JEE Main 2025 (April)
Chapter-wise Qs Bank

Mathematics

Content

Questions

Solutions

Q1. Let α and β be the roots of $x^2+\sqrt{3x}-16=0$, and γ and δ be the roots of $x^2+3x-1=0$. If $P_n=\alpha^n+\beta^n$ and $Q_n=\gamma^n+\delta^n$, then $\frac{\mathrm{P}_{25}+\sqrt{3\mathrm{P}_{24}}}{2\mathrm{P}_{23}}+\frac{\mathrm{Q}_{25}-\mathrm{Q}_{23}}{\mathrm{Q}_{24}}$ is equal to

(1) 3

(2) 4

(3) 5

(4) 7

Q2. For $t>-1$, let α_t and β_t be the roots of the equation
$\left((t+2)^{\frac{1}{7}}-1\right)x^2+\left((t+2)^{\frac{1}{6}}-1\right)x+\left((t+2)^{\frac{1}{21}}-1\right)=0$
If $\lim_{t\to-1^+}\alpha_t=a$ and $\lim_{t\to-1^+}\beta_t=b$, then $72(a+b)^2$ is equal to ________.

Q3. Let $P_n=\alpha^n+\beta^n, n\in\mathbf{N}$. If $P_{10}=123, P_9=76, P_8=47$ and $P_1=1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(1) $x^2-x+1=0$

(2) $x^2+x-1=0$

(3) $x^2-x-1=0$

(4) $x^2+x+1=0$

Q4. If the set of all $\mathrm{a}\in\mathrm{R}-\{1\}$, for which the roots of the equation $(1-a)x^2+2(a-3)x+9=0$ are positive is $(-\infty,-\alpha]\cup[\beta,\gamma)$, then $2\alpha+\beta+\gamma$ is equal to _______ .

Q5. Consider the equation $\mathrm{x}^2+4\mathrm{x}-\mathrm{n}=0$, where $\mathrm{n}\in[20,100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

(1) 7

(2) 8

(3) 6

(4) 5

Q6. Let the set of all values of $\mathrm{p}\in\mathbb{R}$, for which both the roots of the equation $x^2-(p+2)x+(2p+9)=0$ are negative real numbers, be the interval $(\alpha,\beta]$. Then $\beta-2\alpha$ is equal to

(1) 0

(2) 9

(3) 5

(4) 20

Q7. The number of real roots of the equation $\mathrm{x}|\mathrm{x}-2|+3|\mathrm{x}-3|+1=0$ is :

(1) 4

(2) 2

(3) 1

(4) 3

Q8. The sum of the squares of the roots of $|\mathrm{x}+2|^2+|\mathrm{x}-2|-2=0$ and the squares of the roots of $x^2-2|x-3|-5=0$, is

(1) 26

(2) 36

(3) 30

(4) 24

Q1. Let $z \in C$ be such that $\frac{z^2+3i}{z-2+i} = 2 + 3i$. Then the sum of all possible values of z^2 is

(1) $19 - 2\mathrm{i}$

(2) $-19 - 2\mathrm{i}$

(3) $19 + 2i$

(4) $-19 + 2i$

Q2. Let the product of $\omega_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $\omega_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ be $\alpha + i\beta$, $\mathrm{i} = \sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha + \beta$ respectively.

(1) 140

(2) 130

(3) 160

(4) 150

Q3. Among the statements
(S1) : The set $\{\mathrm{z} \in \mathbb{C} - \{-\mathrm{i}\} : |\mathrm{z}| = 1$ and $\frac{\mathrm{z}-\mathrm{i}}{\mathrm{z}+\mathrm{i}}$ is purely real$\}$ contains exactly two elements, and (S2) : The set $\{\mathrm{z} \in \mathbb{C} - \{-1\} : |\mathrm{z}| = 1$ and $\frac{\mathrm{z}-1}{\mathrm{z}+1}$ is purely imaginary contains infinitely many elements.

(1) both are incorrect

(2) only (S1) is correct

(3) only (S2) is correct

(4) both are correct

Q4. Let $\mathrm{A} =$
$\left\{\theta \in [0, 2\pi] : 1 + 10\,\mathrm{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0\right\}$.
Then $\sum_{\theta \in A} \theta^2$ is equal to

(1) $\frac{21}{4}\pi^2$

(2) $8\pi^2$

(3) $\frac{27}{4}\pi^2$

(4) $6\pi^2$

Q5. Let $A = \{z \in C : |z - 2 - i| = 3\}$, $B = \{z \in C : \mathrm{Re}(z - iz) = 2\}$ and $S = A \cap B$. Then $\sum_{z \in S} |z|^2$ is equal to ________ .

Q6. Let z be a complex number such that $|z| = 1$. If $\frac{2+\mathrm{k}^2\mathrm{z}}{\mathrm{k}+\bar{\mathrm{z}}} = \mathrm{kz}, \mathrm{k} \in \mathbf{R}$, then the maximum distance of $\mathrm{k} + \mathrm{ik}^2$ from the circle $|\mathrm{z} - (1 + 2\mathrm{i})| = 1$ is:

(1) $\sqrt{5} + 1$

(2) 2

(3) 3

(4) $\sqrt{3} + 1$

Q7. If $z_1, z_2, z_3 \in C$ are the vertices of an equilateral triangle, whose centroid is z_0, then $\sum_{\mathrm{k}=1}^{3} (\mathrm{z}_\mathrm{k} - \mathrm{z}_0)^2$ is equal to

(1) 0

(2) 1

(3) i

(4) $-$i

Q8. If the locus of $z \in \mathrm{C}$, such that
$\mathrm{Re}\left(\frac{z-1}{2z+\mathrm{i}}\right) + \mathrm{Re}\left(\frac{\bar{z}-1}{2\bar{z}-\mathrm{i}}\right) = 2$
is a circle of radius r and center (a, b) then $\frac{15ab}{r^2}$ is equal to :

(1) 24

(2) 12

(3) 18

(4) 16

Q9. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{\mathrm{k}=1}^{\mathrm{n}} \left(\alpha^\mathrm{k} + \frac{1}{\alpha^\mathrm{k}}\right)^2 = 20$, then n is equal to

Q1. Let $a_1, a_2, a_3 \ldots$ be in an A.P. such that $\sum_{\mathrm{k}=1}^{12} \mathrm{a}_{2\mathrm{k}-1} = -\frac{72}{5}\mathrm{a}_1, \mathrm{a}_1 \neq 0$. If $\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{a}_{\mathrm{k}} = 0$, then n is:

(1) 11

(2) 10

(3) 18

(4) 17

Q2. Let a_{n} be the n^{th} term of an A. P.
If $S_n = a_1 + a_2 + a_3 + \ldots + a_n = 700, a_6 = 7$ and $S_7 = 7$, then a_{n} is equal to :

(1) 56

(2) 65

(3) 64

(4) 70

Q3. Let $a_1, a_2, a_3, \ldots$ be a G. P. of increasing positive numbers. If $\mathrm{a}_3\mathrm{a}_5 = 729$ and $\mathrm{a}_2 + \mathrm{a}_4 = \frac{111}{4}$, then $24\left(a_1 + a_2 + a_3\right)$ is equal to

(1) 131

(2) 130

(3) 129

(4) 128

Q4. The sum $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \ldots$ upto ∞ terms, is equal to

(1) $6e$

(2) $4e$

(3) $3e$

(4) $2e$

Q5. The number of terms of an A.P. is even; the sum of all the odd terms is 24 , the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

(1) 4

(2) 10

(3) 6

(4) 8

Q6. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP's in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

(1) 600

(2) 450

(3) 630

(4) 540

Q7. Let x_1, x_2, x_3, x_4 be in a geometric progression. 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3 x_4 then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1x_2x_3x_4)$ is :

(1) 72

(2) 18

(3) 36

(4) 216

Q8. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309 , then the sum of its first nine terms is :

(1) 760

(2) 755

(3) 750

(4) 757

Q9. If the sum of the first 10 terms of the series $\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \ldots$ is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to______

Q10. The sum $1 + 3 + 11 + 25 + 45 + 71 + ..$ upto 20 terms, is equal to

(1) 7240

(2) 7130

(3) 6982

(4) 8124

Q11. $1+3+5^2+7+9^2+\ldots$ upto 40 terms is equal to

(1) 43890

(2) 41880

(3) 33980

(4) 40870

Q12. If the sum of the first 20 terms of the series
$\frac{4.1}{4+3.1^2+1^4}+\frac{4.2}{4+3.2^2+2^4}+\frac{4.3}{4+3.3^2+3^4}+\frac{4.4}{4+3.4^2+4^4}+\ldots$
is $\frac{m}{n}$, where m and n are coprime, then $m+n$ is equal to :-

(1) 423

(2) 420

(3) 421

(4) 422

Q13. Let $A=\{1,6,11,16,\ldots\}$ and $B=\{9,16,23,30,\ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A\cup B)$ is

(1) 3814

(2) 4027

(3) 3761

(4) 4003

Q14. If $\dfrac{1}{1^4}+\dfrac{1}{2^4}+\dfrac{1}{3^4}+\ldots\ldots\infty=\dfrac{\pi^4}{90}$,
$\dfrac{1}{1^4}+\dfrac{1}{3^4}+\dfrac{1}{5^4}+\ldots\ldots\infty=\alpha$,
$\dfrac{1}{2^4}+\dfrac{1}{4^4}+\dfrac{1}{6^4}+\ldots\ldots\infty=\beta$,
then $\frac{\alpha}{\beta}$ is equal to

(1) 23

(2) 18

(3) 15

(4) 14

Q1. The number of sequences of ten terms, whose terms are either 0 or 1 or 2 , that contain exactly five 1 s and exactly three 2 s , is equal to

(1) 360

(2) 45

(3) 2520

(4) 1820

Q2. Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O . In the first quadrant, $\mathrm{P}_1, \mathrm{P}_2, \ldots . \mathrm{P}_{12}$ are 12 points on line L_1 and $Q_1, Q_2, \ldots . . Q_9$ are 9 points on line L_2. Then the total number of triangles, that can be formed having vertices at three of the 22 points $\mathrm{O}, \mathrm{P}_1, \mathrm{P}_2, \ldots \mathrm{P}_{12}$, $\mathrm{Q}_1, \mathrm{Q}_2, \ldots . \mathrm{Q}_9$, is:

(1) 1080

(2) 1134

(3) 1026

(4) 1188

Q3. From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is

(1) 165

(2) 155

(3) 145

(4) 135

Q4. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

(1) 230

(2) 220

(3) 200

(4) 210

Q5. If the number of seven-digit numbers, such that the sum of their digits is even, is $m \cdot n \cdot 10^{\mathrm{n}}$; $m, n \in \{1,2,3, \ldots, 9\}$, then $m+n$ is equal to ________

Q6. Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P. If $p+q=126$, then the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{n}=1$ is :

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

(3) $\frac{\sqrt{7}}{4}$

(4) $\frac{1}{\sqrt{2}}$

Q7. For $n \geq 2$, let S_n denote the set of all subsets of $\{1, 2 \ldots\ldots, n\}$ with no two consecutive numbers. For example $\{1,3,5\} \in S_6$, but $\{1,2,4\} \notin S_6$. Then $n\left(S_5\right)$ is equal to ________

Q8. The largest $n \in N$ such that 3^n divides 50 ! is:

(1) 21

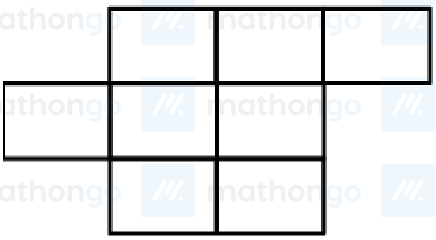
(2) 22

(3) 20

(4) 23

Q9. Let m and n, $(m < n)$ be two 2-digit numbers. Then the total numbers of pairs (m, n), such that $\gcd(m, n)=6$, is ________

Q10. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :



(1) 5880

(2) 960

(3) 840

(4) 5760

Q1. The term independent of x in the expansion of $\left(\frac{(x+1)}{(x^{2/3}+1-x^{1/3})}-\frac{(x+1)}{(x-x^{1/2})}\right)^{10}$, $x>1$ is:

(1) 210

(2) 150

(3) 240

(4) 120

Q2. The remainder when $\left((64)^{(64)}\right)^{(64)}$ is divided by 7 is equal to

(1) 4

(2) 1

(3) 3

(4) 6

Q3. The sum of all rational terms in the expansion of $(2+\sqrt{3})^8$ is

(1) 16923

(2) 3763

(3) 33845

(4) 18817

Q4. For an integer n ≥ 2, if the arithmetic mean of all coefficients in the binomial expansion of $(x+y)^{2n-3}$ is 16 , then the distance of the point $P\left(2n-1, n^2-4n\right)$ from the line $x+y=8$ is:

(1) $\sqrt{2}$

(2) $2\sqrt{2}$

(3) $5\sqrt{2}$

(4) $3\sqrt{2}$

Q5. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, n $\in$ N, if the ratio of 15^{at} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then the value of ${}^nC_3$ is:

(1) 4060

(2) 1040

(3) 2300

(4) 4960

Q6. The product of the last two digits of $(1919)^{1919}$ is

Q7. The number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1016}$ is

(1) 127

(2) 130

(3) 129

(4) 128

Q8. If $\sum_{\mathrm{r}=0}^{10}\left(\frac{10^{\mathrm{r}+1}-1}{10^{\mathrm{r}}}\right) \cdot {}^{11}\mathrm{C}_{\mathrm{r}+1}=\frac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :

(1) 15

(2) 11

(3) 24

(4) 20

Q9. If $\sum_{\mathrm{r}=1}^{9}\left(\frac{\mathrm{r}+3}{2^{\mathrm{r}}}\right) \cdot {}^{9}\mathrm{C}_{\mathrm{r}}=\alpha\left(\frac{3}{2}\right)^{9}-\beta, \quad \alpha, \beta \in \mathrm{N}, \quad$ then $(\alpha+\beta)^{2}$ is equal to

(1) 27

(2) 9

(3) 81

(4) 18

Q10. Let $\left(1+x+x^{2}\right)^{10}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots .+a_{20} x^{20}$. If $\left(a_{1}+a_{3}+a_{5}+\ldots .+a_{19}\right)-11 \mathrm{a}_{2}=121 \mathrm{k}$, then k is equal to _____.

Q11. If $1^{2} \cdot\left({}^{15} C_{1}\right)+2^{2} \cdot\left({}^{15} C_{2}\right)+3^{2} \cdot\left({}^{15} C_{3}\right)+\ldots+15^{2} \cdot\left({}^{15} C_{15}\right)=2^{m} \cdot 3^{n} \cdot 5^{k}$, where $m, n, k \in \mathbf{N}$, then $\mathrm{m}+\mathrm{n}+\mathrm{k}$ is equal to :

(1) 19

(2) 21

(3) 18

(4) 20

Q12. The sum of the series

$2 \times 1 \times {}^{20}C_4 - 3 \times 2 \times {}^{20}C_5 + 4 \times 3 \times {}^{20}C_6 - 5 \times 4 \times {}^{20}C_7 + \ldots + 18 \times 17 \times {}^{20}C_{20}$, is equal to

Q1. Let the mean and the standard deviation of the observation $2, 3, 3, 4, 5, 7$, a, b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

(1) 1

(2) $\frac{3}{4}$

(3) 2

(4) $\frac{1}{2}$

Q2. Let the Mean and Variance of five observations $x_1 = 1, x_2 = 3, x_3 = a, x_4 = 7$ and $x_5 = b, a > b$, be 5 and 10 respectively. Then the Variance of the observations $n + x_n, n = 1, 2, \ldots\ldots.5$ is

(1) 17

(2) 16.4

(3) 17.4

(4) 16

Q3. The mean and standard deviation of 100 observations are 40 and 5.1 , respectively, By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu + \sigma)$ is equal to

(1) 445

(2) 451

(3) 447

(4) 449

Q4. If the mean and the variance of $6, 4, a, 8, b, 12, 10$, 13 are 9 and 9.25 respectively, then $a + b + ab$ is equal to :

(1) 105

(2) 103

(3) 100

(4) 106

Q1. Let $A=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$. If for some $\theta\in(0,\pi)$, $A^2=A^T$, then the sum of the diagonal elements of the matrix $(\mathrm{A}+\mathrm{I})^3+(\mathrm{A}-\mathrm{I})^3-6\,\mathrm{A}$ is equal to ______.

Q2. The number of singular matrices of order 2 , whose elements are from the set $\{2,3,6,9\}$ is

Q3. Let $A=\begin{bmatrix}\alpha & -1\\ 6 & \beta\end{bmatrix}$, $\alpha>0$, such that $\det(A)=0$ and $\alpha+\beta=1$. If I denotes 2×2 identity matrix, then the matrix $(1+\mathrm{A})^8$ is:

(1) $\begin{bmatrix}4 & -1\\ 6 & -1\end{bmatrix}$

(2) $\begin{bmatrix}257 & -64\\ 514 & -127\end{bmatrix}$

(3) $\begin{bmatrix}1025 & -511\\ 2024 & -1024\end{bmatrix}$

(4) $\begin{bmatrix}766 & -255\\ 1530 & -509\end{bmatrix}$

Q4. Let α be a solution of $x^2+x+1=0$, and for some a and b in $\mathbb{R}$, $[4\quad \mathrm{a}\quad \mathrm{b}]\begin{bmatrix}1 & 16 & 13\\ -1 & -1 & 2\\ -2 & -14 & -8\end{bmatrix}=[0\quad 0\quad 0]$. If $\frac{4}{\alpha^4}+\frac{\mathrm{m}}{\alpha^{\mathrm{a}}}+\frac{\mathrm{n}}{\alpha^{\mathrm{b}}}=3$, then $\mathrm{m}+\mathrm{n}$ is equal to

(1) 3

(2) 11

(3) 7

(4) 8

Q5. Let A be a 3×3 real matrix such that $A^2(A-2I)-4(\mathrm{A}-\mathrm{I})=\mathrm{O}$, where I and O are the identity and null matrices, respectively. If $A^5=\alpha A^2+\beta A+\gamma I$, where α,β and γ are real constants, then $\alpha+\beta+\gamma$ is equal to:

(1) 12

(2) 20

(3) 76

(4) 4

Q6. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for n ≥ 3. Then the sum of all the elements of A^{50} is :-

(1) 53

(2) 52

(3) 39

(4) 44

Q7. Let a $\in \mathbf{R}$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3.
If $\det((a+1)\operatorname{adj}((a-1)A))$ is $2^m 3^n, m, n \in \{0, 1, 2, \ldots .20\}$, then m + n is equal to :

(1) 14

(2) 17

(3) 15

(4) 16

Q8. Let A be a matrix of order 3×3 and $|A| = 5$. If $|2\operatorname{adj}(3\text{ A}\operatorname{adj}(2\text{ A}))| = 2^\alpha \cdot 3^\beta \cdot 5^\gamma \alpha, \beta, \gamma \in \mathrm{N}$ then $\alpha + \beta + \gamma$ is equal to

(1) 25

(2) 26

(3) 27

(4) 28

Q9. Let I be the identity matrix of order 3×3 and for the matrix $\mathrm{A} = \begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}$, $|\mathrm{A}| = -1$. Let B be the inverse of the matrix $\operatorname{adj}\left(\mathrm{A}\operatorname{adj}\left(\mathrm{A}^2\right)\right)$. Then $|(\lambda B + 1)|$ is equal to _____

Q10. Let A be a 3×3 matrix such that $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} \mathrm{A}))| = 81$. If $\mathrm{S} = \left\{\mathrm{n} \in \mathbb{Z} : (|\operatorname{adj}(\operatorname{adj} A)|)^{\frac{(n-1)^2}{2}} = |A|^{(3n^2 - 5n - 4)}\right\}$, then $\sum_{n \in S} \left|A^{(n^2+n)}\right|$ is equal to

(1) 866

(2) 750

(3) 820

(4) 732

Q11. Let $A=\begin{bmatrix}2 & 2+p & 2+p+q\\ 4 & 6+2p & 8+3p+2q\\ 6 & 12+3p & 20+6p+3q\end{bmatrix}$.

If $\det(\operatorname{adj}(\operatorname{adj}(3\,\mathrm{A})))=2^{\mathrm{m}}\cdot 3^{\mathrm{n}}, \mathrm{m}, \mathrm{n}\in\mathrm{N}$, then $\mathrm{m}+\mathrm{n}$ is equal to

(1) 22

(2) 24

(3) 26

(4) 20

Q1. Let the system of equations
$x+5y-z=1$
$4x+3y-3z=7$
$24x+y+\lambda z=\mu$
$\lambda, \mu \in \mathrm{R}$, have infinitely many solutions. Then the number of the solutions of this system, If x, y, z are integers and satisfy $7 \leq x+y+z \leq 77$, is

(1) 3

(2) 6

(3) 5

(4) 4

Q2. If $y(x)=\begin{vmatrix} \sin x & \cos x & \sin x+\cos x+1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$, $x \in \mathbb{R}$, then $\frac{d^2 y}{dx^2}+y$ is equal to

(1) -1

(2) 28

(3) 27

(4) 1

Q3. If the system of linear equations
$3x+y+\beta z=3$
$2x+\alpha y-z=-3$
$x+2y+z=4$
has infinitely many solutions, then the value of $22\beta-9\alpha$ is :

(1) 49

(2) 31

(3) 43

(4) 37

Q4. If the system of equation
$2x+\lambda y+3z=5$
$3x+2y-z=7$
$4x+5y+\mu z=9$
has infinitely many solutions, then $\left(\lambda^2+\mu^2\right)$ is equal to :

(1) 22

(2) 18

(3) 26

(4) 30

Q5. Let the system of equations :

$2x + 3y + 5z = 9$
$7x + 3y - 2z = 8$
$12x + 3y - (4 + \lambda)z = 16 - \mu$

have infinitely many solutions. Then the radius of the circle centred at (λ, μ) and touching the line $4x = 3y$ is

(1) $\frac{17}{5}$

(2) $\frac{7}{5}$

(3) 7

(4) $\frac{21}{5}$

Q1. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, the n is equal to

Q2. If the probability that the random variable X takes the value x is given by $P(X=x)=k(x+1)3^{-x}$, $\mathrm{x}=0,1,2,3 \ldots \ldots$, where k is a constant, then $\mathrm{P}(\mathrm{X} \geq 3)$ is equal to

(1) $\frac{7}{27}$

(2) $\frac{4}{9}$

(3) $\frac{8}{27}$

(4) $\frac{1}{9}$

Q3. The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is:

(1) $\frac{129}{182}$

(2) $\frac{103}{182}$

(3) $\frac{17}{26}$

(4) $\frac{19}{26}$

Q4. Given three indentical bags each containing 10 balls, whose colours are as follows :

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the balls is Green, the probability that it is from bag III is q , then the value of $\left(\frac{1}{\mathrm{p}}+\frac{1}{\mathrm{q}}\right)$ is :

(1) 6

(2) 9

(3) 7

(4) 8

Q5. A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{\mathrm{m}}{\mathrm{n}}$, $\operatorname{gcd}(\mathrm{m}, \mathrm{n})=1$, then $\mathrm{n}^2-\mathrm{m}^2$ is equal to :

(1) 80

(2) 60

(3) 72

(4) 64

Q6. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is

(1) $\frac{11}{15}$

(2) $\frac{28}{75}$

(3) $\frac{2}{15}$

(4) $\frac{3}{5}$

Q7. Let a random variable X take values $0, 1, 2, 3$ with $\mathrm{P}(\mathrm{X}=0)=\mathrm{P}(\mathrm{X}=1)=\mathrm{p}, \mathrm{P}(\mathrm{X}=2)=\mathrm{P}(\mathrm{X}=3)$ and $\mathrm{E}\left(\mathrm{X}^2\right)=2\mathrm{E}(\mathrm{X})$. Then the value of $8\mathrm{p}-1$ is :

(1) 0

(2) 2

(3) 1

(4) 3

Q8. Three distinct numbers are selected randomly from the set $\{1, 2, 3, \ldots\ldots, 40\}$. If the probability, that the selected numbers are in an increasing G.P. is $\frac{m}{n}$, $\gcd(m, n)=1$, then $m+n$ is equal to _____.

Q9. All five letter words are made using all the letters $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$ and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n. Let the probability $\mathrm{P}\left(\mathrm{W}_{\mathrm{n}}\right)$ of choosing the word W_{n} satisfy $\mathrm{P}\left(\mathrm{W}_{\mathrm{n}}\right)=2\mathrm{P}\left(\mathrm{W}_{\mathrm{n}-1}\right), \mathrm{n}>1$.
If $\mathrm{P}(\mathrm{CDBEA})=\frac{2^\alpha}{2^\beta-1}, \alpha, \beta \in \mathbb{N}$, then $\alpha+\beta$ is equal to : _______

Q10. If A and B are two events such that $P(A)=0.7$, $\mathrm{P}(\mathrm{B})=0.4$ and $\mathrm{P}(\mathrm{A} \cap \overline{\mathrm{B}})=0.5$, where $\overline{\mathrm{B}}$ denotes the complement of B, then $P(B \mid (A \cup \bar{B}))$ is equal:-

(1) $\frac{1}{4}$

(2) $\frac{1}{2}$

(3) $\frac{1}{6}$

(4) $\frac{1}{3}$

Q1. Let $A = \{-2, -1, 0, 1, 2, 3\}$. let R be a relation on A defined by xRy if and only if $y = \max\{x, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + \mathrm{m} + \mathrm{n}$ is equal to

(1) 12

(2) 11

(3) 13

(4) 14

Q2. Let $\mathrm{A} = \{1, 2, 3, \ldots, 10\}$ and R be a relation on A such that $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) : \mathrm{a} = 2\,\mathrm{b} + 1\}$. Let $(\mathrm{a}_1, \mathrm{a}_2)$, $(a_2, a_3), (a_3, a_4), \ldots, (a_k, a_{k+1})$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k , for which such a sequence exists, is equal to :

(1) 6

(2) 7

(3) 5

(4) 8

Q3. Let A be the set of all functions $f : \mathbf{Z} \rightarrow \mathbf{Z}$ and R be a relation on A such that $\mathrm{R} = \{(\mathrm{f}, \mathrm{g}) : f(0) = \mathrm{g}(1)$ and $f(1) = \mathrm{g}(0)\}$. Then R is:

(1) Symmetric and transitive but not reflective

(2) Symmetric but neither reflective nor transitive

(3) Reflexive but neither symmetric nor transitive

(4) Transitive but neither reflexive nor symmetric

Q4. Let $A = \{-3, -2, -1, 0, 1, 2, 3,\}. Let R$ be a relation on A defined by xRy if and only if $0 \leq x^2 + 2y \leq 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. then $l + m$ is equal to

(1) 19

(2) 20

(3) 17

(4) 18

Q5. Let A $= \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by xRy if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + \mathrm{mn}$ is equal to :-

(1) 18

(2) 17

(3) 15

(4) 16

Q6. The number of relations on the set A $= \{1, 2, 3\}$ containing at most 6 elements including $(1, 2)$, which are reflexive and transitive but not symmetric, is ________

Q7. Let $A = \{0, 1, 2, 3, 4, 5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if max $\{x, y\} \in \{3, 4\}$. Then among the statements (S_1) : The number of elements in R is 18 , and (S_2) : The relation R is symmetric but neither reflexive nor transitive

(1) both are true

(2) both are false

(3) only (S_2) is true

(4) only (S_1) is true

Q1. Let f be a function such that $f(x)+3f\left(\frac{24}{x}\right)=4x, x \neq 0$. Then $f(3)+f(8)$ is equal to

(1) 11

(2) 10

(3) 12

(4) 13

Q2. Let $f(x)+2f\left(\frac{1}{x}\right)=x^2+5$ and $2g(x)-3g\left(\frac{1}{2}\right)=x, x>0$. If $\alpha=\int_1^2 f(x)dx$, and $\beta=\int_1^2 g(x)dx$, then the value of $9\alpha+\beta$ is:

(1) 1

(2) 0

(3) 10

(4) 11

Q3. If the domain of the function
$f(x)=\frac{1}{\sqrt{10+3x-x^2}}+\frac{1}{\sqrt{x+|x|}}$ is (a,b), then $(1+a)^2+b^2$ is equal to :

(1) 26

(2) 29

(3) 25

(4) 30

Q4. If the domain of the function
$f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha,\beta)$
then $\alpha^2+4\beta$ is equal to

(1) 5

(2) 4

(3) 3

(4) 7

Q5. If the domain of the function $f(x)=\log_7\left(1-\log_4\left(x^2-9x+18\right)\right)$ is $(\alpha,\beta)\cup(\gamma,\delta)$, then $\alpha+\beta+\gamma+\delta$ is equal to

(1) 18

(2) 16

(3) 15

(4) 17

Q6. Let the domain of the function

$f(x)=\cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha,\beta]$ and the domain of $\mathrm{g(x)}=\log_2(2-6\log_{27}(2\mathrm{x}+5))$ be (γ,δ). Then $|7(\alpha+\beta)+4(\gamma+\delta)|$ is equal to ________

Q7. Let $f,\ \mathrm{g}:(1,\infty)\to\mathbb{R}$ be defined as $f(\mathrm{x})=\frac{2x+3}{5x+2}$ and $g(x)=\frac{2-3x}{1-x}$. If the range of the function $f\circ g:[2,4]\to\mathbb{R}$ is $[\alpha,\beta]$, then $\frac{1}{\beta-\alpha}$ is equal to

(1) 68

(2) 29

(3) 2

(4) 56

Q8. Consider the sets $\mathrm{A}=\left\{(\mathrm{x},\mathrm{y})\in\mathbb{R}\times\mathbb{R}:\mathrm{x}^2+\mathrm{y}^2=25\right\}$, $\mathrm{B}=\left\{(\mathrm{x},\mathrm{y})\in\mathbb{R}\times\mathbb{R}:\mathrm{x}^2+9\mathrm{y}^2=144\right\}$, $\mathrm{C}=\{(\mathrm{x},\mathrm{y})\in\mathbb{Z}\times\mathbb{Z}:x^2+y^2\leq 4\}$, and $D=A\cap B$. The total number of one-one functions from the set D to the set C is:

(1) 15120

(2) 19320

(3) 17160

(4) 18290

Q9. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function satisfying $f(0)=1$ and $f(2\mathrm{x})-f(\mathrm{x})=\mathrm{x}$ for all $\mathrm{x}\in\mathbb{R}$. If $\lim_{n\to\infty}\left\{f(x)-f\left(\frac{x}{2^n}\right)\right\}=G(x)$, then $\sum_{r=1}^{10}G\left(r^2\right)$ is equal to

(1) 540

(2) 385

(3) 420

(4) 215

Q10. Let the domains of the functions

$\mathrm{f(x)}=\log_4\log_3\log_7\left(8-\log_2\left(\mathrm{x}^2+4\mathrm{x}+5\right)\right)$ and $g(x)=\sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α,β) and $[\gamma,\delta]$, respectively. Then

$\alpha^2+\beta^2+\gamma^2+\delta^2$ is equal to :-

(1) 15

(2) 13

(3) 16

(4) 14

Q11. If the range of the function $f(x)=\frac{5-x}{x^2-3x+2}, x \neq 1,2$, is $(-\infty, \alpha] \cup [\beta, \infty)$, then $\alpha^2+\beta^2$ is equal to :

(1) 190

(2) 192

(3) 188

(4) 194

Q1. If the function $f(x)=\frac{\tan(\tan x)-\sin(\sin x)}{\tan x-\sin x}$ is continuous at $\mathrm{x}=0$, then $f(0)$ is equal to ________

Q2. If $\lim_{x\to 0}\frac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then $(a+b)$ is equal to :

(1) $\frac{1}{2}$

(2) 0

(3) $\frac{3}{4}$

(4) -1

Q3.
$\lim_{x\to 0^+}\frac{\tan\left(5(x)^{\frac{1}{3}}\right)\log_e\left(1+3x^2\right)}{\left(\tan^{-1}3\sqrt{x}\right)^2\left(e^{5(x)^{\frac{4}{3}}}-1\right)}$ is equal to

(1) $\frac{1}{15}$

(2) 1

(3) $\frac{1}{3}$

(4) $\frac{5}{3}$

Q4. For $\alpha,\beta,\gamma,\in\mathbf{R}$, if $\lim_{x\to 0}\frac{x^2\sin\alpha x+(\gamma-1)e^{x^2}}{\sin 2x-\beta x}=3$, then $\beta+\gamma-\alpha$ is equal to:

(1) 7

(2) 4

(3) 6

(4) -1

Q5. Given below are two statements :

Statement I : $\lim_{x\to 0}\left(\frac{\tan^{-1}x+\log_e\sqrt{\frac{1+x}{1-x}}-2x}{x^5}\right)=\frac{2}{5}$

Statement II : $\lim_{\mathrm{x}\to 1}\left(\mathrm{x}^{\frac{2}{1-\mathrm{x}}}\right)=\frac{1}{\mathrm{e}^2}$

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Q6. If $\operatorname{Lim}_{x \rightarrow 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}=p$, then $96 \log_e p$ is equal to ______

Q7. If $\lim_{x \rightarrow 1^{+}} \frac{(x-1)(6+\lambda \cos (x-1))+\mu \sin (1-x)}{(x-1)^3}=-1$, where $\lambda, \mu \in \mathbb{R}$, then $\lambda+\mu$ is equal to

(1) 18

(2) 20

(3) 19

(4) 17

Q1. Let m and n be the number of points at which the function $f(\mathrm{x})=\max\left\{\mathrm{x}, \mathrm{x}^3, \mathrm{x}^5, \ldots, \mathrm{x}^{21}\right\}, \mathrm{x} \in \mathbb{R}$, is not differentiable and not continuous, respectively. Then $\mathrm{m}+\mathrm{n}$ is equal to ________ .

Q2. Let $f(\mathrm{x})=\begin{cases}(1+\mathrm{ax})^{1/\mathrm{x}} & , \quad \mathrm{x}<0 \\ 1+\mathrm{b} & , \quad \mathrm{x}=0 \\ \frac{(\mathrm{x}+4)^{1/2}-2}{(\mathrm{x}+\mathrm{c})^{1/3}-2} & ,\end{cases}$

be continuous at $x=0$. Then $e^a bc$ is equal to

(1) 64

(2) 72

(3) 48

(4) 36

Q3. The number of points of discontinuity of the function $f(\mathrm{x})=\left[\frac{\mathrm{x}^2}{2}\right]-[\sqrt{\mathrm{x}}], \mathrm{x} \in[0,4]$, where $[\cdot]$ denotes the greatest integer function is ________

Q1. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $(\sin x \cos y)(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin \mathrm{y})(f(2\mathrm{x} + 2\mathrm{y}) + f(2\mathrm{x} - 2\mathrm{y}))$, for all $\mathrm{x}, \mathrm{y} \in \mathbf{R}$.
If $f'(0) = \frac{1}{2}$, then the value of $24 f''\left(\frac{5\pi}{3}\right)$ is:

(1) 2

(2) -3

(3) 3

(4) -2

Q1. If the function $f(x)=2x^3-9\mathrm{a}\mathrm{x}^2+12\mathrm{a}^2\mathrm{x}+1$, where $\mathrm{a}>0$, attains its local maximum and local minimum values at p and q, respectively, such that $\mathrm{p}^2=\mathrm{q}$, then $f(3)$ is equal to:

(1) 55

(2) 10

(3) 23

(4) 37

Q2. Let $x=-1$ and $x=2$ be the critical points of the function $\mathrm{f}(\mathrm{x})=\mathrm{x}^3+\mathrm{a}\mathrm{x}^2+\mathrm{b}\log_{\mathrm{c}}|\mathrm{x}|+1, \mathrm{x}\neq 0$. Let m and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2,-\frac{1}{2}\right]$. Then $|\mathrm{M}+m|$ is equal to (Take $\log_{\mathrm{c}} 2=0.7$):

(1) 21.1

(2) 19.8

(3) 22.1

(4) 20.9

Q3. The shortest distance between the curves $y^2=8\mathrm{x}$ and $x^2+y^2+12y+35=0$ is :

(1) $2\sqrt{3}-1$

(2) $\sqrt{2}$

(3) $3\sqrt{2}-1$

(4) $2\sqrt{2}-1$

Q4. Let $\mathrm{f}:\mathrm{R}\to\mathrm{R}$ be a function defined by $f(x)=||x+2|-2|x||$. If m is the number of points of local minima and n is the number of points of local maxima of f, then $m+n$ is

(1) 5

(2) 3

(3) 2

(4) 4

Q5. Let the function $f(x)=\frac{x}{3}+\frac{3}{x}+3, x\neq 0$ be strictly increasing in $(-\infty,\alpha_1)\cup(\alpha_2,\infty)$ and strictly decreasing in $(\alpha_3,\alpha_4)\cup(\alpha_4,\alpha_5)$. Then $\sum_{\mathrm{i}=1}^{5}\alpha_{\mathrm{i}}^2$ is equal to :-

(1) 48

(2) 28

(3) 40

(4) 36

Q6. Let $\mathrm{a} > 0$. If the function $\mathrm{f(x)} = 6\mathrm{x}^3 - 45\mathrm{ax}^2 + 108\mathrm{a}^2\mathrm{x} + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1 x_2 = 54$, then $\mathrm{a} + \mathrm{x}_1 + \mathrm{x}_2$ is equal to :-

(1) 15

(2) 18

(3) 24

(4) 13

Q7. Let $f : \mathbf{R} \to \mathbf{R}$ be a polynomial function of degree four having extreme values at $x = 4$ and $x = 5$. If $\lim_{x \to 0} \frac{f(x)}{x^2} = 5$, then $f(2)$ is equal to :

(1) 12

(2) 10

(3) 8

(4) 14

Q8. Let f be a differentiable function on $\mathbf{R}$ such that $\mathrm{f}(2) = 1$, $f'(2) = 4$. Let $\lim_{x \to 0} (f(2+x))^{3/x} = e^{\alpha}$. Then the number of times the curve $y = 4x^3 - 4x^2 - 4(\alpha - 7)x - \alpha$ meets x -axis is :-

(1) 2

(2) 1

(3) 0

(4) 3

Q1. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^{9}} dx =$
$\frac{1}{m}\left(\left(\sqrt{1+x^2}+x\right)^n\left(n\sqrt{1+x^2}-x\right)\right)+C$ where C
is the constant of integration and $m, n \in N$, then m + n is equal to

Q2. Let $f(x)=\int x^3\sqrt{3-x^2}dx$. If $5f(\sqrt{2})=-4$, then $f(1)$ is equal to

(1) $-\frac{2\sqrt{2}}{5}$

(2) $-\frac{8\sqrt{2}}{5}$

(3) $-\frac{4\sqrt{2}}{5}$

(4) $-\frac{6\sqrt{2}}{5}$

Q3. If $\int\left(\frac{1}{x}+\frac{1}{x^3}\right)\left(\sqrt[23]{3x^{-24}+x^{-26}}\right)dx = -\frac{\alpha}{3(\alpha+1)}\left(3x^\beta+x^\gamma\right)^{\frac{\alpha+1}{\alpha}}+C, x>0,$ $(\alpha, \beta, \gamma \in Z)$, where C is the constant of integration, then $\alpha+\beta+\gamma$ is equal to ________ .

Q1. The integral $\int_0^\pi \frac{8x dx}{4\cos^2 x+\sin^2 x}$ is equal to

(1) $2\pi^2$

(2) $4\pi^2$

(3) π^2

(4) $\frac{3\pi^2}{2}$

Q2. The integral $\int_0^\pi \frac{(x+3)\sin x}{1+3\cos^2 x}dx$ is equal to :

(1) $\frac{\pi}{\sqrt{3}}(\pi+1)$

(2) $\frac{\pi}{\sqrt{3}}(\pi+2)$

(3) $\frac{\pi}{3\sqrt{3}}(\pi+6)$

(4) $\frac{\pi}{2\sqrt{3}}(\pi+4)$

Q3. Let [.] denote the greatest integer function. If $\int_0^{e^3}\left[\frac{1}{\mathrm{e}^{\mathrm{x}-1}}\right]\mathrm{dx}=\alpha-\log_{\mathrm{e}}2$, then α^3 is equal to ________ .

Q4. Let (a,b) be the point of intersection of the curve $x^2=2y$ and the straight line $y-2x-6=0$ in the second quadrant. Then the integral $I=\int_a^b \frac{9x^2}{1+5^x}dx$ is equal to :

(1) 24

(2) 27

(3) 18

(4) 21

Q5. $4\int_0^1\left(\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\right)dx-3\log_e(\sqrt{3})$ is equal to :

(1) $2+\sqrt{2}+\log_e(1+\sqrt{2})$

(2) $2-\sqrt{2}-\log_e(1+\sqrt{2})$

(3) $2+\sqrt{2}-\log_e(1+\sqrt{2})$

(4) $2-\sqrt{2}+\log_e(1+\sqrt{2})$

Q6. Let the domain of the function

$f(\mathrm{x})=\log_2\log_4\log_6(3+4x-x^2)$ be $(\mathrm{a},\ \mathrm{b})$. If $\int_0^{\mathrm{b}-\mathrm{a}}[\mathrm{x}^2]\,\mathrm{dx}=\mathrm{p}-\sqrt{\mathrm{q}}-\sqrt{\mathrm{r}},\mathrm{p},\mathrm{q},\mathrm{r}\in\mathbb{N},\gcd(\mathrm{p},\mathrm{q},\mathrm{r})=1,$

where $[\cdot]$ is the greatest integer function, then $\mathrm{p}+\mathrm{q}+\mathrm{r}$ is equal to

(1) 10

(2) 8

(3) 11

(4) 9

Q7. Let $f(x)$ be a positive function and $I_1=\int_{-\frac{1}{2}}^{1} 2x f(2x(1-2x))dx$ and $I_2=\int_{-1}^{2} f(x(1-x))dx$. Then the value of $\frac{I_2}{I_1}$ is equal to ________

(1) 9

(2) 6

(3) 12

(4) 4

Q8. The value of $\int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x+(\sqrt{|x|-x})e^{-x}}{e^x+e^{-x}} dx$ is equal to

(1) $3-\frac{2\sqrt{2}}{3}$

(2) $2+\frac{2\sqrt{2}}{3}$

(3) $1-\frac{2\sqrt{2}}{3}$

(4) $1+\frac{2\sqrt{2}}{3}$

Q9. The integral $\int_{-1}^{\frac{3}{2}} \left(\left|\pi^2 x \sin(\pi x)\right|\right) dx$ is equal to:

(1) $3+2\pi$

(2) $4+\pi$

(3) $1+3\pi$

(4) $2+3\pi$

Q1. The area of the region $\{(x, y) : |x - y| \leq y \leq 4\sqrt{x}\}$ is

(1) 512

(2) $\frac{1024}{3}$

(3) $\frac{512}{3}$

(4) $\frac{2048}{3}$

Q2. If the area of the region
$\left\{(x, y) : \left|4 - x^2\right| \leq y \leq x^2, y \leq 4, x \geq 0\right\}$
is $\left(\frac{80\sqrt{2}}{\alpha} - \beta\right)$, $\boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbf{N}$, then $\alpha + \beta$ is equal to _______.

Q3. The area of the region bounded by the curve $y = \max\{|x|, x|x - 2|\}$, then x-axis and the lines $x = -2$ and $x = 4$ is equal to _______ .

Q4. If the area of the region $\{(x, y) : |x - 5| \leq y \leq 4\sqrt{x}\}$ is A , then 3 A is equal to _____ .

Q5. If the area of the region bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ is equal to α, then 6α equals

(1) 250

(2) 210

(3) 240

(4) 220

Q6. If the area of the region $\left\{(\mathrm{x}, \mathrm{y}) : 1 + \mathrm{x}^2 \leq \mathrm{y} \leq \min\{\mathrm{x} + 7, 11 - 3\mathrm{x}\}\right\}$ is A , then 3 A is equal to

(1) 50

(2) 49

(3) 46

(4) 47

Q7. Let the area of the bounded region $\left\{(x, y) : 0 \leq 9x \leq y^2, y \geq 3x - 6\right\}$ be A. Then 6 A is equal to ________

Q1. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 3\left(\tan^2 x\right) y + 3y = \sec^2 x$ $y(0) = \frac{1}{3} + e^3$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{3}$

(2) $\frac{4}{3}$

(3) $\frac{4}{3} + \mathrm{e}^3$

(4) $\frac{2}{3} + \mathrm{e}^3$

Q2. Let $f : \mathbf{R} \to \mathbf{R}$ be a thrice differentiable odd function satisfying $f'(\mathrm{x}) \geq 0, f'(\mathrm{x}) = f(\mathrm{x}), f(0) = 0, f'(0) = 3$. Then $9f\left(\log_{\mathrm{c}} 3\right)$ is equal to _______.

Q3. Let g be a differentiable function such that $\int_0^x g(t)dt = x - \int_0^x \mathrm{t}g(t)dt, x \geq 0$ and let $y = y(x)$ satisfy the differential equation $\frac{dy}{dx} - y\tan x = 2(x+1)\sec x g(x), x \in \left[0, \frac{\pi}{2}\right)$. If $y(0) = 0$, then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{2\pi}{3\sqrt{3}}$

(2) $\frac{4\pi}{3}$

(3) $\frac{2\pi}{3}$

(4) $\frac{4\pi}{3\sqrt{3}}$

Q4. Let $f : [0, \infty) \to \mathbb{R}$ be differentiable function such that $f(\mathrm{x}) = 1 - 2\mathrm{x} + \int_0^x e^{x-t} f(t)\mathrm{dt}$ for all $\mathrm{x} \in [0, \infty)$. Then the area of the region bounded by $\mathrm{y} = f(\mathrm{x})$ and the coordinate axes is

(1) $\sqrt{5}$

(2) $\frac{1}{2}$

(3) $\sqrt{2}$

(4) 2

Q5. If a curve $y = y(x)$ passes through the point $\left(1, \frac{\pi}{2}\right)$ and satisfies the differential equation $\left(7x^4 \cot y - e^x \operatorname{cosec} y\right) \frac{dx}{dy} = x^5, x \geq 1$, then at $x = 2$, the value of cosy is:

(1) $\frac{2\mathrm{e}^2 - \mathrm{e}}{64}$

(2) $\frac{2\mathrm{e}^2 + \mathrm{e}}{64}$

(3) $\frac{2\mathrm{e}^2 - \mathrm{e}}{128}$

(4) $\frac{2e^2+e}{128}$

Q6. Let $f(x)=x-1$ and $g(x)=e^x$ for $x\in\mathbb{R}$. If $\frac{dy}{dx}=\left(e^{-2\sqrt{x}}g(f(f(x)))-\frac{y}{\sqrt{x}}\right)$, $y(0)=0$, then $y(1)$ is :-

(1) $\frac{1-e^2}{e^4}$

(2) $\frac{2e-1}{e^3}$

(3) $\frac{e-1}{e^4}$

(4) $\frac{1-e^3}{e^4}$

Q7. Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function, If $10\int_1^{\mathrm{x}} f(\mathrm{t})\mathrm{dt}=5\mathrm{x}f(\mathrm{x})-\mathrm{x}^5-9$ for all $\mathrm{x}\geq 1$, then the value of $f(3)$ is :

(1) 18

(2) 32

(3) 22

(4) 26

Q8. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+2y\sec^2 x=2\sec^2 x+3\tan x\cdot\sec^2 x$ such that $\mathrm{y}(0)=\frac{5}{4}$. Then $12\left(\mathrm{y}\left(\frac{\pi}{4}\right)-\mathrm{e}^{-2}\right)$ is equal to ________.

Q9. Let $y=y(x)$ be the solution curve of the differential equation $x\left(x^2+e^x\right)dy+\left(e^x(x-2)y-x^3\right)dx=0, x>0$ passing through the point $(1,0)$. Then $y(2)$ is equal to :

(1) $\frac{4}{4-\mathrm{e}^2}$

(2) $\frac{2}{2+\mathrm{e}^2}$

(3) $\frac{2}{2-\mathrm{e}^2}$

(4) $\frac{4}{4+\mathrm{e}^2}$

Q10. Let $y=y(x)$ be the solution of the differential equation $\left(x^2+1\right)y'-2xy=\left(x^4+2x^2+1\right)\cos x$, $y(0)=1$. Then $\int_{-3}^{3} y(x)dx$ is :

(1) 24

(2) 36

(3) 30

(4) 18

Q1. Let $\mathrm{A}(4,-2), \mathrm{B}(1,1)$ and $\mathrm{C}(9,-3)$ be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE , formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is ______ .

Q2. Let the area of the triangle formed by a straight Line L: $\mathrm{x}+\mathrm{by}+\mathrm{c}=0$ with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x -axis, then the value of $\mathrm{b}^{2}+\mathrm{c}^{2}$ is:

(1) 90

(2) 93

(3) 97

(4) 83

Q3. Let the equation $\mathrm{x}(\mathrm{x}+2)(12-\mathrm{k})=2$ have equal roots. Then the distance of the point $\left(\mathrm{k}, \frac{\mathrm{k}}{2}\right)$ from the line $3x+4y+5=0$ is

(1) 15

(2) $5\sqrt{3}$

(3) $15\sqrt{5}$

(4) 12

Q4. Let the three sides of a triangle are on the lines $4x-7y+10=0, x+y=5$ and $7x+4y=15$. Then the distance of its orthocentre from the orthocentre of the triangle formed by the lines $x=0, y=0$ and $x+y=1$ is

(1) 5

(2) $\sqrt{5}$

(3) $\sqrt{20}$

(4) 20

Q5. Let ABC be the triangle such that the equations of lines AB and AC be $3y-x=2$ and $x+y=2$, respectively, and the points B and C lie on x -axis. If P is the orthocentre of the triangle ABC, then the area of the triangle PBC is equal to

(1) 4

(2) 10

(3) 8

(4) 6

Q6. If the orthocentre of the triangle formed by the lines $y=x+1, y=4x-8$ and $y=mx+c$ is at $(3,-1)$, then $\mathrm{m}-\mathrm{c}$ is :

(1) 0

(2) -2

(3) 4

(4) 2

Q7. A line passing through the point $\mathrm{P}(\mathrm{a}, \theta)$ makes an acute angle α with the positive x -axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2-\sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2 \tan^2 \alpha-2\sqrt{3}$ is

(1) 4

(2) 6

(3) 5

(4) 8

Q8. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $\mathrm{L}_1: 2\mathrm{x}+\mathrm{y}+6=0$ and $\mathrm{L}_2: 4\mathrm{x}+2\mathrm{y}-\mathrm{p}=0, \mathrm{p}>0$, at the points A and B, respectively. If $AB=\frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to

(1) 5

(2) 4

(3) 2

(4) 3

Q9. Consider the lines $\mathrm{x}(3\lambda+1)+\mathrm{y}(7\lambda+2)=17\lambda+5$, λ being a parameter, all passing through a point P . One of these lines (say L) is farthest from the origin. If the distance of L from the point $(3,6)$ is d, then the value of d^2 is

(1) 20

(2) 30

(3) 10

(4) 15

Q10. Let a be the length of a side of a square OABC with O being the origin. Its side OA makes an acute angle α with the positive x-axis and the equations of its diagonals are $(\sqrt{3}+1)x+(\sqrt{3}-1)y=0$ and $(\sqrt{3}-1)x-(\sqrt{3}+1)y+8\sqrt{3}=0$. Then a^2 is equal to

(1) 48

(2) 32

(3) 16

(4) 24

Q1. The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to ______.

Q2. If the four distinct points $(4, 6), (-1, 5), (0, 0)$ and $(\mathrm{k}, 3\mathrm{k})$ lie on a circle of radius r , then $10\mathrm{k} + \mathrm{r}^2$ is equal to

(1) 32

(2) 33

(3) 34

(4) 35

Q3. Let C_1 be the circle in the third quadrant of radius 3 , that touches both coordinate axes. Let C_2 be the circle with centre $(1, 3)$ that touches C_1 externally at the point (α, β). If $(\beta - \alpha)^2 = \frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :

(1) 9

(2) 13

(3) 22

(4) 31

Q1. The axis of a parabola is the line $y = x$ and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point $(1, \mathrm{k})$ lies on the parabola, then a possible value of k is :-

(1) 4

(2) 9

(3) 3

(4) 8

Q2. Let P be the parabola, whose focus is $(-2, 1)$ and directrix is $2x + y + 2 = 0$. Then the sum of the ordinates of the points on P , whose abscissa is -2 , is

(1) $\frac{3}{2}$

(2) $\frac{5}{2}$

(3) $\frac{1}{4}$

(4) $\frac{3}{4}$

Q3. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x-axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

(1) 15

(2) 25

(3) 30

(4) 20

Q4. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be $(1, -4)$. If the focus of the parabola divides the chord PQ in the ratio m : n, $\gcd(m, n) = 1$, then $m^2 + n^2$ is equal to :

(1) 17

(2) 10

(3) 37

(4) 26

Q5. The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

(1) $\frac{7\sqrt{2}}{2}$

(2) $\frac{7\sqrt{2}}{16}$

(3) $\frac{7\sqrt{2}}{4}$

(4) $\frac{7\sqrt{2}}{8}$

Q6. A line passing through the point $\mathrm{A}(-2,0)$, touches the parabola $P: y^{2}=x-2$ at the point B in the first quadrant. The area, of the region bounded by the line AB , parabola P and the x -axis, is :-

(1) $\frac{7}{3}$

(2) 2

(3) $\frac{8}{3}$

(4) 3

Q7. Let r be the radius of the circle, which touches x -axis at point $(\mathrm{a}, 0), \mathrm{a}<0$ and the parabola $\mathrm{y}^{2}=9 \mathrm{x}$ at the point $(4,6)$. Then r is equal to ________

Q1. If S and S' are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then $\min\left(SP.S'\mathrm{P}\right)+\max\left(\mathrm{SP.S'P}\right)$ is equal to :

(1) $3(1+\sqrt{2})$

(2) $3(6+\sqrt{2})$

(3) 9

(4) 27

Q2. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{4}{\sqrt{17}}$

(2) $\frac{\sqrt{3}}{16}$

(3) $\frac{3}{\sqrt{19}}$

(4) $\frac{\sqrt{5}}{7}$

Q3. A line passing through the point $\mathrm{P}(\sqrt{5},\sqrt{5})$ intersects the ellipse $\frac{\mathrm{x}^2}{36}+\frac{\mathrm{y}^2}{25}=1$ at A and B such that $(PA).(PB)$ is maximum. Then $5\left(PA^2+PB^2\right)$ is equal to :

(1) 218

(2) 377

(3) 290

(4) 338

Q4. The length of the latus-rectum of the ellipse, whose foci are $(2,5)$ and $(2,-3)$ and eccentricity is $\frac{4}{5}$, is

(1) $\frac{6}{5}$

(2) $\frac{50}{3}$

(3) $\frac{10}{3}$

(4) $\frac{18}{5}$

Q5. The centre of a circle C is at the centre of the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle $PF_1 F_2$ is 30 and the length of the major axis of E is 17 , then the distance between the foci of E is :

(1) 26

(2) 13

(3) 12

(4) $\frac{13}{2}$

Q6. Let
$A=\{(\alpha, \beta) \in \mathbf{R} \times \mathbf{R}:|\alpha-1| \leq 4$ and $|\beta-5| \leq 6\}$
and
$B=\left\{(\alpha, \beta) \in \mathbf{R} \times \mathbf{R}: 16(\alpha-2)^2+9(\beta-6)^2 \leq 144\right\}$

(1) $\mathrm{B} \subset \mathrm{A}$

(2) $\mathrm{A} \cup \mathrm{B}=\{(\mathrm{x}, \mathrm{y}):-4 \leq \mathrm{x} \leq 4,-1 \leq \mathrm{y} \leq 11\}$

(3) neither $\mathrm{A} \subset \mathrm{B}$ nor $\mathrm{B} \subset \mathrm{A}$

(4) $A \subset B$

Q7. Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be 10 . If its eccentricity is the minimum value of the function $f(\mathrm{t})=\mathrm{t}^2+\mathrm{t}+\frac{11}{12}, \mathrm{t} \in \mathbf{R}$, then $\mathrm{a}^2+\mathrm{b}^2$ is equal to :

(1) 125

(2) 126

(3) 120

(4) 115

Q8. Let the ellipse $3x^2+\mathrm{p}y^2=4$ pass through the centre C of the circle $x^2+y^2-2x-4y-11=0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6 f_1 f_2-r$ is equal to

(1) 74

(2) 68

(3) 70

(4) 78

Q9. Let for two distinct values of p the lines $y=x+p$ touch the ellipse E : $\frac{\mathrm{x}^2}{4^2}+\frac{\mathrm{y}^2}{3^2}=1$ at the points A and B . Let the line $\mathrm{y}=\mathrm{x}$ intersect E at the points C and D. Then the area of the quadrilateral $ABCD$ is equal to

(1) 36

(2) 24

(3) 48

(4) 20

Q10. Let C be the circle of minimum area enclosing the ellipse $E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricity $\frac{1}{2}$ and foci $(\pm 2, 0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length 29 is parallel to the major axis of E and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is :

(1) $6(3+\sqrt{2})$

(2) $8(3+\sqrt{2})$

(3) $62+\sqrt{3}$

(4) $82+\sqrt{3}$

Q11. Let C be the circle $\mathrm{x}^2+(\mathrm{y}-1)^2=2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on x -axis and y -axis respectively. Let the straight line $x+y=3$ touch the curves C, E_1 and E_2 at $P\left(x_1, y_1\right), Q\left(x_2, y_2\right)$ and $R\left(x_3, y_3\right)$ respectively. Given that P is the mid-point of the line segment QR and $PQ=\frac{2\sqrt{2}}{3}$, the value of $9\left(x_1 y_1+x_2 y_2+x_3 y_3\right)$ is equal to ______ .

Q1. Let one focus of the hyperbola $\mathrm{H}: \frac{\mathrm{x}^2}{\mathrm{a}^2}-\frac{\mathrm{y}^2}{\mathrm{b}^2}=1$ be at $(\sqrt{10}, 0)$ and the corresponding directrix be $\mathrm{x}=\frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H , then $9\left(\mathrm{e}^2+l\right)$ is equal to:

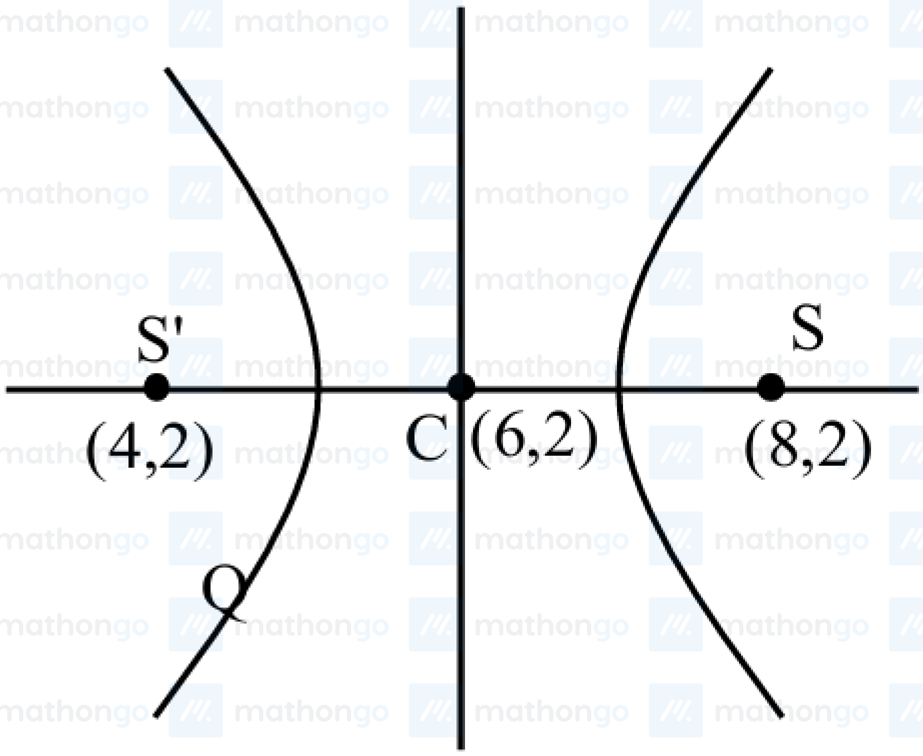
(1) 14

(2) 15

(3) 16

(4) 12

Q2. If the equation of the hyperbola with foci $(4,2)$ and $(8,2)$ is $3x^2-y^2-\alpha x+\beta y+\gamma=0$, then $\alpha+\beta+\gamma$ is equal to _____.



Q3. Let e_1 and e_2 be the eccentricities of the ellipse $\frac{\mathrm{x}^2}{\mathrm{b}^2}+\frac{\mathrm{y}^2}{25}=1$ and the hyperbola $\frac{\mathrm{x}^2}{16}-\frac{\mathrm{y}^2}{\mathrm{b}^2}=1$, respectively. If $\mathrm{b}<5$ and $\mathrm{e}_1 \mathrm{e}_2=1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is :

(1) $\frac{4}{5}$

(2) $\frac{3}{5}$

(3) $\frac{\sqrt{7}}{4}$

(4) $\frac{\sqrt{3}}{2}$

Q4. Let the product of the focal distances of the point $\mathrm{P}(4, 2\sqrt{3})$ on the hyperbola $\mathrm{H}: \frac{\mathrm{x}^2}{\mathrm{a}^2} - \frac{\mathrm{y}^2}{\mathrm{b}^2} = 1$ be 32 . Let the length of the conjugate axis of H be p and the length of its latus rectum be q . Then $\mathrm{p}^2 + \mathrm{q}^2$ is equal to

______

Q5. Let the sum of the focal distances of the point $\mathrm{P}(4, 3)$ on the hyperbola $\mathrm{H}: \frac{\mathrm{x}^2}{\mathrm{a}^2} - \frac{\mathrm{y}^2}{\mathrm{b}^2} = 1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distances of the point P is m , then $9l^2 + 6\ \mathrm{m}$ is equal to :-

(1) 184

(2) 186

(3) 185

(4) 187

Q6. Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having one of its focus at $\mathrm{P}(-3, 0)$. If the latus ractum through its other focus subtends a right angle at P and $a^2b^2 = \alpha\sqrt{2} - \beta, \alpha, \beta \in \mathbb{N}$.

Q7. Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be 2a and 2b, respectively, and one focus and the corresponding directrix of this hyperbola be $(-5, 0)$ and $5x + 9 = 0$, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p, then $4p$ is equal to

Q1. If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\operatorname{cosec}^2\theta - 2(\sqrt{3}-1)\operatorname{cosec}\theta - 4 = 0$, is equal to

(1) 6

(2) 8

(3) 10

(4) 7

Q2. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + (2-\sqrt{6})\cos\theta - \sqrt{3} = 0$, is equal to:

(1) 12

(2) 6

(3) 8

(4) 10

Q3. The number of solutions of the equation $2x + 3\tan x = \pi, x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ is

(1) 6

(2) 5

(3) 4

(4) 3

Q4. The number of solutions of equation $(4-\sqrt{3})\sin x - 2\sqrt{3}\cos^2 x = -\frac{4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$ is

(1) 4

(2) 3

(3) 6

(4) 5

Q5. If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\operatorname{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is:

(1) $\frac{2}{5}$

(2) $\frac{3}{4}$

(3) $\frac{3}{5}$

(4) $\frac{1}{5}$

Q6. If for $\theta \in \left[-\frac{\pi}{3}, 0\right]$, the points $(x, y) = \left(3\tan\left(\theta + \frac{\pi}{3}\right), 2\tan\left(\theta + \frac{\pi}{6}\right)\right)$ lie on $xy + \alpha x + \beta y + \gamma = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 80

(2) 72

(3) 96

(4) 75

Q7. The number of solutions of the equation
$\cos 2\theta \cos \frac{\theta}{2} + \cos \frac{5\theta}{2} = 2\cos^3 \frac{5\theta}{2}$
in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is :

(1) 7

(2) 5

(3) 6

(4) 7

Q1. The value of $\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right)-\cot^{-1}$ $\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$ is equal to

(1) $\pi-\frac{5}{4}$

(2) $\pi-\frac{3}{2}$

(3) $\pi+\frac{3}{2}$

(4) $\pi+\frac{5}{2}$

Q2. If $y=\cos\left(\frac{\pi}{3}+\cos^{-1}\frac{x}{2}\right)$, then $(x-y)^2+3y^2$ is equal to ______.

Q3. Considering the principal values of the inverse trigonometric functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2}<x<\frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{\pi}{4}+\sin^{-1}x$

(2) $\frac{\pi}{6}+\sin^{-1}x$

(3) $\frac{-5\pi}{6}-\sin^{-1}x$

(4) $\frac{5\pi}{6}-\sin^{-1}x$

Q4. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)+\ldots.$ is :-

(1) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$

(2) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$

(3) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$

(4) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

Q1. Let $ABCD$ be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5,6 and 7 square units respectively. Then the area (in square units) of the $\triangle$BCD is equal to :

(1) $\sqrt{340}$

(2) 12

(3) $\sqrt{110}$

(4) $7\sqrt{3}$

Q2. Let the angle $\theta, 0<\theta<\frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\overrightarrow{c}=3\hat{a}+6\hat{b}+9(\hat{a}\times\hat{b})$, then the value of $9(\overrightarrow{c}\cdot\hat{a})-3(\overrightarrow{c}\cdot\hat{b})$ is

(1) 31

(2) 27

(3) 29

(4) 24

Q3. Let the three sides of a triangle ABC be given by the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to ______

Q4. Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|}=\sqrt{2}+1$. Then $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2}$ is :

(1) $2+4\sqrt{2}$

(2) $1+\sqrt{2}$

(3) $2+\sqrt{2}$

(4) $4+2\sqrt{2}$

Q5. If $\vec{a}$ is nonzero vector such that its projections on the vectors $2\hat{\imath}-\hat{\jmath}+2\hat{k}$, $\hat{\imath}+2\hat{\jmath}-2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is:

(1) $\frac{1}{\sqrt{155}}(-7\hat{i}+9\hat{j}+5\hat{k})$

(2) $\frac{1}{\sqrt{155}}(-7\hat{i}+9\hat{j}-5\hat{k})$

(3) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}+5\hat{k})$

(4) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}-5\hat{k})$

Q6. Let $\vec{a} = 2\hat{i} - 3\hat{j} + k, \vec{b} = 3\hat{i} + 2\hat{j} + 5k$ and a vector $\vec{c}$ be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12k$ and $\vec{a} \cdot \vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a} \cdot \vec{d}|$ is equal to :

(1) 18

(2) 12

(3) 9

(4) 15

Q7. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$, If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to _______ .

Q8. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}, \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 4$. Then $|(\vec{a} \times \vec{d})|^2$ is equal to ______ .

Q9. Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}, \lambda > 0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $\left|\vec{v}_1\right|^2 + \left|\vec{v}_2\right|^2$ is equal to

(1) $\frac{23}{2}$

(2) 14

(3) $\frac{25}{2}$

(4) 10

Q10. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is :

(1) $\frac{1}{\sqrt{5}}(\hat{j} - 2\hat{k})$

(2) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$

(3) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(4) $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

Q1. Each of the angles β and γ that a given line makes with the positive y - and z -axes, respectively, is half of the angle that this line makes with the positive x -axes. Then the sum of all possible values of the angle β is

(1) $\frac{3\pi}{4}$

(2) π

(3) $\frac{\pi}{2}$

(4) $\frac{3\pi}{2}$

Q2. The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$ and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{23}{\sqrt{38}}$

(2) $\frac{21}{\sqrt{57}}$

(3) $\frac{23}{\sqrt{57}}$

(4) $\frac{21}{\sqrt{38}}$

Q3. Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is

(1) 42

(2) 46

(3) 48

(4) 40

Q4. If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is

(1) $\frac{3}{2}$

(2) $-\frac{3}{2}$

(3) 3

(4) -3

Q5. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}, QR=5$ and the coordinates of the point P be $(0,2,3)$. If the area of the triangle PQR is $\frac{m}{n}$ then :

(1) $\mathrm{m}-5\sqrt{21}\mathrm{n}=0$

(2) $2\mathrm{~m}-5\sqrt{21}\mathrm{n}=0$

(3) $5\mathrm{~m}-2\sqrt{21}\mathrm{n}=0$

(4) $5m-21\sqrt{2}n=0$

Q6. If the image of the point $\mathrm{P}(1,0,3)$ in the line joining the points $\mathrm{A}(4,7,1)$ and $\mathrm{B}(3,5,3)$ is $\mathrm{Q}(\alpha,\beta,\gamma)$, then $\alpha+\beta+\gamma$ is equal to

(1) $\frac{47}{3}$

(2) $\frac{46}{3}$

(3) 18

(4) 13

Q7. Let a line passing through the point $(4,1,0)$ intersect the line $L_1; \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ at the point $\mathrm{A}\quad(\alpha,\beta,\gamma)$ and the line $L_2: x-6=y=-z+4$ at the point $B(a,b,c)$.

Then $\left|\begin{array}{ccc}1 & 0 & 1\\ \alpha & \beta & \gamma\\ \mathrm{a} & \mathrm{b} & \mathrm{c}\end{array}\right|$ is equal to

(1) 8

(2) 16

(3) 12

(4) 6

Q8. Line L_1 passes through the point $(1,2,3)$ and is parallel to Z -axis. Line L_2 passes through the point $(\lambda,5,6)$ and is parallel to y-axis. Let for $\lambda=\lambda_1,\lambda_2,\lambda_2<\lambda_1$, the shortest distance between the two lines be 3 . Then the square of the distance of the point $(\lambda_1,\lambda_2,7)$ from the line L_1 is

(1) 40

(2) 32

(3) 25

(4) 37

Q9. The distance of the point $(7,10,11)$ from the line $\frac{x-4}{1}=\frac{y-4}{0}=\frac{z-2}{3}$ along the line $\frac{x-9}{2}=\frac{y-13}{3}=\frac{z-17}{6}$ is

(1) 18

(2) 14

(3) 12

(4) 16

Q10. Let A and B be two distinct points on the line L : $\frac{\mathrm{x}-6}{3}=\frac{\mathrm{y}-7}{2}=\frac{\mathrm{z}-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1,2,3)$ on the line L . If O is the origin, then $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is equal to:

(1) 49

(2) 47

(3) 21

(4) 62

Q11. Let the values of p , for which the shortest distance between the lines $\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$ and $\overrightarrow{\mathrm{r}}=(\mathrm{p}\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}})+\lambda(2\hat{\mathrm{i}}+3\hat{\mathrm{j}}+4\hat{\mathrm{k}})$ is $\frac{1}{\sqrt{6}}$, be a, b, $(a<b)$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :-

(1) 9

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) 18

Q12. Let A be the point of intersection of the lines L_1 : $\frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}$ and L_2 : $\frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$. Let B and C be the point on the lines L_1 and L_2 respectively such that $\mathrm{AB}=\mathrm{AC}=\sqrt{15}$. Then the square of the area of the triangle ABC is :

(1) 54

(2) 63

(3) 57

(4) 60

Q13. Let the line L pass through $(1,1,1)$ and intersect the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z}{1}$. Then, which of the following points lies on the line L ?

(1) $(4,22,7)$

(2) $(5,4,3)$

(3) $(10,-29,-50)$

(4) $(7,15,13)$

Q14. Consider the lines $\mathrm{L}_1 : \mathrm{x}-1=\mathrm{y}-2=\mathrm{z}$ and $\mathrm{L}_2 : \mathrm{x}-2=\mathrm{y}=\mathrm{z}-1$. Let the feet of the perpendiculars from the point $\mathrm{P}(5,1,-3)$ on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A , then $4\,\mathrm{A}^2$ is equal to :

(1) 139

(2) 147

(3) 151

(4) 143

Q15. Let the values of λ for which the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-\lambda}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points $(0,0),(\lambda_1,\lambda_2)$ and (λ_2,λ_1) is

(1) $\frac{5\sqrt{2}}{3}$

(2) 4

(3) $\frac{\sqrt{2}}{3}$

(4) 3

Q16. Let the area of the triangle formed by the lines $x+2=y-1=z$, $\frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}$ and $\frac{x}{-3}=\frac{y-3}{3}=\frac{z-2}{1}$ be A. Then A^2 is equal to ________

Q17. If the equation of the line passing through the point $\left(0,-\frac{1}{2},0\right)$ and perpendicular to the lines
$\vec{r}=\lambda(\hat{i}+a\hat{j}+b\hat{k})$ and
$\overrightarrow{\mathrm{r}}=(\hat{\mathrm{i}}-\hat{\mathrm{j}}-6\hat{\mathrm{k}})+\mu(-b\hat{\mathrm{i}}+\mathrm{a}\hat{\mathrm{j}}+5\hat{\mathrm{k}})$
is $\frac{\mathrm{x}-1}{-2}=\frac{\mathrm{y}+4}{\mathrm{d}}=\frac{\mathrm{z}-\mathrm{c}}{-4}$, then $\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d}$ is equal to :

(1) 10

(2) 14

(3) 13

(4) 12

Quadratic Equation

1. (3) **2.** (98) **3.** (2) **4.** (7) **5.** (3) **6.** (3) **7.** (3) **8.** (2)

Complex Number

1. (2) **2.** (2) **3.** (3) **4.** (1) **5.** (22) **6.** (1) **7.** (1) **8.** (3)
9. (11)

Sequences and Series

1. (1) **2.** (3) **3.** (3) **4.** (4) **5.** (1) **6.** (4) **7.** (4) **8.** (4)
9. (441) **10.** (1) **11.** (2) **12.** (3) **13.** (3) **14.** (3)

Permutation Combination

1. (3) **2.** (2) **3.** (2) **4.** (4) **5.** (14) **6.** (4) **7.** (13) **8.** (2)
9. (64) **10.** (4)

Binomial Theorem

1. (1) **2.** (2) **3.** (4) **4.** (4) **5.** (3) **6.** (63) **7.** (4) **8.** (4)
9. (3) **10.** (239) **11.** (1) **12.** (34)

Statistics

1. (1) **2.** (4) **3.** (4) **4.** (2)

Matrices

1. (6) **2.** (36) **3.** (4) **4.** (2) **5.** (1) **6.** (1) **7.** (4) **8.** (3)
9. (38) **10.** (4) **11.** (2)

Determinants

1. (1) **2.** (1) **3.** (2) **4.** (3) **5.** (2)

Probability

1. (2) **2.** (4) **3.** (1) **4.** (3) **5.** (1) **6.** (2) **7.** (2) **8.** (4949)
9. (183) **10.** (1)

Sets and Relations

1. (1) **2.** (3) **3.** (2) **4.** (4) **5.** (2) **6.** (5) **7.** (3)

Functions

1. (1) **2.** (4) **3.** (1) **4.** (2) **5.** (1) **6.** (96) **7.** (4) **8.** (3)
9. (2) **10.** (1) **11.** (4)

Limits

1. (2) **2.** (1) **3.** (3) **4.** (1) **5.** (4) **6.** (32) **7.** (1)

Continuity and Differentiability

1. (3) **2.** (3) **3.** (8)

Differentiation

1. (2)

Application of Derivatives

1. (4) **2.** (1) **3.** (4) **4.** (2) **5.** (4) **6.** (2) **7.** (2) **8.** (1)

Indefinite Integration

1. (379) **2.** (4) **3.** (19)

Definite Integration

1. (1) **2.** (3) **3.** (8) **4.** (1) **5.** (2) **6.** (1) **7.** (4) **8.** (4)
9. (3)

Area Under Curves

1. (2) **2.** (22) **3.** (12) **4.** (368) **5.** (1) **6.** (1) **7.** (15)

Differential Equations

1. (2) **2.** (36) **3.** (2) **4.** (2) **5.** (3) **6.** (3) **7.** (2) **8.** (21)
9. (4) **10.** (1)

Straight Lines

1. (3) **2.** (3) **3.** (1) **4.** (2) **5.** (4) **6.** (1) **7.** (1) **8.** (4)
9. (1) **10.** (1)

Circle

1. (768) **2.** (4) **3.** (3)

Parabola

1. (2) **2.** (1) **3.** (1) **4.** (1) **5.** (4) **6.** (3) **7.** (30)

Ellipse

1. (4) **2.** (1) **3.** (4) **4.** (4) **5.** (2) **6.** (1) **7.** (2) **8.** (3)
9. (2) **10.** (4) **11.** (46)

Hyperbola

1. (3) **2.** (141) **3.** (2) **4.** (120) **5.** (3) **6.** (1944) **7.** (189)

Trigonometric Equations

1. (1) **2.** (3) **3.** (2) **4.** (4) **5.** (1) **6.** (4) **7.** (1)

Inverse Trigonometric Functions

1. (1) **2.** (3) **3.** (2) **4.** (4)

Vector Algebra

1. (3) **2.** (3) **3.** (164) **4.** (3) **5.** (3) **6.** (4) **7.** (5) **8.** (128)
9. (2) **10.** (4)

Three Dimensional Geometry

1. (1) **2.** (1) **3.** (2) **4.** (4) **5.** (2) **6.** (2) **7.** (1) **8.** (3)
9. (2) **10.** (2) **11.** (3) **12.** (1) **13.** (4) **14.** (2) **15.** (1) **16.** (56)
17. (2)

Q1. $x^2+\sqrt{3}\mathrm{x}-16=0<\beta$
(3) $\mathrm{P_n}+\sqrt{3}\mathrm{P_{n-1}}-16\mathrm{P_{n-2}}=0$
$\mathrm{P_{25}}+\sqrt{3}\mathrm{P_{24}}-16\mathrm{P_{23}}=0$
$\therefore \frac{\mathrm{P_{25}}+\sqrt{3}\mathrm{P_{24}}}{2\mathrm{P_{23}}}=8$
Similarly
$x^2+3x-1=0<\sum_\delta^\gamma \quad Q_n=\gamma^n+\delta^n$
$\mathrm{Q_{25}}-\mathrm{Q_{23}}=\gamma^{25}+\delta^{25}-\gamma^{23}-\delta^{23}$
$=\gamma^{23}\left(\gamma^2-1\right)+\delta^{23}\left(\delta^2-1\right)$
$=\gamma^{23}(-3\gamma)+\delta^{23}(-3\gamma)$
$=-3\left[\gamma^{24}+\delta^{24}\right]$
$=-3\mathrm{Q_{24}}$
$\therefore \frac{\mathrm{Q_{25}}-\mathrm{Q_{23}}}{\mathrm{Q_{24}}}=-3$
$\frac{\mathrm{P_{25}}+\sqrt{3}\mathrm{P_{24}}}{2\mathrm{P_{23}}}+\frac{\mathrm{Q_{25}}-\mathrm{Q_{23}}}{\mathrm{Q_{24}}}=8-3=5$

Q2.
(98) $a+b=\lim_{t\to-1^+}(\alpha+\beta)=\lim_{t\to-1^+}-\frac{(t+2)^{\frac{1}{6}}-1}{(t+2)^{\frac{1}{7}}-1}$
let $t+2=y$
$a+b=\lim_{y\to1^+}\frac{y^{1/6}-1}{y^{1/7}-1}=\frac{7}{6}$
$72(a+b)^2=72\frac{49}{36}=98$

Q3. $\alpha^{10}+\beta^{10}=123$
(2) $\alpha+\beta=1$
$\alpha^9+\beta^9=76$
$\alpha^8+\beta^8=47$
$P_{10}=P_9+P_8$
$x^2=x+1\Rightarrow x^2-x-1=0$
$\alpha+\beta=1,\alpha\beta=-1$
$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{-1}=-1,\frac{1}{\alpha\beta}=-1$

Q4. Both the roots are positive
(7) $D\geq0$
$4(a-3)^2-4\times9(1-a)\geq0$
$a^2-6a+9-9+9a\geq0$
$a^2+3a\geq0$
$a(a+3)\geq0$
a $\in(-\infty,-3]\cup[0,\infty)$ ...(i)

$-\frac{b}{2a}>0$

$\frac{2(a-3)}{2(a-1)}>0$

$a \in(-\infty, 1) \cup(3, \infty)$ ...(ii)

$f(0)=9>0$

Equation (i) $\cap$ (ii)

$a \in(-\infty,-3] \cup[0,1)$

$2\alpha+\beta+\gamma-6+0+1=7$

Q5. $x^2+4x+4=n+4$

(3) $(x+2)^2=n+4$

$x=-2 \pm \sqrt{n+4}$

$\because 20 \leq n \leq 100$

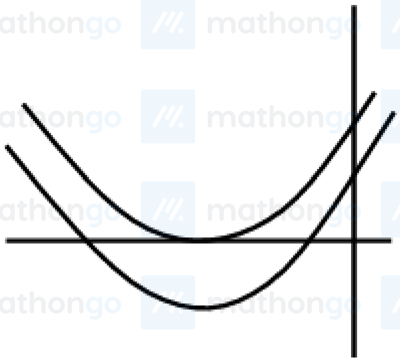
$\sqrt{24} \leq \sqrt{n+4} \leq \sqrt{104}$

$\Rightarrow \sqrt{n+4} \in\{5,6,7,8,9,10\}$

$\therefore$' 6 ' integral values of ' n ' are possible

Q6. Using location of roots :

(3)



(i) $D \geq 0$

(ii) $\frac{-b}{2a}<0$

(iii) a. $f(0)>0$

$(p+2)^2-4(2p+9) \geq 0$

$(p+4)(p-8) \geq 0 \quad p+2<0 \quad 2p+9>0$

Intersection $p \in\left(-\frac{9}{2},-4\right]$

$\therefore \beta-2\alpha=-4+9=5$

Q7. (3)

(I) $x<2$

$-x^2+2x-3x+9+1=0$

$\Rightarrow x^2+x-10=0$

$\Rightarrow x=\frac{-1+\sqrt{41}}{2}, \frac{-1-\sqrt{41}}{2}$

$\times \quad \surd$

(II) $2 \leq x<3$

$\Rightarrow x^2-2x-3x+9+1=0$

$\Rightarrow x^2-5x+10=0$

$\mathrm{D}<0$

(III) $x \geq 3$

$x^2-2x+3x-9+2=0$

$\Rightarrow x^2+x-8=0$

$x=\frac{-1+\sqrt{32}}{2}, \frac{-1-\sqrt{32}}{2}$

$\times \quad \times$

1 real roots

Q8. (2)

$|\mathrm{x}-2|^2+2|\mathrm{x}-2|-|\mathrm{x}-2|-2=0$

$\Rightarrow(|\mathrm{x}-2|+2)(|\mathrm{x}-2|-1)=0$

$\Rightarrow|\mathrm{x}-2|=1$

$\Rightarrow \mathrm{x}=2 \pm 1=3,1$

$\Rightarrow$ sum of square of roots $=9+1=10$

$\mathrm{x}^2-2|\mathrm{x}-3|-5=0$

Case-I $\mathrm{x}-3 \geq 0$

$\Rightarrow \mathrm{x}^2-2\mathrm{x}+1=0$

$\Rightarrow(\mathrm{x}-1)^2=0$

$\Rightarrow \mathrm{x}=1$

But $x \geq 3$

$\Rightarrow \mathrm{x} \in \phi$

Case-II $\mathrm{x}-3<0$

$x^2+2x-11=0, D>0 \Rightarrow$ Real \& distinct roots

$f(x)=x^2+2x-11$

$\mathrm{f}(3)>0, \frac{-\mathrm{p}}{2\mathrm{a}}=-1<3$

$\Rightarrow$ both roots <3, both roots acceptable

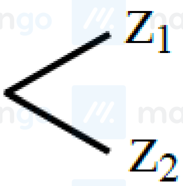
Sum of square of roots $=(\alpha+\beta)^2-2\alpha\beta$

$=4+22=26$

$\Rightarrow$ Final sum $=10+26=36$

Q1. $z^2 + 3i = z(2 + 3i) - 7 - 4i$

(2)



$\mathrm{z}^2 - \mathrm{z}(2 + 3\mathrm{i}) + 7 + 7\mathrm{i} = 0$

$z_1^2 + z_2^2 = (z_1 + z_2)^2 - 2z_1z_2$

$= 4 - 9 + 12i - 14 - 14i$

$= -19 - 2i$

Q2. $\omega_1 = (8\sin\theta + 7\cos\theta) + i(\sin\theta + 4\cos\theta)$

(2) $\omega_2 = (\sin\theta + 4\cos\theta) + i(8\sin\theta + 7\cos\theta)$

$\omega_1\omega_2 = 8\sin^2\theta + 7\sin\theta\cos\theta + 32\sin\theta\cos\theta +$

$28\cos^2\theta - 8\sin^2\theta - 32\sin\theta\cos\theta - 7\sin\theta\cos\theta$

$- 28\cos^2\theta + i\left(\sin^2\theta + 4\sin\theta\cos\theta + 4\sin\theta\cos\theta\right.$

$+ 16\cos^2\theta + 64\sin^2\theta + 56\sin\theta\cos\theta + 56\sin\theta$

$\left.\cos\theta + 49\cos^2\theta\right)$

$\omega_1\omega_2 = 0 + i\left(65\sin^2\theta + 120\sin\theta\cos\theta + 65\cos^2\theta\right)$

$\alpha + \beta = 65 + 60\sin 2q$

$\alpha + \beta|_{\max} = 125$

$\alpha + \beta|_{\min} = 5$

$\text{Ans.} = 125 + 5 = 130$

option (2)

Q3. $\mathrm{S}_1 : |\mathrm{z}| = 1, \dfrac{\mathrm{z} - \mathrm{i}}{\mathrm{z} + \mathrm{i}} = \dfrac{\bar{\mathrm{z}} + \mathrm{i}}{\bar{\mathrm{z}} - \mathrm{i}}$

(3)

$\Rightarrow (\mathrm{z} - \mathrm{i})(\bar{\mathrm{z}} - \mathrm{i}) = (\mathrm{z} + \mathrm{i})(\bar{\mathrm{z}} + \mathrm{i})$

$|\mathrm{z}|^2 - \mathrm{i}(\mathrm{z} + \bar{\mathrm{z}}) - 1 = |\mathrm{z}|^2 + \mathrm{i}(\mathrm{z} + \bar{\mathrm{z}}) - 1$

$\mathrm{i}(\mathrm{z} + \bar{\mathrm{z}}) = 0$

$\mathrm{z} + \bar{\mathrm{z}} = 2\cos\theta = 0 \Rightarrow \cos\theta = 0$

$\mathrm{z} = 0 + 0\mathrm{i}, |\mathrm{z}| \neq 1$

$\mathrm{S}_1 : \dfrac{\mathrm{z} - 1}{\mathrm{z} + 1} + \dfrac{\bar{\mathrm{z}} - 1}{\bar{\mathrm{z}} + 1} = 0$

$(\mathrm{z} - 1)(\bar{\mathrm{z}} + 1) + (\mathrm{z} + 1)(\bar{\mathrm{z}} - 1) = 0$

$\Rightarrow |\mathrm{z}|^2 + (\mathrm{z} - \bar{\mathrm{z}}) - 1 + |\mathrm{z}|^2 + (\mathrm{z} - \bar{\mathrm{z}}) - 1 = 0$

$|\mathrm{z}|^2 = 1$

Q4. $1 + 10\,\mathrm{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0$

(1)

$\therefore \mathrm{z} + \bar{\mathrm{z}} = 2\,\mathrm{Re}(\mathrm{z})$

$\frac{2\cos\theta + \mathrm{i}\sin\theta}{\cos\theta - 3\mathrm{i}\sin\theta} + \frac{2\cos\theta - \mathrm{i}\sin\theta}{\cos\theta + 3\mathrm{i}\sin\theta} = 2 \times \left(\frac{-1}{10}\right)$

$\frac{\left(2\cos^2\theta - 3\sin^2\theta\right) + \left(2\cos^2\theta\right) - \left(3\sin^2\theta\right)}{\cos^2\theta + 9\sin^2\theta} = \frac{-2}{10}$

$\Rightarrow \frac{2\cos^2\theta - 3\sin^2\theta}{\cos^2\theta + 9\sin^2\theta} = \frac{-1}{10}$

$\Rightarrow 20\cos^2\theta - 30\sin^2\theta = -\cos^2\theta - 9\sin^2\theta$

$21\cos^2\theta - 21\sin^2\theta = 0$

$\Rightarrow \cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \sum \theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16} = \frac{84\pi^2}{16} = \frac{21\pi^2}{4}$

Q5. Let $z = x + iy$

(22)

$A : |z - 2 - i| = 3$

$|(x-2) + (y-1)i| = 3$

$(x-2)^2 + (y-1)^2 = 9. \ldots (1)$

$B = \mathrm{Re}(z - iz) = 2$

$\mathrm{Re}((x+y) + i(y-x)) = 2$

$x + y = 2 \ldots. (2)$

On solving (1) and (2) we get

$x = \frac{3 \pm \sqrt{17}}{2}, y = \frac{1 \mp \sqrt{17}}{2}$

$\sum_{z \in \mathrm{s}} |z|^2 = \frac{1}{4}[2 \times 26 + 2 \times 18]$

$\Rightarrow \frac{88}{4} = 22$

Q6. $\frac{2 + \mathrm{k}^2\mathrm{z}}{\mathrm{k} + \bar{\mathrm{z}}} = \mathrm{kz}$

(1)

$|\mathrm{z}|^2\mathrm{k} = 2$

$\mathrm{k} = 2$

point $\mathrm{p}(2,4)$; center $(1,2)$

distance from circle

$(x-1)^2 + (y-2)^2 = 1$ is max.

if $(\mathrm{OP} + \mathrm{r}) = \sqrt{1+4} + 1 = \sqrt{5} + 1$

Q7. $z_1 + z_2 + z_3 = 3z_0$

(1) $(z_1 + z_2 + z_3)^2 = 9z_0^2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 + 2\left(z_1^2 + z_2^2 + z_3^2\right) = 9z_0^2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = 3z_6^2$

$\sum_{k=1}^{3} (z_k - z_0)^2 = (z_1 - z_0)^2 + (z_2 - z_0)^2 + (z_3 - z_0)^2$

$= z_1^2 + z_2^2 + z_3^2 + 3z_0^2 - 2(z_1 + z_2 + z_3)z_0$

$= 6z_0^2 - 6z_0^2$

$= 0$

Q8. $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) + \operatorname{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right) = 2$

(3)

Here, $\frac{z-1}{2z+i} = \left(\frac{\bar{\bar{z}}-1}{2\bar{z}-i}\right) = 2$

$= \operatorname{Re}\left(\frac{z-1}{2z+i}\right) + \operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 2$

$= 2\operatorname{Re}\left(\frac{z-1}{2z+1}\right) = 2 \Rightarrow \operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$

Let $z = x + iy$

$\operatorname{Re}\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right) = 1 \Rightarrow \operatorname{Re}\left[\frac{((x-1)+iy)(2x-i(y+1)}{(2x+i(2y+1)(2x-i(2y+1))}\right] = 1$

$\Rightarrow \frac{2x(x-1)+y(2y+1)}{4x^2+(2y+1)^2} = 1$

$\Rightarrow 2x^2 - 2x + 2y^2 + y = 4x^2 + 4y^2 + 1 + 4y$

$\Rightarrow 2x^2 + 2y^2 + 3y + 2x + 1 = 0$

$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

centre $= \left(\frac{-1}{2}, \frac{-3}{4}\right), r = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$

$a = \frac{-1}{2}, b = \frac{-3}{4}, r^2 = \frac{5}{16}$

$15\frac{ab}{r^2} = 15 \times \left(\frac{-1}{2}\right) \times \left(\frac{-3}{4}\right) \times \frac{16}{5} = 18$

Q9. $\alpha = \omega$

(11) $\therefore \left(\omega^k + \frac{1}{\omega^k}\right)^2 = \omega^{2k} + \frac{1}{\omega^{2k}} + 2$

$= \omega^{2k} + \omega^k + 2 \quad \because \omega^{3k} = 1$

$\therefore \sum_{k=1}^{n} \left(\omega^{2k} + \omega^k + 2\right) = 20$

$\Rightarrow \left(\omega^2 + \omega^4 + \omega^6 + \ldots + \omega^{2n}\right) + \left(\omega + \omega^2 + \omega^3 + \ldots +\right.$
$\left.\omega^n\right) + 2n = 20$

Now if $n = 3m, \quad m \in I$

Then $0 + 0 + 2n = 20 \Rightarrow n = 10$ (not satisfy) if $n = 3m + 1$, then

$\omega^2 + \omega + 2\mathrm{n} = 20$

$-1 + 2\mathrm{n} = 20 \Rightarrow \mathrm{n} = \frac{21}{2}(\text{ not possible })$

if $n = 3m + 2$,

$\left(\omega^8 + \omega^{10}\right) + \left(\omega^4 + \omega^5\right) + 2\mathrm{n} = 20$

$\Rightarrow \left(\omega^2 + \omega\right) + \left(\omega + \omega^2\right) + 2\mathrm{n} = 20$

$2\mathrm{n} = 22$

$\mathrm{n} = 11$ satisfy $\mathrm{n} = 3\ \mathrm{m} + 2$

$\therefore \mathrm{n} = 11$

Q1. Let $a_1 = a$, common difference $= d$

(1) $a_1 + a_3 + a_5 + \ldots\ldots + a_{23} = -\frac{72}{5}a$

$\frac{12}{2}[2a + 11 \times 2d] = -\frac{72}{5}a$

$12a + 132d = -\frac{72}{5}a$

$132a + 132 \times 5d = 0$

$\mathrm{a} = -5\ \mathrm{d}$

$\frac{\mathrm{n}}{2}(2\mathrm{a} + (\mathrm{n}-1)\mathrm{d}) = 0 \Rightarrow -10\ \mathrm{d} + \mathrm{nd} - \mathrm{d} = 0$

$\mathrm{n} = 11$

Q2. $\mathrm{S_n} = 700 = \frac{\mathrm{n}}{2}[2\mathrm{a} + (\mathrm{n}-1)\mathrm{d}]$...(i)

(3) $a_6 = 7 \Rightarrow a + 5d = 7$...(ii)

$\mathrm{S_7} = 7 \Rightarrow \frac{7}{2}(2\mathrm{a} + 6\ \mathrm{d}) = 7$

$a + 3d = 1$...(iii)

Solve (ii) and (iii)

$\frac{\mathrm{n}}{2}(-16 + 3n - 3) = 700 \Rightarrow 3n^2 - 19n - 1400 = 0$

$(3n + 56)(\mathrm{n} - 25) = 0$

$\therefore \mathrm{a_{25}} = \mathrm{a} + 24\ \mathrm{d} = -8 + 24 \times 3$

$= -8 + 72$

=64

Q3. Let the I$^{\text{st}}$ term of G.P. be a \& common ratio be r

(3) $\mathrm{a_3 a_5} = \mathrm{ar^2} \cdot \mathrm{ar^4} = 729$

$= \mathrm{a^2 r^6} = 729$

$= \mathrm{ar^3} = 27 \quad \ldots.(i)$

$\mathrm{a_2} + \mathrm{a_4} = \mathrm{ar} + \mathrm{ar^3} = \frac{111}{4}$

$= \mathrm{ar} = \frac{3}{4} \quad \ldots.(ii)$

(i) $\div$ (ii)

$\frac{\mathrm{ar^3}}{\mathrm{ar}} = \frac{27}{3/4}$

$\mathrm{r^2} = 36$

$\mathrm{r} = 6$

from (ii)

$a(6) = \frac{3}{4} \Rightarrow a = \frac{1}{8}$

Now, $24\left(a_1 + a_2 + a_3\right)$

$= 24\left(a + ar + a^2\right)$

$= 24a\left(1 + r + r^2\right)$

$= 24 \times \frac{1}{8}(1 + 6 + 36)$

$= 3(43)$

$= 129$

Q4.
(4)

$$S = 1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \ldots$$
$$= \sum_{r=1}^{\infty} \frac{r^2}{r!}$$
$$= \sum_{r=1}^{\infty} \frac{(r-1+1)}{(r-1)!} = \sum_{r=2}^{\infty} \frac{1}{(r-2)!} + \sum_{r=1}^{\infty} \frac{1}{(r-1)!}$$
$$= 2e$$

Q5.
(1)

$a_2 + a_4 + \ldots + a_n = 30$ ...(1)

$a_1 + a_3 + \ldots + a_{n-1} = 24$ ...(2)

$(1) - (2)$

$(a_2 - a_1) + (a_4 - a_3) \ldots (a_n - a_{n-1}) = 6$

$\Rightarrow \frac{n}{2} d = 6 \Rightarrow nd = 12$

$a_n - a_1 = (n-1)d = \frac{21}{2}$

$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}, n = 8$

Sum of odd terms $= \frac{4}{2}[2a + (4-1)3] = 24$

$\Rightarrow a = \frac{3}{2}$

A.P. $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms $= 4$

Q6.
(4)

Let $A(a-d, a, a+d)$ $\quad B(b-D, b, b+D)$

$a = 12$ $\quad b = 12$

$p = 12\left(144 - d^2\right)$

$q = 12\left(144 - D^2\right)$

$\frac{p+q}{p-q} = \frac{19}{5}$

$\frac{p}{q} = \frac{24}{14} = \frac{12}{7}$

$\frac{144 - d^2}{144 - (d^2 + 6d + 9)} = \frac{12}{7}$

$1008 - 7d^2 = -12d^2 - 72d + 1620$

$5d^2 + 72d - 612 = 0$

$d = 6$

$D = 9$

$p - q = 12\left(D^2 - d^3\right)$

$= 12(81 - 36)$

$= 12(45)$

$= 540$

option (4)

Q7. $x_1, x_2, x_3, x_4 \rightarrow$ G.P.

(4) Let a, ar, $ar^2, ar^3 \rightarrow$ G.P.

Now $a-2, ar-7, a^2-9, ar^3-5 \rightarrow$ A.P.

$2(ar-7) = a-2+ar^2-9$....(i)

$2\left(ar^2-9\right) = ar-7+ar^3-5$....(ii)

Solving $r = 2, a = -3$

$\therefore$ Product $= x_1, x_2, x_3, x_4 = a^4r^6 = 81 \times 64$

Solving $r = 2, a = -3$

$\therefore$ Product $= x_1, x_2, x_3, x_4 = a^4r^6 = 81 \times 64$

Q8. $ar + ar^3 + ar^5 = 21, \quad ar^7 + ar^9 + ar^{11} = 15309$

(4) $\Rightarrow ar\left(1+r^2+r^4\right) = 21, \quad ar^7\left(1+r^2+r^4\right) = 15309$

$\text{Eq}^n\left(\overline{2}\right) \div \text{eq}^n(1) \qquad (2)\,{}^{(2)}_{(1)}$

$$\Rightarrow \frac{a \cdot r^7}{ar} = \frac{15309}{21} \Rightarrow r^6 = 729$$

$$\Rightarrow \frac{a \cdot \left(r^9-1\right)}{r-1} = \frac{\frac{7}{91}(19683-1)}{2} = \frac{7 \times 19682}{91 \times 2}$$

$$= \frac{9841}{13} = 757$$

Q9. $T_r = \dfrac{4.r}{1+4.r^4}$

(441)

$$T_r = \frac{4.r}{\left(2r^2+2r+1\right)\left(2r^2-2r+1\right)}$$

$$T_r = \frac{\left(2r^2+2r+1\right)-\left(2r^2-2r+1\right)}{\left(2r^2+2r+1\right)\left(2r^2-2r+1\right)}$$

$$T_r = \frac{1}{2r^2-2r+1} - \frac{1}{2r^2+2r+1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$m + n = 441$

Q10. Given sum is

(1) $S_n = 1+3+11+25+45+71+\ldots+T_n$

First order differences are in A.P.

Thus, we can assume that

$T_n = an^2 + bn + c$

$$\text{Solving}\left\{\begin{array}{c} T_1 = 1 = a+b+c \\ T_2 = 3 = 4a+2b+c \\ T_3 = 11 = 9a+3b+c \end{array}\right\},$$

we get $a = 3,\ b = -7, c = 5$

Hence, general term of given series is

$T_n = 3n^2 - 7n + 5$

Hence, required sum equals

$\sum_{n=1}^{n=20}(3n^2 - 7n + 5) = 3\left(\frac{20\cdot21\cdot41}{6}\right) - 7\left(\frac{20\cdot21}{2}\right) + 5(20) = 7240$

Q11. $\left(1^2 + 5^2 + 9^2 + \ldots\ldots \text{ upto 20 terms }\right) + (3 + 7 + 11 + \ldots\text{upto 20 terms})$

(2)

$$= \sum_{r=1}^{20}(4r-3)^2 + \sum_{r=1}^{20}(4r-1)$$

$$= \sum_{r=1}^{20}(4r-3)^2 + (4r-1)$$

$$= 4\sum_{r=1}^{20}\left(4r^2 - 5r + 2\right)$$

$$= 16\sum_{r=1}^{20} r^2 - 20\sum_{r=1}^{20} r + 8\sum_{r=1}^{20} 1 = 41880$$

Q12. $\sum_{r=1}^{20}\frac{4r}{4+3r^2+r^4}$

(3)

$$\sum_{r=1}^{20}\frac{4r}{(r^2+r+2)(r^2-r+2)}$$

$$2\sum_{r=1}^{20}\left(\frac{1}{r^2-r+2} - \frac{1}{r^2+r+2}\right)$$

$$2\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$\frac{1}{4} - \frac{1}{8}$$

$$\frac{1}{8} - \frac{1}{14}$$

$$\left(\frac{1}{382} - \frac{1}{422}\right)$$

$$= 2\left(\frac{1}{2} - \frac{1}{422}\right)$$

$$= \frac{420}{422}$$

$$= \frac{210}{211}$$

option (3)

Q13. $A=\{1,6,11,16,21,26,31,36,41,46,51,56,61,$
(3) $66,71,76,81,86,91,\ldots\ldots\}$

$B=\{9,16,23,30,37,44,51,58,65,72,79,86,$
$93,100,\ldots\ldots\}$

$A\cap B=\{16,51,86,\ldots\ldots\}$

For set 'A' $\Rightarrow T_{2025}=1+(2025-1)(5)=10121$

For set ' $B' \Rightarrow T_{2025}=9+(2025-1)(7)=14177$

So, for $(A\cap B)\Rightarrow T_n=16+(n-1)(35)\leq 10121$

$$(n-1)\leq \frac{10121-16}{35}=288.71$$

$n\leq 289.71\Rightarrow n=289$

$\therefore n(A\cup B)=n(A)+n(B)-n(A\cap B)$
$=2025+2025-289=3761$

Q14. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\infty=\frac{\pi^4}{90}\ldots.(i)$
(3)

$$\beta=\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots,$$

$$=\frac{1}{16}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\right],$$

$=\frac{1}{16}\times\frac{\pi^4}{90}$

using (ii) ________...(ii)

$$\alpha=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots\infty$$

$$\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+\ldots\ldots\right)$$

$$-\left(\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots\ldots\right)$$

$\alpha=\frac{\pi^4}{90}-\frac{1}{16}\times\frac{\pi^4}{90}$ [using (i) and (ii)]

$$\alpha=\frac{16-1}{16\times 90}\times\pi^4=\frac{15}{16\times 90}\pi^4=\frac{\pi^4}{96}$$

$$\therefore\frac{\alpha}{\beta}=\frac{\frac{\pi^4}{96}}{\frac{\pi^4}{16\times 90}}=\frac{16\times 90}{96}=15$$

Q1. 11111 222 00

(3) No. of sequences $= \frac{10!}{5!3!2!} = 2520$

Note : Sequence can start with 0.

Q2. Total number of Δ are

(2) $= {}^9C_1{}^{12}C_2 + {}^9C_2{}^{12}C_1 + {}^1C_1{}^9C_1{}^{12}C_1$

$= 594 + 432 + 108$

$= 1134$

Q3. 7 Batsmen \& 6 Bowlers

(2) To select 10 players including atleast

4 Batsmen \& 4 Bowlers

Captain \& vice-captain already selected

No. of ways $= {}^6C_5 \times {}^5C_3 + {}^6C_4 \times {}^5C_4 + {}^6C_3 \times {}^5C_5$

$= 6 \times 10 + 15 \times 5 + 20 \times 1$

$= 60 + 75 + 20 = 155$

Q4. ${}^{12}C_3 - {}^5C_3 = 210$

(4)

Q5. Total 7 digit nos. $= 9000000$

(14) 7 digit nos. having sum of digits

Even $= 4500000$

$= 9.5 \cdot 10^5$

$m = 9, n = 5$

$m + n = 14$

Q6. Total trangles $\Longrightarrow = {}^hC_3$

(4) Total auadrilaterals $= {}^hC_4 = q$

${}^nC_3 + {}^nC_4 = 126 \Rightarrow {}^{n+1}C_4 = 126$

$\Rightarrow n + 1 = 9 \Rightarrow n = 8$

$$\frac{x^2}{16} + \frac{y^2}{n} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{8} = 1$$

$$e = \sqrt{1 - \frac{8}{16}} = \sqrt{\frac{8}{16}} = \frac{1}{\sqrt{2}}$$

Q7. $A = \{1, 2, 3, 4, 5 \ldots\ldots n\}$

(13) No. of subsets having r elements such that no two are consecutive is $= {}^{n-r+1}C_r$

for $n = 5$, no. of ways $= {}^{6\text{-}r}C_r$

Subsets having no element $= 1$

Subsets having exactly 1 element $= {}^5C_1 = 5$

Subsets having exactly 2 element $= {}^4C_2 = 6$

Subsets having exactly 3 element $= {}^3C_3 = 1$

$\Rightarrow 5 + 6 + 1 + 1 = 13$

Q8. $2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$

(2) $\mathrm{B}=\left[\frac{50}{3}\right]+\left[\frac{50}{3^{2}}\right]+\left[\frac{50}{3^{3}}\right]+\left[\frac{50}{3^{4}}\right]$

$=16+5+1$

$=2$

Maximum value of n is 22

Q9. Let $\mathrm{m}=6\mathrm{a}, \mathrm{n}=6\ \mathrm{b}$

(64) $\mathrm{m}<\mathrm{n} \Rightarrow \mathrm{a}<\mathrm{b}$

where a& b are co-prime numbers also since $m \& n$ are 2 digit nos, so

$10 \leq \mathrm{m} \leq 99 \& 10 \leq \mathrm{n} \leq 99$

i.e. $2 \leq \mathrm{a} \leq 16 \& 2 \leq \mathrm{b} \leq 16$

($\because$ a is integer)

Now

$2 \leq \mathrm{a}<\mathrm{b} \leq 16 \&$ a& b are co-prime

$\therefore$ if

$\mathrm{a}=2,\ \mathrm{b}=3,5,7,9,11,13,15$

$a=3, b=4,5,7,8,10,11,13,14,16$

$a=4, b=5,7,9,11,13,15$

$a=5, b=6,7,8,9,11,12,13,14,16$

$\mathrm{a}=6,\ \mathrm{b}=7,11,13$

$\mathrm{a}=7,\ \mathrm{b}=8,9,10,11,12,13,15,16$

$\mathrm{a}=8,\ \mathrm{b}=9,11,13,15$

$a=9, b=10,11,13,14,16$

$\mathrm{a}=10,\ \mathrm{b}=11,13$

$\mathrm{a}=11,\ \mathrm{b}=12,13,14,15,16$

$\mathrm{a}=12,\ \mathrm{b}=13$

$a=13, b=14,15,16$

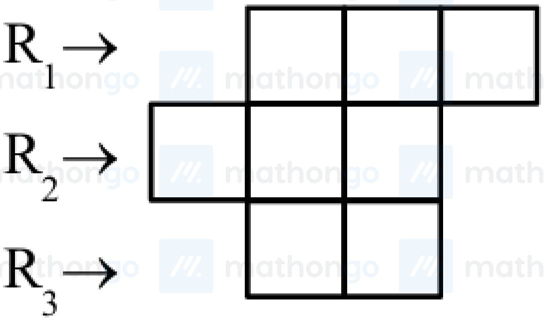
$\mathrm{a}=14,\ \mathrm{b}=15$

$\mathrm{a}=15,\ \mathrm{b}=16$

64 ordered pairs

Q10.

(4)



$= \text{Total } - [(\text{Allin}_1 \text{ and } R_3) + (\text{ All in } R_2 \text{ and } R_3) +$
$(\text{ All in } R_1 \text{ and } R_2)]$
$= {}^8C_5 \cdot \lfloor 5 - \{\underline{5} + \lfloor 5 + {}^6C_5 \cdot \underline{5}\}$
$= \lfloor 5(56 - 1 - 1 - 6) = 120(48)$
$= 5760$

Q1.
(1)
$$\left(\frac{(x+1)}{\left(x^{\frac{2}{3}}+1-x^{\frac{1}{3}}\right)}-\frac{(x-1)}{\left(x-x^{\frac{1}{2}}\right)}\right)^{10}$$
$$=\left(\left(x^{\frac{1}{3}}+1\right)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$
$$=\left(x^{\frac{1}{3}}-\frac{1}{\sqrt{x}}\right)^{10}$$
$T_{r+1}={}^{10}C_r(x)^{\frac{10-r}{3}}(-1)^r(x)^{-\frac{r}{2}}$
$\frac{10-r}{3}-\frac{r}{2}=0$
$(20-2r)-3r=0$
$r^{10}=4$
$\Rightarrow {}^{10}C_4(-1)^4=210$

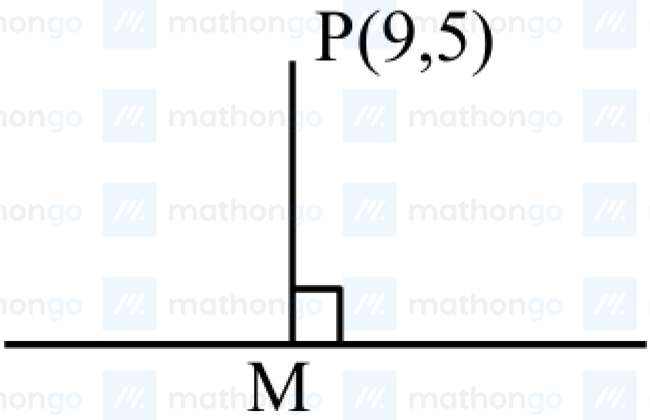
Q2. Let $\mathrm{N}=\left((64)^{64}\right)^{64}$
(2) $\mathbf{N}=(64)^{64^2}$
$\mathbf{N}=(1+63)^{64^2}$, let $64^2=\mathrm{n}$

Expanding by binomial
$\mathrm{N}=(1+63)^{\mathrm{n}}=1+{}^{\mathrm{n}}\mathrm{C}_1 63+{}^{\mathrm{n}}\mathrm{C}_2(63)^2+\ldots.$
$=1+63\lambda=1+7(9\lambda)$

Remainder when divided by 7 is 1

Q3. $S=(2+\sqrt{3})^8$
(4) For sum of rational terms
$={}^8\mathrm{C}_0(2)^8+{}^8\mathrm{C}_2(2)^6\cdot(\sqrt{3})^2+{}^8\mathrm{C}_4(2)^4(\sqrt{3})^4$
$+\ {}^8\mathrm{C}_6(2)^2(\sqrt{3})^6+{}^8\mathrm{C}_8(\sqrt{3})^8$
$=2^8+28\times 2^6\cdot 3+70.2^4\cdot 9+28.2^2.27+81$
$=256+5376+10080+3024+81$
$=18817$

Q4.
(4) No. of terms in $(x+y)^{(2n-3)}\Rightarrow\left[\begin{array}{c}(2n-3+1)\\(2n-2)\end{array}\right].$
$\therefore$ sum of all coefficients $=2^{2n-3}$
(Put $x=y=1$)
$\therefore$ Arithmetic mean of all coefficients
$=\left(\frac{2^{2n-3}}{2n-2}\right)=16$
$\Rightarrow 2^{2n-3}=2^5(n-1)\Rightarrow n=5$
$\therefore P\left(2n-1,n^2-4n\right)=(9,5)$



$x+y=8$

$\therefore PM=\left|\frac{9+5-8}{\sqrt{2}}\right|=\frac{6}{\sqrt{2}}=\frac{3\times 2}{\sqrt{2}}=3\sqrt{2}$

Q5. (3)

$T_{r+1}={}^{n}C_r\left(2^{1/3}\right)^{n-r}\left(\frac{1}{3^{1/3}}\right)^{r}$

$r=14$

$T_{15}={}^{n}C_{14}\left(2^{1/3}\right)^{n-14}\left(\frac{1}{3^{1/3}}\right)^{14}$

$T'_{15}=15^{h}$ term from last is $(n-13)^{h}$ term from beginning.

$T'_{15}={}^{n}C_{n-14}\left(2^{1/3}\right)^{14}\left(\frac{1}{3^{1/3}}\right)^{n-14}$

$\Rightarrow \frac{T_{15}}{T'_{15}}=\frac{{}^{n}C_{14}\left(2^{1/3}\right)^{n-14}\left(\frac{1}{3^{1/3}}\right)^{14}}{{}^{n}C_{n-14}\left(2^{1/3}\right)^{14}\left(\frac{1}{3^{1/3}}\right)^{n-14}}=\frac{1}{6}$

$=\left(2^{1/3}\right)^{n-28}\left(3^{1/3}\right)^{n-28}=\frac{1}{6}$

$=6^{\frac{n-28}{3}}=6^{-1}$

So, ${}^{n}C_3={}^{25}C_3=2300$

Q6. (63)

$(1919)^{1919}=(1920-1)^{1919}$

$={}^{1919}C_0(1920)^{1919}-{}^{1919}C_1(1920)^{1918}+\ldots.$

$+{}^{1919}C_{1918}(1920)^{1}-{}^{1919}C_{1919}$

$=100\lambda+1919\times 1920-1$

$=100\lambda+3684480-1$

$=100\lambda+\ldots\ldots\ldots..79$ (last two digit)

$\Rightarrow$ Number having last two digit 79

$\therefore$ Product of last two digit 63

Q7. $T_r = {}^{1016}C_r(5)^{\frac{1016-r}{2}} 7^{\frac{r}{8}}$
(4) $\Rightarrow r = 0, 8, 16, 24, \ldots, 1016$
$1016 = 0 + (n-1)8$
$\Rightarrow n - 1 = \frac{1016}{8} = 127$
So, $n = 128$.

Q8. $\sum_{r=0}^{10}\left(\frac{1r^{r-1}-1}{10^r}\right){}^{11}C_{r+1}$
(4)
$= \sum_{r=0}^{10}\left(10 - \frac{1}{10^r}\right){}^{11}C_{r+1}$
$= 10\sum_{r=0}^{10} C_{r+1} - 10\sum\left({}^{11}C_{r+1}\left(\frac{1}{10}\right)^{r+1}\right)$
$= 10\left[{}^{11}C_1 + {}^{11}C_2 + \ldots\ldots + {}^{11}C_{11}\right]$
$- 10\left[{}^{11}C_1\left(\frac{1}{10}\right)^1 + {}^{11}C_2\left(\frac{1}{10}\right)^2 + \ldots\ldots + {}^{11}C_{11}\left(\frac{1}{10}\right)^{11}\right]$
$= 10\left[2^{11} - 1\right] - 10\left[\left(1 + \frac{1}{10}\right)^{11} - 1\right]$
$= 10(2)^{11} - 10 - \frac{11^{11}}{10^{10}} + 10$
$= \frac{(20)^{11} - 11^{11}}{10^{10}}$
$\therefore \alpha = 20$

Q9. $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot {}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta, \alpha, \beta \in N$
(3) Now,
$\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot {}^9C_r = \sum_{r=1}^{9}\left(\frac{r}{2^r}\right)\cdot {}^9C_r + \sum_{r=1}^{9}\left(\frac{3}{2^r}\right)\cdot {}^9C_r$
$= \sum_{r=1}^{9}\left(\frac{9}{2^r}\right)\cdot {}^8C_{r-1} + 3\sum_{r=1}^{9} {}^9C_r\left(\frac{1}{2}\right)^r \left[U\sin\ g\frac{{}^9C_r}{{}^8C_{r-1}} = \frac{9}{r}\right]$
$= \frac{9}{2}\sum_{r=1}^{9} {}^8C_{r-1}\left(\frac{1}{2}\right)^{r-1} + 3\left(\sum_{r=0}^{9}\left({}^9C_r\left(\frac{1}{2}\right)^r\right) - 1\right)$
$= \frac{9}{2}\left(1 + \frac{1}{2}\right)^8 + 3\left(\left(1 + \frac{1}{2}\right)^9 - 1\right)$
$= \frac{9}{2}\cdot\left(\frac{3}{2}\right)^8 + 3\left(\frac{3}{2}\right)^9 - 3 = 6\cdot\left(\frac{3}{2}\right)^9 - 3$
Hence, $\alpha = 6, \beta = 3$
Thus $(\alpha + \beta)^2 = 81$

Q10. $\left(1 + x + x^2\right)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$
(239) $\therefore 3^{10} = a_0 + a_1 + a_2 + \ldots + a_{20}$ ...(i)
$1 = a_0 - a_1 + a_2 \ldots + a_{20}$ ...(ii)

(i) - (ii) $\Rightarrow a_1 + a_3 + \ldots + a_{19} = \frac{3^{10}-1}{2} = 29524$

Also $\{1 + x(1+x)\}^{10} = 1$

$+ {}^{10}C_1 x(1+x) + {}^{10}C_2 x^2 (1+x)^2 + \ldots$

$\therefore a_2 = {}^{10}C_1 + {}^{10}C_2 = 55$

$\therefore \frac{(a_1+a_3+\ldots+a_{19})-11a_2}{121} = 239$

Q11. $\sum_{r=1}^{15} r^2 \left({}^{15}C_r\right) \Rightarrow 15 \sum_{r=1}^{15} r\,{}^{14}C_{r-1}$

(1)

$15 \sum_{r=1}^{15} (r-1+1)\,{}^{14}C_{r-1}$

$15 \cdot 14 \sum_{r=1}^{15} {}^{13}C_{r-2} + 15 \sum_{r=1}^{15} {}^{14}C_{r-1}$

$15 \cdot 14 \cdot 2^{13} + 15 \cdot 2^{14}$

$3^1 \cdot 2^{13}(70+10)$

$3^1 \cdot 2^{13} \cdot 80$

$3^1 \cdot 5^1 \cdot 2^{17}$

$m = 17 n = 1 \quad k = 1$

option (1)

Q12. $(1-x)^{20} = {}^{20}C_0 - {}^{20}C_1 x + {}^{20}C_2 x^2 \ldots\ldots + {}^{20}C_{20} x^{20}$

(34) $\frac{(1-x)^{20}}{x^2} = \frac{{}^{20}C_0}{x^2} - \frac{{}^{20}C_1}{x} + {}^{20}C_2 - {}^{20}C_3 x + {}^{20}C_4 x^2 \ldots$

Diff twice and put $x = 1$

$= 6 - {}^{20}C_1(2) + A$

$A = 40 - 6 = 34$

Q1.
(1)

$$\frac{24+a+b}{8}=4$$
$a+b=8$
$$2=\frac{4+1+1+0+1+9+(a-4)^2+(b-4)^2}{8}$$
$16=48+a^2+b^2-8a-8b$
$a^2+b^2=32$
$32=2ab$
$ab=16$
$a=4b=4$
$\text{mode }=4$
$$\text{mean deviation }=\frac{2+1+1+0+1+3+0+0}{8}=1$$
option (1)

Q2.
(4)

$$\bar{x}=\frac{\sum x_i}{n}=\frac{1+3+a+7+b}{5}=5$$
$a+b=14$
$$\sigma^2=\frac{\sum x_i^2}{n}-(\bar{x})^2$$
$$\Rightarrow \frac{1^2+3^2+a^2+7^2+b^2}{5}-25=10$$
$a^2+b^2=116$
$a>b \quad a=10 \quad b=4$
$n+x_n: 2,5,13,11,9$
$$\sigma^2=\frac{2^2+5^2+13^2+11^2+9^2}{5}-\left(\frac{2+5+13+11+9}{5}\right)^2$$
$=80-64=16$
option 4

Q3.
(4)

$$\text{Actual means }=\mu=\frac{100(40)-50+40}{100}$$
$$\mu=40-\frac{1}{10}=39.9$$
Incorrect variance
$$(5.1)^2=\frac{\sum \mathrm{x}_{\mathrm{i}}^2}{100}-(\overline{\mathrm{x}})^2$$
$\sum \mathrm{x}_{\mathrm{i}}^2=100\times\left(40^2\right)+100(5.1)^2$
$\sum \mathrm{x}_{\mathrm{i}}^2=16\times10^4+(5.1)^2\times100=162601$
$$\sigma^2=\frac{\sum \mathrm{x}_{\mathrm{i}}^2-50^2+40^2}{100}-(\mu)^2$$
$\sigma^2=1617.01-(39.9)^2=25$
$\sigma=5$
$10(\mu+\sigma)=10(39.9+5)$
$=10\times44.9=449$

Q4. $\because$ mean $= 9$

(2) $\therefore 53 + a + b = 72$

$\Rightarrow a + b = 19$

$\because \sigma^2 = \dfrac{37}{4}$ and $(\overline{X})^2 + \sigma^2 = \dfrac{\sum x_1^2}{N}$

$\Rightarrow 81 + \dfrac{37}{4} = \dfrac{529 + a^2 + b^2}{8}$

$\Rightarrow 648 + 74 = 529 + a^2 + b^2$

$\Rightarrow a^2 + b^2 = 193$

$\because a + b = 19 \Rightarrow a^2 + b^2 + 2ab = 361$

$\Rightarrow 2ab = 168$

$\Rightarrow ab = 84$

$\therefore a + b + ab = 103$

Q1. $\because$ A is orthogonal matrix
(6) $\therefore A^T = A^{-1}$

$\Rightarrow A^2 = A^{-1} \left(\because \mathrm{A}^2 = \mathrm{A}^{\mathrm{T}}\right)$

$\Rightarrow A^3 = I$

let $B = (A+I)^3 + (A-I)^3 - 6A$

$= 2\left(A^3 + 3A\right) - 6A$

$= 2A^3$

$$B = 2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Now sum of diagonal elements $= 2+2+2 = 6$

Q2. $\begin{vmatrix} \mathrm{a} & \mathrm{d} \\ \mathrm{b} & \mathrm{c} \end{vmatrix} = \mathrm{ad} - \mathrm{bc} \Rightarrow \mathrm{ad} = \mathrm{bc}$
(36)

Case-I Exactly 1 no. is used

$\Rightarrow$ All singular $= {}^4\mathrm{C}_1$

Case-II Exactly 2 no. is used

$\Rightarrow {}^4\mathrm{C}_2 \times 2 \times 2$

Case-III Exactly 3 no. is used

None will be singular

Case-IV Exactly 4 No. is used

ad=bc

$\Rightarrow 2 \times 9 = 3 \times 6$

$\begin{vmatrix} 9 & - \\ - & 2 \end{vmatrix} \Rightarrow {}^4\mathrm{C}_1 \times 21$

Total $= 36$

Q3. $|A| = 0$

(4) $\alpha\beta + 6 = 0$

$\alpha\beta = -6$

$\alpha + \beta = 1$

$\Rightarrow \alpha = 3, \beta = -2$

$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$

$\therefore A^2 = A$

$A = A^2 = A^3 = A^4 = A^5$

$(I + A)^8$

$= I + {}^8C_1\, A^7 + {}^8C_2\, A^6 + \ldots\ldots + {}^8C_8\, A^8$

$= I + A\left({}^8C_1 + {}^8C_2 + \ldots\ldots + {}^8C_8\right)$

$= I + A\left(2^8 - 1\right)$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 765 & -255 \\ 1530 & -510 \end{bmatrix}$

$= \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Q4. $x^2 + x + 1 = 0$

(2) α is root

$\therefore \alpha^2 + \alpha + 1 = 0$

$\Rightarrow \alpha = \omega$ as ω^2 [cube root of unity]

also

$\begin{bmatrix} 4 - a - 2b & 64 - a - 14b & 52 + 2a - 8b \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

$\therefore a + 2b = 4$

$a + 14b = 64$

$\Rightarrow 12b = 60 \Rightarrow b = 5$

$\Rightarrow a = -6$

$\therefore \dfrac{4}{\alpha^4} + \dfrac{m}{\alpha^{-6}} + \dfrac{n}{\alpha^5} = 3$

$\Rightarrow \dfrac{4}{\omega} + \dfrac{m}{1} + \dfrac{n}{\omega^2} = 3$

$\Rightarrow 4\omega^2 + m + n\omega = 3$

$\Rightarrow 4\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right) + m + n\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = 3$

$\therefore -2 + m - \dfrac{n}{2} = 3 \ldots.(1)$

$\& \dfrac{-4\sqrt{3}}{2} + \dfrac{n\sqrt{3}}{2} = 0$

$\therefore n = 4$

$m = 7$

$\therefore\ m + n = 11$

Q5.
(1)

$A^3-2A^2-4A+4I=0$
$A^3=2A^2+4A-4I$
$A^4=2A^3+4A^2-4A$
$=2\left(2A^2+4A-4I\right)+4A^2-4A$
$A^4=8A^2+4A-8I$
$A^5=8A^3+4A^2-8A$
$=8\left(2A^2+4A-4I\right)+4A^2-8A$
$A^5=20A^2+24A-32I$
$\therefore \alpha=20, \beta=24, \gamma=-32$
$\therefore \alpha+\beta+\gamma=12$

Q6.
(1)

$\mathrm{A}^{50}=\mathrm{A}^{48}+\mathrm{A}^2-\mathrm{I}$
$=\mathrm{A}^{46}+2\left(\mathrm{A}^2-\mathrm{I}\right)$
$=\mathrm{A}^{44}+3\left(\mathrm{A}^2-\mathrm{I}\right)$
$=\mathrm{A}^2+24\left(\mathrm{A}^2-\mathrm{I}\right)$
$=25\mathrm{A}^2-24\mathrm{I}$
$=25\begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix}-24\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
$=\begin{bmatrix}1&0&0\\25&1&0\\25&0&1\end{bmatrix}$

Sum $=53$

option (1)

Q7.
(4)

$\mathrm{A}=\begin{bmatrix}1&\mathrm{a}&1\\2&1&0\\\mathrm{a}&1&2\end{bmatrix}-\mathrm{I}=\begin{bmatrix}0&\mathrm{a}&1\\2&0&0\\\mathrm{a}&1&1\end{bmatrix}$
$|\mathrm{A}|=-4 \Rightarrow 2-2\mathrm{a}=-4 \Rightarrow \mathrm{a}=3$
$|(\mathrm{a}+1)\operatorname{adj}(\mathrm{a}-1)\mathrm{A}|=|4\operatorname{adj}3\mathrm{A}|$
$=4^3|\operatorname{adj}3\mathrm{A}|$
$=4^3\times|3\mathrm{A}|^{3-1}=64|3\mathrm{A}|^2$
$=64\times\left(3^3\right)^2|\mathrm{A}|^2$
$=2^6\times3^6\times16$
$2^{\mathrm{m}}\times3^{\mathrm{n}}=2^{10}\times3^6$
$\therefore \mathrm{m}=10, \mathrm{n}=6$
$\Rightarrow \mathrm{m}+\mathrm{n}=16$

Q8. $|2\,\mathrm{adj}(3\,\mathrm{A}\,\mathrm{adj}(2\,\mathrm{A}))|$

(3)
$2^3 \cdot |3\,\mathrm{A}\,\mathrm{adj}(2\,\mathrm{A})|^2$

$2^3 \cdot \left(3^3\right)^2 \cdot |\,\mathrm{A}|^2 \cdot |\,\mathrm{adj}(2\,\mathrm{A})|^2$

$2^3 \cdot 3^6 \cdot |\,\mathrm{A}|^2 \cdot \left(|2\,\mathrm{A}|^2\right)^2$

$2^3 \cdot 3^6 \cdot |\,\mathrm{A}|^2\left[\left(2^3\right)^2 \cdot |\,\mathrm{A}|^2\right]^2$

$2^3 \cdot 3^6 \cdot |\,\mathrm{A}|^2 \cdot 2^{12} \cdot |\,\mathrm{A}|^4$

$2^{15} \cdot 3^6 \cdot |\,\mathrm{A}|^6$

$2^{15} \cdot 3^6 \cdot 5^6 = 2^\alpha \cdot 3^\beta \cdot 5^\gamma$

$\alpha = 15, \quad \beta = 6, \quad \gamma = 6$

$\alpha + \beta + \gamma = 27$

Q9.

(38)
$$|A| = \begin{vmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{vmatrix} = -1$$

$\lambda(16) - 2(-34) + 3(-39) = -1$

$16\lambda = 48 \Rightarrow \lambda = 3$

$\mathrm{B}^{-1} = \mathrm{adj}\left(\mathrm{A} \cdot \mathrm{adj}\left(\,\mathrm{A}^2\right)\right)$

Let $\mathrm{C} = \mathrm{A} \cdot \mathrm{adj}\left(\,\mathrm{A}^2\right)$

$AC = A^2\,\mathrm{adj}\left(A^2\right) = |A|^2 \cdot I = I \Rightarrow C = A^{-1}$

Now $B^{-1} = \mathrm{adj}\left(A^{-1}\right) = B = \mathrm{adj}(A)$

Now $\lambda \mathrm{B} + \mathrm{I} \Rightarrow 3\,\mathrm{B} + \mathrm{I}$

Let $\mathrm{P} = 3\,\mathrm{B} + \mathrm{I}$

$\mathrm{P} \ = 3\,\mathrm{adj}(\,\mathrm{A}) + \mathrm{I}$

$\mathrm{AP} = 3\,\mathrm{A}\mathrm{adj}(\,\mathrm{A}) + \mathrm{A}$

$\mathrm{AP} = 3|\,\mathrm{A}| \cdot \mathrm{I} + \mathrm{A}$

$\mathrm{AP} = \mathrm{A} - 3\mathrm{I}$

$|\mathrm{AP}| = |\mathrm{A} - 3\mathrm{I}|$

$$|\mathrm{A}| \cdot |\mathrm{P}| = \begin{vmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{vmatrix} = 38$$

$|\mathrm{P}| = -38$

Q10. $|\operatorname{adj}(\operatorname{adj})(\operatorname{adj}A)| = 81$

(4) $\Rightarrow |\operatorname{adj}A|^4 = 81$

$\Rightarrow |\operatorname{adj}A| = 3$

$\Rightarrow |A|^2 = 3$

$\Rightarrow |A| = \sqrt{3}$

$\left(|A|^4\right)^{\frac{(n-1)^2}{2}} = |A|^{n^2-5n-4}$

$\Rightarrow 2(n-1)^2 = 3n^2 - 5n - 4$

$\Rightarrow 2n^2 - 4n + 2 = 3n^2 - 5n - 4$

$\Rightarrow n^2 - n - 6 = 0$

$\Rightarrow (n-3)(n+2) = 0$

$\Rightarrow n = 3, -2$

$\sum_{n \in s} \left|A^{n^2+n}\right|$

$= \left|A^2\right| + \left|A^{12}\right|$

$= 3 + 36 = 3 + 729 = 732$

Q11.

(2) $|A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$

$C_3 \rightarrow C_3 - C_2 - C_1 \times \frac{q}{2}$

Then $C_3 \rightarrow C_2 - C_1 X \left(1 + \frac{p}{2}\right)$

$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$

$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$

$|\operatorname{adj}(\operatorname{adj}(3A))| = |3A|^{(3-1)^2} = |3A|^4$

$= \left(3^3|A|\right)^4 = \left(3^3 \times 2^3\right)^4 = 2^{12} \times 3^{12}$

$\Rightarrow m + n = 24$

Q1. For infinitely many solution

(1) $\Delta = 0$

$$\begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$

$\Rightarrow -17\lambda + 3 - 4 \times 72 - 4 = 0$

$\Rightarrow 17\lambda = -289$

$$\Rightarrow \boxed{\lambda = -17}$$

$\Delta 1 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0 \qquad \boxed{\mu = 45}$$

$\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$

$\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$

$\Rightarrow 12\mu = 540$

$x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y - 17z = 45$

Let $z = 1$

$x + 5y = 1 + \lambda] \times 4$

$4x + 3y = 7 + 3\lambda$

$4x + 20y = 4 + 4\lambda$

$$\frac{-17y = 3 - \lambda}{-17}$$

$$\mathrm{y} = \frac{\lambda - 3}{17}, \mathrm{x} = 1 + \lambda - \frac{5\lambda - 15}{17}$$

$$= \frac{32 - 12\lambda}{17}$$

$$7 \leq \frac{\lambda - 3}{17} + \frac{32 + 12\lambda}{17} + \lambda \leq 77$$

$$7 \leq \frac{30\lambda + 29}{17} \leq 77$$

$3 \leq \lambda \leq 42$

$\lambda = 3, 20, 37$

Q2. $C_3 \to C_3 - C_1$
(1)
$$y(x)=\begin{vmatrix}\sin x & \cos x & 1+\cos x\\ 27 & 28 & 0\\ 1 & 1 & 0\end{vmatrix}$$
$y(x)=-(1+\cos x)$
$\frac{dy}{dx}=\sin x$
$\frac{d^2y}{dx^2}=\cos x$
$\frac{d^2y}{dx^2}+y=-1$

Q3.
(2)
$$\Delta=\begin{vmatrix}3 & 1 & \beta\\ 2 & \alpha & -1\\ 1 & 2 & 1\end{vmatrix}=0$$
$3\alpha+4\beta-\alpha\beta+3=0$
$$\Delta_3=\begin{vmatrix}3 & 1 & 3\\ 2 & \alpha & -3\\ 1 & 2 & 4\end{vmatrix}=0$$
$9\alpha+19=0$
$\alpha=\frac{-19}{9},\beta=\frac{6}{11}$
$\Rightarrow 22\beta-9\alpha=31$

Q4.
(3)
$$\Delta=0\Rightarrow\begin{vmatrix}2 & \lambda & 3\\ 3 & 2 & -1\\ 4 & 5 & \mu\end{vmatrix}=0$$
$\Rightarrow 2(2\mu+5)+\lambda(-4-3\mu)+3(7)=0$
$\Rightarrow 4\mu-3\lambda\mu-4\lambda+31=0$ ...(1)
$$\Delta_3=0\Rightarrow\begin{vmatrix}2 & \lambda & 5\\ 3 & 2 & 7\\ 4 & 5 & 9\end{vmatrix}=0$$
$\Rightarrow 2(-17)+\lambda(1)+5(7)=0$
$\Rightarrow \lambda=-1$
from equation (1)
$4\mu+3\mu+4+31=0\Rightarrow\mu=-5$
$\therefore \lambda^2+\mu^2=26$

Q5.
(2) $\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(\lambda+4) \end{vmatrix} = 0$

$\Rightarrow 12(-21) - 3(-39) - (\lambda+4)(-15) = 0$

$\Rightarrow -252 + 117 + 15(1+4) = 0$

$\Rightarrow 15\lambda + 177 - 252 = 0$

$\Rightarrow 15\lambda - 75 = 0 \Rightarrow \lambda = 5$

$\begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 7 \\ \mu-8 & 0 & 7 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0$

$\Rightarrow 7 - 7(\mu-8) = 0 \Rightarrow 1 - (\mu-8) = 0 \Rightarrow \mu = 9$

$\Rightarrow$ centre of circle $(5, 9)$

radius $=$ length of $\perp$ from centre $(5, \quad 9) =$

$\left|\dfrac{20-27}{5}\right| = \dfrac{7}{5}$

Q1. n cards are drawn \& are found all spade, thus remaining spades $= 13 - x$ remaining total cards $= 52 - x$
(2) Now given that P(lost card is spade)$= \frac{11}{50}$

i.e. $\frac{{}^{13-n}C_1}{{}^{52-n}C_1} = \frac{11}{50}$

$50(13 - n) = 11(52 - n)$

$39n = 78$

$n = 2$

Q2. $\sum_{x=0}^{\infty} k(x+1)3^{-x} = 1$
(4) $\Rightarrow \frac{1}{k} = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots$(i)

$\frac{1}{3k} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots\ldots$(ii)

(i)- (ii) $\Rightarrow \frac{1}{k} - \frac{1}{3k} = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots$

$\Rightarrow k = \frac{4}{9}$

$P(x \geq 3) = 1 - P(x = 0) - P(x = 1) - P(x = 2)$

$= 1 - k\left(1 + \frac{2}{3} + \frac{3}{9}\right) = \frac{1}{9}$

Q3. 3 engineering $+ 1$ doctor $+ 8$ Prof $\rightarrow {}^4C_3 \cdot {}^2C_1 \cdot {}^{10}C_8$
(1) $= 360$

3 engineering $+ 2$ doctors $+ 7$ Prof $\rightarrow {}^4C_3 \cdot {}^2C_2 \cdot {}^{10}C_7$
$= 480$

4 engineering $+ 1$ doctor $+ 7$ Prof $\rightarrow {}^4C_4 \cdot {}^2C_1 \cdot {}^{10}C_7$
$= 240$

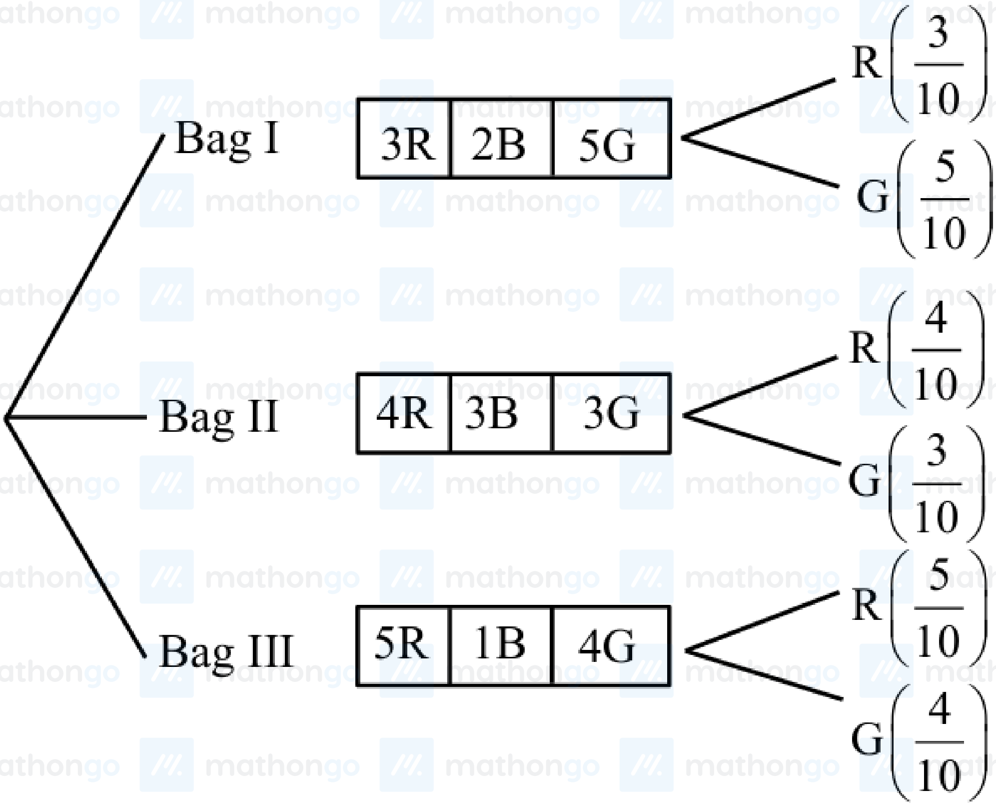
4 engineering $+ 2$ doctors $+ 6$ Prof $\rightarrow {}^4C_4 \cdot {}^2C_2 \cdot {}^{10}C_6$
$= 210$

Total $= 1290$

Req. probability $= \frac{1290}{{}^{16}C_{12}} = \frac{1290}{1820} = \frac{129}{182}$

Q4.

(3)



$$p = P\left(\frac{B_I}{R}\right) = \frac{\frac{1}{3}\left(\frac{3}{10}\right)}{\frac{1}{3}\left(\frac{3}{10}+\frac{4}{10}+\frac{5}{10}\right)} = \frac{1}{4}$$

$$q = \left(\frac{B_{III}}{G}\right) = \frac{\frac{1}{3}\left(\frac{4}{10}\right)}{\frac{1}{3}\left(\frac{5}{10}+\frac{3}{10}+\frac{4}{10}\right)} = \frac{1}{3}$$

$$\therefore \frac{1}{p} + \frac{1}{q} = 7$$

Q5.

(1)

$$P(H) = \underbrace{\frac{19}{20}}_{\text{Selection of unbiased}} \times \underbrace{\frac{1}{2}}_{\text{Head occurs}} + \underbrace{\frac{1}{20}}_{\text{Selection of biased coin}} \times \underbrace{1}_{\text{Head occurs}}$$

$$\text{Required probability } = \frac{\frac{19}{20} \times \frac{1}{2}}{\frac{19}{20} \times \frac{1}{2} + \frac{1}{20} \times 1} = \frac{19}{21}$$

$$\therefore \frac{m}{n} = \frac{19}{21}$$

$$\Rightarrow n^2 - m^2 = 441 - 361 = 80$$

Q6.

(2)

x	$\mathrm{x}=0$	$\mathrm{x}=1$	$\mathrm{x}=2$
$\mathrm{P(x)}$	$\frac{{}^{7}C_2}{{}^{10}C_2}$	$\frac{{}^{7}C_1^{3}C_1}{{}^{10}C_2}$	${}^{310}C_2$

$\mu=\Sigma \mathrm{x_i P\left(x_i\right)}=0+\frac{7}{15}+\frac{2}{15}=\frac{3}{5}$

$\text{Variance }(\mathrm{x})=\Sigma \mathrm{P_i}\left(\mathrm{x_i}-\mu\right)^2=\frac{28}{75}$

Q7. $2p+2q=\frac{1}{2}$

(2) $p+q$

$E\left(x^2\right)=\sum_{i=0}^3 x_i^2 p\left(x_i\right)=0 \cdot p+1 \cdot p+4 \cdot q+9q$

$=p+13q$

$E(x)=\sum_{i=0}^3 x_i^2 p\left(x_i\right)=0 \cdot p+1 \cdot p+2q+3q=p+5q$

$p+13q=2(p+5q)$

$p=3q$

So, $q=\frac{1}{8} \& p=\frac{3}{8}$

So, $8p-1=2$ Option (2)

Q8. $1 \leq \mathrm{a}<\mathrm{ar}<\mathrm{ar}^2 \leq 40$

(4949) (If $r \in N$)

If $r=2$

$1 \leq a<2a<4a \leq 40$

$a \in\{1, \ldots \ldots ., 10\}$ ________ (10 GP)

If $r=3$

$1 \leq a<3a<9a \leq 40$

$\mathrm{a} \in\{1,2,3,4\}$ ________ ________ (4 GP)

If $\mathrm{r}=4$

$1 \leq a<4a<16a \leq 40$

$a \in\{1,2\}$ ________ ________ (2 GP)

If $r=5$

$1 \leq a<5a<25a \leq 40$

$a \in\{1\}$ ________ ________ (1 GP)

If $r=6$

$1 \leq a<6a<36a \leq 40$

$\mathrm{a} \in\{1\}$ ________ ________ (1 GP)

$\left(\mathrm{P}=\frac{18}{9880}=\frac{9}{4940}\right)$ as per NTA for $\mathrm{r} \in \mathrm{N}$

$\mathrm{m}+\mathrm{n}=4949$

If $\underline{r} \notin$ N (also possible)

$r = \frac{3}{2}$

$ar^2 = \frac{9a}{4}; a = 4k$

$$\left.\begin{array}{l}(4,6,9)\\(8,12,18)\\(12,18,27)\\(16,24,36)\end{array}\right\} 4GP$$

$r = \frac{5}{2} \quad ar^2 = \frac{25a}{4}; a = 4k$

$(4,10,25)\ldots\ldots\ldots(1)GP$

$r = \frac{4}{3} \quad ar^2 = \frac{16a}{9} \rightarrow a = 9k$

$(9,12,16),(18,24,32)\ldots\ldots\ldots(2)GP$

$r = \frac{5}{3} \quad ar^2 = \frac{25a}{9}; a = 9k$

$(9,15,25)\ldots\ldots\ldots\ldots(1)GP$

$r = \frac{5}{4} \quad ar^2 = \frac{25a}{16}; a = 16k$

$(16,20,25)\ldots\ldots\ldots\ldots(1)GP$

$r = \frac{6}{5} \quad ar^2 = \frac{36a}{25}; a = 25k$

$(25,30,36)\ldots\ldots\ldots\ldots(1)GP$

$\text{Total} = 18 + 10 = 28$

$P = \frac{28}{{}^{40}C_3} = \frac{28}{9880} = \frac{7}{2470}$

$m + n = 2477$

Q9. **(183)**

Let $P(W_1) = x$

$$\sum_{i=1}^{120} P(W_i) = 1$$

$$x + 2x + 2^2x + 2^3x + \ldots + 2^{119}x = 1$$

$$\frac{x\left(2^{120}-1\right)}{(2-1)} = 1 \Rightarrow x = \frac{1}{2^{120}-1} \qquad \ldots(i)$$

Rank of CDBEA

$$A\ \_\ \_\ \_\ \_ = \lfloor 4 = 24$$

$$B\ \_\ \_\ \_\ \_ = \lfloor 4 = 24$$

$$C\ A\ \_\ \_\ \_\ \_ = \lfloor 3 = 6$$

$$C\ B\ \_\ \_\ \_\ \_ = \lfloor 3 = 6$$

$$C\ D\ A\ \_\ \_\ \_ = \lfloor 2 = 2$$

$$\overline{\qquad\qquad\qquad\qquad 64}$$

$C\,D\,B\,A\,E = 1$

$C\,D\,B\,E\,A = 1$

So, $P(W_{64}) = 2P(W_{63}) = \ldots = 2^{63}P(W_1)$

$= \dfrac{2^{63}}{2^{120} - 1}$

$\alpha + \beta = 63 + 120 = 183$

Q10. (1)

$P(A) = \dfrac{7}{10}, P(B) = \dfrac{4}{10}$

$P(A \cup \overline{B}) = \dfrac{5}{10}$

$P\left(\dfrac{B}{A \cup \overline{B}}\right) = \dfrac{P(B \cap (A \cup \overline{B}))}{P(A \cup \overline{B})}$

$= \dfrac{P((B \cap \overline{B}) \cup (B \cap A))}{P(A \cup \overline{B})} = \dfrac{P(A \cap B)}{P(A \cup \overline{B})}$

$= \dfrac{P(A) - P(A \cap \overline{B})}{P(A) + P(\overline{B}) - P(A \cap \overline{B})} = \dfrac{\frac{7}{10} - \frac{5}{10}}{\frac{7}{10} + \left(1 - \frac{4}{10}\right) - \frac{5}{10}}$

$= \dfrac{2}{8} = \dfrac{1}{4}$

Q1. (1) $A=\{-2,-1,0,1,2,3\}$
$R=\{(-2,1),(-1,1),(0,1),(1,1),(2,2),(3,3)\}$
$\ell=6$
$m=3$
$n=3$
$\ell+m+n=12$

Q2. (3) $a=2b+1$
$2b=a-1$
$R=\{(3,1),(5,2),\ldots,(99,49)\}$
Let $(2m+1,m),(2\lambda-1,\lambda)$ are such ordered pairs.
According to the condition
$m=2\lambda-1 \Rightarrow m=$ odd number
$\Rightarrow 1^{\text{st}}$ element of ordered pair (a,b)
$a=2(2\lambda-1)+1=4\lambda-1$
Hence $a\in\{3,7,\ldots,99\}$
$\Rightarrow \lambda\in\{1,2,\ldots,25\}$
$\Rightarrow$ set of sequence
$$\left\{(4\lambda-1,2\lambda-1),(2\lambda-1,\lambda-1),\left(\lambda-1,\frac{\lambda-2}{2}\right),\ldots\ldots\right\}$$
2^{nd} element of each ordered pair $=\dfrac{\lambda-2^{r-2}}{2^{r-2}}$
For maximum number of ordered pairs in such sequence
$\dfrac{\lambda-2^{r-2}}{2^{r-2}}=1$ or $2; 1\leq\lambda\leq 25$
$\lambda=2^{r-1}$ or $\lambda=3.2^{r-2}$
Case-I : $\lambda=2r-1$
$\lambda=2,2^2,2^3,2^4$
$r=2,3,4,5$
Hence maximum value of r is 5 when $\lambda=16$
Case-II : $\lambda=3.2^{r-2}$
$\lambda=3,6,12,24$
$r=2,\quad 3,\quad 4,5$
Hence maximum value of r is 5 when $\lambda=24$

Q3. (2) $R=\{(f,g): f(0)=g(1) \text{ and } f(1)=g(0)\}$
Reflexive: $(f,f)\in R$
$=f(0)=f(1)$ and $f(1)=f(0)\rightarrow$ must hold
$\Rightarrow$ but this is not true for all function
so not reflexive
Symmetric: If $(f,g)\in R\Rightarrow(g,f)\in R$
Now, $g(0)=f(1)$ and $g(1)=f(0)\rightarrow$ true
$\therefore$ symmetric
Transitive : If$(f,g)\in R$ and $(g,h)\in R$

$\Rightarrow (\mathrm{f}, \mathrm{h}) \in \mathrm{R}$

Now $(f, g) \in R \Rightarrow f(0)=g(1)$ and $f(1)=g(0)$

$(\mathrm{g}, \mathrm{h}) \in \mathrm{R} \Rightarrow \mathrm{g}(0)=\mathrm{h}(1)$ and $\mathrm{g}(1)=\mathrm{h}(0)$

For $(f, h) \in R$ we need $f(0)=h(1)$ and $f(1)=h(0)$

Now $f(0)=g(1)=h(0)$ and $f(1)=g(0)=h(1)$

Hence not transitive

Q4. $\mathrm{A}=\{-3,-2,-1,0,1,2,3\}$

(4) $-2\mathrm{y} \leq \mathrm{x}^2 \leq 4-2\mathrm{y}$

$y=-3 \quad 6 \leq x^2 \leq 10 \quad \Rightarrow x \in\{-3,3\}$

$y=-2 \quad 4 \leq x^2 \leq 8 \quad \Rightarrow x \in\{-2,2\}$

$y=-1 \quad 2 \leq x^2 \leq 6 \quad \Rightarrow x \in\{-2,2\}$

$y=0 \quad 0 \leq x^2 \leq 4 \quad \Rightarrow x \in\{-2,-1,0,1,2\}$

$y=1 \quad -2 \leq x^2 \leq 2 \quad \Rightarrow x \in\{-1,0,1\}$

$y=2 \quad -4 \leq x^2 \leq 0 \quad \Rightarrow x \in\{0\}$

$y=3 \quad -6 \leq x^2 \leq -2 \quad \Rightarrow$ No x-Exist

$\mathrm{R}=\{(-3,-3)(-3,3),(-2,-2)(-2,2)(-1,-2)(-1,2)$
$(0,-2)(0,-1)(0,0)(0,1)(0,2)(1,-1)(1,0)(1,1)$
$(2,0)\}$

$\therefore \ell=15$

To make it reflexive we will add

$\{(-1,-1),(2,2),(3,3)\} \quad \therefore \mathrm{m}=3$

$\therefore \ell+\mathrm{m}=15+3=18$

Q5. $2\mathrm{x}-\mathrm{y}=0$

(2) $\{0,0\}\{-1,-2\}\{1,2\}$

$2\mathrm{x}-\mathrm{y}=1$

$\{0,-1\}\{1,1\}\{2,3\}\{-1,-3\}$

Total $(0,0)(-1,-2),(1,2)(0,-1),(1,1)(2,3)(-1,-3)$

Reflexive $\quad \mathrm{m}=5 \quad \& \ell=7$

Symm. $\quad \mathrm{n}=5 \quad \ell+\mathrm{m}+\mathrm{n}=17$

option (2)

Q6. $A=\{1,2,3\}$

(5) $(1,1),(2,2),(3,3),(1,2) \in R$

Remaining elements are

$(2,1),(2,3),(1,3),(3,1),(3,2)$

(1) If relation contains exactly 4 elements = 1 way

(2) if relation contains exactly 5 elements

It can be $(1,3),(3,2) \Rightarrow 2$ ways

(3) If relation contain exactly 6 elements

It can be $((2,3),(1,3)),((1,3),(3,2)),((3,1),(3,2))$

$\Rightarrow 3$ ways.

Total $=6$ ways

Q7. $A = \{0, 1, 2, 3, 4, 5\}$

(3) $\mathrm{R} \equiv \{(0, 3), (3, 0), (0, 4), (4, 0), (1, 3), (3, 1), (1, 4), (4, 1), (2, 3), (3, 2), (2, 4), (4, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

Total 16 elements

Not reflexive as $(0, 0), \ldots\ldots, (2, 2) \notin \mathrm{R}$

Symmetric $\because \forall$ all a,b

$(a, b) \& (b, a) \in R$

Not transitive $\because (0, 3), (3, 1) \in R$

but $(0, 1) \notin \mathrm{R}$

$\Rightarrow$ Only S_2 correct

Q1. $f(x)+3f\left(\frac{24}{x}\right)=4x$

(1) Put $x=3 \quad f(3)+3f(8)=12$

Put $x=8 \quad f(8)+3f(3)=32$

Add both $\quad 4(\mathrm{f}(3)+\mathrm{f}(8))=44$

$f(3)+f(8)=11$

Q2. $f(x)+2f\left(\frac{1}{x}\right)=x^2+5$

(4) $f\left(\frac{1}{x}\right)+2f(x)=\frac{1}{x^2}+5$

$f(x)=\frac{2}{3x^2}-\frac{x^2}{3}+\frac{5}{3}$

$\alpha=\int_1^2\left(\frac{2}{3x^2}-\frac{x^2}{3}+\frac{5}{3}\right)\mathrm{dx}$

$\left(-\frac{2}{3x}-\frac{x^3}{9}+\frac{5x}{3}\right)_1^2$

$-\frac{1}{3}-\frac{8}{9}+\frac{10}{3}+\frac{2}{3}+\frac{1}{9}-\frac{5}{3}$

$\alpha=2-\frac{7}{9}=\frac{11}{9}$

$2g(x)-3g\left(\frac{1}{2}\right)=x$

$\mathrm{g}\left(\frac{1}{2}\right)=-\frac{1}{2}$

$\mathrm{g(x)}=\frac{\mathrm{x}}{2}-\frac{3}{4}$

$\beta=\int_1^2\left(\frac{x}{2}-\frac{3}{4}\right)dx$

$\left(\frac{x^2}{4}-\frac{3x}{4}\right)_1^2=1-\frac{3}{2}-\frac{1}{4}+\frac{3}{4}=0$

$9\alpha+\beta=11$

option (4)

Q3. $\mathrm{x}+|\mathrm{x}|>0 \quad \Rightarrow \mathrm{x}\in(0,\infty)$ ...(1)

(1) $\&10+3x-x^2>0$

$\Rightarrow x^2-3x-10<0$

$\Rightarrow x\in(-2,5)$ ...(2)

from (1)&(2) $\quad \mathrm{x}\in(0,5)$

$\therefore \mathrm{a}=0 \& \mathrm{~b}=5$

$\therefore \left(1+\mathrm{a}^2\right)+\mathrm{b}^2=1+25=26$

Q4. Given function is

(2) $f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$

For domain, the conditions are

$\frac{2x-3}{5+4x}>0$ and $\left|\frac{4+3x}{2-x}\right|\leq 1$

Now, $\frac{2x-3}{5+4x}>0 \Rightarrow x\in\left(-\infty,-\frac{5}{4}\right)\cup\left[\frac{3}{2},\infty\right)$

and $\quad -1\leq\frac{4+3x}{2-x}\leq 1$

$\Rightarrow\left(-1\leq\frac{4+3x}{2-x}\right)\cap\left(\frac{4+3x}{2-x}\leq 1\right)$

$\Rightarrow\left(\frac{6+2x}{2-x}\geq 0\right)\cap\left(\frac{2+4x}{2-x}\leq 0\right)$

$\Rightarrow\frac{6+2x}{2-x}\cdot\frac{2+4x}{2-x}\leq 0$

$\Rightarrow x\in\left[-3,-\frac{1}{2}\right]$

Hence, we get the domain of f as $x\in\left[-3,-\frac{5}{4}\right)$ This means that $\alpha=-3,\beta=-\frac{5}{4}$ Thus, $\alpha^2+4\beta=9-5=4$

Q5. (1)

Domain $1-\log_4\left(x^2-9x+18\right)>0$

Also $x^2-9x+18>0$

$(x-3)(x-6)>0$

$x\in(-\infty,3)\cup(6,\infty)$ ...(1)

also $x^2-9x+18<4$

$x^2-9x+14<0$

$x\in(2,7)$ ...(2)

$(1)\cap(2)\quad(2,3)\cup(6,7)=(\alpha,\beta)\cup(\gamma,\delta)$

$\Rightarrow\alpha+\beta+\gamma+\delta=18$

Q6. (96)

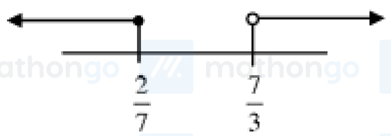
$f(x)=\cos^{-1}\left(\frac{4x+5}{3x-7}\right)$

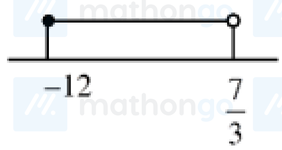
$\Rightarrow -1\leq\left(\frac{4x+5}{3x-7}\right)\leq 1$

$\left(\frac{4x+5}{3x-7}\right)\geq -1$

$\frac{4x+5+3x-7}{3x-7}\geq 0$

$\Rightarrow\frac{7x-2}{3x-7}\geq 0$





$x\in\left(-\infty,\frac{2}{7}\right]\cup\left(\frac{7}{3},\infty\right)$

$\&\frac{4x+5}{3x-7}\leq 1\Rightarrow\frac{x+12}{3x-7}\leq 0$

$\therefore$ Domain of $f(x)$ is

$\left[-12,\frac{2}{7}\right]\alpha=-12,\beta=\frac{2}{7}$

$g(x)=\log_2(2-6\log_{27}(2x+5))$

Domain

$2-6\log_{27}(2x+5)>0$

$\Rightarrow 6\log_{27}(2\mathrm{x}+5)<2$

$\Rightarrow \log_{27}(2\mathrm{x}+5)<\frac{1}{3}$

$\Rightarrow 2\mathrm{x}+5<3$

$\Rightarrow \mathrm{x}<-1$

$\& 2x+5>0 \Rightarrow x>-\frac{5}{2}$

Domain is $\mathrm{x}\in\left(-\frac{5}{2},-1\right)$

$$\gamma=-\frac{5}{2},\delta=-1$$

$$|7(\alpha+\beta)+4(\gamma+\delta)|=\left|7\left(-12+\frac{2}{7}+4\left(-\frac{5}{2}-1\right)\right|\right.$$

$$|-82-14|=96$$

Q7. $\operatorname{fog}(x)=f(g(x))$

(4)

$$=f\left(\frac{2-3x}{1-x}\right)=\frac{2\left(\frac{2-3x}{1-x}\right)+3}{5\left(\frac{2-3x}{1-x}\right)+2}$$

$$=\frac{4-6x+3-3x}{10-15x+2-2x}=\left(\frac{7-9x}{12-17x}\right)$$

$$\therefore\left[\begin{array}{l}12-7x\neq 0\\ \quad x\neq\frac{12}{17}\end{array}\right.$$

$$\left[\begin{array}{l}\operatorname{fog}(2)=\frac{7-9(2)}{12-17(2)}=\frac{-11}{-22}=\frac{1}{2}\\ \operatorname{fog}(4)=\frac{7-9(4)}{12-17(4)}=\frac{-29}{-56}=\frac{29}{56}\end{array}\right.$$

$$\text{Range of fog}:[\alpha,\beta]=\left[\frac{1}{2},\frac{29}{56}\right]$$

$$\therefore(\beta-\alpha)=\frac{29}{56}-\frac{1}{2}=\frac{29-28}{56}=\frac{1}{56}$$

$$\frac{1}{(\beta-\alpha)}=56$$

Q8. $\mathrm{A}:\mathrm{x}^2+\mathrm{y}^2=25\ldots(i)$

(3) $\mathrm{B}:\frac{x^2}{144}+\frac{y^2}{16}=1\ldots(ii)$

C: $x^2+y^2\leq 4\ldots(iii)$

Solve (1) \& (2)

$$x^2+9\left(25-x^2\right)=144$$

$$-8x^2=144-225=-81$$

$$x=\pm\frac{9}{2\sqrt{2}}$$

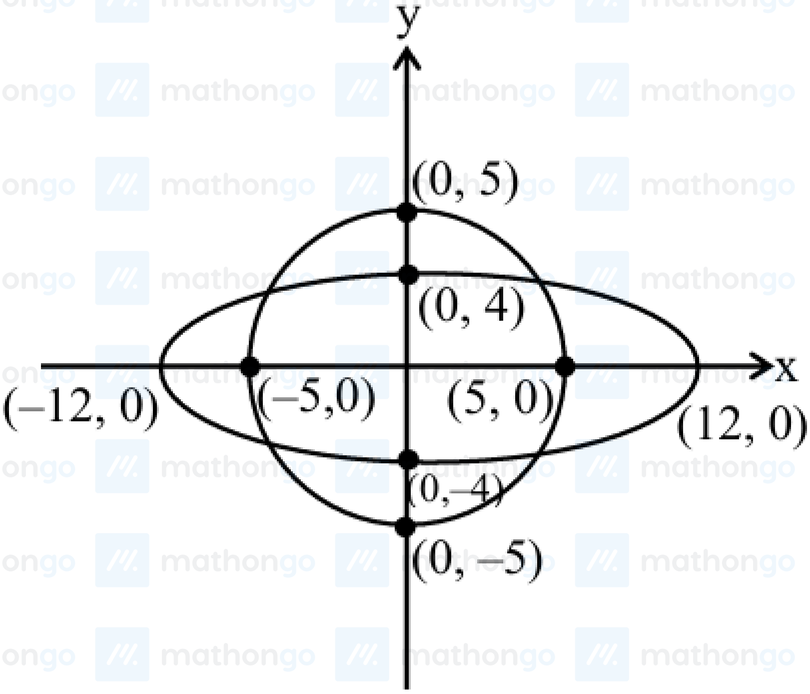
$\mathrm{By}(1)\Rightarrow y=\pm\sqrt{25-x^2}$

$=\pm\sqrt{25-\frac{81}{8}}=\pm\frac{\sqrt{119}}{2\sqrt{2}}$

$\therefore \mathrm{D}=\mathrm{A}\cap\ \mathrm{B}=$

$\left\{\left(\frac{9}{2\sqrt{2}}, \frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{9}{2\sqrt{2}}, -\frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{-9}{2\sqrt{2}}, \frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{-9}{2\sqrt{2}}, \frac{-\sqrt{119}}{2\sqrt{2}}\right)\right\}$

No. of elements in set $D = 4$



$\because C = \{(x, y) \in \mathrm{Z} \times \mathrm{Z} : \mathrm{x}^2 + \mathrm{y}^2 \leq 4\}$
$= \{(0,2), (2,0), (0,-2), (-2,0), (1,1), (-1,-1),$
$(1,-1), (-1,1), (1,0), (0,1), (-1,0), (0,-1),$
$(0,0)\}$

No. of elements in set $\mathrm{C} = 13$

Total no. of one-one function from

Set D to sec $C \Rightarrow 13 \times 12 \times 11 \times 10 = 17160$

Q9. $f(2x)-f(x)=x$

(2) $f(x)-f\left(\frac{x}{2}\right)=\frac{x}{2}$

$\mathrm{f}\left(\frac{\mathrm{x}}{2}\right)-\mathrm{f}\left(\frac{\mathrm{x}}{4}\right)=\frac{\mathrm{x}}{4}$

$\mathrm{f}\left(\frac{\mathrm{x}}{4}\right)-\mathrm{f}\left(\frac{\mathrm{x}}{8}\right)=\frac{\mathrm{x}}{8}$

$f\left(\frac{x}{2^{n-1}}\right)-f\left(\frac{x}{2^n}\right)=\frac{x}{2^n}$

$f(2x)-f\left(\frac{x}{2^n}\right)=x\left\{\frac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}}\right\}$

$f(x)-f\left(\frac{x}{2^n}\right)=2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$

$\mathrm{f}(\mathrm{x})+\mathrm{x}-\mathrm{f}\left(\frac{\mathrm{x}}{2^{\mathrm{n}}}\right)=2\mathrm{x}\left(1-\left(\frac{1}{2}\right)^{\mathrm{n}+1}\right)$

$\lim_{n\to\infty}\left(f(x)-f\left(\frac{x}{2^n}\right)\right)=\lim_{n\to\infty}\left(2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)-x\right)$

$G(x)=x$

$\sum_{r=1}^{10}G\left(r^2\right)=\sum_{r=1}^{10}r^2=385$

Q10. $\log_3\left(\log_7\left(8-\log_2\left(\mathrm{x}^2+4\mathrm{x}+5\right)\right)\right)>0$

(1) $\log_2\left(\mathrm{x}^2+4\mathrm{x}+5\right)<1$

$\mathrm{x}^2+4\mathrm{x}+3<0$

$\Rightarrow \mathrm{x}\in(-3,-1)$

$-1\leq\frac{7\mathrm{x}+10}{\mathrm{x}-2}\leq 1$

$\Rightarrow \mathrm{x}\in[-2,-1]$

$\alpha=-3, \beta=-1, \gamma=-2, \delta=-1$

$\alpha^2+\beta^2+\gamma^2+\delta^2=15$

option (1)

Q11. $y=\frac{5-x}{x^2-3x+2}$

(4) $yx^2-3xy+2y+x-5=0$

$yz^2+(-3y+1)x+(2y-5)=0$

Case I : If $y=0$ (Accepted)

$\Rightarrow x=5$

Case II : If $y\neq 0$

$\mathrm{D} \geq 0$

$(-3y+1)^2 - 4(y)(2y-5) \geq 0$

$9y^2 + 1 - 6y - 8y^2 + 20y \geq 0$

$\mathrm{y}^2 + 14y + 1 \geq 0$

$(y+7)^2 - 48 \geq 0$

$|y+7| \geq 4\sqrt{3}$

$\Rightarrow y + 7 \geq 4\sqrt{3}$ or $\mathrm{y} + 7 \leq -4\sqrt{3}$

$\Rightarrow \mathrm{y} \geq 4\sqrt{3} - 7$ or $\mathrm{y} \leq -4\sqrt{3} - 7$

From Case I and Case II

$\mathrm{y} \in (-\infty, -4\sqrt{3} - 7] \cup [4\sqrt{3} - 7, \infty)$

So $\alpha = -4\sqrt{3} - 7$

$\beta = 4\sqrt{3} - 7$

$$\begin{aligned} \Rightarrow a^2 + b^2 &= (-4\sqrt{3} - 7)^2 + (4\sqrt{3} - 7)^2 \\ &= 2(48 + 49) \\ &= 194 \end{aligned}$$

Q1. $\lim_{x\to 0}\frac{\frac{\tan(\tan x)-\tan x}{\tan^3 x}\frac{\tan^3 x}{x^3}+\frac{\tan x-\sin x}{x^3}+\frac{\sin x-\sin(\sin x)}{\sin^3 x}\frac{\sin^3 x}{x^3}}{\frac{\tan x-\sin x}{x^3}}$

(2)

$=\frac{\frac{1}{3}+\frac{1}{2}+\frac{1}{6}}{\frac{1}{2}}=2$

Q2. $\lim_{x\to 0}\frac{\cos 2x+a\cos 4x-b}{x^4}=\text{finite}$

(1)

$$L=\frac{\left\{1-\frac{(2x)^2}{\underline{2}}+\frac{(2x)^4}{\underline{4}\cdots\}\}+a\left\{1-\frac{(4x)^2}{\underline{2}}+\frac{(4x)^4}{\lfloor 4}\cdots\right\}-b}\right.}{x^4}$$

$$L=\frac{(1+a-b)-x^2(2+8a)+x^4\left(\frac{2}{3}+\frac{32}{3}a\right)+x^6()\ldots}{x^4}$$

$\therefore 1+\mathrm{a}-\mathrm{b}=0$ and $2+8\mathrm{a}=0\Rightarrow \mathrm{a}=-\frac{1}{4}$

$\mathrm{b}=\mathrm{a}+1$

$=-\frac{1}{4}+1=\frac{3}{4}$

$\therefore \mathrm{a}+\mathrm{b}=-\frac{1}{4}+\frac{3}{4}=\frac{1}{2}$

Q3. $\lim_{x\to 0^{\cdot}}\left(\frac{\tan\left(5\mathrm{x}^{1/3}\right)}{5\mathrm{x}^{1/3}}\right)\cdot\left(\frac{(3\sqrt{\mathrm{x}})^2}{\left(\tan^{-1}3\sqrt{\mathrm{x}}\right)^2}\right)\left(\frac{\ell\left(1+3\mathrm{x}^2\right)}{3\mathrm{x}^2}\right)\left(\frac{5\mathrm{x}^{4/3}}{\mathrm{e}^{5x^4}-1}\right)\times\frac{5\mathrm{x}^{1/3}\cdot 3\mathrm{x}^2}{5\mathrm{x}^{4/3}\cdot 9\mathrm{x}}$

(3)

$=\frac{1}{3}$

Q4. $\lim_{x\to 10}\frac{x^2(\alpha x)+(\gamma-1)\left(1+\frac{x^2}{1}\right)}{2x-\frac{8x^3}{6}-\beta x}=3$

(1)

$\lim_{x\to 0}\frac{(\gamma-1)+(\gamma-1)x^2+\alpha x^3}{(2-\beta)x-\frac{4}{3}x^3}=3$

$\gamma-1,\beta=2,\frac{-3\alpha}{4}=+3\Rightarrow\alpha=-4$

$\beta+\gamma-\alpha=7$

Q5. $\lim_{x\to 0}\frac{\tan^{-1}x+\frac{1}{2}[\ln(1+x)-\ln(1-x)]-2x}{x^5}$

(4)

$=\lim_{x\to 0}\frac{\left(x-\frac{x^3}{3}+\frac{x^5}{5}\ldots\right)+\frac{1}{2}\left[x-\frac{x^2}{2}+\frac{x^3}{3}\ldots-\left(-x-\frac{x^2}{2}-\frac{x^3}{3}\ldots\right)\right]-2x}{x^5}$

$=\lim_{x\to 0}\frac{2x+\frac{2x^5}{5}\ldots-2x}{x^5}=\frac{2}{5}$

$\lim_{x\to 1}x^{\frac{2}{(1-x)}}=e^{\lim_{x\to l}\left(\frac{2}{(1-x)}\right)(x-1)}=e^{-2}$

$\Rightarrow$ Both statements correct

Q6. $P=\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$
(32)
$\Rightarrow \mathrm{P}=\mathrm{e}^{\lim_{x\to 0}\left(\frac{\tan x-\mathrm{x}}{\mathrm{x}^3}\right)}$

$=\mathrm{e}^{\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+\ldots\ldots-x\right)}{x^3}}$

$=e^{1/3}$

$\therefore 96\log_{\mathrm{e}}^{\mathrm{p}}=96\times\frac{1}{3}=32$

Q7. Put $x=1+h$
(1)

$$\lim_{h\to 0}\frac{h(6+\lambda\cosh)-\mu\sinh}{h^3}=-1$$

$$\lim_{h\to 0}\frac{h\left(6+\lambda\left(1-\frac{h^2}{2!}\right)\right)-\mu\left(h-\frac{h^3}{3!}\right)}{h^3}=-1$$

$$6+\lambda-\mu=0 \text{ and } -\frac{\lambda}{2}+\frac{\mu}{6}=-1$$

$$\lambda+\mu=18$$

Q1.
(3) $f(x)=\begin{cases} x, & x<-1 \\ x^{21}, & -1\leq x<0 \\ x, & 0\leq x<1 \\ x^{21}, & x\geq 1 \end{cases}$

$f(x)$ is continuous everywhere.

$\therefore \mathrm{n}=0$

$\mathrm{f}'(\mathrm{x})=\begin{cases} 1, & \mathrm{x}<-1 \\ 21\mathrm{x}^{20}, & -1\leq \mathrm{x}<0 \\ 1, & 0<\mathrm{x}<1 \\ 21\mathrm{x}^{20}, & \mathrm{x}\geq 1 \end{cases}$

$\therefore$ f(x) is non-differentiable at $\mathrm{x}=-1,0,1$

$\therefore \mathrm{m}=3$

$\mathrm{m}+\mathrm{n}=3$

Q2. $f\left(0^{-}\right)=\mathrm{e}^{\lim_{x\to 0}\frac{\mathrm{ax}}{\mathrm{x}}}=\mathrm{e}^{\mathrm{a}}$
(3) $\mathrm{f}(0)=1+\mathrm{b}$

$\mathrm{f}\left(0^{+}\right)=\dfrac{\frac{1}{2\sqrt{\mathrm{x}+4}}}{\frac{1}{3}(\mathrm{x}+\mathrm{c})^{-\frac{2}{3}}}=\dfrac{\frac{1}{2(2)}}{\frac{1}{3}\cdot \mathrm{c}^{-\frac{2}{3}}}$

$=\dfrac{3}{4}\mathrm{c}^{2/3}$

Also at $\mathrm{x}=0$;

$c^{1/3}=2\Rightarrow c=8$

So $\mathrm{f}\left(0^{+}\right)=\frac{3}{4}(8)^{2/3}=3$

Now, $\mathrm{e}^{\mathrm{a}}=\mathrm{b}+1=3$

$\mathrm{e}^{\mathrm{a}}.\,\mathrm{b}\cdot\mathrm{c}=3\cdot 2\cdot 8=48$

Q3. Check for $\left[\frac{x^2}{2}\right]$ and $[\sqrt{x}]$ becomes integers.
(8) $\{0,1,\sqrt{2},2,\sqrt{6},\sqrt{8},\sqrt{10},\sqrt{12},\sqrt{14},4\}$

Continuous at 0^{+}, continuous at 4^{-} $\left[\frac{x^2}{2}\right]=[\sqrt{x}]$, occurs at $x=\sqrt{2}$

$\Rightarrow$ Not continuous

Q1. (2) $(\sin x \cos y)(f(2x+2y) - f(2x-2y)) = (\cos x \sin y)$
$(f(2x+2y) + f(2x-2y))$
$f(2x+2y)(\sin(x-y)) = f(2x-2y)\sin(x+y)$
$$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$$
Put $2x+2y = m, 2x-2y = n$
$$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = \mathrm{K}$$
$$\Rightarrow f(m) = K\sin\left(\frac{m}{2}\right)$$
$$\therefore f(x) = K\sin\left(\frac{x}{2}\right)$$
$$f'(x) = \frac{K}{2}\cos\left(\frac{x}{2}\right)$$
$P_{\text{Put}}\ x = 0; \frac{1}{2} = \frac{K}{2} \Rightarrow K = 1$
$$f'(x) = \frac{1}{2}\cos\frac{x}{2}$$
$$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$$
$$4f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$$
$$= \frac{-24}{8} = -3$$

Q1. $f'(x) = 6x^2 - 18ax + 12a^2$

(4) $f'(x) = 6\left(x^2 - 3ax + 2a^2\right)$

roots are $a, 2a$

$\mathrm{p}^2 = \mathrm{q} \Rightarrow \mathrm{a}^2 = 2\mathrm{a}$

$a = 2$

$f(x) = 2x^3 - 18x^2 + 48x + 1$

$f(3) = 37$

Q2. $\mathrm{f(x) = x^2 + ax^2 + b\ell n|x| + 1, \quad x \neq 0}$

(1) $\mathrm{f'(x) = 3x^2 + 2ax + \dfrac{b}{x}}$

$\mathrm{f'(-1) = 3 - 2a - b = 0}$

$\mathrm{f'(-2) = 12 + 4a - \dfrac{b}{2} = 0}$

$\mathrm{a = \dfrac{-9}{2},\ b = 12}$

$\mathrm{f'(x) = 3x^2 - 9x + \dfrac{12}{x} = \dfrac{3(x+1)(x+2)^2}{x}}$

Max. at $\mathrm{n = -1}$

$f(x) = x^2 - \frac{9}{2}x^2 + 12\ln|x| + 1$

$\mathrm{f(-1) = -1 - \dfrac{9}{2} + 1 = -\dfrac{9}{2}}$

$\mathrm{M = -4.5}$

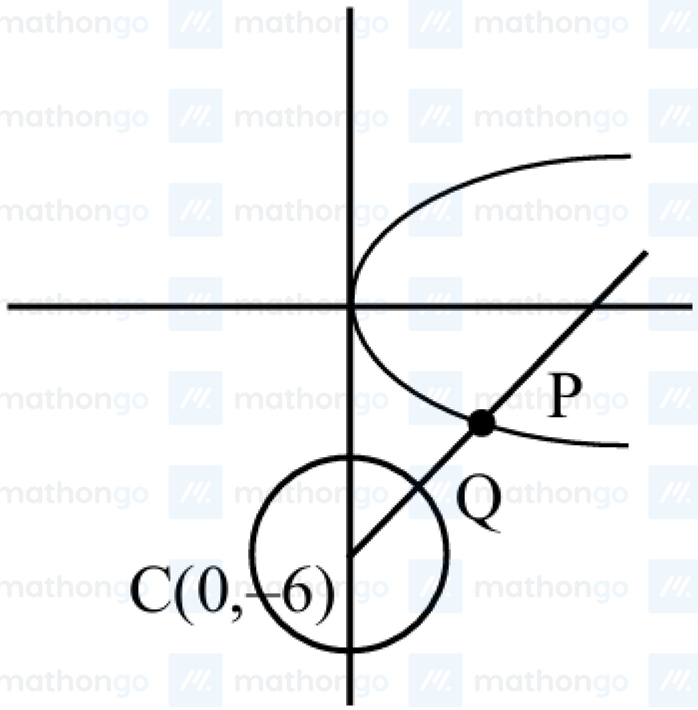
Min. value at $\mathrm{x = -2}$

$\mathrm{f(-2) = -8 - 18 + 12\ell n2 + 1}$

$\mathrm{m = -25 + 12\ell n2 = -16.6}$

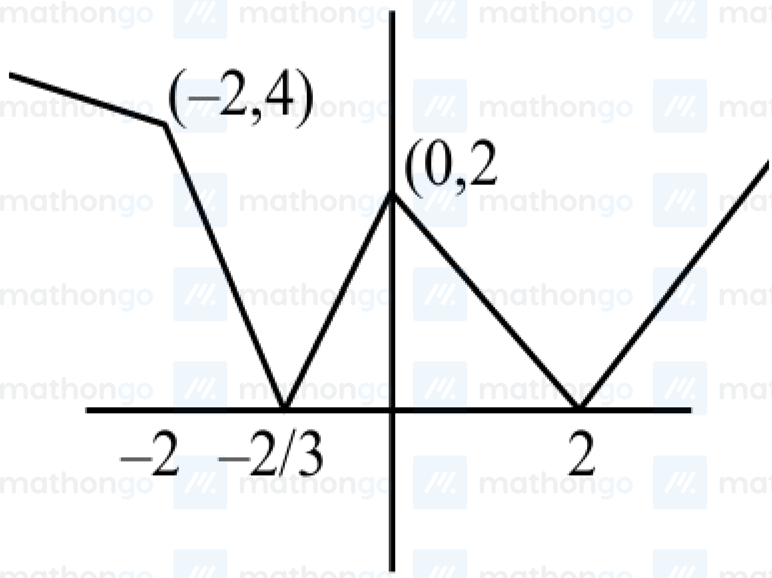
$\mathrm{|M + m| = 21.1}$

Q3.
(4)



Equation of normal to parabola
$y^2 = 8x$ is $y = mx - 4m - 2m^3$
passes through $(0, -6)$ we get
$-6 = -4\text{ m} - 2\text{ m}^3$
$\Rightarrow \text{ m}^3 + 2\text{ m} - 3 = 0$
$\Rightarrow (\text{ m} - 1)\left(\text{m}^2 + \text{m} + 3\right) = 0 \Rightarrow \text{ m} = -1$
$\text{P} = \left(\text{am}^2, -2\text{am}\right) = (2, -4)$
$\therefore$ Shortest distance $= \text{PC} - \text{r}$
$= (2\sqrt{2} - 1)$

Q4. $f(x) = \|x + 2| - 2|x\|$
(2) Critical points, $0, -2, 2, -\frac{2}{3}$



No. of maxima $= 1$

No. of minima $= 2$

option (2)

Q5.
(4)
$f(x)=\dfrac{x}{3}+\dfrac{3}{x}+3, x\neq 0$

$f'(x)=\dfrac{1}{3}-\dfrac{3}{x^2}=0 \quad \Rightarrow x=\pm 3$

$f'(x)=\dfrac{x^2-3}{3x^2}$

$\mathrm{f}'(\mathrm{x})>0 \forall(-\infty,-3)\cup(3,\infty) \rightarrow$ increasing

$\mathrm{f}'(\mathrm{x})<0 \forall(-3,0)\cup(0,3) \rightarrow$ decreasing

$\sum_{i=1}^{5}\alpha_i^2=(-3)^2+(3)^2+(-3)^2+(0)^2+(3)^2$

$=36$

Q6.
(2)
$f'(x)=18x^2-90ax+108a^2=0$

$x=2a \& x=3a$

$x_1=2a \quad x_2=3a$

$x_1x_2=54$

$6a^2=54$

$a=3$

$a+x_1+x_2$

$3+2\times 3+3\times 3=18$

option (2)

Q7.
(2)
$\lim\limits_{x\rightarrow 0}\dfrac{f(x)}{x^2}=5$

$\lim\limits_{x\rightarrow 0}\dfrac{ax^4+bx^3+cx^2+dx+e)}{x^2}=5$

$c=5$ and $d=e=0$

$f(x)=ax^4+bx^3+5x^2$

$f'(x)=4ax^3+3bx^2+10x$

$=x\left(4ax^2+3bx+10\right)$

has extremes at 4 and so $f'(4)=0 \& f'(5)=0$

so $\mathrm{a}=\frac{1}{8}$& $\mathrm{b}=\frac{-3}{2}$

so $f(2)=\frac{1}{8}\times 2^4-\frac{3}{2}\times 2^3+5\times 2^2$

$=2-12+20=10$

Q8.
(1)
$\lim\limits_{x\rightarrow 0}(f(2+x))^{\frac{3}{x}}$

$\lim\limits_{\mathrm{e}^{x\rightarrow 0}}\dfrac{(f(2+x)-1)3}{x}$

$\mathrm{e}^{3f'(2)}=(e)^{12}=(e)^a \Rightarrow a=12$

$y=4x^3-4x^2-4(a-7)x-a$

$y=4x^3-4x^2-20x-12$

roots $x=-1,-1,3$

option (1)

Q1. rationalise
(379)

$$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9} \times \frac{\left(\sqrt{1+x^2}+x\right)^9}{\left(\sqrt{1+x^2}+x\right)^9} dx$$

$$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{1} dx$$

Put $\sqrt{1+x^2}+x=t$

$\left(\frac{x}{\sqrt{1+x^2}}+1\right) dx = dt$

$dx = \frac{dt}{t}\sqrt{1+x^2}$

Now as $\sqrt{1+x^2}+x=t$

so $\sqrt{1+x^2}-x=\frac{1}{t}$

$\therefore \sqrt{1+x^2}=\frac{1}{2}\left(t+\frac{1}{t}\right)$

Thus $I=\int t^{19}\cdot\frac{dt}{t}\cdot\frac{1}{2}\left(t+\frac{1}{t}\right)$

$\Rightarrow \frac{1}{2}\int\left(t^{19}+t^{17}\right) dt$

$=\frac{1}{2}\left(\frac{t^{20}}{20}+\frac{t^{18}}{18}\right)+C$

$=\frac{t^{19}}{360}\left[9t+\frac{10}{t}\right]+C$

$=\frac{t^{19}}{360}\left[9\left(t+\frac{1}{t}\right)+\frac{1}{t}\right]+C$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[9\left(2\sqrt{1+x^2}\right)+\left(\sqrt{1+x^2}-x\right)\right]+C$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[19\sqrt{1+x^2}-x\right]+C$

$\therefore\ m=360, n=19$

$\therefore\ m+n=379$

Q2.
(4)

Let $3 - x^2 = t^2$

$+ xdx = -tdt$

$f(x) = \int (3 - t^2) \cdot t(-tdt) + c$

$= \int (t^4 - 3t^2)\, dt + c$

$= \frac{t^5}{5} - t^3 + c$

$f(x) = \frac{(3 - x^2)^{5/2}}{5} - (3 - x^2)^{3/2} + c$

$f(\sqrt{2}) = \frac{1}{5} - 1 + c = -\frac{4}{5}$

$c = 0$

$f(1) = \frac{2^{5/2}}{5} - 2^{3/2}$

$= 2^{1/2}\left(\frac{4}{5} - 2\right)$

$f(1) - \frac{-6\sqrt{2}}{5}$

Q3.
(19)

$\int \left(\frac{1}{x^2} + \frac{1}{x^4}\right)\left(\frac{3}{x} + \frac{1}{x^3}\right)^{\frac{1}{23}} dx$

using $t = \frac{3}{x} + \frac{1}{x^3} \Rightarrow dt = -3\left(\frac{1}{x^2} + \frac{1}{x^4}\right) dx$

$\int \frac{t^{1/23} dt}{-3} = \frac{t^{24/23}}{\left(\frac{24}{23}\right)(-3)} + C$

$\Rightarrow \alpha = 23 \beta = -1 \gamma = -3$

$\alpha + \beta + \gamma = 19$

Q1. $I=\int_0^\pi \frac{8xdx}{4\cos^2 x+\sin^2 x}$
(1)
$I=\int_0^\pi \frac{8(\pi-x)dx}{4\cos^2 x+\sin^2 x}$
$2I=8\pi\int_0^\pi \frac{dx}{4\cos^2 x+\sin^2 x}$
$2I=8\pi\times 2\int_0^{\pi/2} \frac{\sec^2 x}{4+\tan^2 x}dx$
$I=8\pi\int_0^\infty \frac{dt}{4+t^2}=8\pi\times\frac{1}{2}\left(\tan^{-1}\frac{t}{2}\right]_0^\infty$
$=4\pi\times\frac{\pi}{2}=2\pi^2$
option (1)

Q2. $I=\int_0^\pi \frac{(x+3)\sin x}{1+3\cos^2 x}dx$
(3)
$I=\int_0^\pi \frac{(\pi-x+3)\sin x}{(1+3\cos^2 x)}dx$
$2I=\int_0^\pi \frac{(\pi+6)\sin x\cdot dx}{(1+3\cos^2 x)}=2\int_0^{\pi/2} \frac{(\pi+6)\sin x}{(1+3\cos^2 x)}$
$I=\int_0^{\pi/2} \frac{(\pi+6)\sin x\cdot dx}{(1+3\cos^2 x)}=\frac{\pi}{3\sqrt{3}}(\pi+6)$
$\sqrt{3}\cos x=t$
$\sqrt{3}\sin x=dt$

Q3. $f(x)=\frac{1}{e^{x-1}}=e^{1-x}$
(8)

$f(x)=2$	$f(x)=1$
$\frac{1}{e^{x-1}}=2$	$x=1$
$x=1-\ln 2$	

$f(0)=e^1=2.71$
$f\left(e^3\right)=e^{1-e^3}\in(0,1)$
$I=\int_0^{1-\ell n2} 2dx+\int_{1-\ell n2}^1 1dx+\int_1^{e^3} 0dx$
$=2(1-\ell n 2-0)+1(1-1+\ln 2)+0$
$\alpha-\ln 2=2-\ln 2$
$\alpha=2$
$\alpha^3=8$

Q4. $x^2=2y\&y=2x+6$
(1) $x^2=4x+12$
$x^2-4x-12=0\Rightarrow \begin{array}{r|l} x=6 & \text{if } x=-2 \\ y=18 & y=2\end{array}$
$\therefore(6,18)\&(-2,2)$

Here $(6,18)$ Rejected because (a,b) lies in 2^{nd} quadrant

$\therefore \mathrm{a}=-2 \& \mathrm{~b}=2$

$$\therefore \mathrm{I}=\int_{-2}^{2} \frac{9 \mathrm{x}^{2}}{1+5^{\mathrm{x}}} \mathrm{dx}=\int_{-2}^{2} \frac{9 \cdot 5^{\mathrm{x}} \cdot \mathrm{x}^{2}}{1+5^{\mathrm{x}}} \mathrm{dx}$$

$$\therefore 2 \mathrm{I}=\int_{-2}^{2} 9 \mathrm{x}^{2} \mathrm{dx}=18 \int_{0}^{2} \mathrm{x}^{2} \mathrm{dx}=18\left(\frac{\mathrm{x}^{3}}{3}\right)_{0}^{2}$$

$2 \mathrm{I}=48$

$\therefore \mathrm{I}=24$

Q5. **(2)**

$$4 \int_{0}^{1} \frac{1}{\sqrt{3+\mathrm{x}^{2}}+\sqrt{1+\mathrm{x}^{2}}} \mathrm{dx}-3 \ln \sqrt{3}$$

$$=4 \int_{0}^{1} \frac{\sqrt{3+\mathrm{x}^{2}}-\sqrt{1+\mathrm{x}^{2}}}{\left(3+\mathrm{x}^{2}\right)-\left(1-\mathrm{x}^{2}\right)} \mathrm{dx}-\frac{3}{2} \ln 3$$

$$= 2\left[\left\{\frac{\mathrm{x}}{2} \sqrt{3+\mathrm{x}^{2}}+\frac{3}{2} \ln \left(\mathrm{x}+\sqrt{3+\mathrm{x}^{2}}\right)\right\}_{0}^{1}\right.$$
$$\left.-\left\{\frac{\mathrm{x}}{2} \sqrt{1+\mathrm{x}^{2}}+\frac{1}{2} \ln \left(\mathrm{x}+\sqrt{1+\mathrm{x}^{2}}\right)\right\}_{0}^{1}\right]-\frac{3}{2} \ln 3$$

$$=2\left[\left\{\frac{1}{2} \sqrt{4}+\frac{3}{2} \ln (1+\sqrt{4})\right\}-\left\{0+\frac{3}{2} \ln \sqrt{3}\right\}\right.$$
$$\left.-\left\{\frac{1}{2} \sqrt{2}+\frac{1}{2} \ln (1+\sqrt{2})\right\}+\left\{0+\frac{1}{2}(0)\right\}\right]-\frac{3}{2} \ln 3$$

$$=2\left[1+\frac{3}{2} \ln 3-\frac{3}{4} \ln 3-\frac{1}{\sqrt{2}}-\frac{1}{2} \ln (1+\sqrt{2})\right]-\frac{3}{2} \ln 3$$

$$=2+3 \ln 3-\frac{3}{2} \ln 3-\sqrt{2}-\ln (1+\sqrt{2})-\frac{3}{2} \ln 3$$

$$=2-\sqrt{2}-\ln (1+\sqrt{2})$$

Q6. **(1)**

$\log_{4} \log_{6}\left(3+4 x-x^{2}\right)>0$

$\log_{6}\left(3+4 x-x^{2}\right)>1$

$3+4 x-x^{2}>6$

$x^{2}-4 x+3<0$

$(x-1)(x-3)<0$

$x \in(1,3)$

so $a=1 \& b=3$

$$\Rightarrow \int_{0}^{2}\left[x^{2}\right] d x=?$$

$$I=\int_{0}^{1}\left[x^{2}\right] d x+\int_{1}^{\sqrt{2}}\left[x^{2}\right] d x+\int_{\sqrt{2}}^{\sqrt{3}}\left[x^{2}\right] d x+\int_{\sqrt{3}}^{\sqrt{4}}\left[x^{2}\right] d x$$

$$=0+\left.|x|\right|_{1}^{\sqrt{2}}+2\left.|x|\right|_{\sqrt{2}}^{\sqrt{3}}+3\left.|x|\right|_{\sqrt{3}}^{\sqrt{4}}$$

$$=(\sqrt{2}-1)+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})$$

$$=5-\sqrt{2}-\sqrt{3} \Rightarrow \mathrm{p}+\mathrm{q}+\mathrm{r}=10$$

Q7. $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$
(4)

$\Rightarrow 2x = t \Rightarrow 2dx = dt \quad \Rightarrow I_1 = \frac{1}{2}\int_{-1}^{2} tf(t(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2} (1-t)f(1-t)(1-(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2} f\left(t(1-t)dt - \int_{-1}^{2} tf(t(1-t)dt\right.$

$\Rightarrow 2I_1 = I_2 - 2I_1$
$\Rightarrow 4I_1 = I_2$
$\Rightarrow \frac{I_2}{I_1} = 4$

Q8. $I = \int_{-1}^{1} \frac{(1+\sqrt{|-x|-(-x)})e^{-x} + (\sqrt{|-x|-(-x)})e^{-(-x)}}{e^{-x}+e^{-(-x)}}dx$
(4)

$\Rightarrow I = \int_{-1}^{1} \frac{(1+\sqrt{|x|+x})e^{-x} + (\sqrt{|x|+x})e^{x}}{e^{x}+e^{x}}dx$

$\Rightarrow 2I = \int_{-1}^{1} \frac{(1+\sqrt{|x|+x}+\sqrt{|x|-x})\,(e^{x}+e^{-x})}{(e^{x}+e^{-x})}dx$

$\Rightarrow 2I = \int_{-1}^{1} (1+\sqrt{|x|+x}+\sqrt{|x|-x})dx$

$\Rightarrow 2I = 2\int_{0}^{1} (1+\sqrt{|x|+x}+\sqrt{|x|-x})dx$

$\Rightarrow 2I = 2\int_{0}^{1} (1+\sqrt{2x}+\sqrt{0})dx$

$\Rightarrow I = \int_{0}^{1} (1+\sqrt{2x})dx = \left[x + \frac{2\sqrt{2}}{3}x^{3/2}\right]_0^1$

$\Rightarrow I = \frac{2\sqrt{2}}{3} + 1$

Q9. Let, $I = \pi^2 \int_{-1}^{3/2} |x\sin\pi x|dx$
(3)

$= \pi^2\left\{\int_{-1}^{1} x\sin\pi x dx - \int_{1}^{3/2} x\sin\pi x dx\right\}$

$= \pi^2\left\{2\int_{0}^{1} x\sin\pi x dx - \int_{-1}^{3/2} x\sin\pi x dx\right\}$

Consider

$$\int \mathrm{x} \sin \pi \mathrm{x} \mathrm{dx}$$

$$-\mathrm{x} \cdot \frac{1}{\pi} \cos \pi \mathrm{x}+\int 1 \cdot \frac{1}{\pi} \cos \pi \mathrm{x} \mathrm{dx}$$

$$=-\frac{\mathrm{x}}{\pi} \cos \pi \mathrm{x}+\frac{\sin \pi \mathrm{x}}{\pi^2}$$

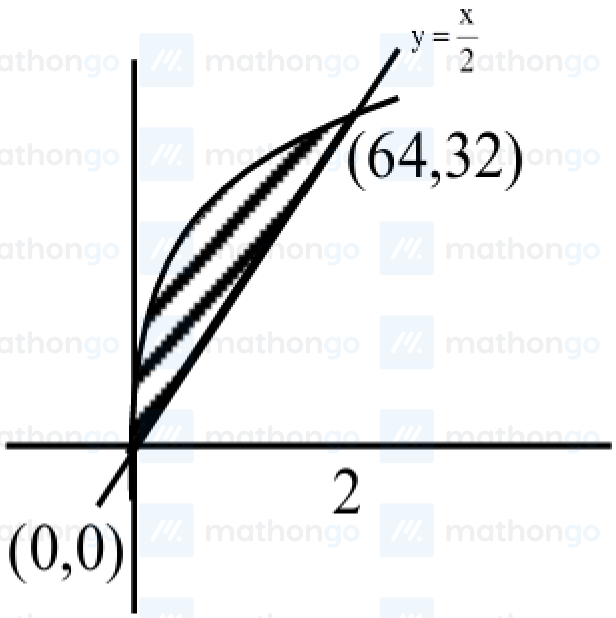
$$\mathrm{I}=\pi^2\left\{2\left(-\frac{\mathrm{x}}{\pi} \cos \pi \mathrm{x}+\frac{\sin \pi \mathrm{x}}{\pi^2}\right)_0^1-\left(-\frac{\mathrm{x}}{\pi} \cos \pi \mathrm{x}+\frac{\sin \pi \mathrm{x}}{\pi^2}\right)_1^{3/2}\right\}$$

$$=\pi^2\left\{\frac{2}{\pi}-\left(-\frac{1}{\pi^2}-\frac{1}{\pi}\right)\right\}$$

$$=\pi^2\left\{\frac{3}{\pi}+\frac{1}{\pi^2}\right\}$$

$$=3\pi+1$$

Q1.
(2)



$|x-y| \leq y \leq 4\sqrt{x}$

Now $y=|x-y|$

$y^2=(x-y)^2$

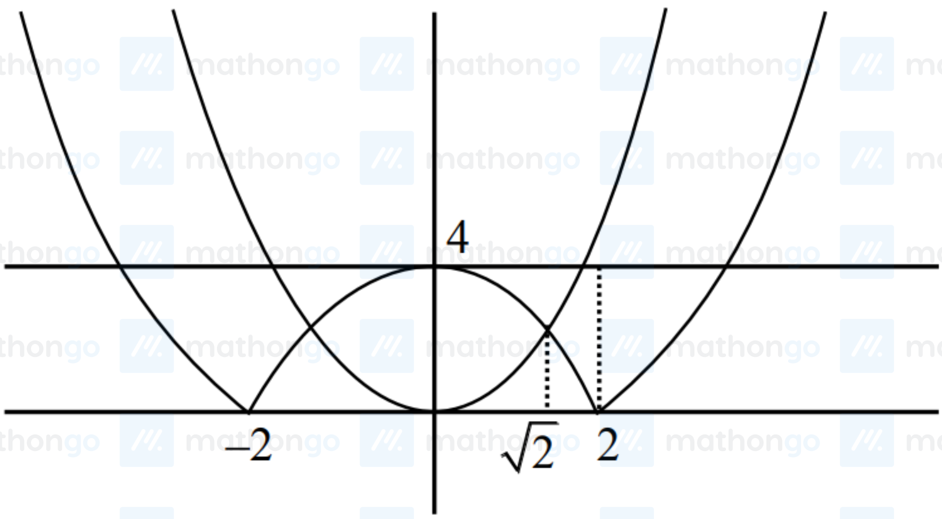
$\Rightarrow y=\frac{x}{2}$ and $x=0$

Now area $=\int_0^{64}\left(4\sqrt{x}-\frac{x}{2}\right) dx$

$=\left[\frac{4x^{3/2}}{3/2}-\frac{x^2}{4}\right]_0^{64}=\frac{8}{3}\cdot 8^3-\frac{64^2}{4}=64^2\left(\frac{1}{12}\right)$

$=\frac{1024}{3}$

Q2.
(22)



$A=\int_0^4 \sqrt{4+y}dy-\int_0^2 \sqrt{4-y}dy-\int_2^4 \sqrt{y}dy$

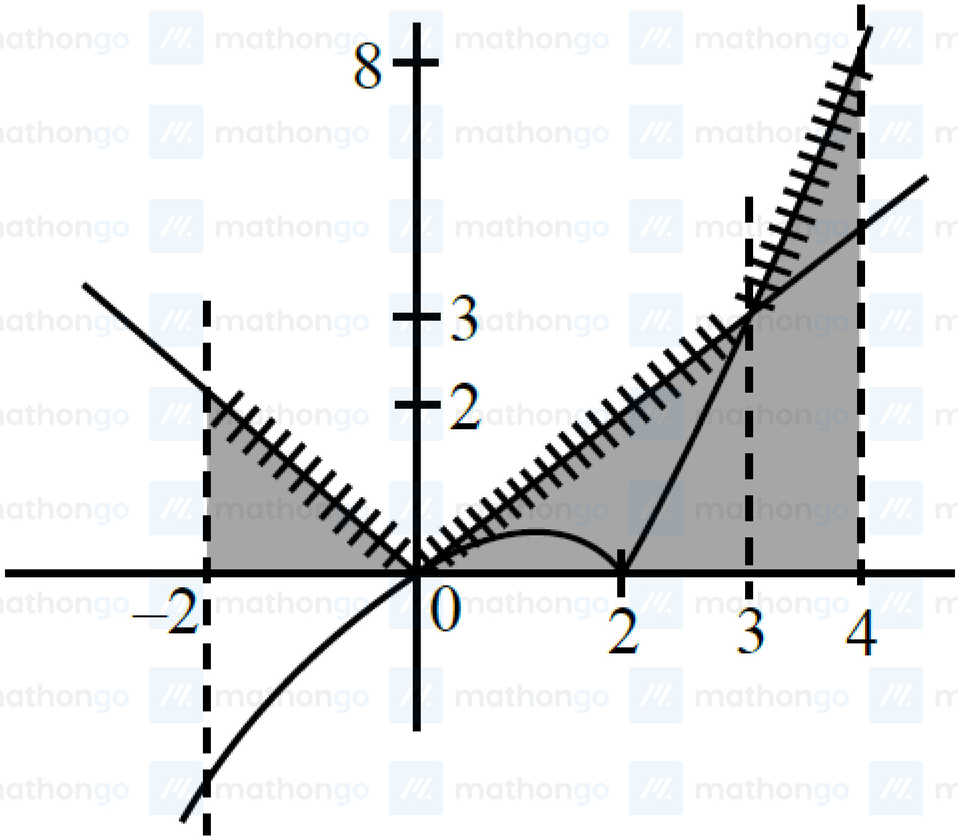
$$= \left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4 + \left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 - \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$\frac{80\sqrt{2}}{3} - 16 = \frac{40\sqrt{2}}{3} - 16$

$\alpha = 6, \beta = 16$

$\alpha + \beta = 22$

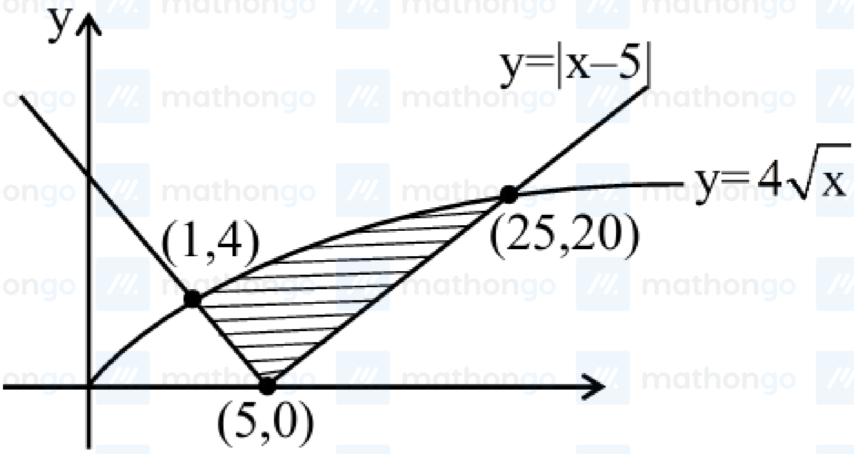
Q3.
(12)



Required Area $= \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 11$

$= 12$

Q4.
(368)



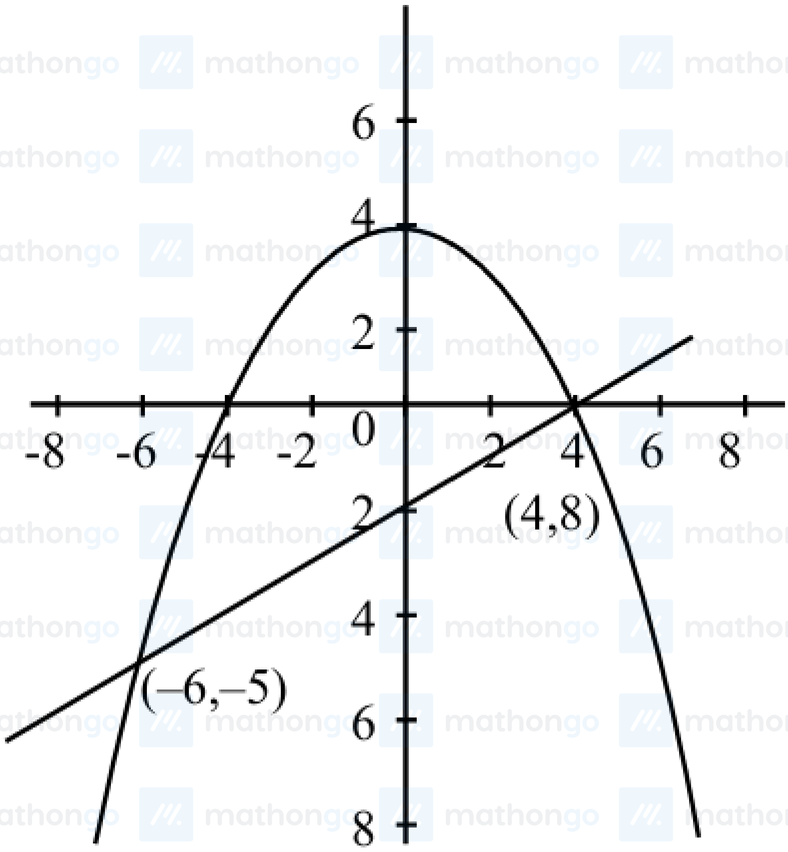
$$A = \int_1^{25} 4\sqrt{x}dx - \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 20 \times 20$$

$$A = \left[\frac{4x^{3/2}}{\frac{3}{2}}\right]^{25} - 8 - 200$$

$$A = \frac{8}{3}(125 - 1) - 208$$

$$A = \frac{368}{3} \Rightarrow 3A = 368$$

Q5.
(1)



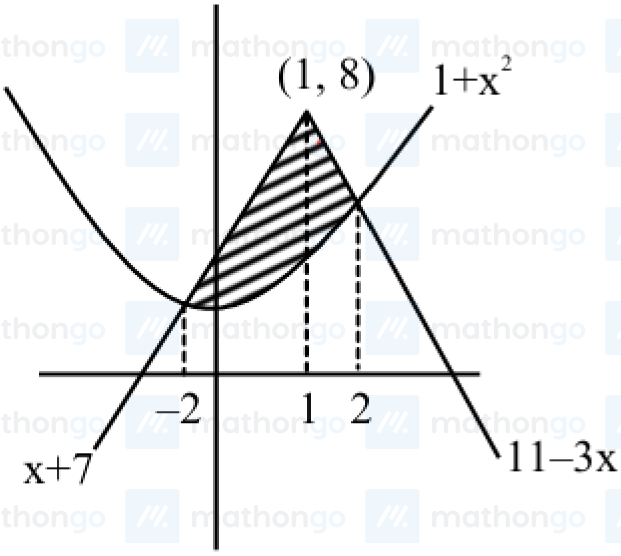
$$\text{Area } = \int_{-6}^{4}\left\{\left(4-\frac{x^2}{4}\right)-\left(\frac{x-4}{2}\right)\right\}dx$$

$$=\int_{-6}^{4}\left\{-\frac{x^2}{4}-\frac{x-6}{2}\right\}dx$$

$$\alpha=-\frac{x^3}{12}-\frac{x^2}{4}+6x\Big|_{-6}^{4}=\frac{125}{3}$$

$$\therefore 6\alpha=250$$

Q6.
(1)

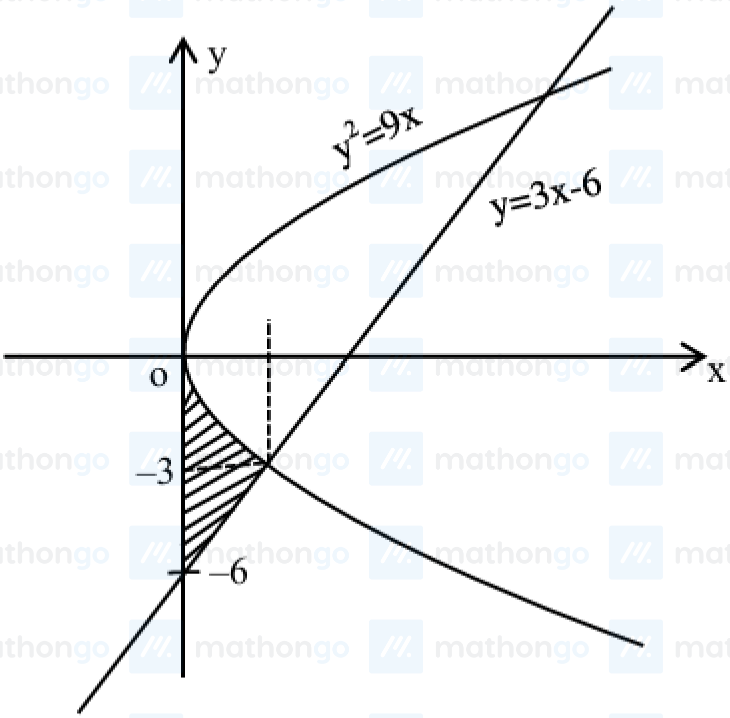


$$A=\int_{-2}^{1}\left(x+7-x^2-1\right)dx+\int_{1}^{2}\left(11+3x-x^2-1\right)dx$$

$$=\left[\frac{x^2}{2}+6x-\frac{x^3}{3}\right]_{-2}^{1}+\left[10x-\frac{3x^2}{2}-\frac{x^3}{3}\right]_{1}^{2}$$

$$=\frac{50}{3}\Rightarrow 3A=50 \quad \text{Option (1)}$$

Q7. $0\leq 9x\leq y^2 \& y\geq 3x-6$
(15)



$$A = \text{ Required Area } = \left[\int_0^1 (-3\sqrt{x})dx - \int_0^1 (3x-6)dx\right]$$

$$A = -3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^1 - \left(\frac{3x^2}{2} - 6x\right)\Bigg|_0^1$$

$$A = -2[1-0]\left[\frac{3}{2} - 6\right]$$

$$A = -2 - \frac{3}{2} + 6 = \frac{5}{2} \text{ Sq. unit}$$

$$\therefore 6A = 6 \times \frac{5}{2} = 15$$

Q1. $\dfrac{dy}{dx}+3\left(\sec^2 x\right)y=\sec^2 x, y(0)=\dfrac{1}{3}+e^3$
(2)
If $=e^{3\int \sec^2 x dx}=e^{3\tan x}$

$\therefore$ Solution is

$e^{3\tan x}y=\int e^{3\tan x}\sec^2 x dx$

$e^{3\tan x}y=\dfrac{e^{3\tan x}}{3}+c$

$\because y(0)=\dfrac{1}{3}+e^3 \Rightarrow c=e^3$

$\therefore y\left(\dfrac{\pi}{4}\right)=\dfrac{\frac{e^3}{3}+e^3}{e^3}=\dfrac{4}{3}$

Q2. $\mathrm{f}''(\mathrm{x})=\mathrm{f}(\mathrm{x})$
(36)
$\Rightarrow \mathrm{f}'(\mathrm{x})\cdot\mathrm{f}''(\mathrm{x})=\mathrm{f}'(\mathrm{x})\cdot\mathrm{f}(\mathrm{x})$

$\Rightarrow \dfrac{\left(\mathrm{f}'(\mathrm{x})\right)^2}{2}=\dfrac{(\mathrm{f}(\mathrm{x}))^2}{2}+\mathrm{C}$

$\Rightarrow \left(\mathrm{f}'(\mathrm{x})\right)^2=(\mathrm{f}(\mathrm{x}))^2+\mathrm{C}'$

$\mathrm{f}(0)=0, \mathrm{f}'(0)=3 \quad \Rightarrow \mathrm{C}'=9$

$\therefore \left(\mathrm{f}'(\mathrm{x})\right)^2=(\mathrm{f}(\mathrm{x}))^2+9$

$\mathrm{f}'(\mathrm{x})=\sqrt{(\mathrm{f}(\mathrm{x}))^2+9} \quad \because \mathrm{f}'(\mathrm{x})\geq 0$

$\int \frac{\mathrm{dy}}{\sqrt{\mathrm{y}^2+9}}=\int \mathrm{dx} \Rightarrow \ln\left|\mathrm{y}+\sqrt{\mathrm{y}^2+9}\right|=\mathrm{x}+\mathrm{C}$

$\Rightarrow \mathrm{f}(0)=0 \Rightarrow \mathrm{C}=\ln 3$

$\Rightarrow \mathrm{y}+\sqrt{\mathrm{y}^2+9}=3\mathrm{e}^{\mathrm{x}}$

at $x=\ln 3; y=4$

$\therefore 9\mathrm{f}(\ln 3)=36$

Q3. Diff. w.r.t. x
(2)
$g(x)=1-xg(x)$

$g(x)=\dfrac{1}{1+x}$

so $\dfrac{dy}{dx}-y\tan x=2\sec x$

$IF=e^{-\int \tan dx}=e^{\log\cos x}=\cos x$

solution of D.E.

$y\cos x=\int 2dx+c$

$y\cos x=2x+c$

$y(0)=0$

c = 0

$y = \frac{2x}{\cos x}$

$y = 2x \sec x$

$y\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\pi}{3} \cdot 2 = \frac{4\pi}{3}$

Q4.
(2)

$y = 1 - 2x + e^x \int_0^x e^{-t} f(t) dt$

$\frac{dy}{dx} = -2 + e^{-x} \cdot e^x f(x) + e^x \int_0^x e^{-t} f(t) dt$

$\frac{dy}{dx} = -2 + y + y + 2x - 1$

$\frac{dy}{dx} - 2y = (2x - 3)$

$ye^{-2x} = \int (2x - 3) dx \cdot e^{-2x}$

$ye^{-2x} = \frac{-(2x-3)}{2} e^{-2x} + \int e^{-2x} dx$

$ye^{-2x} = \frac{-(2x-3)}{2} e^{-2x} - \frac{1}{2} e^{-2x} + c$

$f(0) = 1 \Rightarrow c = 1 - \frac{3}{2} + \frac{1}{2} = 0$

$y = -\frac{(2x-3)}{2} - \frac{1}{2}$

$y = -x + 1$

$x + y = 1$

area $= \frac{1}{2}(1)(1) = \frac{1}{2}$

Q5.
(3)

$$\frac{dy}{dx}=\frac{7\cot y}{x}-\frac{e^x\operatorname{cosec} y}{x^5}$$

$$\frac{dy}{dx}=\frac{7\cot y}{\sin y\cdot x}-\frac{e^x}{\sin yx^5}$$

$$\sin y\frac{dy}{dx}-\cos y\cdot\frac{7}{x}=\frac{-e^x}{x^5}$$

let $-\cos y=t$

$$\sin y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}+\frac{7t}{x}=\frac{-e^x}{x^5}$$

I.F. $=x^7$

$$t.\,x^7=-\int x^2e^xdx$$

$$\operatorname{cosy}x^7=x^2e^x-2\int xe^xdx$$

$$\operatorname{cosy}x^7=x^2e^x-2xe^x+2e^x+c$$

$$x=1,y=\frac{\pi}{2},c=-e$$

$$\operatorname{cosy}=\frac{2e^2-e}{128}$$

option (3)

Q6.
(3)

$$f(x)=x-1$$

$$f(f(x))=f(x)-1=x-1-1=x-2$$

$$g(f(f(x)))=e^{x-2}$$

$$\therefore\frac{dy}{dx}=e^{-2\sqrt{x}}\times e^{x-2}-\frac{1}{\sqrt{x}}y$$

$$\frac{dy}{dx}+\frac{1}{\sqrt{x}}y=e^{x-2\sqrt{x}-2}\text{ which is L.D.E}$$

I.F. $=e^{\int\frac{dy}{\sqrt{x}}}=e^{2\sqrt{x}}$

Its solution is

$$y\times e^{2\sqrt{x}}=\int e^{2\sqrt{x}}\times e^{x-2\sqrt{x}-2}dx+c$$

$$y\times e^{2\sqrt{x}}=\int e^{x-2}dx+c$$

$$y\times e^{2\sqrt{x}}=e^{x-2}+c$$

$\mathrm{Given\ x=0,y=0\Rightarrow 0=e^{-2}+c\quad;c=-e^{-2}}$

$\therefore\mathrm{y\times e^{2\sqrt{x}}=e^{x-2}-e^{-2}}$

$\mathrm{when\ x=1,y\times e^2=e^{-1}-e^{-2}}$

$$y=\frac{e^{-1}-e^{-2}}{e^2}=\frac{\frac{1}{e}-\frac{1}{e^2}}{e^2}=\frac{e^2-e}{e^5}=\frac{e-1}{e^4}$$

Option (1) is correct

Q7. (2) $10\frac{\mathrm{d}}{\mathrm{dx}}\int_1^x \mathrm{f(t)dt} = \frac{\mathrm{d}}{\mathrm{dx}}\left(5\mathrm{xf(x)} - \mathrm{x}^5 - 9\right)$

$\Rightarrow 10f(x) = 5f(x) + 5\mathrm{xf}'(\mathrm{x}) - 5\mathrm{x}^4$

$\Rightarrow \mathrm{f(x)} + \mathrm{x}^4 = \mathrm{xf}'(\mathrm{x})$

$\Rightarrow \mathrm{y} + \mathrm{x}^4 = \mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}}$

$\Rightarrow \frac{dy}{dx} + y\left(-\frac{1}{x}\right) = x^3$

$\Rightarrow ye^{\int -\frac{1}{x}dx} = \int x^3 e^{\int -\frac{1}{x}dx} + c$

$\Rightarrow \mathrm{ye}^{-\operatorname{tn}|x|} = \int \mathrm{x}^3\mathrm{e}^{-\operatorname{tr}|x|} + \mathrm{c}$

$\Rightarrow \frac{\mathrm{y}}{|\mathrm{x}|} = \int \frac{\mathrm{x}^3}{|\mathrm{x}|} + \mathrm{c}$

$\Rightarrow \frac{\mathrm{y}}{\mathrm{x}} = \int \mathrm{x}^2 + \mathrm{c}$

$\Rightarrow \frac{\mathrm{y}}{\mathrm{x}} = \frac{\mathrm{x}^3}{3} + \mathrm{c}$

Put $x = 1$ in given equation

$\Rightarrow 0 = 5\mathrm{f}(1) - 1 - 9 \Rightarrow \mathrm{f}(1) = 2$

$\Rightarrow \frac{2}{1} = \frac{1}{3} + \mathrm{c} \Rightarrow \mathrm{c} = \frac{5}{3}$

$\Rightarrow \frac{\mathrm{f}(3)}{3} = \frac{27}{3} + \frac{5}{3}$

$\Rightarrow \mathrm{f}(3) = 32$

Q8. (21) I.F. $= \mathrm{e}^{\int 2\sec^2 x dx}$

$= \mathrm{e}^{2\tan x}$

Solution of diff. eq.

$$y \cdot e^{2\tan x} = \int e^{2\tan x}\left(2\sec^2 x + 3\tan x \cdot \sec^2 x\right) dx$$

$y \cdot e^{2\tan x} = \int e^{2\tan x} \cdot \left(2\sec^2 x\right) dx + \int e^{2\tan x} \cdot \left(3\tan x \cdot \sec^2 x\right) dx$

y. $\mathrm{e}^{2\tan x} = \mathrm{e}^{2\tan x} \cdot 2\tan x - \int \mathrm{e}^{2\tan x} \cdot 2\sec^2 x \times 2\tan x dx + \int \mathrm{e}^{2\tan x} \cdot 3\tan x \cdot \sec^2 x dx$

$y \cdot e^{2\tan x} = 2\tan x \cdot e^{2\tan x} - \int \mathrm{e}^{2\tan x} \cdot \tan x \sec^2 x dx$

$y.e^{2\tan x} = 2\tan x \cdot e^{2\tan x} - \frac{\tan x \cdot e^{2\tan x}}{2} + \frac{e^{2\tan x}}{4} + C$

$y = 2\tan x - \frac{\tan x}{2} + \frac{1}{4} + \mathrm{Ce}^{-2\tan x}$

$x = 0, y = \frac{5}{4}$

$c = 1$

$\mathrm{y}\left(\frac{\pi}{4}\right) = \frac{7}{4} + \mathrm{e}^{-2}$

Then $12\left(y\left(\frac{\pi}{4}\right) - \mathrm{e}^{-2}\right) = 12\left(\frac{7}{4}\right) = 21$

Q9. $x\left(x^2+e^x\right)dy+\left(e^x(x-2)y-x^3\right)dx=0$

(4) $x\left(x^2+e^x\right)\frac{dy}{dx}+e^x(x-2)y=x^3$

$$\frac{dy}{dx}+\frac{e^x(x-2)}{x\left(x^2+e^x\right)}y=\frac{x^2}{x^2+e^x}$$

$$\text{I.F.}=e^{\int\frac{e^x(x-2)}{x\left(x^2+e^x\right)}dx}=e^{\int\frac{e^x\left(\frac{1}{x^2}-\frac{2}{x^2}\right)dx}{\left(1+\frac{e^x}{x^2}\right)}dx}$$

Let $1+\frac{e^x}{x^2}=t\Rightarrow\frac{x^2e^x-e^x2x}{x^4}dx=dt$

$\Rightarrow$ I.F. $e^{\ln\left(1+\frac{e^2}{x^2}\right)}=1+\frac{e^x}{x^2}$

Now $y\left(1+\frac{e^x}{x^2}\right)=\int\frac{x^2}{x^2+e^x}\cdot\frac{x^2+e^x}{x^2}dx+C$

$y\left(1+\frac{e^x}{x^2}\right)=x+C$

Passing through $(1,0)$

$\Rightarrow C=-1$

$$y=\frac{x-1}{1+\frac{e^x}{x^2}}$$

$$y(2)=\frac{1}{1+\frac{e^2}{4}}=\frac{4}{4+e^2}$$

Q10. $\left(x^2+1\right)\frac{dy}{dx}-2xy=\left(x^4+2x^2+1\right)\cos x$

(1)

$$\frac{dy}{dx}-\left(\frac{2x}{x^2+1}\right)y=\frac{\left(x^2+1\right)^2\cos x}{\mathrm{cx}^2+1}=\left(\mathrm{x}^2+1\right)\cos x$$

(Linear D.E)

$$P=\frac{-2x}{x^2+1},Q=\left(x^2+1\right)\cos x$$

$$\text{I.F}=e^{\int Pdx}=e^{\int\frac{-2x}{x^2+1}dx}=\frac{1}{x^2+1}$$

$$y\cdot\frac{1}{x^2+1}=\int\left(x^2+1\right)\cos x\cdot\frac{1}{x^2+1}dx$$

$\frac{y}{x^2+1}=\sin x+c\Rightarrow y\cos=1\Rightarrow c=1$

$y=\left(x^2+1\right)(\sin x+1)$

$$\int_{-3}^{3}ydx=\int_{-3}^{3}\left(x^2+1\right)(\sin x+1)$$

$$dx=\int_{-3}^{3}x^2\sin x+x^2\sin x+1dx$$

$$\Rightarrow\int_{-3}^{3}x^2\sin xdx+\int_{-3}^{3}x^2dx+\int_{-3}^{3}\sin xdx+\int_{-3}^{3}1dx$$

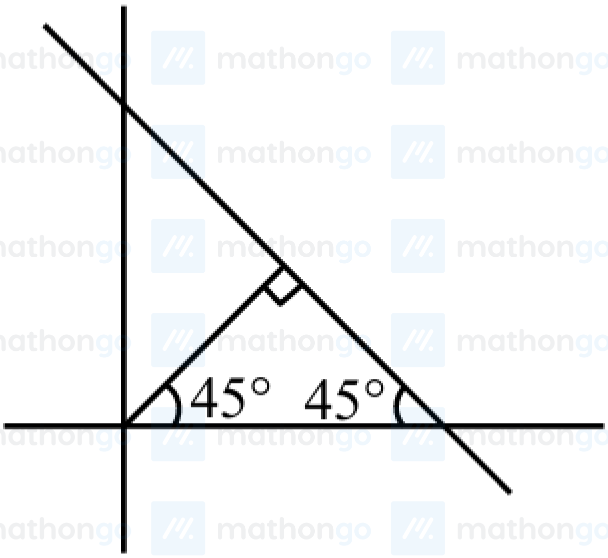
$=0+18+0+6=24$

Q1.
(3) Area of $\triangle ABC = \frac{1}{2}\begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$

$= 6$ square units

Maximum area of AFDE $= \frac{1}{2} \times 6 = 3$ sq. units

Q2. $\frac{x}{-c} + \frac{y}{-c/b} = 1$
(3)



$\therefore$ area of triangle $= \frac{1}{2}\left|\frac{c^2}{b}\right| = 48$

$\left|\frac{c^2}{b}\right| = 96$

$\because -c = -\frac{c}{b}$

$\Rightarrow b = 1 \quad \therefore c^2 = 96$

$\therefore b^2 + c^2 = 97$

Q3. $\left(x^2 + 2x\right)(12 - k) = 2$

(1) $\lambda x^2 + 2\lambda x - 2 = 0 \quad k \neq 12$ Let $12 - k = \lambda$

$D = 0$

$4\lambda^2 + 8\lambda = 0$

$\lambda = 0$ or $\lambda = -2$

$\Rightarrow 12 - k = -2$

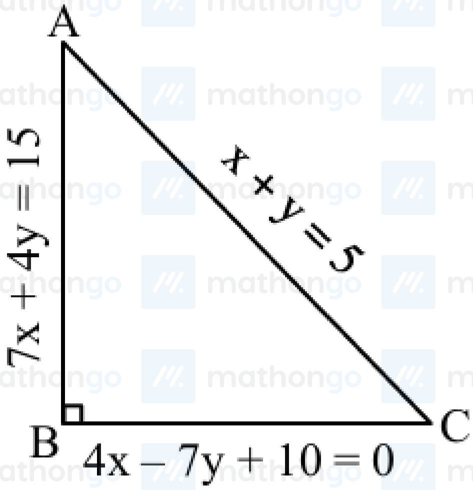
$k = 14$

So $P\left(k, \frac{k}{2}\right) = (14, 7)$

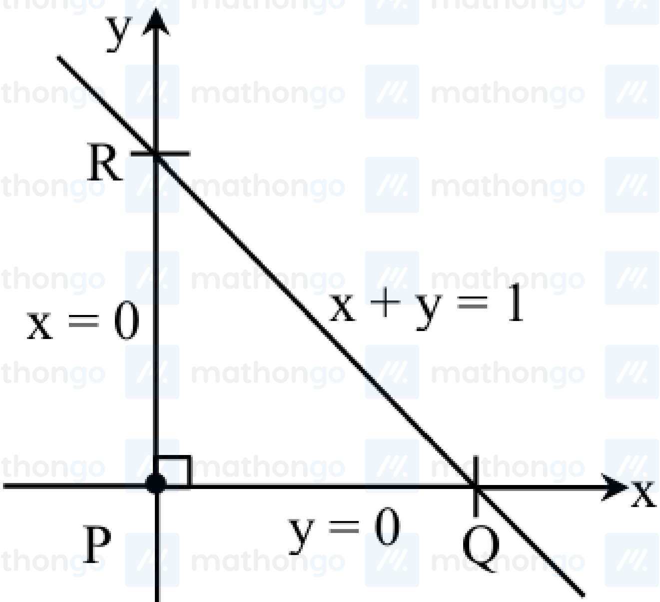
$d = \left|\frac{3\times14+4\times7+5}{5}\right| = 15$

option (1)

Q4.
(2)

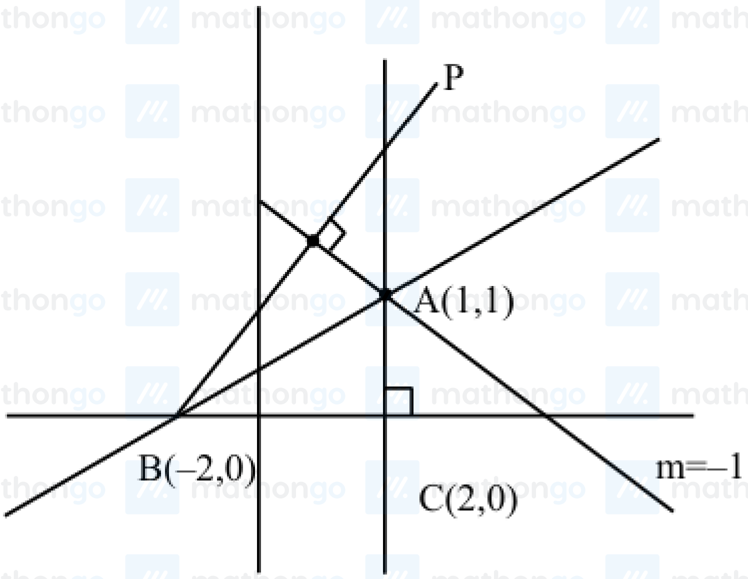


$$\left.\begin{array}{l} 7x + 4y = 15 \\ 7x + 7y + 10 = 0 \end{array}\right\rangle \text{P.O.I } B \equiv (1,2)$$



$$7x + 4y = 15$$
$$7x + 7y + 10 = 0$$
distance between P and B $= \sqrt{5}$

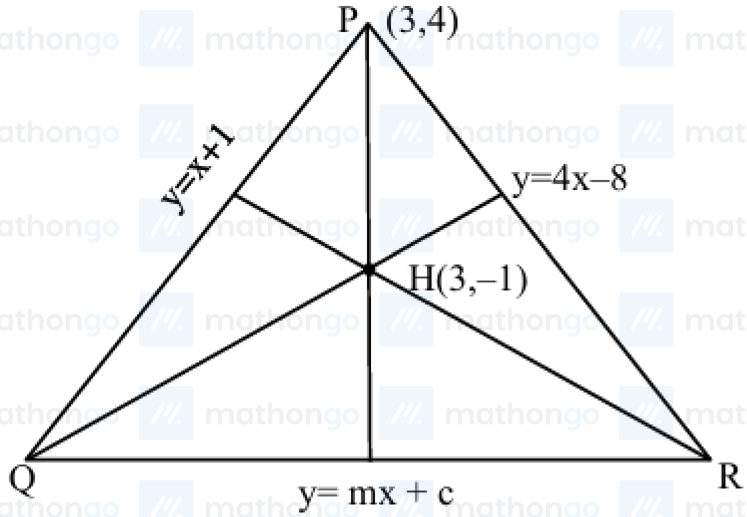
Q5.
(4)



Equation of Altitude AP : $x = 1$
Equation of Altitude BP : $y - 0 = 1(x + 2)$
$\Rightarrow x = 1 \&$
$x - y + 2 = 0$
$P(1,3)$

Area of $\triangle PBC = \frac{1}{2} \times 4 \times 3 = 6$

Q6.
(1)



Solve line PQ \& QR Point Q $\left(\frac{1-c}{m-1}, \frac{1-c}{m-1} + 1\right)$

$m_{2H} = \frac{\frac{1-c}{m-1}+2}{\frac{1-c}{m-1}-3} = \frac{1-c+2m-2}{1-c-3m+3} = -\frac{1}{4}$

$m_{2H} = \frac{\frac{1-c}{m-1}+2}{\frac{1-c}{m-1}-3} = \frac{1-c+2m-2}{1-c-3m+3} = -\frac{1}{4} \ldots(1)$

$\because m_{PH} = \frac{5}{0} \rightarrow \infty$

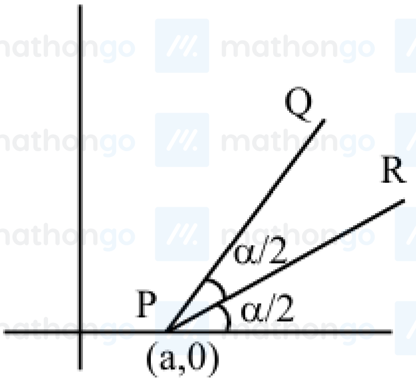
$\Rightarrow$ Slope of line QR $(m) = 0$

Put value of m in equation (1)

$\frac{1-c-2}{1-c+3} = -\frac{1}{4} \Rightarrow c = 0$

so $\mathrm{m} - \mathrm{c} = 0$ Ans.

Q7.
(1)



$\mathrm{m_{PR}} = 2 - \sqrt{3} = \tan 15^\circ$

$\therefore \frac{\alpha}{2} = 15^\circ \quad \Rightarrow \alpha = 30^\circ$

equation of PR :

$y = \tan 15^\circ (x - a)$

$y = (2 - \sqrt{3})(x - a)$

$\perp$ distance from origin $= \frac{1}{\sqrt{2}}$

$$\left|\frac{\sqrt{3}a - 2a}{\sqrt{4 + 3 - 4\sqrt{3} + 1}}\right| = \frac{1}{\sqrt{2}}$$

$$\frac{|a|(2-\sqrt{3})}{2\sqrt{(2-\sqrt{3})}} = \frac{1}{\sqrt{2}}$$

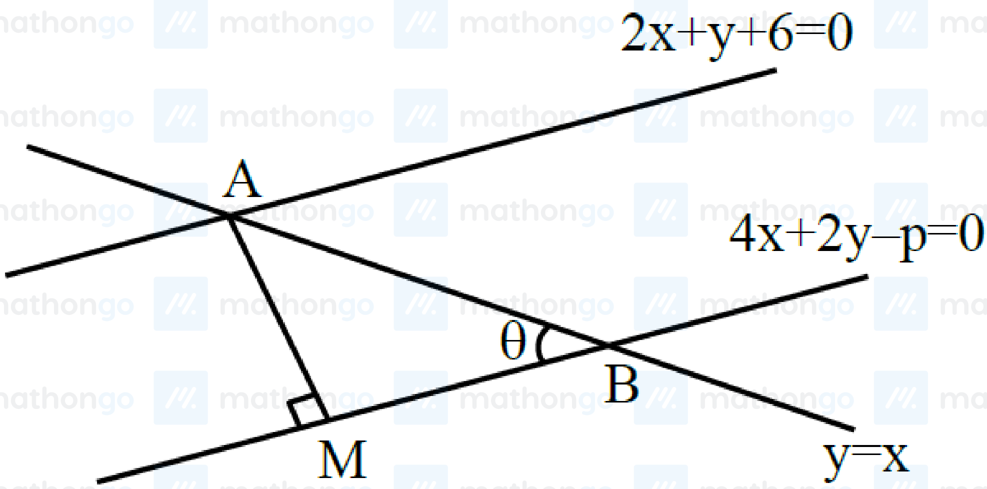
$$|a| = \frac{\sqrt{2}}{\sqrt{2-\sqrt{3}}} = \sqrt{2}(\sqrt{2+\sqrt{3}})$$

$a^2 = 2(2 + \sqrt{3})$

$3a^2 \tan^2 \alpha - 2\sqrt{3}$

$$3 \times (4 + 2\sqrt{3}) \cdot \frac{1}{3} - 2\sqrt{3} = 4$$

Q8.
(4)



Line is $y = x$

$m_1 = 1,\ m_2 = -2$

so $\tan\theta = \left|\frac{1+2}{1-2}\right|$

$\tan\theta = \frac{AM}{BM} = 3$

Q9.
(1)

$x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$

$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

intersection of family of lines

$P(1, 2)$

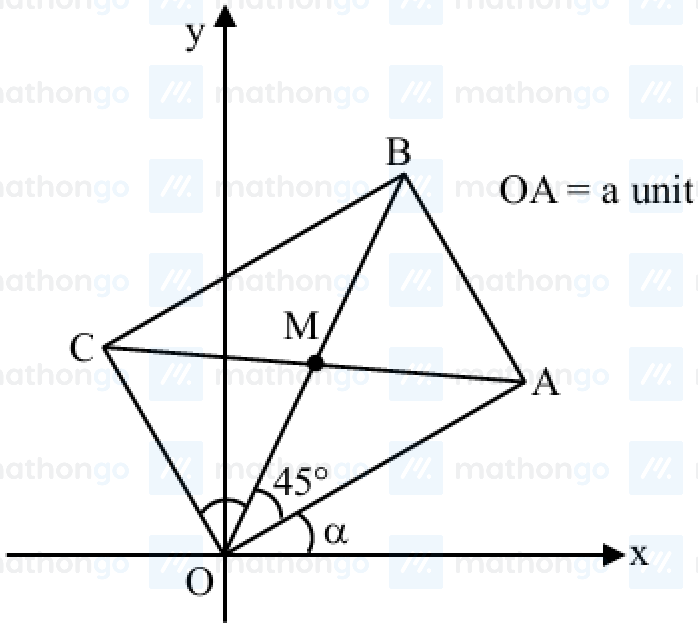
Let $Q(3, 6)$

$d = PQ = \sqrt{2^2 + 4^2} = \sqrt{20}$

$d^2 = 20$

option (1)

Q10.
(1)



Slope of diagonal OB $= \dfrac{\sqrt{3}+1}{1-\sqrt{3}}$

$\therefore = \tan 105^\circ$

$\therefore \alpha = 60^\circ$

$\therefore$ A $(\mathrm{a}\cos 60^\circ, \mathrm{a}\sin 60^\circ)$

$\therefore$ A $\left(\dfrac{\mathrm{a}}{2}, \dfrac{\sqrt{3}\mathrm{a}}{2}\right)$

A Lies on other diagonal

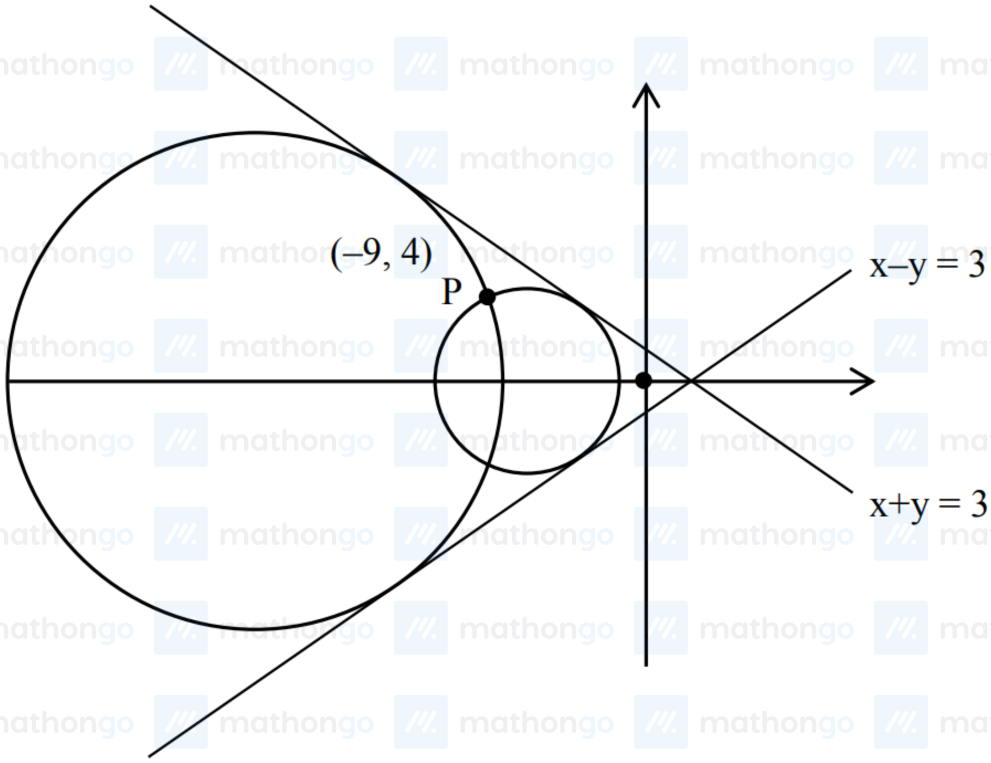
$$\therefore \left(\frac{\sqrt{3}-1}{2}\right) a - \left(\frac{\sqrt{3}+1}{2}\right) \cdot \sqrt{3} a + 8\sqrt{3} = 0$$

$$a\left[\frac{\sqrt{3}-1-3-\sqrt{3}}{2}\right] = -8\sqrt{3}$$

$a = 4\sqrt{3}$

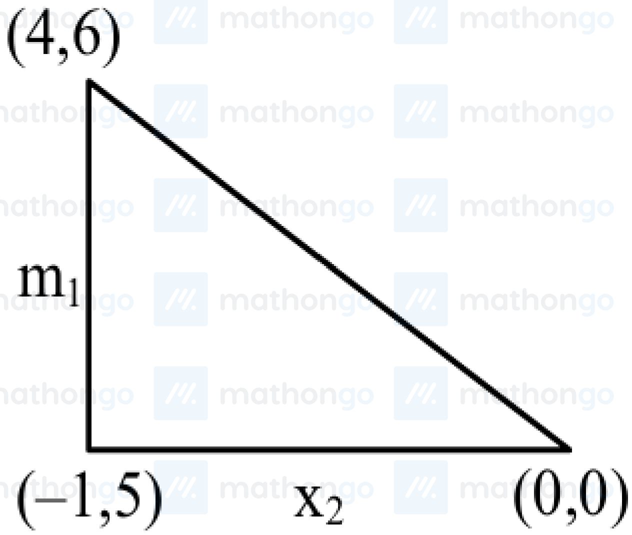
$\therefore a^2 = 48$

Q1.
(768)



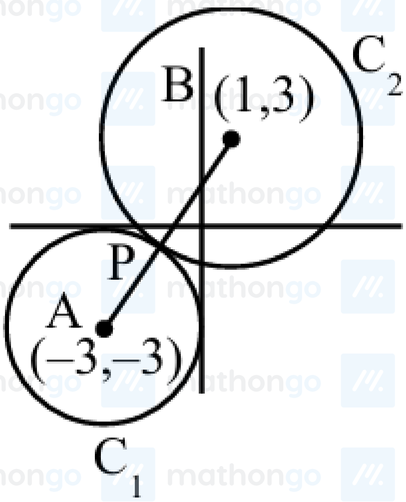
Centre $(\mathrm{a}, 0)$
$\mathrm{r}=\left|\frac{\mathrm{a}-0-3}{\sqrt{2}}\right|$
circle$(x-a)^2+y^2=\left(\frac{a-3}{\sqrt{2}}\right)^2$
passes through $(-9,4)$
$2\left(a^2+18 a+81+16\right)=\left(a^2-6 a+9\right)$
$\mathrm{a}^2+42 \mathrm{a}+185=0$
$(a+37)(a+5)=0$
$\Rightarrow a=-37,-5$
$r_1=\left|\frac{-37-3}{\sqrt{2}}\right|=20 \sqrt{2}$
$\mathrm{r}_2=\left|\frac{-5-3}{\sqrt{2}}\right|=4 \sqrt{2}$
$\left|r_1^2-r_2^2\right|=|800-32|=768$

Q2.
(4)



$m_1 \, m_2 = -1$ so right angle equation circle is
$(x-4)(x-0)+(y-6)(y-0)=0$
$x^2+y^2-4x-6y=0$
$(k, 3k)$ lies on it so
$k^2+9k^2-4k-18k=0$
$10k^2-22k=0$
$k=0, \frac{11}{5}$
$k=0$ is not possible so $k=\frac{11}{5}$
also $r=\sqrt{4+9}=\sqrt{13}$
so $10k+r^2=10 \cdot \frac{11}{5}+(\sqrt{13})^2=35$

Q3. $C_1:(x+3)^2+(y+3)^2=3^2$
(3)



Let C_1 and C_2 has centres $A\,(-3_1-3)$ and $B(1,3)$

$\mathrm{AB}=\sqrt{16+36}=2\sqrt{13}$

$\mathrm{r}_1=3$ and $\mathrm{r}_2=2\sqrt{13}-3$

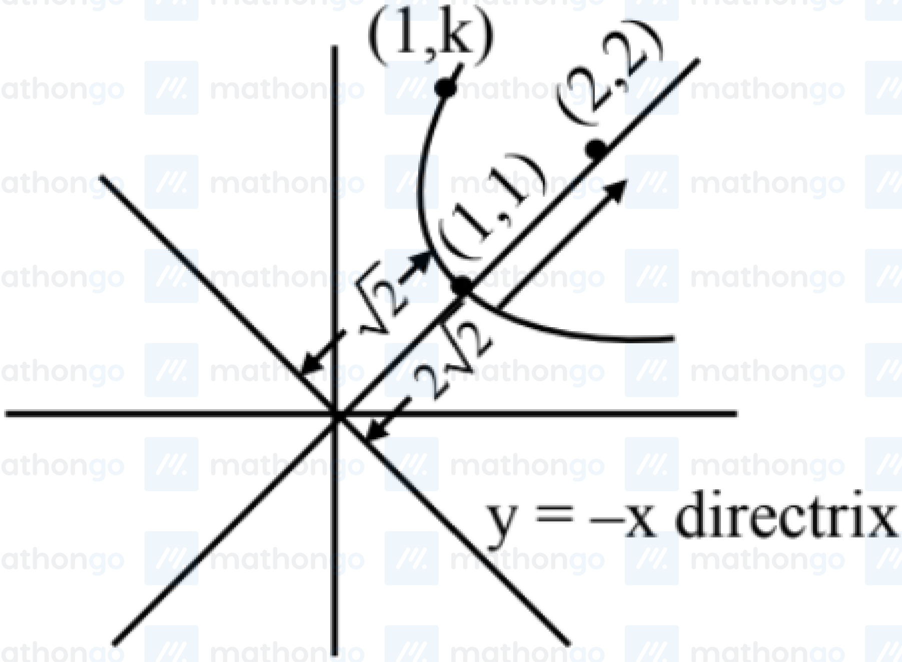
$$\mathrm{P}(\alpha,\beta),\alpha=\frac{\mathrm{r}_1(1)+\mathrm{r}_2(-3)}{\mathrm{r}_1+\mathrm{r}_2},\beta=\frac{\mathrm{r}_1(3)+\mathrm{r}_2(-3)}{\mathrm{r}_1+\mathrm{r}_2}$$

$$\alpha=\frac{3-3(2\sqrt{13}-3)}{2\sqrt{13}},\beta=\frac{18-6\sqrt{13}}{2\sqrt{13}}$$

$$(\beta-\alpha)^2=\left(\frac{6}{2\sqrt{13}}\right)^2$$

$$(\beta-\alpha)^2=\left(\frac{6}{2\sqrt{13}}\right)^2,\ \mathrm{m}+\mathrm{n}=22$$

Q1.
(2)



Directrix $x+y=0$

$\mathrm{PS}=\mathrm{PM}$

$\sqrt{(1-2)^2+(\mathrm{K}-2)^2}=\frac{(1+\mathrm{K})}{\sqrt{2}}$

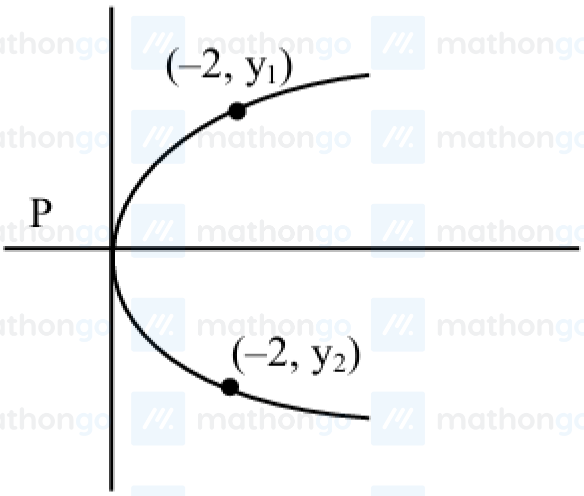
$2\,\mathrm{K}^2+8-8\,\mathrm{K}+2=\mathrm{K}^2+1+2\,\mathrm{K}$

$\mathrm{K}^2-10\,\mathrm{K}+9=0$

$\mathrm{K}=9$

option (2)

Q2. Equation of parabola
(1) $(x+2)^2+(y-1)^2=\left(\frac{2x+y+2}{\sqrt{5}}\right)^2$



$5\left[(x+2)^2+(y-1)^2\right]=(2x+y+2)^2$

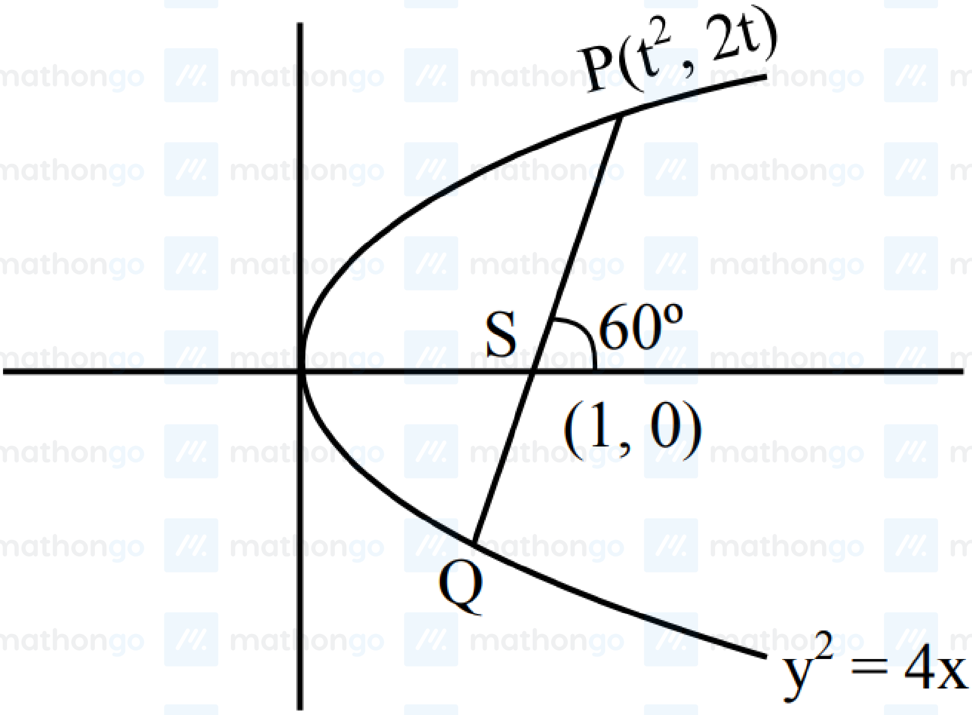
Put $x=-2, 5(y-1)^2=(y-2)^2$

$5\left(y^2-2y+1\right)=y^2-4y+4$

$\Rightarrow 4y^2-6y+1=0 \Rightarrow y_1+y_2=\frac{3}{2}$

Q3.

(1)



$\tan 60^\circ=\frac{2t-0}{t^2-1}=\sqrt{3} \Rightarrow t=\sqrt{3}$

$\therefore P(3,2\sqrt{3})$

Circle :

$(x-1)(x-3)+(y-0)(y-2\sqrt{3})=0$

at $x=0$

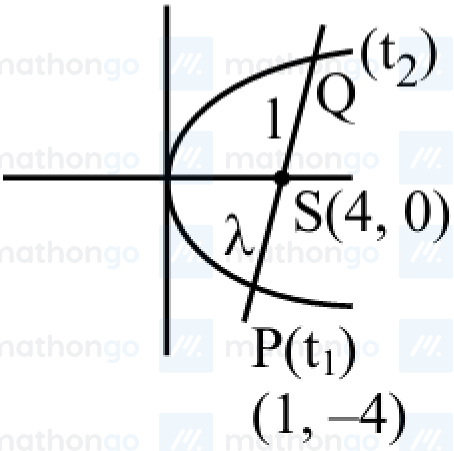
$\Rightarrow 3+y^2-2\sqrt{3}y=0$

$\Rightarrow y=\sqrt{3}=\alpha$

$5\alpha^2=15$

Q4. $y^2 = 16x; a = 4$ focus $S \equiv (4, 0)$

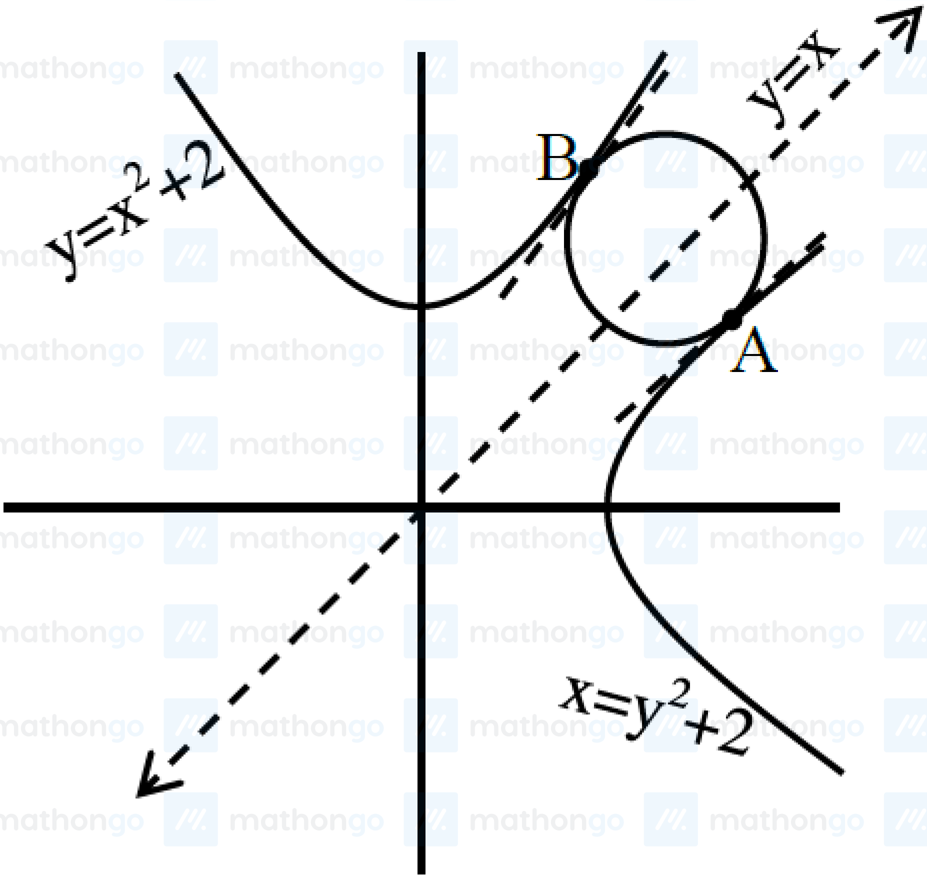
(1)



$2at_1 = -4$
$\Rightarrow 2(4)t_1 = -4$
$\Rightarrow t_1 = -\frac{1}{2}$
$\because t_1 t_2 = -1$
$\Rightarrow t_2 = 2$
$\therefore Q\left(at_2^2, 2at_2\right) = (16, 16)$
Let, S divides PQ internally in $\lambda : 1$ ratio
$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0$
$\lambda = \frac{1}{4} = \frac{m}{n}$
$\therefore m^2 + n^2 = 1 + 16 = 17$

Q5. The given parabolas are symmetric about the line $y=x$

(4)



Tangents at A& B must be parallel to y = x line, so slope of the tangents = 1

$\left(\frac{dy}{dx}\right)_{\min A}=1=\left(\frac{dy}{dx}\right)_{\min B}$

For point B, $\mathrm{y}=\mathrm{x}^2+2$

$$\frac{dy}{dx}=2x=1$$

$$x=\frac{1}{2} \Rightarrow y=\frac{9}{4}$$

$\therefore$ Point $B=\left(\frac{1}{2}, \frac{9}{4}\right) \Rightarrow$ Point A $=\left(\frac{9}{4}, \frac{1}{2}\right)$

$$\mathrm{AB}=\sqrt{\left(\frac{1}{2}-\frac{9}{4}\right)^2+\left(\frac{9}{4}-\frac{1}{2}\right)^2}$$

$$=\sqrt{\frac{98}{16}}=\frac{7\sqrt{2}}{4}$$

Radius $==\frac{7\sqrt{2}}{8}$

Q6. Tangent

(3) $y = m(x+2)$

$y^2 = x - 2$

$(m(n+2))^2 = n - 2$

$m^2x^2 + (4m^2 - 1)\,x + (4m^2 + 2) = 0$

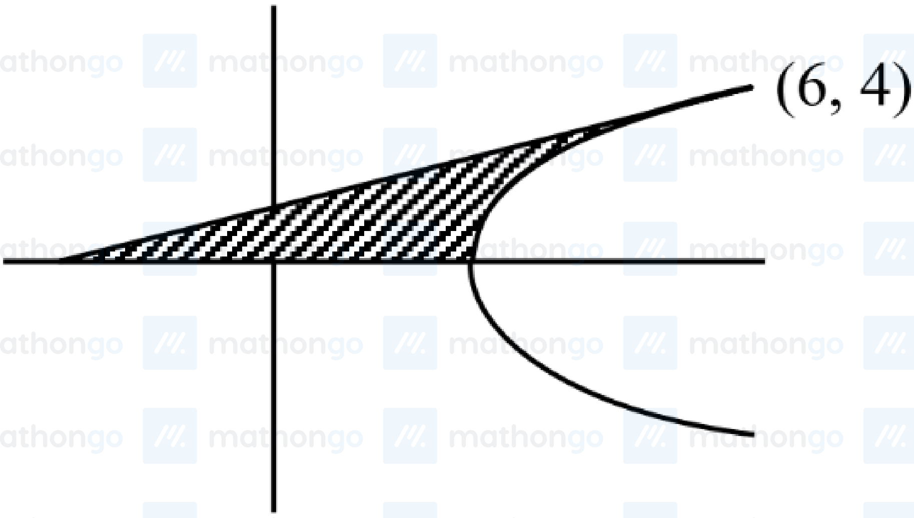
$D = 0$

$(4m^2 - 1)^2 - 4m^2\,(4m^2 + 2) = 0$

$m = \dfrac{1}{4}$

$y = \dfrac{1}{4}(n+2)$

and point of tangency $(6, 2)$

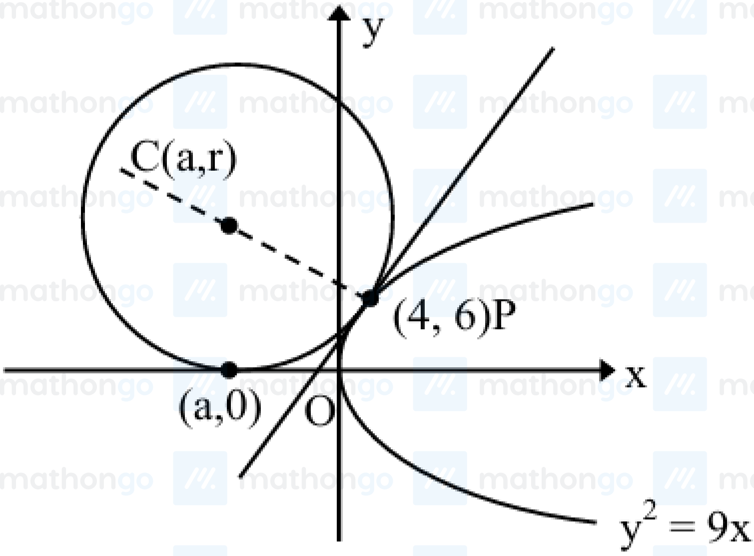


Area $A = \displaystyle\int_0^2 \left((y^2 + 2) - (4y - 2)\right) dy$

$A = \dfrac{8}{3}$

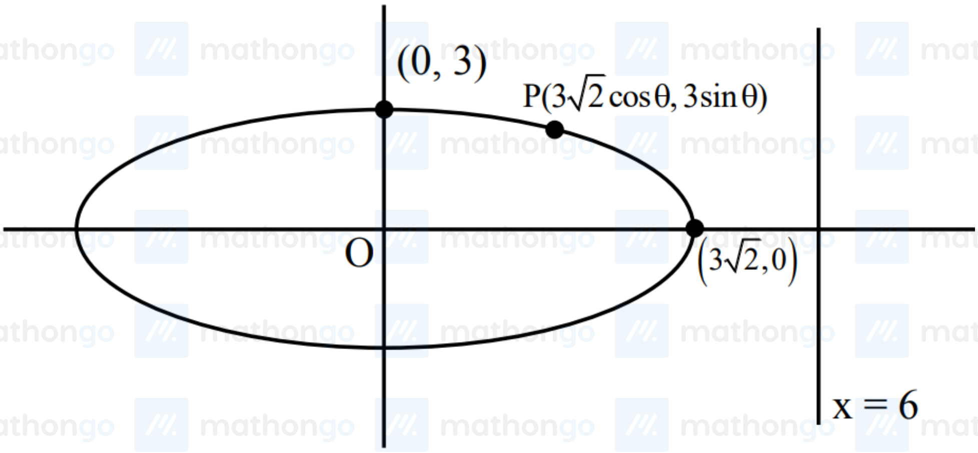
option (3)

Q7.
(30)



$(x-a)^2+(y-r)^2=r^2$
$(4-a)^2+(6-r)^2=r^2$
$16+a^2-8a+36+r^2-12r=r^2$
$a^2-8a-12r+52=0$
Tangent to parabola at $(4,6)$ is
$6.4=9.\left(\frac{x+4}{2}\right)$ i.e. $3x-4y+12=0$
This is also tangent to the circle
$\therefore \quad CP=r$
$$\frac{3a-4r+12}{5}=\pm r$$
$$3\mathrm{a}+12=4\mathrm{r}\pm 5\mathrm{r}\begin{cases}\mathrm{ar}\\ -\mathrm{r}\end{cases}\ldots\ldots(1)$$
equation of circle is
$(x-a)^2+(y-r)^2=r^2$
satsty $\mathrm{P}(4,6) \Rightarrow \mathrm{a}^2-8\mathrm{a}-12r+52=0\ldots\ldots(2)$
From equation (1)
If $\mathrm{a}+4=3\mathrm{r}$ then $\mathrm{a}=+6$ (rejected)
If $3a+12=-r$ then $a=-14$ and $r=30$

Q1.
(4)



$PS + PS' = 2 \times 3\sqrt{2}$
$b^2 = a^2\left(1 - e^2\right) \Rightarrow 9 = 18\left(1 - e^2\right)$
$\Rightarrow e = \frac{1}{\sqrt{2}}$
Directrix $x = \frac{a}{e} = \frac{3\sqrt{2}}{\frac{1}{\sqrt{2}}} = 6$
$PS \cdot PS' = \left|\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta - 6)\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta + 6)\right|$
$= \frac{1}{2}\left|18\cos^2\theta - 36\right|$
$\left(PS \cdot PS'\right)_{max} = 18; (PS \cdot PS)_{min} = 9$
sum $= 27$

Q2.
(1)

$2\,b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

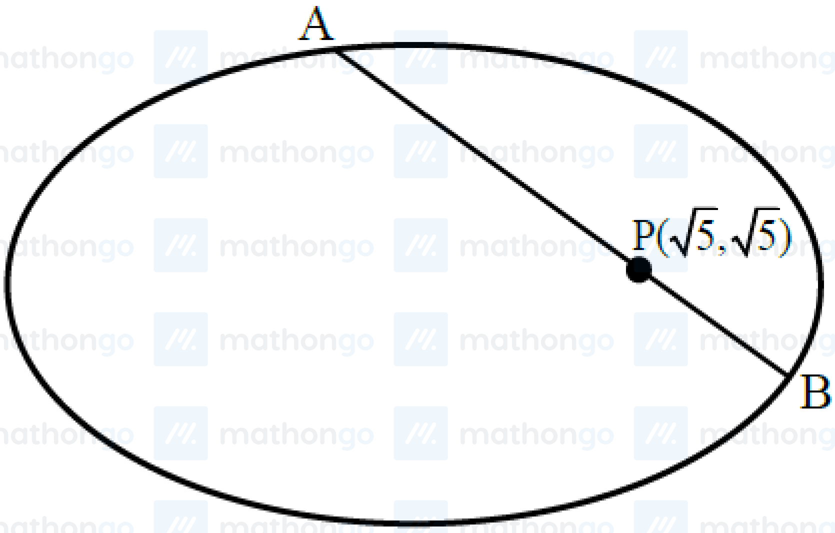
$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2\left(1 + \frac{1}{16}\right) = 1$

$e = \frac{4}{\sqrt{17}}$

Q3. Given ellipse is $\frac{x^2}{36}+\frac{y^2}{25}=1$

(4)



Any point on line AB can be assumed as

$\mathrm{Q}(\sqrt{5}+\mathrm{r}\cos\theta, \sqrt{5}+\mathrm{r}\sin\theta)$

Putting this in equation of ellipse, we get

$25(\sqrt{5}+r\cos\theta)^2+36(\sqrt{5}+r\sin\theta)^2=900$

Simplifying, we get

$\mathrm{r}^2\left(25\cos^2\theta+36\sin^2\theta\right)+2\sqrt{5}\mathrm{r}(25\cos\theta+36\sin\theta)-595=0$

$|\mathrm{r}|=\mathrm{PA}, \mathrm{PB}$

Thus, $\mathrm{PA}\cdot\mathrm{PB}=\dfrac{595}{25\cos^2\theta+36\sin^2\theta}=\dfrac{595}{25+11\sin^2\theta}$

$=$ maximum, if $\sin^2\theta=0$

This means line AB must be parallel to x-axis

$\Rightarrow y_A=y_B=\sqrt{5}$

Putting $\mathrm{y}=\sqrt{5}$ in equation of ellipse, we get

$\frac{x^2}{36}+\frac{1}{5}=1\Rightarrow x^2=36\cdot\frac{4}{5}$

Hence,

$\mathrm{PA}^2+\mathrm{PB}^2=\left(\sqrt{5}-\frac{12}{\sqrt{5}}\right)^2+\left(\sqrt{5}+\frac{12}{\sqrt{5}}\right)^2$

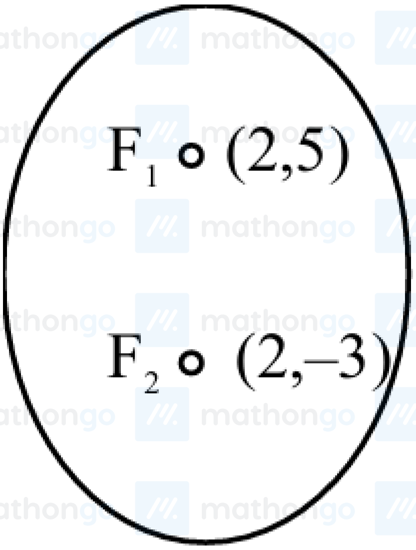
$=2\left(5+\dfrac{144}{5}\right)=\dfrac{338}{5}$

$5\left(\mathrm{PA}^2+\mathrm{PB}^2\right)=338$

Q4. (4)

$2be = 8$

$be = 4$



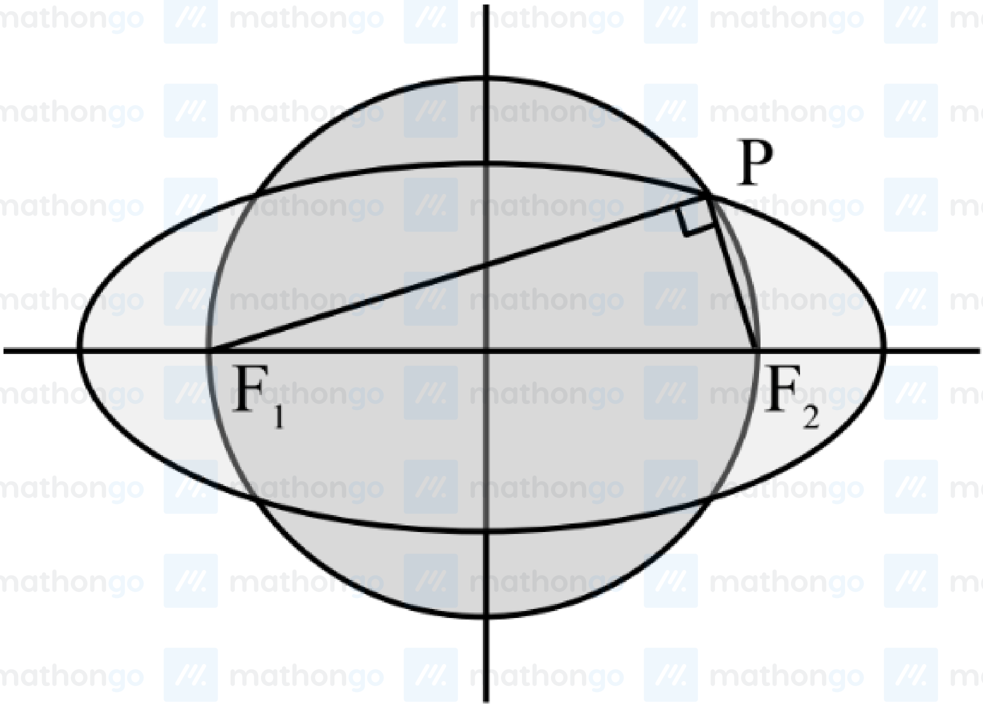
$b\left(\frac{4}{5}\right) = 4 \Rightarrow b = 5$

$\because c^2 = b^2 - a^2$

$16 = 25 - a^2 \Rightarrow a = 3$

L.R. $= \frac{2a^2}{b} = \frac{18}{5}$

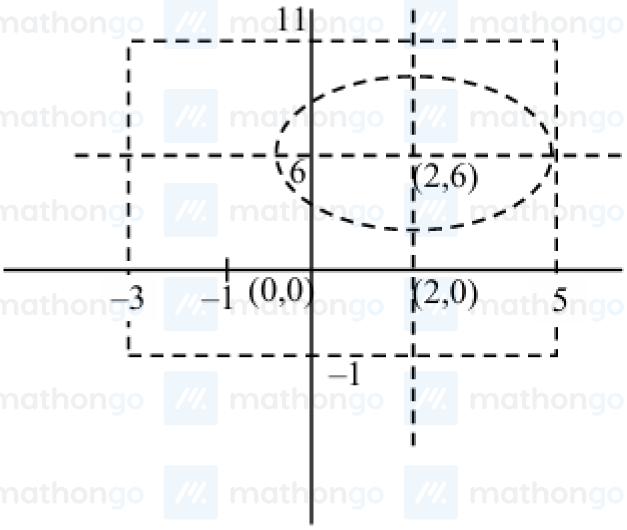
Q5.
(2)



$\frac{1}{2}PF_1 \cdot PF_2 = 30$
$PF_1 + PF_2 = 17$
$PF_1 = 12PF_2 = 5$
$F_1\ F_2 = 13$
option (2)

Q6.
(1)

$A : |x - 1| \le 4$ and $|y - 5| \le 6$
$\Rightarrow -4 \le x - 1 \le 4 \Rightarrow -6 \le y - 5 \le 6$
$\Rightarrow -3 \le x \le 5 \quad \Rightarrow -1 \le y \le 11$
$B : 16(x - 2)^2 + 9(y - 6)^2 \le 144$
$B : \frac{(x-2)^2}{9} + \frac{(y-6)^2}{16} \le 1$



From Diagram $B \subset A$

Q7. (2)

Length of LR $= \frac{2b^2}{a} = 10 \Rightarrow 5a = b^2 ...(1)$

$f(t) = t^2 + t + \frac{11}{12}$

$\frac{df(t)}{dt} = 2t + 1 = 0 \Rightarrow t = \frac{-1}{2}$

Min value of $f(t) = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + \frac{11}{12}$

$= \frac{1}{4}\frac{-1}{2} + \frac{11}{12} = \frac{3-6+11}{12} = \frac{8}{12} = \frac{2}{3} = e$

$e^2 = \frac{1-b^2}{a^2} \Rightarrow \frac{4}{9} = \frac{1-b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{1-4}{a} = \frac{5}{a} \Rightarrow b^2 = \frac{5a^2}{a} ...(2)$

From (1) \& (2)

$5a = \frac{5a^2}{a} \Rightarrow a = 9, \quad b = \sqrt{45} = 3\sqrt{5}$

$\therefore a2 + b2 = 81 + 45 = 126$

Q8. (3)

$E : \frac{x^2}{4/3} + \frac{y^2}{4/P} = 1$

Centre of circle (1,2), radius

$r = \sqrt{1 + 4 + 11}$

$r = 4$

$\because$ E pass from centre $(1, 2)$

$\therefore \frac{3}{4} + P = 1$

$P = \frac{1}{4} \quad \therefore$ vertical ellipse

$e = \sqrt{1 - \frac{4/3}{16}} = \sqrt{1 - \frac{1}{12}} = \sqrt{\frac{11}{12}}$

$\therefore$ Focal distance of $C(h, k)$

$= b \pm ek$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1 F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6 F_1 F_2 - r = 74 - 4 = 70$

Q9. (2)

Point of contact are $\left(\frac{\mp a^2 m}{\sqrt{a^2 m^2 + b^2}}, \frac{\pm b^2}{\sqrt{a^2 m^2 + b^2}}\right)$

$A\left(\frac{-16}{5}, \frac{9}{5}\right) B\left(\frac{16}{5}, \frac{-9}{5}\right)$

Point D is $\left(\frac{12}{5}, \frac{12}{5}\right)$

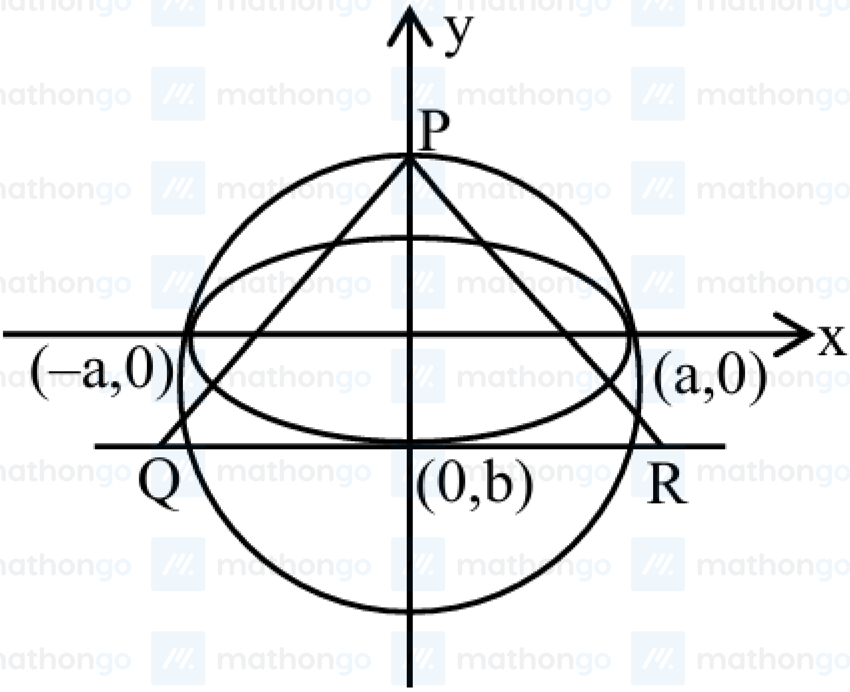
$$\text{Area of ABD} = \frac{1}{2}\begin{vmatrix} -\frac{16}{5} & \frac{9}{5} & 1 \\ \frac{16}{5} & \frac{-9}{5} & 1 \\ \frac{12}{5} & \frac{12}{5} & 1 \end{vmatrix}$$

$= 12$

Area of ABCD is $= 24$

option (2)

Q10.
(4)



Area of $\triangle PQR$

$= \frac{1}{2}(2a)(a \sin \theta + b)$

$\therefore$ maximum area $= a(a+b)$

$= 4(4 + 2\sqrt{3}) = 8(2 + \sqrt{3})$

Q11.
(46)

Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$

$E_2 : \frac{x^2}{c^2} + \frac{y^2}{d^2} = 1, (c < d)$

$C : x^2 + (y-1)^2 = 2$

Equation of tangent at $P(x_1, y_1)$

$xx_1 + y(y_1 - 1) = (y_1 + 1)$

comparing with $x + y = 3$ we get $P(1, 2)$

$\because$ Now parametric equation of $x + y = 3$

$\frac{(x-1)}{\left(\frac{-1}{\sqrt{2}}\right)} = \frac{(y-2)}{\left(\frac{1}{\sqrt{2}}\right)} = \pm\frac{2\sqrt{2}}{3} \quad \left(\because PQ = \frac{2\sqrt{2}}{3}\right)$

On solving we get $Q\left(\frac{5}{3}, \frac{4}{3}\right), R\left(\frac{1}{3}, \frac{8}{3}\right)$

So, $9(x_1y_1 + x_2y_2 + x_3y_3)$

$9\left(2 + \frac{5}{3} \times \frac{4}{3} + \frac{1}{3} \times \frac{8}{3}\right)$

$\Rightarrow 46$

Q1.
(3)

$ae = \sqrt{10}$ and $\frac{a}{e} = \frac{9}{10}$

$\Rightarrow a^2 = 9$ and $e = \frac{\sqrt{10}}{3}$

$(ae)^2 = a^2 + b^2$

$10 = 9 + b^2 \Rightarrow b^2 = 1$

$\ell = \frac{2b^2}{a} = \frac{2(1)}{3}$

Now

$\Rightarrow 9\left(e^2 + \ell\right)$

$= 9\left(\frac{10}{9} + \frac{2}{3}\right)$

$= 10 + 6$

$= 16$

Q2.
(141)

Equation of hyperbola is

$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{4-a^2} = 1$

$\Rightarrow \left(4 - a^2\right)(x-6)^2 - a^2(y-2)^2 = a^2\left(4 - a^2\right)$

comparing with $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, we get $a^2 = 1$ and $\alpha = 36, \beta = 4$ and $\gamma = 101$

$\therefore \alpha + \beta + \gamma = 141$

Q3.
(2)

$e_1^2 = 1 - \frac{b^2}{25} \quad e_2^2 = 1 - \frac{b^2}{16}$

$\therefore e_1^2 e_2^2 = 1$

$\left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

$\Rightarrow 2 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^2}{400} = 1$

$\Rightarrow \frac{9b^2}{400} = \frac{b^4}{400}$

$b^2 = 9$

$\frac{x^2}{9} + \frac{y^2}{25} = 1 \quad \frac{x^2}{16} - \frac{y^2}{9} = 0$

$e_1 \sqrt{1 - \frac{9}{25}}$

$e_1 = \frac{4}{5}$

Focii : - $(0, \pm 4) \quad (\pm 5, 0)$

ellipse passing through all four foci

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Q4. **(120)**

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad \ldots .(1)$

$\mathrm{P}(4,2\sqrt{3})$

$\mathrm{PS}_1 \cdot \mathrm{PS}_2=32$

$\left|\mathrm{PS}_1-\mathrm{PS}_2\right|=2\mathrm{a}$

$\mathrm{P}(4,2\sqrt{3})$ lies on H

$\therefore \frac{16}{\mathrm{a}^2}-\frac{12}{\mathrm{b}^2}=1$

$16b^2-12a^2=a^2b^2 \quad \ldots .(2)$

$\left|\mathrm{PS}_1-\mathrm{PS}_2\right|^2=4\mathrm{a}^2$

$\mathrm{PS}_1^2+\mathrm{PS}_2{}^2-2\mathrm{PS}_1 \cdot \mathrm{PS}_2=4\mathrm{a}^2$

$(\mathrm{ae}-4)^2+12+(\mathrm{ae}+4)^2+12-64=4\mathrm{a}^2$

$2\mathrm{a}^2\mathrm{e}^2-8=4\mathrm{a}^2$

$\mathrm{a}^2+\mathrm{b}^2-4=2\mathrm{a}^2$

$\mathrm{b}^2-\mathrm{a}^2=4$

$(2)\&(3) \Rightarrow 16\left(a^2+4\right)-12a^2=a^2\left(a^2+4\right)$

$\Rightarrow 16a^2+64-12a^2=a^4+4a^2$

$\Rightarrow a^4=64$

$\Rightarrow a^2=8$

$\therefore \mathrm{b}^2=12$

$\mathrm{p}^2+\mathrm{q}^2=4\,\mathrm{b}^2+\frac{4\,\mathrm{b}^4}{\mathrm{a}^2}$

$=120$

Q5. **(3)**

$ex+a+ex-a=8\sqrt{\frac{5}{3}}$

$2ex=8\sqrt{\frac{5}{3}}$

$2e \times 4=8\sqrt{\frac{5}{3}}$

$e=\sqrt{\frac{5}{3}}$

$b^2=a^2\left(\left(\frac{\sqrt{5}}{3}\right)^2-1\right)$

$b^2=\frac{2}{3}a^2$

$\frac{16}{\mathrm{a}^2}-\frac{9}{\mathrm{b}^2}=1$

and $b^2=\frac{2}{3}a^2$

$\Rightarrow \mathrm{a}^2=\frac{5}{2} \quad \mathrm{b}^2=\frac{5}{3}$

Now,

$\ell = \frac{2\,b^2}{a}$

$\ell^2 = \frac{4\,b^4}{a^2}$

$9\ell^2 = 36 \times \frac{25}{9 \times 5} \times 2$

$9\ell^2 = 40$

$m = (ex + a)(ex - a)$

$m = e^2x^2 - a^2$

$= \frac{5}{3} \times 16 - \frac{5}{2} = \frac{145}{6}$

$= 6\,m = 145$

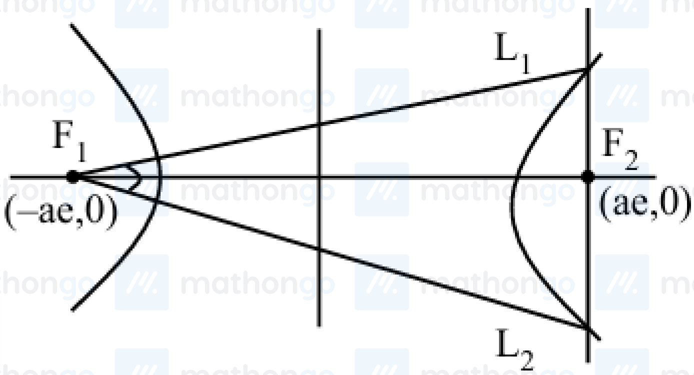
$9\ell^2 + 6\,m$

$40 + 145 = 185$

option (3)

Q6. $f_1 \equiv (-ae, 0) \equiv P(-3, 0)$

(1944) $\Rightarrow ae = 3$



$\tan 45^\circ = \frac{b^2/a}{2ae}$

$2ae = \frac{b^2}{a}$

$b^2 = 6a$

Also $a^2e^2 = a^2 + b^2$

$9 = a^2 + 6a$

$a^2 + 6a - 9 = 0$

$a = -3 \pm 3\sqrt{2} = -3(1 \pm \sqrt{2})$

$\therefore a^2b^2 = a^2 \cdot 6a = 6a^3$

$= 6(135\sqrt{2} - 189)$

$\alpha = 810$ and $\beta = 1134$

$\therefore \alpha + \beta = 1944$

Q7. Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(189) Directrix: $x = \frac{-9}{5}$ and corresponding foci $(-5, 0)$

$\Rightarrow -\frac{a}{e} = -\frac{9}{5}$ and $-ae = -5$

$\Rightarrow \frac{9e^2}{5} = 5 \Rightarrow e = \sqrt{\frac{25}{9}} = \frac{5}{3} \Rightarrow a = 3$

$\therefore \mathrm{b}^2 = \mathrm{a}^2\left(\mathrm{e}^2 - 1\right) = 9\left(\frac{25}{9} - 1\right) = 16$

Hyperbola $\frac{\mathrm{x}^2}{9} - \frac{\mathrm{y}^2}{16} = 1$

$(\alpha, 2\sqrt{5})$ lie on it

$\Rightarrow \frac{\alpha^2}{9} - \frac{20}{16} = 1 \Rightarrow \alpha^2 = \frac{36}{16} \times 9 = \frac{81}{4}$

Product for distance of $(\mathrm{x}_1 \mathrm{y}_1)$ from the two foci

$= (ex_1 + a)\,|ex_1 - a|$

$= e^2 x_1^2 - a^2$

For $(\alpha, 2\sqrt{5}) \Rightarrow \mathrm{P} = \frac{25}{9} \cdot \frac{81}{4} - 9 = \frac{189}{4}$

$4\mathrm{P} = 189$

Q1.
(1) $\operatorname{cosec}\theta=\dfrac{2(\sqrt{3}-1)\pm\sqrt{4(3+1-2\sqrt{3})+16\sqrt{3}}}{2\sqrt{3}}$

$=\dfrac{2(\sqrt{3}-1)\pm\sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$

$=\dfrac{2(\sqrt{3}-1)\pm(2+2\sqrt{3})}{2\sqrt{3}}$

$\operatorname{cosec}\theta=2$ or $\frac{-2}{\sqrt{3}}$

$\therefore \sin\theta=\frac{1}{2}$ or $\frac{-\sqrt{3}}{2}$

$\therefore \sin\theta=\frac{1}{2}$ has 3 solutions \& also $\sin\theta=\frac{-\sqrt{3}}{2}$

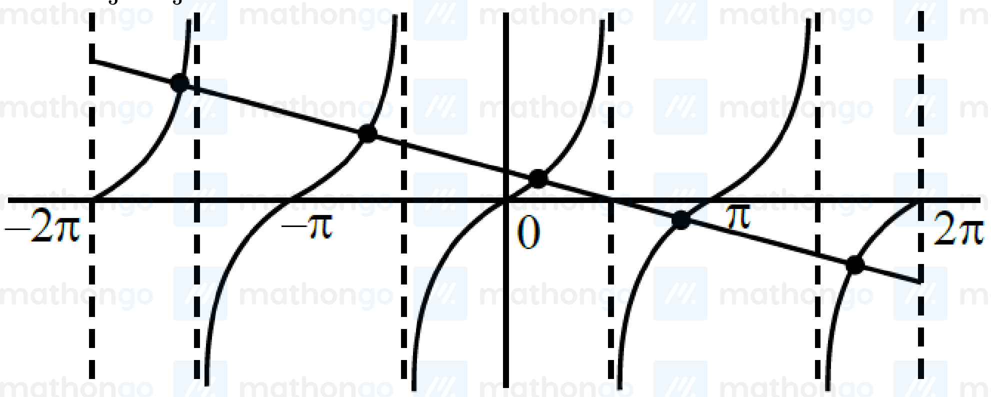
has 3 solutions in $\left[\frac{-7\pi}{6},\frac{4\pi}{3}\right]$

Q2. $2\sqrt{2}\cos^2\theta+2\cos\theta-\sqrt{6}\cos\theta-\sqrt{3}=0$
(3) $(2\cos\theta-\sqrt{3})(\sqrt{2}\cos\theta+1)=0$

$\cos\theta=\dfrac{\sqrt{3}}{2},\dfrac{-1}{\sqrt{2}}$

Number of solution $=8$

Q3. $\tan \mathrm{x}=\frac{\pi}{3}-\frac{2\mathrm{x}}{3}$
(2)



5 solutions

Q4. $(4-\sqrt{3})\sin x-2\sqrt{3}\cos^2 x=\dfrac{-4}{1+\sqrt{3}}, x\in\left[-2\pi,\dfrac{5\pi}{2}\right]$
(4)

$\Rightarrow(4-\sqrt{3})\sin x-2\sqrt{3}\left(1-\sin^2 x\right)=2(1-\sqrt{3})$

$\Rightarrow 2\sqrt{3}\sin^2 x+4\sin x-\sqrt{3}\sin x-2=0$

$\Rightarrow(2\sin x-1)(\sqrt{3}\sin x+2)=0$

$\Rightarrow \sin x=\dfrac{1}{2}$

$\therefore$ Number of solution $=5$

Q5. $10\left(\sin^2\theta\right)^2+15\left(1-\sin^2\theta\right)^2=6$

(1) Let $\sin^2\theta=\mathrm{t} \Rightarrow 10\mathrm{t}^2+15(1-\mathrm{t})^2=16$

$10\mathrm{t}^2+15-30\mathrm{t}+15\mathrm{t}^2=6$

$25\mathrm{t}^2-30\mathrm{t}+9=0$

$(5\mathrm{t}-3)^2=0$

$\sin^2\theta=\frac{3}{5}$ and $\cos^2\theta=\frac{2}{5}$

$$\frac{27\times\frac{125}{27}+8+\frac{125}{8}}{16\left(\frac{5}{2}\right)^4}=\frac{250}{125\times5}=\frac{2}{5}$$

Q6. $x=3\left(\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\right)$

(4)

$x-\sqrt{3}\tan\theta=3\tan\theta+3\sqrt{3}$

$\tan\theta=\frac{x-3\sqrt{3}}{3+\sqrt{3}x}\ldots(1)$

$$2\left(\frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{\tan\theta}{\sqrt{3}}}=y\right)$$

$2(\sqrt{3}\tan\theta+1)=y(\sqrt{3}-\tan\theta)\ldots.$

using (1) and (2)

$$2\left(\frac{x-3\sqrt{3}}{\sqrt{3}+x}+1\right)=y\left(\sqrt{3}-\frac{(x-3\sqrt{3})}{\sqrt{3}(\sqrt{3}+x)}\right)$$

$2\sqrt{3}(x-3\sqrt{3}+x+\sqrt{3})=y(3(\sqrt{3}+x)-x+3\sqrt{3})$

$4\sqrt{3}x-12=y(2x+6\sqrt{3})$

$xy-2\sqrt{3}x+3\sqrt{3}y-6=0$

$\Rightarrow\alpha=-2\sqrt{3},\beta=3\sqrt{3},\gamma=-6$

$\alpha^2+\beta^2+\gamma^2=12+27+36=75$

Q7. $\cos2\theta\cos\frac{\theta}{2}+\cos\frac{5\theta}{2}=2\cos^3\frac{5\theta}{2}$

(1)

$$\frac{1}{2}\left(2\cos2\theta\cos\frac{\theta}{2}\right)+\cos\frac{50}{2}$$

$$=\frac{1}{2}\left(\cos\frac{15\theta}{2}+3\cos\frac{5\theta}{2}\right)$$

or solving

$\cos\frac{3\theta}{2}=\cos\frac{15\theta}{2}$

$\cos\frac{15\theta}{2}-\cos\frac{3\theta}{2}=0$

$2\sin30\sin\frac{9\theta}{2}=0$

$3\theta = \mathrm{n}\pi$ or $\frac{9\theta}{2} = \mathrm{m}\pi$

$$\theta = \frac{\mathrm{n}\pi}{3} \quad \theta = \frac{2\,\mathrm{m}\pi}{9}$$

$$\theta = \left\{-\frac{\pi}{2}, \frac{\pi}{3}, 0\right\}$$

$$\theta = \left\{-\frac{4\pi}{9}, \frac{-2\pi}{9}, \frac{4\pi}{9}, \frac{2\pi}{9}\right\}$$

Q1.
(1)
$$\cot^{-1}\left(\frac{|\sec 2|-1}{\tan 2}\right)-\cot^{-1}\left(\frac{\left|\sec\frac{1}{2}\right|+1}{\tan\frac{1}{2}}\right)$$
$$=\cot^{-1}\left(\frac{-1-\cos 2}{\sin 2}\right)-\cot^{-1}\left(\frac{1+\cos\frac{1}{2}}{\sin\frac{1}{2}}\right)$$
$$=\pi-\cot^{-1}(\cot 1)-\cot^{-1}\left(\cot\frac{1}{4}\right)$$
$$=\pi-1-\frac{1}{4}=\pi-\frac{5}{4}$$

Q2.
(3)
$$y=\cos\left(\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{x}{2}\right)$$
$$y=\frac{1}{2}\times\frac{x}{2}-\sqrt{1-\frac{1}{4}}\sqrt{1-\frac{x^2}{4}}$$
$$4y=x-\sqrt{3}\sqrt{4-x^2}$$
$$3\left(4-x^2\right)=x^2+16y^2-8xy$$
$$12-3x^2=x^2+16y^2-8xy$$
$$4x^2+16y^2-8xy=12$$
$$x^2+4y^2-2xy=3$$
$$x^2+y^2-2xy-3y^2=3$$
$$(x-y)^2+3y^2=3$$

Q3.
(2)
$$\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right),\ \frac{-1}{2}<x<\frac{1}{\sqrt{2}}$$
$$\Rightarrow \text{ Let } \sin^{-1}(x)=\theta \quad \frac{-\pi}{6}<\theta<\frac{\pi}{4}$$
$$\Rightarrow x=\sin\theta,\ \text{then}$$
$$\Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta\right)$$
$$\Rightarrow \sin^{-1}\left(\sin\left(\theta+\frac{\pi}{6}\right)\right)=\theta+\frac{\pi}{6}$$
$$\Rightarrow \sin^{-1}(x)+\frac{\pi}{6}$$

Q4.
(4)

$$T_n = \tan^{-1}\left(\frac{4}{4n^2+3}\right)$$

$$T_n = \tan^{-1}\left(\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$$

$$T_n = \tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$$

$$T_1+T_2+\ldots+T_n = \tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$$

$$S_\infty = \frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$$

option (4)

Q1. $\text{Ar}(\triangle BCD)$

(3) $= \sqrt{(\text{Ar}(\triangle \text{ABC}))^2 + (\text{Ar}(\text{ACD}))^2 + (\text{Ar}(\triangle \text{ADB}))^2}$

$= \sqrt{5^2 + 6^2 + 7^2}$

$= \sqrt{110}$

Q2. $\vec{c} = 3\vec{a} + 6\vec{b} + 9(\vec{a} \times \vec{b})$

(3)

$\sin^{-1}\left(\frac{\sqrt{65}}{9}\right) \Rightarrow \sin\theta = \frac{\sqrt{65}}{9} \Rightarrow \cos\theta = \frac{4}{9}$

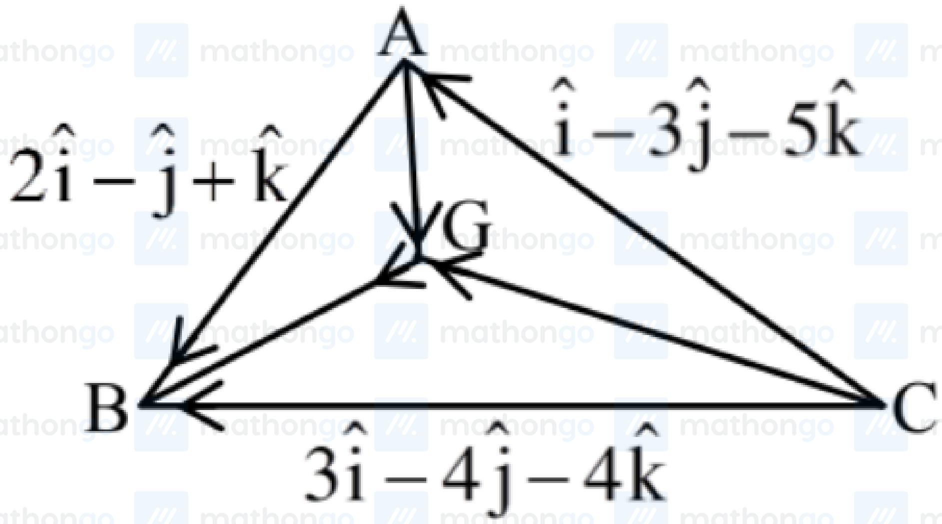
$\vec{c} \cdot \vec{a} = 3|\vec{a}|^2 + 6\vec{a} \cdot \vec{b} = 3 + \frac{6 \cdot 4}{9} = \frac{51}{9}$

$\vec{c} \cdot \vec{a} = 3\vec{a} \cdot \vec{b} + 6|\vec{b}|^2 = \frac{3 \cdot 4}{9} + 6 = \frac{22}{3}$

$\therefore 9(\vec{c} \cdot \vec{a}) - 3(\vec{c} \cdot \vec{b}) = 51 - 22 = 29$

Q3.

(164)



By given data

$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{CB}$

Let pv of $\vec{A}$ are $\vec{O}$ then

$\overrightarrow{AB} = \vec{B} - \vec{A}$

i.e. *pv* of $\vec{B} - = 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{CA} = \vec{A} - \vec{C}$

i.e. pv of $\vec{C} = -(\hat{i} - 3\hat{j} - 5\hat{k})$

Now pv of centroid

$(\vec{G}) - = \frac{\vec{A} + \vec{B} + \vec{C}}{3} = \frac{\vec{0} + (2, -1, 1) + (-1, 3, 5)}{3}$

$\vec{G} = \frac{1}{3}(\hat{i} + 2\hat{j} + 6\hat{k})$

Now $\overrightarrow{AG} = \frac{1}{3}(\hat{i} + 2\hat{j} + 6\hat{k})$

$\Rightarrow |\overrightarrow{AG}|^2 = \frac{1}{9} \times 41$

$\overrightarrow{BG} = \left(\frac{1}{3} - 2\right)\hat{i} + \left(\frac{2}{3} + 1\right)\hat{j} + (2-1)\hat{k}$

$\Rightarrow |\overrightarrow{BG}|^2 = \frac{59}{9}$

$\overrightarrow{CG} = \left(\frac{1}{3} + 1\right)\hat{i} + \left(\frac{2}{3} - 3\right)\hat{j} + (2-5)\hat{k}$

$\Rightarrow |\overrightarrow{CG}|^2 = \frac{146}{9}$

Now

$6\left[|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2\right] = 6 \times \left[\frac{41}{9} + \frac{59}{9} + \frac{146}{9}\right]$

$= 6 \times \frac{246}{9} = 164$

Q4. $\frac{|\bar{a}+\bar{b}|+|\bar{a}-\bar{b}|}{|\bar{a}+\bar{b}|-|\bar{a}-\bar{b}|} = \sqrt{2} + 1$

(3) Apply componendo and dividendo

$\Rightarrow \frac{2|\bar{a}+\bar{b}|}{2|\bar{a}-\bar{b}|} = \frac{\sqrt{2}+2}{\sqrt{2}}$

$\Rightarrow |\bar{a}+\bar{b}| = (1+\sqrt{2})|\bar{a}-\bar{b}|$

$\Rightarrow |\bar{a}+\bar{b}|^2 = (3+2\sqrt{2})|\bar{a}-\bar{b}|^2$

$\Rightarrow 2|\bar{a}|^2 + 2\bar{a}\cdot\bar{b} = (3+2\sqrt{2})\left(2|\bar{a}|^2 - 2\bar{a}\cdot\bar{b}\right)$

$\Rightarrow 2|\bar{a}|^2(2+2\sqrt{2}) = 2\bar{a}\cdot\bar{b}(4+2\sqrt{2})$

$\Rightarrow \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|^2} = \frac{2+2\sqrt{2}}{4+2\sqrt{2}} = \frac{1}{\sqrt{2}}$

Now

$\frac{|\bar{a}+\bar{b}|^2}{|\bar{a}|^2} = 1 + \frac{|\bar{b}|^2}{|\bar{a}|^2} + \frac{2\bar{a}\cdot\bar{b}}{|\bar{a}|^2}$

$= 1 + 1 + 2\left(\frac{1}{\sqrt{2}}\right) = 2 + \sqrt{2}$

Q5. Let $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

(3) $a_1^2 + a_2^2 + a_3^2 = 1$

Let $\bar{b} = 2\vec{i} - \hat{j} + 2\hat{k}, \bar{c} = \vec{i} - 2\hat{j} - 2\hat{k}$

$\bar{d} = \hat{k}$

$\frac{\bar{a}\cdot\bar{b}}{|b|} = \frac{\bar{a}\cdot\bar{c}}{|c|} = \frac{\bar{a}\cdot\bar{d}}{|d|}$

$\frac{2a_1 - a_2 + 2a_3}{3} = \frac{a_1 + 2a_2 - 2a_3}{3} = a_3$

By solving

$a_1 = \frac{7}{\sqrt{155}}, a_2 = \frac{9}{\sqrt{155}}, a_3 = \frac{5}{\sqrt{155}}$

Q6. $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

(4)

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix}$$

$= -17\hat{i} - 7\hat{j} + 13\hat{k}$

$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3j + 12\hat{k}$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{c} \times \vec{b}) = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (\vec{a} \times \vec{b})$

$= (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17i - 7\hat{j} + 13\hat{k})$

$\vec{b} \times \vec{c} = -\hat{i} + 4\hat{j} - \hat{k}$

$\therefore \vec{a} \cdot \vec{d} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (2i - 3j + k) \cdot (-\hat{i} + 4\hat{j} - \hat{k})$

$= -2 - 12 - 1 = -15$

$\therefore |\vec{a} \cdot \vec{d}| = 15$

Q7. $\vec{a} \times \vec{d} - \vec{b} \times \vec{d} = 0$

(5)

$(\vec{a} - \vec{b}) \times \vec{d} = 0$

$\vec{d} = t(\vec{a} - \vec{b})$

$\vec{d} = t(-2\hat{i} - \hat{j} + 2\hat{k})$

$|\vec{d}| = 1$

$|t| = \frac{1}{3}$

$\vec{c} \cdot \vec{a} = 0$

$\lambda + \mu = 0$

$\mu = -\lambda$

$\vec{c} = \lambda(\hat{j} - \hat{k}), |\vec{c}|^2 = 2\lambda^2$

$\vec{c} \cdot \hat{d} = 1$

$t(-2, -1, 2) \cdot \lambda(0, 1, -1) = 1$

$\lambda t = \frac{-1}{3} \Rightarrow \lambda^2 = 1$

$|3\lambda\,\hat{d} + \mu c|^2 = 9\lambda^2|\hat{d}|^2 + \mu^2|\vec{c}|^2 + 6\lambda\mu(\hat{d} \cdot \vec{c})$

$= 3\lambda^2 + 2\lambda^4$

$= 5$

Q8. $\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$-and $\vec{a} \cdot \vec{d} = 4$

(128)

$\Rightarrow \vec{d} = \lambda(\vec{b} - \vec{c}) = \lambda(\hat{i} - 2\hat{j} + \hat{k})$

$\because \vec{a} \cdot \vec{d} = 4 \Rightarrow \lambda = -2$

Also. $|\vec{a} \times \vec{d}|^2 + |\vec{a} \cdot \vec{d}|^2 = |\vec{a}|^2|\vec{d}|^2$

$\Rightarrow |\vec{a} \times \vec{d}|^2 = 6 \times 4 \times 6 - 16 = 128$

Q9. $\overrightarrow{u} = 3\hat{i} - \hat{j}, \overrightarrow{v} = 2\hat{i} + \hat{j} - \lambda\hat{k},$
(2)
$$\Rightarrow \frac{\overrightarrow{u} \cdot \overrightarrow{v}}{|\overrightarrow{u}||\overrightarrow{v}|} = \cos\theta$$
$$\Rightarrow \frac{5}{\sqrt{10}\sqrt{5+\lambda^2}} = \frac{\sqrt{5}}{2\sqrt{7}}$$
$$\Rightarrow \lambda^2 = 9 \Rightarrow \lambda = 3 (\because \lambda > 0)$$
$$\overrightarrow{v} = \overrightarrow{v}_1 + \overrightarrow{v}_2$$
$$\Rightarrow |\overrightarrow{v}|^2 = \overrightarrow{v}_1^2 + \overrightarrow{v}_2^2 + 2\overrightarrow{v}_1 \cdot \overrightarrow{v}_2$$
$$\Rightarrow 14 = \overrightarrow{v}_1^2 + \overrightarrow{v}_2^2 + 0 \quad \left(\because \overrightarrow{v}_1 \perp \overrightarrow{v}_2\right)$$
$$\Rightarrow \left|\overrightarrow{v}_1^2\right| + \left|\overrightarrow{v}_2^2\right| = 14$$

Q10. Let vector $\vec{p}$ in plane of $\vec{a} \& \vec{b} = K(\vec{a} + \lambda\vec{b})$
(4) $\overrightarrow{p} \perp \overrightarrow{a} = \overrightarrow{p} \cdot \overrightarrow{a} = 0$
$$\Rightarrow K(\overrightarrow{a} + \lambda\overrightarrow{b}) \cdot \overrightarrow{a} = 0$$
$$\Rightarrow \overrightarrow{a} \cdot \overrightarrow{a} + \lambda\overrightarrow{b} \cdot \overrightarrow{a} = 0$$
$$\Rightarrow 6 + \lambda(3) = 0$$
$$\Rightarrow \lambda = -2$$
$$\Rightarrow \overrightarrow{p} = (-3\hat{i} + 3\hat{k})$$
Unit vector $\rightarrow \pm\frac{(-\hat{i}+\hat{k})}{\sqrt{2}}$

Q1. $\beta = \frac{\alpha}{2}, \gamma = \frac{\alpha}{2}$

(1) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + 2\cos^2\frac{\alpha}{2} = 1$

$\cos^2\alpha + \cos\alpha = 0$

$\cos\alpha(\cos\alpha + 1) = 0$

$\cos\alpha = 0, -1$

$\alpha = \frac{\pi}{2}, \pi$

Now $\beta = \frac{\alpha}{2} \Rightarrow \frac{\pi}{4}, \frac{\pi}{2}$

so sum is $\frac{3\pi}{4}$

Q2. $L_1 : (7\hat{i} + 6\hat{j} + 2k) + \lambda(-3\hat{i} + 2\hat{j} + 4k)$

(1) $L_2 : (5\hat{i} + 3\hat{j} + 4k) + \lambda(2\hat{i} + \hat{j} + 3k)$

Distance between skew lines

$$= \frac{(2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 2\hat{\mathrm{k}}) \cdot (2\hat{\mathrm{i}} + 17\hat{\mathrm{j}} - 7\hat{\mathrm{k}})}{\sqrt{342}}$$

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

Q3. $\mathrm{A}(3, \alpha, 3) \& B(-3, -7, \beta)$

(2) $\overrightarrow{\mathrm{BA}} = 6\hat{\mathrm{i}} + (\alpha + 7)\hat{\mathrm{j}} + (3 - \beta)\hat{\mathrm{k}}$

$$\overrightarrow{\mathrm{p}} \times \overrightarrow{\mathrm{q}} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$\frac{|\overrightarrow{\mathrm{BA}} \cdot (\overrightarrow{\mathrm{p}} \times \overrightarrow{\mathrm{q}})|}{|\overrightarrow{\mathrm{p}} \times \overrightarrow{\mathrm{q}}|} = 3\sqrt{30}$

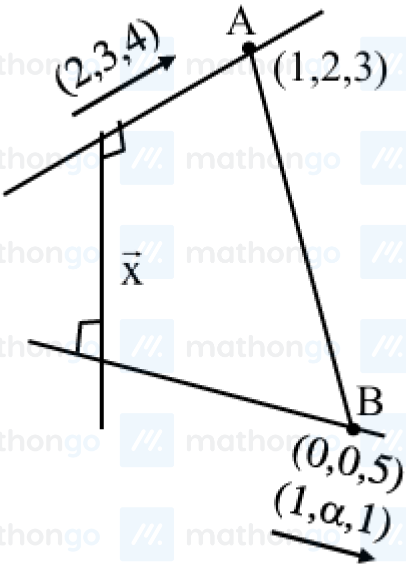
$36 + 15(\alpha + 7) - 3(3 - \beta) = (3\sqrt{30})^2$

$36 + 15\alpha + 105 - 9 + 3\beta = 270$

$15\alpha + 3\beta = 138$

$5\alpha + \beta = 46$

Q4.
(4)



$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$L_1 : \frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$$

$$\vec{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & \alpha & 1 \end{vmatrix} = \hat{\mathrm{i}}(3-4\alpha) - \hat{\mathrm{j}}(-2) + \hat{\mathrm{k}}(2\alpha - 3)$$

$$\text{S.D.} = \left|\frac{\overrightarrow{\mathrm{BA}} \cdot \overrightarrow{\mathrm{n}}}{|\overrightarrow{\mathrm{n}}|}\right| = \left|\frac{(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - 2\hat{\mathrm{k}}) \cdot \vec{n}}{|\overrightarrow{\mathrm{n}}|}\right|$$

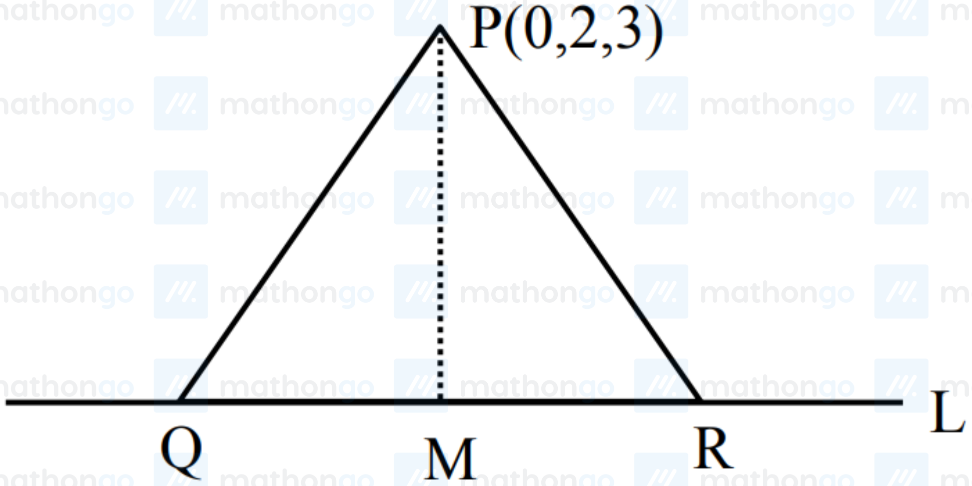
$$\Rightarrow 6(13 - 8\alpha)^2 = 25\left((4\alpha - 3)^2 + (2\alpha - 3)^2 + 16\right)$$

$$6\left(64a^2 - 280\alpha + 169\right) = 25\left(20\alpha^2 - 36\alpha + 34\right)$$

$$\Rightarrow 116\alpha^2 + 348\alpha - 164 = 0$$

$$\alpha_1 + \alpha_2 = \frac{-348}{116} = -3$$

Q5.
(2)



QR = 5

$M(5\lambda-3, 2\lambda+1, 3\lambda-4)$

Drs of PM $\Rightarrow 5\lambda-3, 2\lambda-1, 3\lambda-7$

Drs of line $L \Rightarrow 5,2,3$

$PM \perp L$

$\Rightarrow (5\lambda-3)5+(2\lambda-1)2+(3\lambda-7)3=0$

$\Rightarrow \lambda=1$

$\therefore M(2,3,-1)$

$PM=\sqrt{4+1+16}=\sqrt{21}$

Area $=\frac{1}{2}\times 5\times\sqrt{21}=\frac{m}{n}$

$2m-5\sqrt{21}n=0$

Q6.
(2)

$P(1,0,3)$

$A(4,7,1), B(3,5,3)$

Line AB $\Rightarrow \frac{x-3}{1}=\frac{y-5}{2}=\frac{z-3}{-2}=\lambda$

Let foot of perpendicular of P on AB be

$R\equiv(\lambda+3, 2\lambda+5, -2\lambda+3)$

$\Rightarrow(\lambda+3-1)(1)+(2\lambda+5-0)(2)+(-2\lambda+3-3)$
$(-2)=0$

$\Rightarrow \lambda+2+4\lambda+10+4\lambda=0$

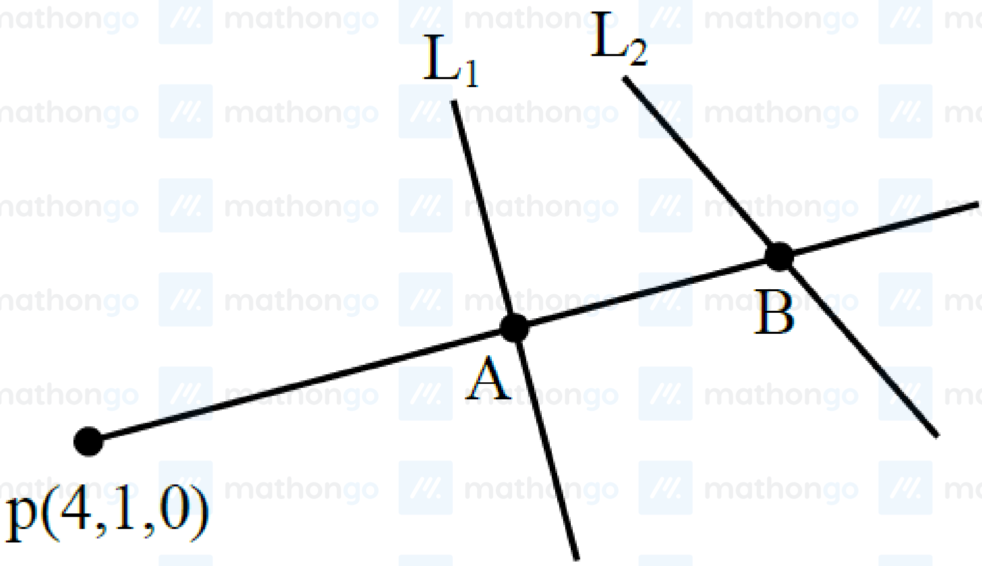
$\Rightarrow \lambda=-\frac{4}{3}$

$\Rightarrow R\equiv\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

$Q\equiv\left(\frac{10}{3}-1,\frac{14}{3}-0,\frac{34}{3}-3\right)\equiv\left(\frac{7}{3},\frac{14}{3},\frac{25}{3}\right)$

$\Rightarrow \alpha+\beta+\gamma=\frac{7+14+25}{3}=\frac{46}{3}$

Q7.
(1)



$L_1 = \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4} = p$

$L_2 = \dfrac{x-6}{1} = \dfrac{y}{1} = \dfrac{z-4}{-1} = q$

$A(2p+1, 3p+2, 4p+3)$

$B(q+6, q, 4-q)$

$\text{D.R. of } PA = 2p-3, 3P+1, 4p+3$

$\text{D.R. of } PB = q+2, q-1, 4-q$

$\dfrac{2p-3}{q+2} = \dfrac{3p+1}{q-1} = \dfrac{4p+3}{4-q}$

$2pq - 2p - 3q + 3 = 3pq + 6p + q + 2$

$\mathrm{pq} + \mathrm{rp} + 4\mathrm{q} - 1 = 0$

$12p - 3pq + 4 - q = 4pq + 3q - 4p - 3$

$7pq - 16p + 4q - 7 = 0$

$8p - 2pq - 12 + 3q = 4pq + 8p + 3q + 6$

$6\mathrm{pq} = -18 \quad \therefore \mathrm{pq} = -3$

$8p + 4q = 4 \quad \Rightarrow 2p + q = 1$

$-21 - 16p + 4q - 7 \quad \Rightarrow 4p - q = -7$

$16\mathrm{p} - 4\mathrm{q} = -28 \quad \therefore \mathrm{p} = -1, \mathrm{q} = 3$

$\mathrm{A}(-1, -1, -1) \quad \mathrm{B}(9, 3, 1)$

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1(-1+9) = 8$$

Q8.
(3)

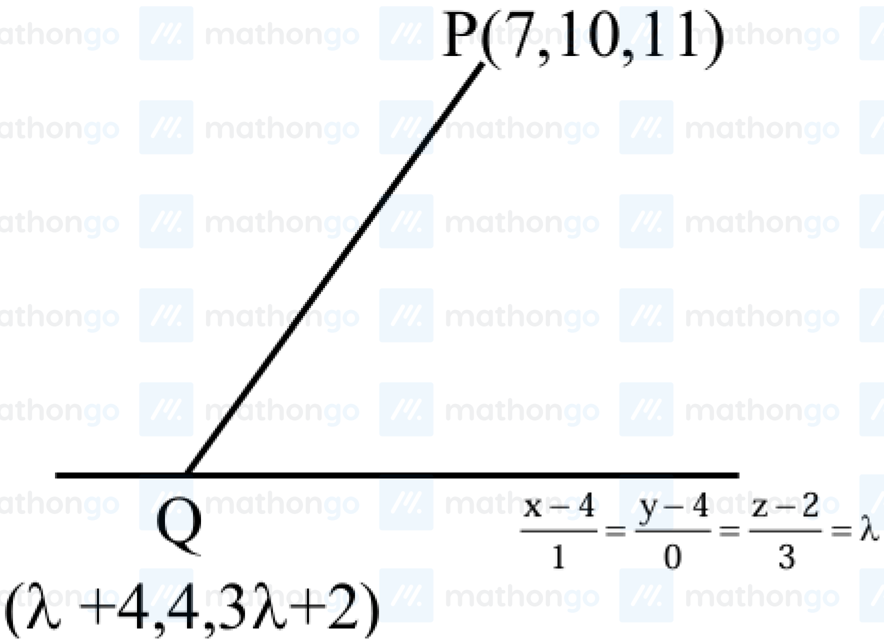
$L_1 \equiv \dfrac{x-1}{0} = \dfrac{y-2}{0} = \dfrac{z-3}{1}$

$L_2 \equiv \dfrac{x-\lambda}{0} = \dfrac{y-5}{1} = \dfrac{z-6}{0}$

$$\mathrm{SD} = \frac{\begin{vmatrix} \lambda-1 & 3 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}$$

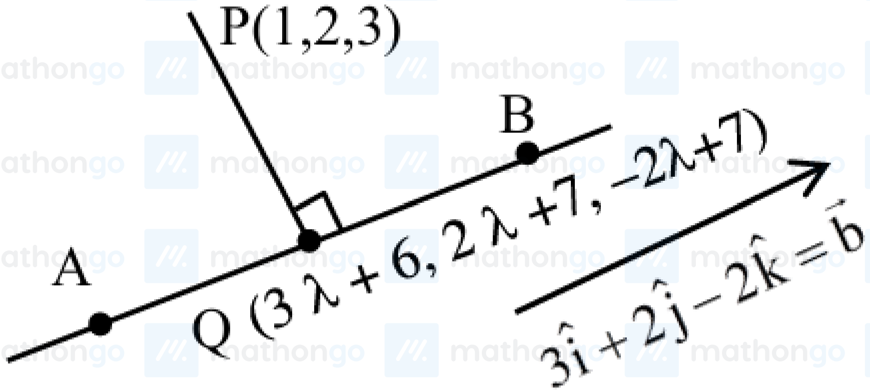
$= |\lambda - 1| = 3$
$\lambda = 4, -2$
$\lambda_1 = 4$
$\lambda_2 = -2$
Let foot of perpendicular from
$\mathrm{P}(4, -2, 7)$ is $\mathrm{Q}(1, 2, \mathrm{t} + 3)$
$\mathrm{So}(3, -4, 4 - \mathrm{t}) \cdot (0, 0, 1) = 0$
$\mathrm{t} = 4$
$\mathrm{So}\,\mathrm{Q}(1, 2, 7)$
$\mathrm{PQ}^2 = 9 + 16$
$\mathrm{PQ}^2 = 25$

Q9.
(2)



$\because$ line PQ is parallel to line $\frac{x-9}{2} = \frac{y-3}{3} = \frac{z-17}{6}$
$$\therefore \frac{\lambda - 3}{2} = \frac{-6}{3} = \frac{3\lambda - 9}{6} \Rightarrow \lambda = -1$$
$Q = (3, 4, -1)$
$\therefore PQ = \sqrt{16 + 36 + 144} = 14$

Q10.
(2)



$\overrightarrow{PQ} \cdot \overrightarrow{b} = 0$
$\Rightarrow 3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$
$\Rightarrow 17\lambda=-17 \Rightarrow \lambda=-1$
$Q(3,5,9)$
Let A $(3\mu+6, 2\mu+7, -2\mu+7)$
$(3\mu+3)^2+(2\mu+2)^2+(-2\mu-2)^2=68$
$\Rightarrow \mu^2+2\mu-3=0 \mu=-3$ or $\mu=1$
A $(-3,1,13)$ and $B(9,9,5)$
$\overrightarrow{OA} \cdot \overrightarrow{OB}=-27+9+65=47$

Q11.
(3)
shortest distance $=\frac{|(\bar{a}-\bar{b})| \cdot(\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|}$
where
$\bar{a}=-\hat{i}+0\hat{j}+0\hat{k}$
$\bar{b}=p\hat{i}+2\hat{j}+\hat{k}$
$\bar{p}=3\hat{i}+4\hat{j}+5\hat{k}-\bar{b}=(-1-p)\hat{i}-2\hat{j}-\hat{k}$
$\bar{q}=2\hat{i}+3\hat{j}+4\hat{k}$
$\frac{1}{16}=\frac{|-1-p+4-1|}{\sqrt{6}}$
$|-p+2|=1$
$p=3$
$\frac{x^2}{1^2}+\frac{y^2}{3^3}=1$
$L \cdot R=\frac{2a^2}{b}=\frac{2 \times 1}{3}=\frac{2}{3}$
option (3)

Q12. Angle between both lines

(1) $\cos\theta = \left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$

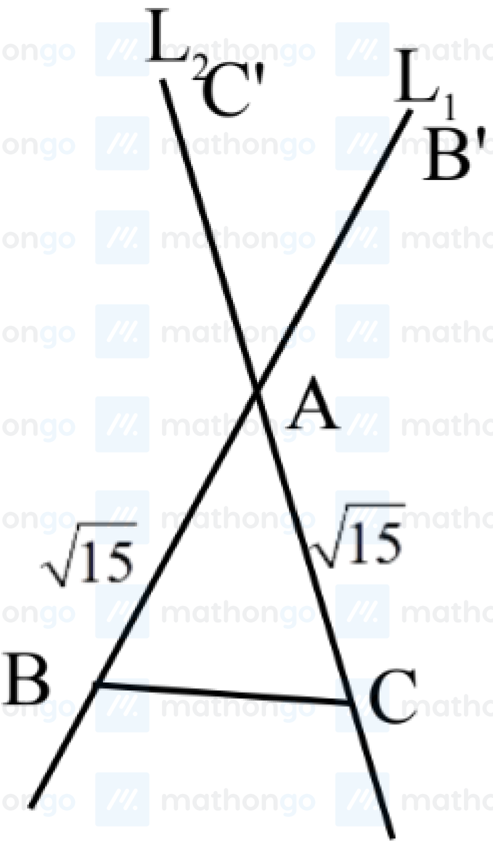
$\sin\theta = \frac{2}{10} = \frac{1}{5}$

$\sin\theta = \frac{\sqrt{24}}{5}$

area $= \frac{1}{2}ab\sin\theta$

$\frac{1}{2}\sqrt{15}\sqrt{15}\frac{\sqrt{24}}{5}$

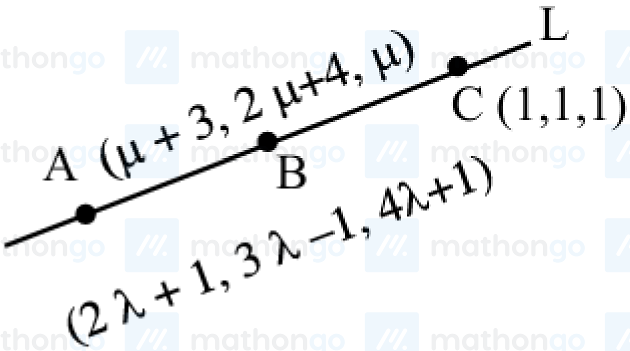
square of area $\frac{15.15.24}{4.25}$



option (1)

Q13.

(4)



Dr's of AC $\Rightarrow 2\lambda, 3\lambda-2, 4\lambda$

Dr's of BC $\Rightarrow \mu+2, 2\mu+3, \mu-1$

$\Rightarrow \frac{\mu+2}{2\lambda} = \frac{2\mu+3}{3\lambda-2} = \frac{\mu-1}{4\lambda}$

$\Rightarrow 2(\mu+2) = \mu-1 \Rightarrow \mu = -5$

$\Rightarrow$ Dr's of BC $\Rightarrow 3, 7, 6$

$\Rightarrow$ equation of L $\Rightarrow \frac{x-1}{3} = \frac{y-1}{7} = \frac{z-1}{6}$

$(7, 15, 13)$ satisfies.

Q14.

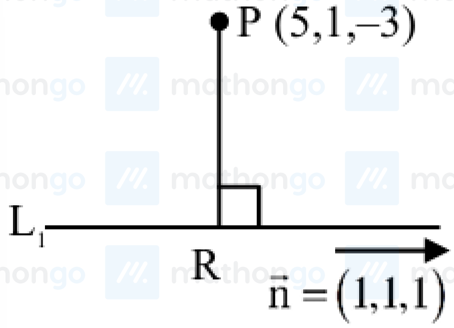
(2)

$L_1 : \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-0}{2}$

Let $Q(\lambda+1, \lambda+2, \lambda)$

$\overrightarrow{PQ} = (\lambda-4, \lambda-1, \lambda+3)$

$\overrightarrow{PQ} \cdot \overrightarrow{m} = 0$



$\Rightarrow \lambda - 4 + \lambda + 1, \lambda + 3 = 0$ $\Rightarrow \boxed{Q(1,2,0)}$

$\Rightarrow 3\lambda = 0$

$\lambda = 0$

$L_2 : \dfrac{x-2}{1} = \dfrac{y-0}{1} = \dfrac{z-1}{2}$

$\boxed{R(2,0,1)}$

Let $\mathrm{R}(\mu+2, \mu, \mu+1) \overrightarrow{\mathrm{PR}} = (\mu-3, \mu-1, \mu+4)$

$\overrightarrow{\mathrm{PR}} \cdot \overrightarrow{\mathrm{n}} = 0$

$\mu - 3 + \mu - 1 + \mu + 4 = 0$

$\neq \mu = 0$

Area of $\triangle \mathrm{PQR}(\mathrm{A}) = \dfrac{1}{2}|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}|$

$\mathrm{A} = \dfrac{1}{2}|(-4\hat{\mathrm{i}} + \hat{\mathrm{j}} + 3\hat{\mathrm{k}}) \times (-3\hat{\mathrm{i}} + \hat{\mathrm{j}} + 4\hat{\mathrm{k}})|$

$\mathrm{A} = \dfrac{1}{2}|7(\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}})|$

$$\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ -4 & 1 & 3 \\ -3 & -1 & 4 \end{vmatrix}$$

$= 7\hat{\mathrm{i}} + 7\hat{\mathrm{j}} + 7\hat{\mathrm{k}}$

$4\,\mathrm{A}^2 = 49 \times 3 = 147$

Q15. $\overrightarrow{\mathrm{p}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 4\hat{\mathrm{k}}, \overrightarrow{\mathrm{q}} = 3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$

(1)

$\Rightarrow \overrightarrow{\mathrm{p}} \times \overrightarrow{\mathrm{q}} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - \hat{\mathrm{k}}$

$\mathrm{A} \equiv (1,2,3) B \equiv (\lambda, 4, 5)$

Shortest Distance $= \left|\dfrac{\overrightarrow{\mathrm{AB}} \cdot (\overrightarrow{\mathrm{P}} \times \overrightarrow{\mathrm{q}})}{|\overrightarrow{\mathrm{P}} \times \overrightarrow{\mathrm{q}}|}\right|$

$\dfrac{1}{\sqrt{6}} = \left|\dfrac{((\lambda-1)\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}}\right|$

$\Rightarrow |-\lambda + 1 + 4 - 2| = 1 \Rightarrow |\lambda - 3| = 1$

$\Rightarrow \lambda = 3 \pm 1 = 4, 2$

Radius of circle passing through points

$(0,0), (4,2) \& (2,4)$

$= \dfrac{\mathrm{abc}}{4\Delta} = \dfrac{\sqrt{20} \times \sqrt{20} \times \sqrt{8}}{4 \times \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{vmatrix}} = \dfrac{20 \times 2\sqrt{2}}{2 \times 12}$

$= \dfrac{5\sqrt{2}}{3}$

Q16. $\mathrm{L}_1 : \mathrm{x} + 2 = \mathrm{y} - 1 = \mathrm{z} = \ell$

(56) $\mathrm{L}_2 : \dfrac{\mathrm{x}-3}{5} = \dfrac{\mathrm{y}}{-1} = \dfrac{\mathrm{z}-1}{1} = \mathrm{m}$

$\mathrm{L}_3 : \dfrac{\mathrm{x}}{-3} = \dfrac{\mathrm{y}-3}{5} = \dfrac{\mathrm{z}-2}{1} = \mathrm{n}$

Point of intersection of L_1 and L_2

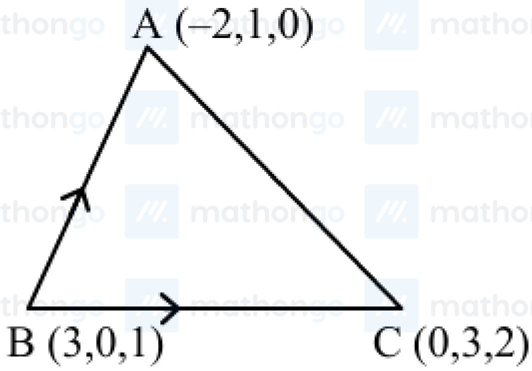
$$\left.\begin{aligned} \ell-2&=5\mathrm{~m}+3 \\ \ell+1&=-\mathrm{m} \\ \ell&=\mathrm{m}+1 \end{aligned}\right\} \ell=0,\ \mathrm{m}=-1 \quad \mathrm{A}(-2,1,0)$$

Point of intersection of L_2 and L_3

$$\left.\begin{aligned} 5\mathrm{~m}+3&=-3\mathrm{n} \\ -\mathrm{m}&=3\mathrm{n}+3 \\ \mathrm{m}+1&=\mathrm{n}+2 \end{aligned}\right\} \mathrm{m}=0, \mathrm{n}=-1, B(3,0,1)$$

Point of intersection L_3 and L_4

$$\left.\begin{aligned} -3\mathrm{n}&=\ell-2 \\ 3\mathrm{n}+3&=\ell+1 \\ \mathrm{n}+2&=\ell \end{aligned}\right\} \ell=2, \mathrm{n}=0, \mathrm{C}(0,3,2)$$



$$\operatorname{Ar}(\Delta \mathrm{ABC})=\left|\frac{1}{2}\right| \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ -5 & 1 & -1 \\ -3 & 3 & 1 \end{vmatrix}$$

$$\mathrm{A}=\frac{1}{2}|\hat{\mathrm{i}}(4)-\hat{\mathrm{j}}(-8)+\hat{\mathrm{k}}(-12)|$$

$$\mathrm{A}=\frac{1}{2} \sqrt{16+64+144}=\sqrt{56}$$

$$\mathrm{A}^2=56$$

Q17. Line is $\perp^{\mathrm{r}}$ to 2 line $\Rightarrow$ line will be parallel to

(2) $(i+a\hat{j}+b\hat{k}) \times(-b\hat{i}+a\hat{j}+5\hat{k})$

Parallel vector along the required line is

$\hat{\mathrm{i}}(5\mathrm{a}-\mathrm{ab})-\hat{\mathrm{j}}\left(\mathrm{b}^2+5\right)+\hat{\mathrm{k}}(\mathrm{a}+\mathrm{ab})$

Dr's of required line $\alpha(5a-ab),-\left(b^2+5\right),(a+ab)$

Also Dr's of required line $\alpha-2,\ \mathrm{d},-4$

$$\therefore \boxed{\frac{5\mathrm{a}-\mathrm{ab}}{-2}=\frac{-\left(\mathrm{b}^2+5\right)}{\mathrm{d}}=\frac{\mathrm{a}+\mathrm{ab}}{-4}} \quad .....(1)$$

Also point $\left(0, \frac{-1}{2}, 0\right)$ will lie on $\frac{\mathrm{x}-1}{-2}=\frac{\mathrm{y}+4}{\mathrm{d}}=\frac{\mathrm{z}-\mathrm{c}}{-4}$

$\frac{0-1}{-2}=\frac{\frac{-1}{2}+4}{d}=\frac{0-c}{-4} \Rightarrow d=7, c=2$

From (1) $\frac{5a-ab}{-2}=\frac{-b^2-5}{7}=\frac{a+ab}{-4}$

$\frac{5a-ab}{-2}=\frac{a+ab}{-4};\frac{-b^2-5}{7}=\frac{a+ab}{-4}$

$$\begin{array}{r|l} -20a+4ab=-2a-2ab & 4b^2+20=70+7ab \\ 18a=6ab & 36+20=70+21a \\ b=3 & 56=28a \Rightarrow a=2 \end{array}$$

$a+b+c+d=2+3+2+7=14$