated on the paper.

C. Sentence Construction

1. Use the simplest language and structure possible to convey clearly and unambiguously the meaning of the question.
2. Split down even relatively short sentences if they contain a lot of condensed information.
3. Do not use the passive if it can be avoided because it can make a sentence impersonal and complex. Avoid also using the conditional form 9Sentence starting with 'if') and the double negative.
4. Eliminate superfluous words and any abstract and metaphorical language which is not necessary.
5. Make sure that introductory statements in questions contain only the information which is required for answering those questions relevantly.

D. Specification Grids

1. The writing of test items should be guided by a carefully prepared set of test specifications.
2. The specification grid relates outcomes to content and indicates the relative weight to be given to each of the various areas.
3. Specification grid indicates
    (i) The learning outcomes to be tested
    (ii) The subject matter or content area
    (iii) The assigned weighting to the learning outcomes and content areas in terms of their relative importance.
4. The learning outcomes to be tested include (a) recall of knowledge (b) intellectual abilities or skills (understanding, application etc) (c) general skills (eg. Practical, performance, communication), (d) attitudes, interests, appreciations.

E. Constructing relevant test items

The items used could be either selection type or supply type items. The selection type items present the students with a set of possible responses from which they are to select the most appropriate answer. The supply- type item requires students to create and supply their own answers.

Selection type items include:

Multiple choice, True-False, matching, interpretative exercises. The preparation of good selection-type items is difficult and students can get a proportion of answers correct by guessing.

Supply type items include:

Short answer, Essays (Restricted responses, unrestricted responses). Supply type items are easier to construct but more difficult to score.

1. Use the item types that provide the most direct measures of student performance specified by the learning outcome.
2. Avoid grammatical inconsistencies that eliminate wrong answers.
3. Avoid specific determiners that make certain answers probable (e.g. sometimes) and others impossible (i.e., always).
4. Avoid stereotyped or textbook phrasing of correct answers.
5. Avoid material in an item that aids in answering another item.
6. Avoid tick questions that might cause a knowledgeable student to focus on the wrong aspect of the task.
7. Ensure that the difficulty level matches the intent of the learning outcomes and the age group to be tested.
8. Ensure that there is no disagreement concerning the answer. Typically the answer should be one that experts would agree on the correct or best answer.
9. The number of test item depends on the age of the students tested, the time available for testing, type of test items used and on the type of interpretation to be made.
10. Give due consideration to the best arrangement of the test items, where possible, all items of the same type should be grouped together. The items should be arranged in terms of increasing difficulty.
11. For True-False items make sure that:
    - ❖ Each statement is unequivocally judged true or false.
    - ❖ The statement is brief and stated in simple, clear language.
    - ❖ Negative statements are used sparingly and double negatives are avoided.

- ❖ The statements are free of clues to the answer (i.e., verbal clues, length)
- ❖ There is approximately an equal number of true and false statements.
- ❖ True and False items are arranged in random order.

12. For matching items ensure that
    - ❖ Items are based on homogeneous materials.
    - ❖ The items appear in the same page.
    - ❖ The instructions clearly state the basis for matching and that each response can be used one, more than once or not at all.
13. For multiple-choice items make certain that
    - ❖ The stem of the item present a single, clearly formulated problem
    - ❖ The stem is stated in simple, clear language.
    - ❖ The stem is worded so that there is no repetition of material in the alternations.
    - ❖ The stem is stated in positive form where ever possible.
    - ❖ If negative wording is used in the stem, it is emphasized in bold or by undertaking.
    - ❖ The intended answer is correct or clearly best.
    - ❖ All alternatives are grammatically consistent with the step and parallel in form.
    - ❖ The alternative "all of the above" or "none of the above" are used only when appropriate.
14. For short-answer items ensure that :
    - ❖ The item calls for a single, brief answer
    - ❖ The item has been written as a directed question or a well stated incomplete sentence.
    - ❖ The desire response is related to the main point of the item
    - ❖ Clues to the answer have been avoided (eg 'a' or 'an' length of the blank)
    - ❖ The units and degree of precision is indicated for numerical answer.
15. For Essay question make sure that:
    - ❖ Questions starting with "who", "what", "where", "name", "list" are awaited as these terms limit the response.
    - ❖ Questions demanding higher order skills such as comparing, interpreting, inferring, creating etc are used.

8.4 Open Ended Question

How did we learn mathematics in our school days? For most of us, looking back we may find that Mathematics was all about solving sum and getting the correct answers. There was and is a notion that in Mathematics, every question has only one correct answer.

The moment the teacher has a question, the student becomes absorbed in the search for the right answer. In this process, she tries to recall and blindly follow the methods or procedure that she was taught in order to obtain the correct answer. Even, if the child succeeds in solving the problem, she has not really paid any attention to the thinking process by which solutions are obtained. This does not lead to the understanding that the problem solving activity is supposed to bring about. We need children to break out of this habit of searching for the single answer or the correct method. We need them to think about the problem, to pay attention to processes and strategies of obtaining the solution, and to understand the mathematics underlying the problem.

One of the powerful ways to bring about his change in the students' approach to mathematics is to introduce question which have more than one correct response, which are not product oriented and which require thinking on part of children. Such question can be described as "Open Ended Question".

Why Open Ended Questions?

These questions pose a challenge and engage the students minds. For example, take a problem of what should be added to 22 to get 40? Here is only one answer. Many students and teachers may insist that there is only one method – by subtracting 22 from 40. But we could modify this problem and ask- which numbers would you add to get 40.

___________ + ___________ = 40

___________ + ___________ = 40

___________ + ___________ = 40

In how many different ways can you fill up the blanks?

The situation changes. Firstly, the child is facing a question for which there can be many answers. Different children may suggest different answers. Secondly, there is a new challenge: the child begins to look for all possible answers. Then the child faces the next challenge: finding out exactly how many answers are possible. Finding all possible correct answers is much more challenging than finding the only correct answer. Such questions not only develop the child's reasoning process, but also motivate her to look at different aspects of the same problem.

A problem of this kind gives much more scope to build the child's understanding. Note that one cannot solve this problem by mechanically following a rule but only by understanding. The child begins the pattern of the numbers which add up to 40. For example,

39 + 1

33 + 7

32 + 8

34 + 6

In doing so, she gets a better insight into the structure of addition problem. When she understands addition better, she will also have a way of verifying problems that she solves using the standard method or algorithm.

When a child learns Mathematics through solving such open ended problems, she struggles with exploration and creating strategies, instead of relying on memorization or any predetermined rule to search for the solutions. Solving a problem for which a solution is not immediately apparent gives students a chance to take a risk, which in turn can give them more confidence.

Open ended questions give a chance to every child in the class to express her views because of the possible multiple solutions. The child is not worried if her answer is different from another child's. Both may be correct. She was to rely more and more on her own judgement about whether the answer is right. This too helps in gaining confidence and removing the fear of Mathematics. Hence such questions offer pupils room for their own decision making and a natural way of thinking and problem solving.

8.5 Preparation of Objective Based Questions:

In any evaluation system / process, effective evaluation depends upon the checking material used in the examination. There is a need to develop a new kind of question papers which includes objectivity, comprehensiveness and reliability. Checking this type of test saves the time, labour / energy. It helps in better evaluation of students.

Four type of Objective based questions may be prepared i.e., knowledge based, understanding based, application based and skill based.

1. Knowledge based: Knowledge based questions includes definitions,, formulae, symbols and fill in the blanks. Right and wrong answers can also be determined through multiple choice questions. Students' knowledge is evaluated by following this kind of questions.

(i) Define circle with the help of a diagram.

(ii) What is Circular area?

(iii) What are congruent figures? Define.

(iv) Explain SAS congruency.

(v) Two line segments are congruent if they have __________ length.

(vi) Put (√) mark on correct answer of the MCQ.

(a) Sum of four angles of a quadrilateral is

320^o, 180^o, 360^o, 270^o.

(vii) How many types of quadrilateral are there?

(viii) How many non-collinear points are required to make a quadrilateral?

(ix)

2. Understanding based : In this section questions include number of parts of any geometrical figure, their relation of formula of relation, comparison in parts, difference between two figures, finding answers by measurement, when a figure is changed into another, explaining various shapes with the help of a figure or all the other questions in which explanation of a questions asked. For example.

(i) When two angles are congruent? Explain with the help of a diagram.

(ii) When does a rhombus become square? Explain.

(iii) Name all the pairs of adjacent angles of a parallelogram ABCD.

(iv) In a circle, how many radii can be drawn?

(v) What is the longest chord of a circle called?

(vi) What is the relation between diameter and radius of a circle?

(vii) Two line segments, given below are congruent. If AB = 3.8 cm, find the length of CD.

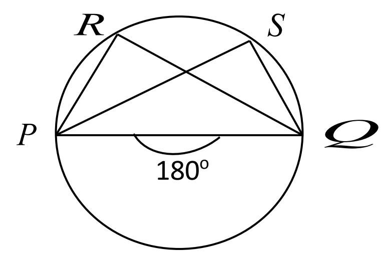
(viii) ABCD is a parallelogram. What name will you give to this quadrilateral after getting additional facts given below:

(a) AB = AD (b) Angle ABC = 90^o (c) AB = BC and Angle DAB = 90^o.

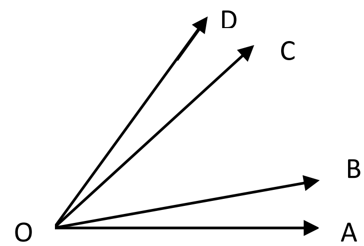
3. Application based : It includes questions based on the use of formula or a rule, finding out the result while looking at a figure or when one result and the rule is given, fill in the blanks can be included. In this type of series of questions,, unknown values are found by the help of known rules, formulae, hints, concepts, equation or figures. For Example.

(i) If Mohan want to buy a quadrilateral plot having angles of measure 45^o, 95^o , 90^o and 135^o . Is it possible to buy such a plot? If not, why?

(ii) What will the radius of a circle if diameter is 28 cm?



(iii) In the figure, is $\angle PRS = \angle PSQ = 90^o$



(iv) In the given figure, $\angle AOB \cong \angle COD$
Can we say $\angle AOC \cong \angle BOD$?

(v) The ratio of four angles of a quadrilateral is 1:2:3:4. Find out the measure of four angles.

(vi) Is it right to say, "two triangles are congruent". Explain with reason.

(vii) Give two examples each of rectangular and square shaped things from your environment.

4. Skill based : In skill based, questions include the construction of a figure, finding an unknown thing with the help of figure; explaining by constructing a figure, constructing a figure and labelling it; finding the shape, size and position of a figure by measurement; making a figure on a paper and finding its area with the help of given data, etc for Example:
   (i) Draw a circle on tracing paper and draw it diameter AB.
   (ii) On a graph paper, construct a rectangle of length 5 cm and breadth 8 cm.
   (iii) Show with the figure, congruency (SAS) of two triangles.
   (iv) With O as Centre and of radius 5 cm, draw a circle.
   (v) Show with figure, congruency of two line segments.
   (vi) Construct a square with side 5.5 cm.
   (vii) By measuring the side of figure PQRS given below, tell the kind of quadrilateral. Put (√) sign on the correct answer.
      (a) Rhombus
      (b) Parallelogram.

8.6 DIAGNOSTIC TEST ON VARIOUS LEARNING POINTS AND ORGNIGINGREMEDIAL TEACHING

i) Meaning: The term diagnosis used in medical sciences has also a role in Educational field. According to R. G. Mishra for a doctor, "Diagnosis means a careful and extensive observation of the patient under controlled conditions with a view to examine symptoms to identify diseases and then suggest remedies". For this purpose he uses a number of sophisticated instruments like Clinical thermometer, microscope, stethoscope etc which help him in accurate observation. So also diagnosis has many similar implications for a teacher. When a teacher finds that during a course of instruction in spite of adopting various modern methods, devices, the pupil is facing recurring difficulties, he wants to diagnose the place of difficulty. This means to discover the specific area of weakness of the pupil in a given course of instruction. After this the teacher has to apply remedial measures.

Educational difficulties may be due to organic defects or functional orders. Educational diagnosis is more concerned with functional disorder. Many times the normal children face great difficulty with various aspects of the school situation. It may be due to faulty habit formation, lack of interest or an educational poor-home environment. This educational diagnosis will circumscribe a large variety of field with students.

The main difference in the use of diagnostic tests between conventional school and modern schools is, the former type of school have limited the idea of diagnosis to certain academic knowledge and skill, where as the later type of school have extended the idea of all round growth and development.

According to F. J. Schonell, "the diagnostic test is constructed not to access level but to reveal difficulties is school subjects. These tests are also helpful to teacher in evaluating classroom teaching.

Again, if the errors, learning difficulties are not checked from the beginning, it will continue to increase in geometrical progression and, it will be very difficult and time consuming to know the strength and weakness at higher level and apply remedial measures.

Mathematics is a subject in which one topic depends on many other topics. If some difficulties are found with pupil at the beginning and it is not cured, then defects will go on increasing day by day and the child will fear to attend the Mathematics class. So, from the beginning the difficulties should be removed by identifying those through diagnostic test.

iii) How to Prepare Diagnostic Test

It is more difficult to prepare a diagnostic test than to prepare an Achievement test. Suppose a teacher marked that most of his pupils are unable to work out problems on addition of fractions, then he will develop a diagnostic test on the addition of fractions observing following steps:

1. Planning of the Tests: When the teacher has decided to construct a diagnostic test on addition of fractions, then he has to make a suitable plan for constructing the test. In this step, he has to consider:
   (a) The class for which he will construct the test

(b) What are the different types of additions of fractions the student have learnt?
(c) Approximate number of questions to be included.
(d) What will be the value of each question?
(e) What will be the method of scoring the test?
(f) What will be the duration of the test?
(g) The experts and mathematics teachers are to be consulted for preparing the test.
(h) How the items will be analysed.
(i) What types of remedial measures will be taken.

2. Preparation of Learning Units and Sub-Units: In consultation with the experience mathematics teachers, experts the teacher will prepare different learning units and sub-units.

Sl. No.	Measure Learning units or concepts	Learning Sub-units or concepts	Examples	No. of Questions
1	Addition of Proper fraction having same denominator	1. Sum of Proper fraction having same numerators and same denominators 2. Sum of two proper fractions having same denominators and different numerators sum being less than one	$\frac{1}{2}+\frac{1}{2}=?$ $\frac{2}{5}+\frac{1}{5}=?$ $\frac{1}{7}+\frac{2}{7}=?$	1 2
2	Proper fractions having different denominators	1. Sum of Proper fractions in which one denominator is the LCM of all denominators	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=?$ $\frac{1}{3}+\frac{1}{12}+\frac{1}{6}=?$	2
3	Improper fractions having same denominators			
4	Mixed fractions			

Like wise different learning units and for each learning unit different sub units will be prepared for each sub – unit. At least one question will be constructed by the teacher. These questions will be subjected to critical revision in consultation with different experts and teachers in Mathematics.

3.Trying out the test : After preparing the test, the teacher will try out the test to :

(a) Modify weak, defective and ambiguous items.
(b) Reject double answer questions.
(c) Determine tentative time limit for final test.
(d) Find out difficulty value of each item
(e) Find out other difficulties if any.

After first try out of the test, the teacher will rectify the defects and will find out difficulty value of each item.

Difficulty value of an item

After scoring the pupils working sheet, all the items of the test will be subjected to item analysis. The main function of item analysis is to find out difficulty value of each item. So for each item, total number of correct responses and total number of omissions will be found out. Then the difficulty value of each item will be calculated by the formula.

$D = \frac{R}{N-U} \text{ x } 100$ where D stands for difficulty value of an item

R is the number of correct answer for the item.

N is the total number of pupils

U is the number of pupils omitting the items.

After finding the difficulty value of all the items, the items will be subjected to critical analysis.

3. Evaluating of Test: In this stage, the validity, reliability, scoring procedure, analysis chart etc will be found out. Then from the analysis chart, the item which has been correctly answered by less than 20 % of students will be sorted out an analysed.

After scoring the test, the teacher will try to identify the type of common errors committee by the students, over whom the test was administered.

Suppose question number 7 has been answered corrected only by 17% of students, then this is a typical question in which most students are weak. So, the teacher has to see what sub content this question carries. Likewise all other questions answered by less than 20% of students will be sorted out and corresponding sub-concepts will be sorted out by the teacher.

iv) Remedial Teaching

When the teacher will come to know, the specific weakness of the pupils, he will take measures to rectify them and will take precautionary measures for beginners so that those errors or concepts may be understood to them. As some medicine cannot be given to every patient suffering from a common disease. Similarly, all students have to be given separate remedial teaching as per their individual requirements.

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