Open quantum systems and quantum thermodynamics

End-Sem 2022

Time - 3 hours

Full marks 100 (20×5)

1. Consider the following two qubit state $\rho_w = \frac{p}{4}\mathbb{I} + (1-p)\,|\psi\rangle\langle\psi|$, where $\mathbb{I}$ is the 4 dimensional identity matrix, p is probability and $|\psi\rangle$ is one of the Bell states (you can take any one of them). By using partial transposition, find the maximum value of p, for which this state is entangled.

2. Consider the following operation

$$\frac{d}{dt}\rho(t) = (\sigma_x.\rho(t).\sigma_x - \rho(t)) + (\sigma_y.\rho(t).\sigma_y - \rho(t)) - \tanh t\,(\sigma_z.\rho(t).\sigma_z - \rho(t)).$$

Find whether this operation is a valid quantum operation or not. Further show whether this operation (if valid) is Markovian or Non-Markovian.

3. Consider the following qubit operation

$$\rho_{11}(t) = \rho_{11}(0)\exp(-\gamma t), \quad \rho_{12}(t) = \rho_{12}(0)\exp(-2\gamma t).$$

The notations has their usual meaning (predict the other terms). Find its Kraus operators. Further show whether it is invertible or not.

4. Solve the following Lindblad equation and find the corresponding Kraus operators

$$\frac{d}{dt}\rho = \gamma(n+1)\left(\sigma_-.\rho.\sigma_+ - \frac{1}{2}\{\sigma_+.\sigma_-, \rho\}\right) + \gamma n\left(\sigma_+.\rho.\sigma_- - \frac{1}{2}\{\sigma_-.\sigma_+, \rho\}\right).$$

Also find the fixed point for this operation.

5. Construct the mathematical representations of 1st and 2nd law of thermodynamics for a reduced quantum system undergoing Lindblad type of evolution. Note: the operations are Markovian with constant Lindblad coefficients and Hamiltonians are time independent, i.e. no external driving is present.

$\langle 0| \quad \langle 1|$

$|0\rangle \; \rho_{11} \quad \rho_{12}$

$|1\rangle \; \rho_{21}$

$\sigma_+\sigma_- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\sigma_-\sigma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ \rho_{21} & 0 \end{pmatrix}$

$(0, 1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = (0$

$\sigma_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}(1 \;\; 0) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 2\rho_{11} & \rho_{12} \\ \rho_{21} & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ \rho_{11} & \rho_{12} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \end{pmatrix}$

for bell state $|\phi^+\rangle$

$$\rho_w = \frac{p}{4} I + (1-p)|\psi\rangle\langle\psi|$$

$|\psi\rangle =$

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$$\frac{p}{4} + \frac{1-p}{2}$$

$$\frac{p+2-2p}{4}$$

$$\frac{2-p}{4}$$

$$\rho_w = p\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + (1-p)\frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\begin{pmatrix}1&0&0&1\end{pmatrix}$$

$$= \frac{p}{4}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + \left(\frac{1-p}{2}\right)\begin{pmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix}$$

$$\begin{pmatrix}\frac{3-p}{4}&0&0&\frac{1-p}{2}\\0&p&0&0\\0&0&1&0\\\frac{1-p}{2}&0&0&\frac{3-p}{4}\end{pmatrix} \xrightarrow{PT} \begin{pmatrix}\boxed{\frac{2-p}{4}}&0&0&\boxed{0}\\0&\frac{p}{4}&\frac{1-p}{2}&0\\0&\frac{1-p}{2}&\frac{p}{4}&0\\\boxed{0}&0&0&\boxed{\frac{2-p}{4}}\end{pmatrix}$$

$$\lambda_1, \lambda_2 = \frac{3-p}{4}$$

$$(1-\lambda)^2 - \left(\frac{1-p}{2}\right)^2 = 0$$

$$1-\lambda = \pm\left(\frac{1-p}{2}\right)$$

$$\lambda = 1 \mp \left(\frac{1-p}{2}\right) \qquad \lambda_1, \lambda_2 = \frac{2-p}{4}$$

$$\lambda_3 = \frac{1+p}{2}, \quad \lambda_4 = \frac{3-p}{2}$$

$$\frac{2-p}{4} < 0$$

$$2 - p < 0$$

$$\boxed{2 < p}$$

$$\left(\frac{p}{4} - \lambda\right) = \pm\left(\frac{1-p}{2}\right)$$

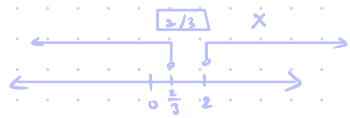
$$\frac{2}{3}$$

$$\lambda = \frac{p}{4} \pm \left(\frac{1-p}{2}\right)$$

$$\underline{p > 2}$$

$$\lambda_3, \lambda_4 = \frac{3p-2}{4}, \ \frac{1}{2} - \frac{p}{4} < 0$$

$$2 < p$$



$$\frac{3p-2}{4} < 0$$

$$p < \frac{2}{3}$$

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for bell state $|\psi^-\rangle$

$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\0\\0\end{pmatrix} - \begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\\0\end{pmatrix}$$

$$\frac{p}{4} - \frac{1}{2} + \frac{p}{2}$$

$$\frac{3p}{4} - \frac{1}{2}$$

$$\frac{p}{4} + \frac{1}{2}(1-p)$$

$$\frac{1}{2} - \frac{p}{4}$$

$$\frac{2-p}{4}$$

$$\rho_w = \frac{p}{4}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + \frac{1}{2}(1-p)\begin{pmatrix}0\\-1\\1\\0\end{pmatrix}\begin{pmatrix}0&-1&1&0\end{pmatrix}$$

$$= \frac{p}{4}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + \frac{1}{2}(1-p)\begin{pmatrix}0&0&0&0\\0&1&-1&0\\0&-1&1&0\\0&0&0&0\end{pmatrix}$$

$$\rho_\omega = \begin{pmatrix} p/4 & 0 & 0 & 0 \\ 0 & (2-p)/4 & -\frac{(1-p)}{2} & 0 \\ 0 & -\frac{(1-p)}{2} & \left(\frac{2-p}{4}\right) & 0 \\ 0 & 0 & 0 & p/4 \end{pmatrix}$$

$p = \frac{2}{3}$

$$\rho_\omega^{T_B} = \begin{pmatrix} p/4 & 0 & 0 & -(1-p)/2 \\ 0 & (2-p)/4 & 0 & 0 \\ 0 & 0 & \left(\frac{2-p}{4}\right) & 0 \\ -\frac{(1-p)}{2} & 0 & 0 & p/4 \end{pmatrix}$$

$$\lambda_1, \lambda_2 = \left(\frac{2-p}{4}\right)$$

$2-p < 0$

$2 < p$

$p > 2$

$$\left(\frac{p}{4} - \lambda\right) = \pm\left(\frac{1-p}{2}\right)$$

$$\lambda = \frac{p}{4} \mp \left(\frac{1-p}{2}\right)$$

$$\boxed{p = \frac{2}{3} - \varepsilon}$$

$\varepsilon \to 0$

$$\lambda_3, \lambda_4 = \frac{3p-2}{4}, \quad \frac{1}{2} - \frac{p}{4}$$

$$\boxed{p < \frac{2}{3}} \qquad \boxed{p \to \frac{2}{3}}$$

$$1 \geq 1-p > \frac{1}{3}$$

$$-1 \leq p-1 < -\frac{1}{3}$$

$$\boxed{0 \leq p < \frac{2}{3}}$$

$\rho_{22}(0) + \rho_{11}(0) = 1$

$\rho_{11}(0) = 1 - \rho_{22}(0)$

3. Consider the following qubit operation

$$\rho_{11}(t) = \rho_{11}(0)\exp(-\gamma t), \quad \rho_{12}(t) = \rho_{12}(0)\exp(-2\gamma t).$$

The notations has their usual meaning (predict the other terms). Find its Kraus operators. Further show whether it is invertible or not.

$$\rho_{22}(t) = \rho_{22}(0)\left[1 - e^{-\gamma t}\right]$$

$$\rho_{11}(0) = 1 - \rho_{22}(0)$$

density matrix property $\rightarrow$ $Tr(\rho) = 1$, PSD, Hermitian

$$\rho = \begin{pmatrix} \rho_{11}(0)e^{-\gamma t} & \rho_{12}(0)e^{-2\gamma t} \\ \rho_{12}(0)^* e^{-2\gamma t} & 1-(1-\rho_{22}(0))e^{-\gamma t} \end{pmatrix}$$

$$\rho_{21}(t) = (\rho_{12}(t))^* = \rho_{12}(0)^* e^{-2\gamma t}$$

$$\rho_{22}(t) = 1 - \rho_{11}(t) = 1 - \rho_{11}(0)e^{-\gamma t} = 1-(1-\rho_{22}(0))e^{-\gamma t}$$

$$|\phi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\Phi_{AB} = |\phi_{AB}\rangle\langle\phi_{AB}|$$

$$= \frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix}$$

$$(\mathbb{I}\otimes\Lambda)\Phi_{AB} = \begin{pmatrix} \Lambda\begin{pmatrix}1/2 & 0\\0&0\end{pmatrix} & \Lambda\begin{pmatrix}0&1/2\\0&0\end{pmatrix} \\ \Lambda\begin{pmatrix}0&0\\1/2&0\end{pmatrix} & \Lambda\begin{pmatrix}0&0\\0&1/2\end{pmatrix}\end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}e^{-\gamma t} & 0 & 0 & \frac{1}{2}e^{-2\gamma t} \\ 0 & 1-e^{-\gamma t} & \frac{1}{2}e^{-2\gamma t} & 1-e^{-\gamma t} \\ 0 & 0 & 0 & 0 \\ 0 & 1-e^{-\gamma t} & 0 & 1-\frac{1}{2}e^{-\gamma t}\end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2}e^{-\gamma t}-\lambda & 0 & 0 & \frac{1}{2}e^{-2\gamma t} \\ 0 & 1-e^{-\gamma t}-\lambda & \frac{1}{2}e^{-2\gamma t} & 1-e^{-\gamma t} \\ 0 & 0 & -\lambda & 0 \\ 0 & 1-e^{-\gamma t} & 0 & \frac{1-e^{-\gamma t}}{2}-\lambda \end{pmatrix}$$

$\lambda_1, \lambda_2 = 0$

$$\left(\frac{1}{2}e^{-\gamma t}-\lambda\right)\left[-\lambda\left[(1-e^{-\gamma t}-\lambda)\left(1-\frac{e^{-\gamma t}}{2}-\lambda\right) - (1-e^{-\gamma t})^2\right]\right] = 0$$

$$\cancel{-\frac{1}{2}e^{-2\gamma t}\left[0\right]} \to 0$$

$$\left[\frac{9}{4}e^{-2\gamma t} - 6e^{-\gamma t} + 4 + 4e^{-2\gamma t} + 8e^{-\gamma t}\right]$$

$$\frac{25}{4}e^{-2\gamma t} + 2e^{-\gamma t} + 4$$

$$25e^{-2\gamma t} + 8e^{-\gamma t} + 16$$

$$(\lambda)\left(\frac{1}{2}e^{-\gamma t}-\lambda\right)\left(\lambda^2 + \lambda\left(\frac{3e^{-\gamma t}}{2}-2\right) \cancel{+1} - e^{-2\gamma t} - 2e^{-\gamma t} \cancel{-1}\right)$$

$$(\lambda)\left(\frac{1}{2}e^{-\gamma t}-\lambda\right)$$

$$\begin{pmatrix} \frac{e^{-\gamma t}}{2}-\lambda & e^{-2\gamma t}/2 \\ e^{-2\gamma t}/2 & 1-e^{-\gamma t}/2-\lambda \end{pmatrix} \to \frac{e^{-\gamma t}}{2} - \lambda - \left(\frac{e^{-2\gamma t}}{4} - \lambda^2\right) - \frac{e^{-4\gamma t}}{4} = 0$$

$$1-2a+a^2+a^4$$

$$(a-1)^2+a^4$$

$$2e^{-\gamma t} - 4\lambda - e^{-2\gamma t} + 4\lambda^2 - e^{-4\gamma t} = 0$$

$$4\lambda^2 - 4\lambda + 2e^{-\gamma t} - e^{-2\gamma t} - e^{-4\gamma t} = 0$$

$$\begin{bmatrix} e^{-\gamma t}/2 & 0 & 0 & e^{-2\gamma t}/2 \\ 0 & 1-e^{-\gamma t} & 0 & 1-e^{-\gamma t} \\ 0 & 0 & 0 & 0 \\ e^{-2\gamma t}/2 & 1-e^{-\gamma t} & 0 & 1-e^{-\gamma t}/2 \end{bmatrix}$$

$$\begin{bmatrix} e^{-\gamma t}/2-\lambda & 0 & 0 & e^{-2\gamma t}/2 \\ 0 & 1-e^{-\gamma t}-\lambda & 0 & 1-e^{-\gamma t} \\ 0 & 0 & -\lambda & 0 \\ e^{-2\gamma t}/2 & 1-e^{-\gamma t} & 0 & 1-e^{-\gamma t}/2-\lambda \end{bmatrix}$$

$$(-\lambda)\left[\begin{vmatrix} e^{-\gamma t}/2-\lambda & 0 & e^{-2\gamma t}/2 \\ 0 & 1-e^{-\gamma t}-\lambda & 1-e^{-\gamma t} \\ e^{-2\gamma t}/2 & 1-e^{-\gamma t} & 1-\frac{e^{-\gamma t}}{2}-\lambda \end{vmatrix}\right]$$

$$\left(\frac{e^{-\gamma t}}{2}-\lambda\right)\Big[(1-e^{-\gamma t}-\lambda)(1-\frac{e^{-\gamma t}}{2}-\lambda)$$

$$-(1-e^{-\gamma t})^2\Big]$$

$$(\lambda)\left[(1-e^{-\gamma t}-\lambda)\left[\left(1-\frac{e^{-\gamma t}}{2}-\lambda\right)\left(\frac{e^{-\gamma t}}{2}-\lambda\right)-(1-e^{-\gamma t})^2\right]\right.$$
$$\left.+\frac{e^{-2\gamma t}}{2}\left(0-\frac{e^{-2\gamma t}}{2}(1-e^{-\gamma t}-\lambda)\right)\right]$$

$$(\lambda)\left[\left(\frac{e^{-\gamma t}}{2}-\lambda\right)\left[(1-e^{-\gamma t}-\lambda)\left(1-\frac{e^{-\gamma t}}{2}-\lambda\right)\right.\right.$$
$$\left.\left.-(1-e^{-\gamma t})^2\right]-\frac{e^{-4\gamma t}}{4}(1-e^{-\gamma t}-\lambda)\right]$$

$$e^{-\gamma t}-1 \quad \frac{e^{-\gamma t}}{2}-1$$

$$\lambda^2+\left(\frac{3}{2}e^{-\gamma t}-2\right)\lambda+(1-e^{-\gamma t})\left(1-\frac{e^{-\gamma t}}{2}\right)-(1-2e^{-\gamma t}+e^{-2\gamma t})$$

$$\lambda^2+\left(\frac{3}{2}e^{-\gamma t}-2\right)\lambda+1-\frac{e^{-\gamma t}}{2}-e^{-\gamma t}+\frac{e^{-2\gamma t}}{2}-(1-2e^{-\gamma t}+e^{-2\gamma t})$$

$$\lambda^2+\left(\frac{3}{2}e^{-\gamma t}-2\right)\lambda+\left(\frac{3}{2}e^{-\gamma t}-\frac{1}{2}e^{-2\gamma t}\right)$$

$$\left(\frac{3}{2}e^{-\gamma t}-2\right)^2-4\left(\frac{3}{2}e^{-\gamma t}-\frac{1}{2}e^{-2\gamma t}\right)$$

$$\frac{9}{4}e^{-2\gamma t}-6e^{-\gamma t}+4-6e^{-\gamma t}+2e^{-2\gamma t}$$

$$\frac{17}{4}e^{-2\gamma t}-12e^{-\gamma t}+4$$

$$\sigma_+|0\rangle = |1\rangle \qquad \sigma_-|0\rangle = 0$$

$$\sigma_+|1\rangle = 0 \qquad \sigma_-|1\rangle = |0\rangle$$

$$\sigma_+ = |1\rangle\langle 0| \qquad \sigma_- = |0\rangle\langle 1|$$

$$\sigma_+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} \qquad \sigma_- = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \sigma_- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_+\sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_-\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\frac{d\rho}{dt} = \begin{pmatrix} \dot{a} & \dot{b} \\ \dot{c} & \dot{d} \end{pmatrix} =$$

$$\sigma_-\rho\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_+\rho\sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$$

$$\{\sigma_+\sigma_-, \rho\} = \sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix} + \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix}$$

$$= \begin{pmatrix} 0 & b \\ c & 2d \end{pmatrix}$$

$$\{\sigma_-\sigma_+, \rho\} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2a & b \\ c & 0 \end{pmatrix}$$

$$\gamma(n+1)\left[\begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 & b \\ c & 2d \end{pmatrix}\right] + \gamma n\left(\begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix} - \begin{pmatrix} a & b/2 \\ c/2 & 0 \end{pmatrix}\right)$$

$$\gamma(n+1)\begin{bmatrix} d & -b/2 \\ -c/2 & -d \end{bmatrix} + \gamma n\begin{bmatrix} -a & -b/2 \\ -c/2 & a \end{bmatrix}$$

$$[\gamma n]\left[\begin{pmatrix} d & -b/2 \\ -c/2 & -d \end{pmatrix} + \begin{pmatrix} -a & -b/2 \\ -c/2 & a \end{pmatrix}\right]$$

$$\gamma n\begin{pmatrix} d-a & -b \\ -c & a-d \end{pmatrix} + \gamma\begin{pmatrix} d & -b/2 \\ -c/2 & -d \end{pmatrix}$$

$$\gamma\begin{bmatrix} (d-a)n+d & -bn-b/2 \\ -cn-c/2 & (a-d)n-d \end{bmatrix} = \begin{bmatrix} \dot{a} & \dot{b} \\ \dot{c} & \dot{d} \end{bmatrix}$$

$$\frac{db}{dt} = -(n+1/2)\gamma\ b$$

$$b = b_0 \exp(-(n+\tfrac{1}{2})\gamma t)$$

$$\int \frac{1}{an+b} = \frac{1}{a}\log(an+b)$$

$$c = c_0 \exp(-(n+\tfrac{1}{2})\gamma t)$$

$$\left.\begin{aligned} \frac{da}{dt} &= ((d-a)n+d)\gamma \\ \frac{dd}{dt} &= ((a-d)n-d)\gamma \end{aligned}\right\} \quad \boxed{a+d=1}$$

$$d = 1-a$$

$$d = 1-a$$

$$\frac{da}{dt} = ((1-2a)n + 1-a)\gamma$$
$$= ((n+1) - (2n+1)a)\gamma$$

$$\int_{a_0}^{a} \frac{+da}{-(n+1)+(2n+1)a} = \int_0^t -\gamma\, dt$$

$$\frac{1}{2n+1}\log((2n+1)a-(n+1))\Big|_{a_0}^{a} = -\gamma t$$

$$\frac{1}{2n+1}\log\left(\frac{(2n+1)a-(n+1)}{(2n+1)a_0-(n+1)}\right) = -\gamma t$$

$$\frac{(2n+1)a-(n+1)}{(2n+1)a_0-(n+1)} = \exp(-\gamma(2n+1)t)$$

$$(2n+1)a-(n+1) = ((2n+1)a_0-(n+1))\exp(-\gamma(2n+1)t)$$

$$a = \left[\frac{1}{2n+1}\right]\left[((2n+1)a_0-(n+1))\exp(-\gamma(2n+1)t)+(n+1)\right]$$

$$d_\infty = 1 - a_\infty$$

$$= 1 - \frac{(n+1)}{2n+1} = \frac{2n+1-n-1}{2n+1} = \frac{n}{2n+1}$$

$$d_0 = 1 - a_0$$

$$d = \left[\frac{1}{2n+1}\right]\left[n - \left((2n+1)a_0 - (n+1)\right)\exp(-\gamma(2n+1)t)\right]$$

$$= \left(\frac{1}{2n+1}\right)\left[n - \left[n - (2n+1)d_0\right]\exp(-\gamma(2n+1)t)\right]$$

steady state calculation

$$\gamma\begin{bmatrix} (d-a)n+d & -bn-b/2 \\ -cn-c/2 & (a-d)n-d \end{bmatrix} = \begin{bmatrix} \dot{a} & \dot{b} \\ \dot{c} & \dot{d} \end{bmatrix} = 0$$

$$d = (a-d)n$$

$$1-a = (a-1+a)n$$

$$1-a = (2a-1)n$$

$$1-a = 2na - n$$

$$1+n = (2n+1)a$$

$$a = \frac{n+1}{2n+1}, \quad d = \frac{n}{2n+1}$$

$$b = 0, \quad c = 0$$

$$\rho_{steady} = \begin{pmatrix} (n+1)/(2n+1) & 0 \\ 0 & n/(2n+1) \end{pmatrix}$$

$$b = b_0 \exp(-(n+\tfrac{1}{2})\gamma t)$$

$$c = c_0 \exp(-(n+\tfrac{1}{2})\gamma t)$$

$$a = \left[\frac{1}{2n+1}\right]\left[\left((2n+1)a_0 - (n+1)\right)\exp(-\gamma(2n+1)t) + (n+1)\right]$$

$$d = \left(\frac{1}{2n+1}\right)\left[n - \left[n - (2n+1)d_0\right]\exp(-\gamma(2n+1)t)\right]$$

$$\text{Chi state} = \begin{pmatrix} \Lambda\begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix} & \Lambda\begin{pmatrix} 0 & 1/2 \\ 0 & 0 \end{pmatrix} \\ \Lambda\begin{pmatrix} 0 & 0 \\ 1/2 & 0 \end{pmatrix} & \Lambda\begin{pmatrix} 0 & 0 \\ 0 & 1/2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{(2n+1)}\left(-\frac{1}{2}\exp(-\gamma(2n+1)t) + n+1\right) & 0 & \left(\frac{n+1}{2n+1}\right)\left(1-\exp(-\gamma(2n+1)t)\right) & \frac{1}{2}\exp(-(n+\frac{1}{2})\gamma t) \\ 0 & \frac{n}{2n+1}\left(1-\exp(-\gamma(2n+1)t)\right) & 0 & \frac{n}{2n+1}\left(1-\exp(-\gamma(2n+1)t)\right) \\ \left(\frac{n+1}{2n+1}\right)\left(1-\exp(-\gamma(2n+1)t)\right) & 0 & \left(\frac{n+1}{2n+1}\right)\left(1-\exp(-\gamma(2n+1)t)\right) & 0 \\ \frac{1}{2}\exp(-(n+\frac{1}{2})\gamma t) & \frac{n}{2n+1}\left(1-\exp(-\gamma(2n+1)t)\right) & 0 & \frac{1}{2n+1}\left(n-\frac{1}{2}e^{-\gamma(2n+1)t}\right) \end{pmatrix}$$

Q.3 Given $\rho_{11}(t) = \rho_{11}(0)\exp(-\gamma t)$, $\rho_{12}(t) = \rho_{12}(0)\exp(-2\gamma t)$

$$\rho_{22}(t) = \rho_{11}(0) + \rho_{22}(0) - \rho_{11}(t)$$

$$= \rho_{11}(0)\left[1 - e^{-\gamma t}\right] + \rho_{22}(0)$$

$$\rho_{21}(t) = \left(\rho_{12}(t)\right)^*$$

$$= \left(\rho_{12}(0)e^{-2\gamma t}\right)^*$$

$$= \rho_{21}(0)\, e^{-2\gamma t}$$

dynamic map: $$\rho(t) = \begin{bmatrix} \rho_{11}(0)e^{-\gamma t} & \rho_{12}(0)e^{-2\gamma t} \\ \rho_{21}(0)e^{-2\gamma t} & \rho_{11}(0)(1-e^{-\gamma t}) + \rho_{22}(0) \end{bmatrix}$$

$$\begin{bmatrix} \Lambda\begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix} & \Lambda\begin{pmatrix} 0 & 1/2 \\ 0 & 0 \end{pmatrix} \\ \Lambda\begin{pmatrix} 0 & 0 \\ 1/2 & 0 \end{pmatrix} & \Lambda\begin{pmatrix} 0 & 0 \\ 0 & 1/2 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{e^{-\gamma t}}{2} & 0 & 0 & e^{-2\gamma t}/2 \\ 0 & \frac{1-e^{-\gamma t}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{e^{-2\gamma t}}{2} & 0 & 0 & 1/2 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} e^{-\gamma t}/2 & e^{-2\gamma t}/2 \\ e^{-2\gamma t}/2 & 1/2 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} (1-e^{-\gamma t})/2 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \lambda_1, \lambda_2 = 0, \frac{(1-e^{-\gamma t})}{2}$$

$$\left(\frac{e^{-\gamma t}}{2} - \lambda\right)\left(\frac{1}{2} - \lambda\right) - \frac{e^{-4\gamma t}}{4} = 0$$

$$\lambda^2 - \frac{1}{2}(1 + e^{-\gamma t})\lambda + \frac{e^{-\gamma t}}{4} - \frac{e^{-4\gamma t}}{4} = 0$$

$$\frac{1}{4}(1 + e^{-\gamma t})^2 - e^{-\gamma t} + e^{-4\gamma t}$$

$$e^{-4\gamma t} - \frac{1}{2}e^{-\gamma t} + e^{-2\gamma t} + \frac{1}{4}$$

$$\lambda = \frac{\frac{1}{2} + e^{-\gamma t} \pm \left(e^{-2\gamma t} + \frac{1}{2}\right)}{2}$$

$$\lambda_3 = \frac{e^{-2\gamma t} + e^{-\gamma t} + 1}{2}$$

$$\lambda_4 = \frac{e^{-\gamma t} - e^{-2\gamma t}}{2}$$

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

$$\begin{matrix} a-\lambda & b \\ c & d-\lambda \end{matrix}$$

$$\lambda^2 - (a+d)\lambda - bc + ad$$

$$\frac{(a+d) \pm \sqrt{(a+d)^2 - 4(-bc+ad)}}{2}$$

$$\underline{bc \geq 0}$$

$$ad - bc \geq 0$$

$$\boxed{ad \geq bc}$$

$$a_{11} a_{22} \geq a_{12} a_{21}$$

Finding the dynamic map, Kraus operators and fixed point of amplitude damping channel from master equation

$$\frac{d\rho}{dt} = \gamma\left(\sigma_- \rho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\right)$$

$\sigma_- = |0\rangle\langle 1|$

$\sigma_+ = |1\rangle\langle 0|$

$$\sigma_+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \sigma_- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_+\sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix} + \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix}$$

$$\begin{pmatrix} \dot{a} & \dot{b} \\ \dot{c} & \dot{d} \end{pmatrix} = \begin{pmatrix} d & -b/2 \\ -c/2 & -d \end{pmatrix}\gamma$$

$\dot{a} + \dot{d} = a_0 + d_0$ $\qquad b = b_0 e^{-\gamma t/2}$

$\dot{d} = -d\gamma$ $\qquad c = c_0 e^{-\gamma t/2}$

$d = d_0 e^{-\gamma t}$

$a = a_0 + d_0(1 - e^{-\gamma t})$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a_0 + d_0(1-e^{-\gamma t}) & b_0 e^{-\gamma t/2} \\ c_0 e^{-\gamma t/2} & d_0 e^{-\gamma t} \end{pmatrix}$$

$p = 1 - e^{-\gamma t}$

$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\gamma t/2} \end{pmatrix}$

$M_2 = \begin{pmatrix} 0 & \sqrt{1-e^{\gamma t}} \\ 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} \Lambda\begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix} & \Lambda\begin{pmatrix} 0 & 1/2 \\ 0 & 0 \end{pmatrix} \\ \Lambda\begin{pmatrix} 0 & 0 \\ 1/2 & 0 \end{pmatrix} & \Lambda\begin{pmatrix} 0 & 0 \\ 0 & 1/2 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-e^{-\gamma t}) & 0 \\ \frac{1}{2}e^{-\gamma t/2} & 0 & 0 & \frac{1}{2}e^{-\gamma t} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} - \lambda & \frac{1}{2}e^{-\gamma t/2} \\ \frac{1}{2}e^{-\gamma t/2} & \frac{1}{2}e^{-\gamma t} - \lambda \end{pmatrix}$$

$$\lambda^2 - \frac{1}{2}(e^{-\gamma t}+1)\lambda + \cancel{\frac{1}{4}e^{-\gamma t}} - \cancel{\frac{1}{4}e^{-\gamma t}}$$

0

$$\boxed{\begin{aligned}\lambda_3 &= 0\\ \lambda_4 &= \frac{1}{2}(e^{-\gamma t}+1)\end{aligned}}$$

$$\lambda_1, \lambda_2 = 0, \ \frac{1}{2}(1-e^{-\gamma t})$$

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-e^{-\gamma t}) & 0 \\ \frac{1}{2}e^{-\gamma t/2} & 0 & 0 & \frac{1}{2}e^{-\gamma t} \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \frac{1}{2}(1-e^{-\gamma t}) \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\begin{pmatrix} \frac{a}{2} + \frac{d}{2}e^{-\gamma t/2} \\ 0 \\ \frac{1}{2}(1-e^{-\gamma t})c \\ \frac{a}{2}e^{-\gamma t/2} + \frac{d}{2}e^{-\gamma t} \end{pmatrix} = \frac{1}{2}(1-e^{-\gamma t}) \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$\boxed{b = 0}$, $c = \text{free}$

$$\frac{a}{2} + \frac{d}{2}e^{-\gamma t/2} = \frac{1}{2}(1-e^{-\gamma t})a$$

$$e^{-\gamma t}a + \frac{d}{2}e^{-\gamma t/2} = 0$$

$$a = -\frac{d}{2}e^{\gamma t/2}$$

$$\frac{1}{2}\left(1-e^{-\gamma t}\right) \leftarrow \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{a}{2} + \frac{d}{2} e^{-\gamma t/2} \\ 0 \\ \frac{1}{2}\left(1-e^{-\gamma t}\right) c \\ \frac{a}{2} e^{-\gamma t/2} + \frac{d}{2} e^{-\gamma t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad a = -d e^{-\gamma t/2}$$

$$c = 0 \qquad \frac{a}{2} + \frac{d}{2} e^{-\gamma t/2} = 0$$

$$-d e^{-\gamma t/2} = a$$

$$\begin{pmatrix} -d e^{-\gamma t/2} \\ 1 \\ 0 \\ d \end{pmatrix} \qquad \begin{pmatrix} a \\ 0 \\ 0 \\ e^{\gamma t/2} a \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ e^{\gamma t/2} \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-e^{-\gamma t}) & 0 \\ \frac{1}{2}e^{-\gamma t/2} & 0 & 0 & \frac{1}{2}e^{-\gamma t} \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & e^{-\gamma t/2} \end{pmatrix}$$

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 0$$

$$\frac{a}{2} + \frac{1}{2}e^{-\gamma t/2} d = 0$$

$$\lambda = 0 \longrightarrow \begin{pmatrix} -e^{-\gamma t/2} d \\ 1 \\ 0 \\ d \end{pmatrix}$$

$$\underline{c = 0}$$

$$a + e^{-\gamma t/2} d = 0$$

$$d = 0$$

$$a = 0$$

$$a = -e^{-\gamma t/2} d$$

b

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-e^{-\gamma t}) & 0 \\ \frac{1}{2}e^{-\gamma t/2} & 0 & 0 & \frac{1}{2}e^{-\gamma t} \end{pmatrix}$$

$$\lambda = \frac{1 \pm e^{-\gamma t}}{2}$$

$\lambda = (1+e^{-\gamma t})/2$

$$\begin{pmatrix} -e^{-\gamma t}/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & -\frac{(1+e^{-\gamma t})}{2} & 0 & 0 \\ 0 & 0 & -e^{-\gamma t} & 0 \\ 1/2\, e^{-\gamma t/2} & 0 & 0 & -1/2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -e^{-\gamma t/2} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ e^{-\gamma t/2} & 0 & 0 & -1 \end{pmatrix} = 0$$

$\lambda = (1+e^{-\gamma t})/2$

$\uparrow$

$$\begin{pmatrix} e^{\gamma t/2} \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -e^{-\gamma t/2} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 0$$

$-e^{-\gamma t/2} a + d = 0$

$d = e^{-\gamma t/2} a$

$a = e^{\gamma t/2} d$

$b = 0$

$c = 0$

$d = \text{free}$

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2\, e^{-\gamma t/2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-e^{-\gamma t}) & 0 \\ \frac{1}{2}e^{-\gamma t/2} & 0 & 0 & \frac{1}{2}e^{-\gamma t} \end{pmatrix}$$

$$\lambda = \frac{1-e^{-\gamma t}}{2}$$

$$\begin{pmatrix} \frac{1}{2}e^{-\gamma t} & 0 & 0 & \frac{1}{2}e^{-\gamma t/2} \\ 0 & -\frac{(1-e^{-\gamma t})}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2}(e^{-\gamma t/2}) & 0 & 0 & e^{-\gamma t}-\frac{1}{2} \end{pmatrix} = 0$$

$$\begin{pmatrix} e^{-\gamma t/2} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e^{-\gamma t/2} & 0 & 0 & 2e^{-\gamma t}-1 \end{pmatrix} = 0$$

$$\begin{pmatrix} e^{-\gamma t/2} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2e^{-\gamma t}-2 \end{pmatrix} = 0$$

$b = 0$
$c = \text{free}$
$d = 0$
$a = 0$

$$\lambda = \frac{(1-e^{-\gamma t})}{2} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ -e^{\gamma t/2} \end{pmatrix}$$

$$\lambda_1, \lambda_2 = 0 \qquad \lambda_3 = \left(\frac{1+e^{-\gamma t}}{2}\right) \qquad \lambda_4 = \left(\frac{1-e^{-\gamma t}}{2}\right)$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} e^{\gamma t/2} \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$K_1, K_2 = 0 \qquad K_3 = \sqrt{\frac{1+e^{-\gamma t}}{2}} \begin{pmatrix} e^{\gamma t/2} & 0 \\ 0 & 1 \end{pmatrix} = \sqrt{\frac{1+e^{\gamma t}}{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-\gamma t/2} \end{pmatrix}$$

$$p = 1 - e^{-\gamma t}$$

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\gamma t/2} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & \sqrt{1-e^{-\gamma t}} \\ 0 & 0 \end{pmatrix}$$

$$K_4 = \sqrt{\frac{1-e^{-\gamma t}}{2}} \begin{pmatrix} -e^{-\gamma t/2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$K_3^\dagger K_3 + K_4^\dagger K_4 = \mathbb{I} \qquad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1+e^{-\gamma t}}{2} \begin{pmatrix} e^{\gamma t} & 0 \\ 0 & 1 \end{pmatrix} + \left(\frac{1-e^{-\gamma t}}{2}\right) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1+e^{-\gamma t}}{2} \begin{pmatrix} e^{\gamma t} & 0 \\ 0 & 1 \end{pmatrix} + \left(\frac{1-e^{-\gamma t}}{2}\right) \begin{pmatrix} e^{-\gamma t} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{e^{\gamma t} + 1}{2} + \frac{e^{-\gamma t} - e^{-2\gamma t}}{2}$$

$$\left(\frac{1+e^{-rt}}{2}\right)\begin{pmatrix} e^{rt} & 0 \\ 0 & 1 \end{pmatrix} + \left(\frac{1-e^{-rt}}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & e^{rt} \end{pmatrix}$$

$$\frac{e^{rt}+1}{2} + \frac{1-e^{-rt}}{2}$$

$$A \longrightarrow (\lambda^3+\lambda^2+\lambda+1)=0$$

$$A^3+A^2+A+I=0$$

$$A^2+A+I+A^{-1}=0$$

$$A^{-1}=-(A^2+A+I)$$

Procedure

1. General master
2. $H(t)$
3. Average energy $\rightarrow$ $Tr(H(t)\rho)$

$Tr(A\rho)$

4. $\frac{d}{dt}(Tr(H(t)\rho)) \rightarrow$ Energy change with time

5. $Tr(\dot{H}\rho) + Tr(H\mathcal{L}(\rho))$

$-P + J$ $\rightarrow$ Heat current

$$\frac{dE}{dt} = -P + J$$

$$J = P + \frac{dE}{dt}$$ $\rightarrow$ first law

$$\frac{d\rho(t)}{dt} = \frac{-i}{\hbar}[\rho(t), H(t)] + \sum \gamma_i \left(L_i \rho L_i^{\dagger} - \frac{1}{2}\{L_i^{\dagger} L_i, \rho\} \right)$$

$$S(\rho) = -Tr(\rho \ln(\rho))$$

$$S(\rho | q) =$$