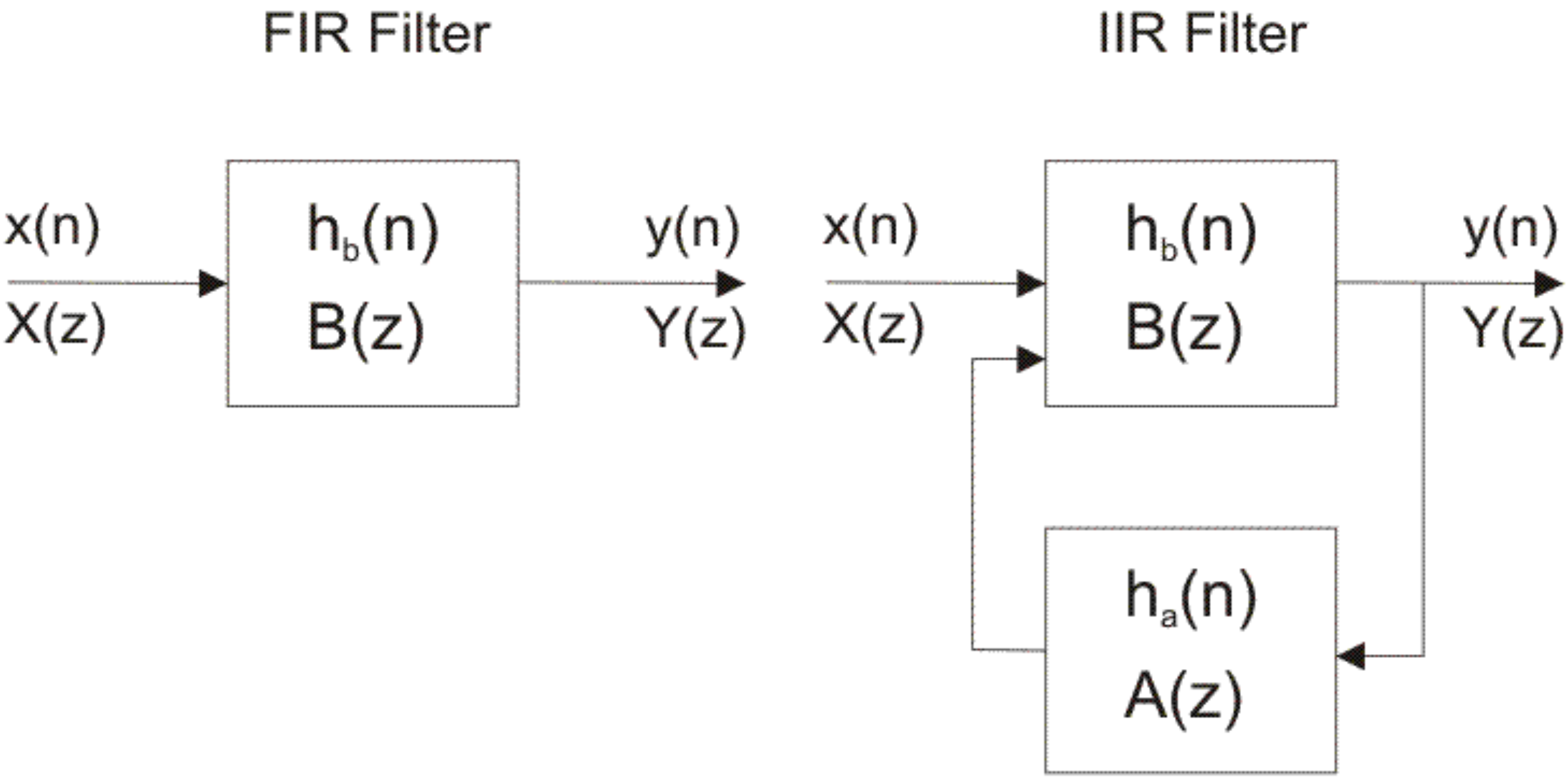
Transfer Function

- defined as the z-transform of the impulse response *h(t)*

$$\boxed{Y(z) = H(z)X(z)}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)}}$$


FIR Filter
IIR Filter
x(n)
X(z)
h_b(n)
B(z)
y(n)
Y(z)
x(n)
X(z)
h_b(n)
B(z)
y(n)
Y(z)
h_a(n)
A(z)

- Transfer function $H(z)$
  - Consider the system shown in Figure 4.

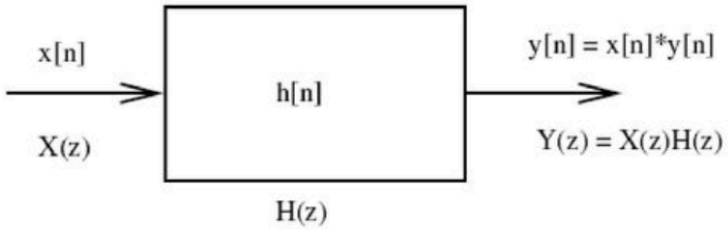


Figure 4: signal - system representation

  - $x[n]$ is the input and $y[n]$ is the output
  - h[n] is the impulse response of the system. Mathematically, this signal-system interaction can be represented as follows

$$y[n] = x[n] * h[n]$$

  - In frequency domain this relation can be written as

$$Y(z) = X(z).H(z)$$

or

$$H(z) = \frac{Y(z)}{X(z)}$$

$H(z)$ is called 'Transfer function' of the given system.
In the time domain if $x[n] = \delta[n]$ then $y[n] = h[n]$,
$h[n]$ is called the 'impulse response' of the system.
Hence, we can say that

$$h[n] \longleftrightarrow H(z)$$

Z-Transform

- converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation

- converts the difference equations in time domain into the algebraic equations in z-domain (*k* represents sample number)

$$X(z) = \sum_{k=0}^{N} x[k]z^{-k} = \sum_{k=0}^{N} x[k](z^{-1})^{k}$$

Z-Transform Vs DFT

Fourier Transform

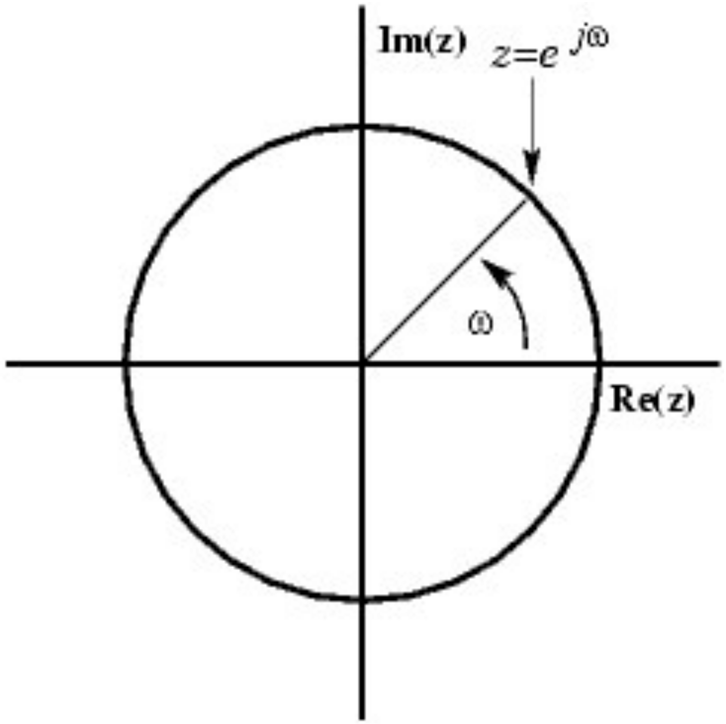
$$X\left(e^{jw}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Z-Transform

- important in identifying system stability
- determines frequency response of a system
- complex number $z = re^{jw}$
- w = *angular* frequency



Z-Transform

- $x(n)$ = [1 2 5 7 0 1]
- $X(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$

$$\boxed{X(z) = \sum_{k=0}^{N} x[k]z^{-k} = \sum_{k=0}^{N} x[k](z^{-1})^{k}}$$

Example

- First Order FIR Filter

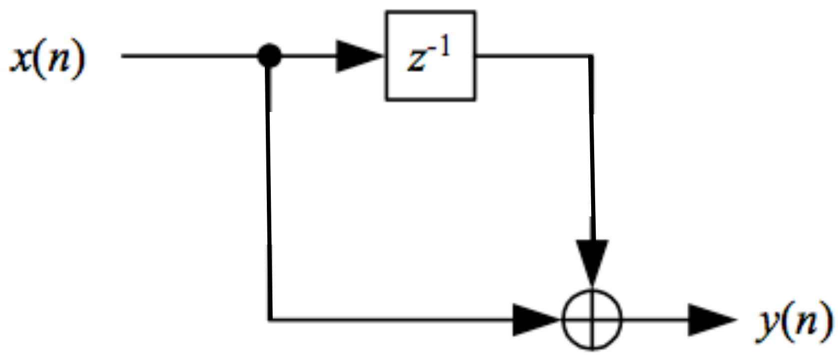
$$y(n) = x(n) + x(n-1)$$

$$Y(z) = X(z) + X(z)(z^{-1})$$

$$Y(z)/X(z) = (1+z^{-1})$$

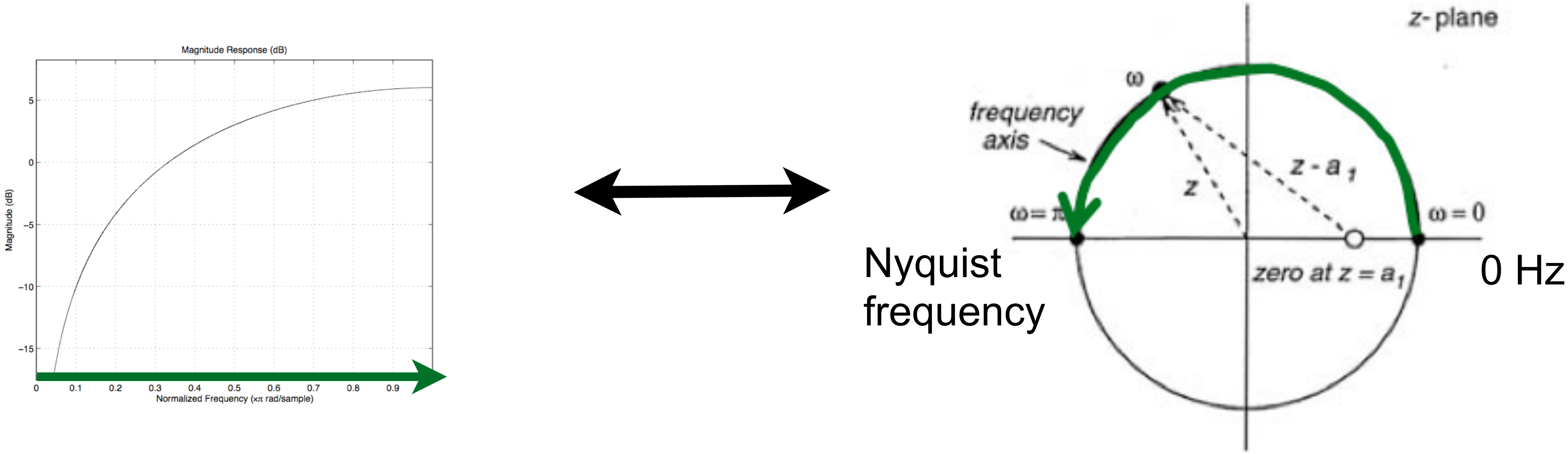
$$H(z) = (1+z^{-1})$$

$$= (z+1)/(z)$$



Poles & Zeros

- Pole-Zero plot = a pole–zero plot is a graphical representation transfer function
- Attenuate = Zeros
- Amplify = Poles



Example

restoola.m

- First Order FIR Filter

$$y(n) = x(n) + x(n-1)$$

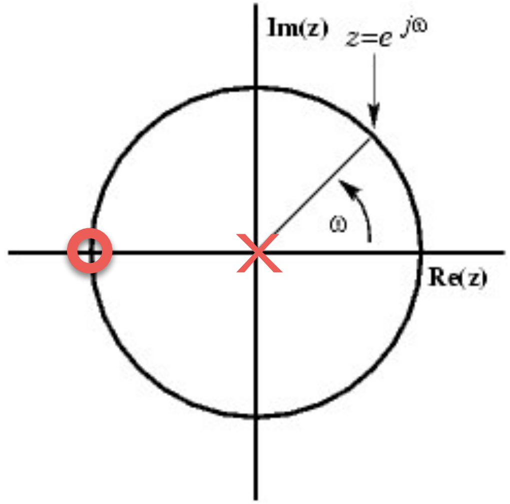
$$Y(z) = X(z) + X(z)(z^{-1})$$

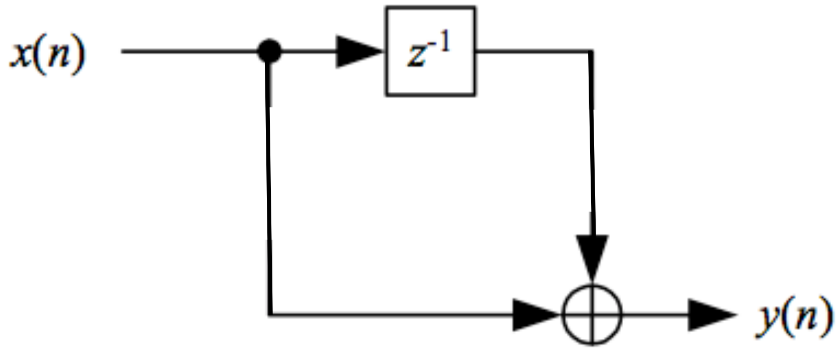
$$Y(z)/X(z) = (1+z^{-1})$$

$$H(z) = (1+z^{-1})$$

$$= (z+1)/(z)$$

fvtool([1, 1],1)

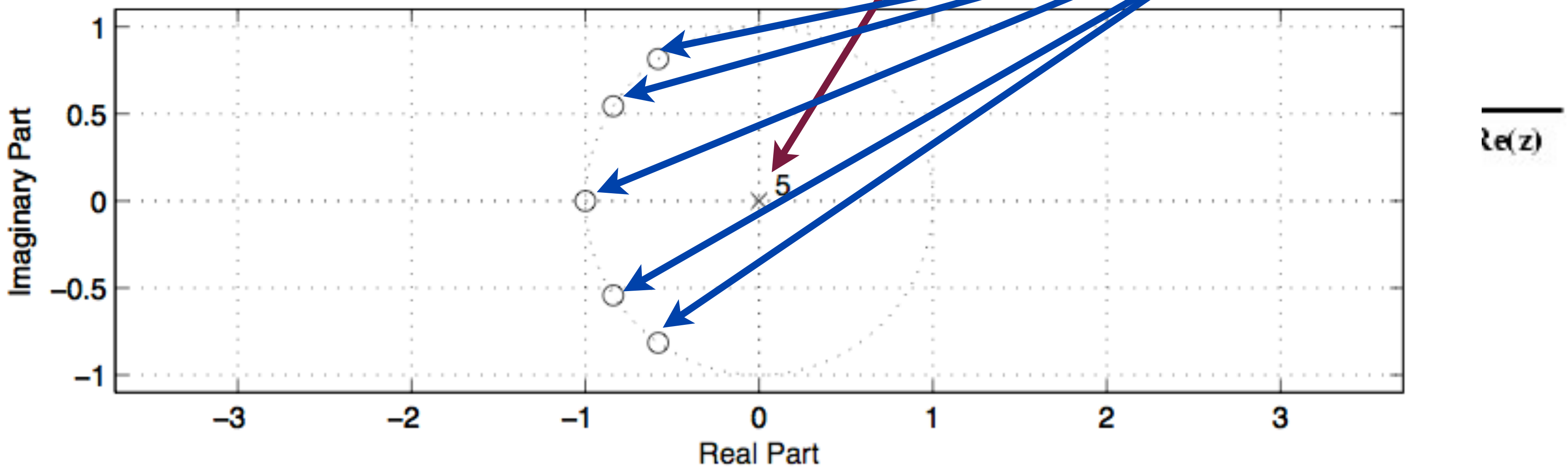




Z-Domain

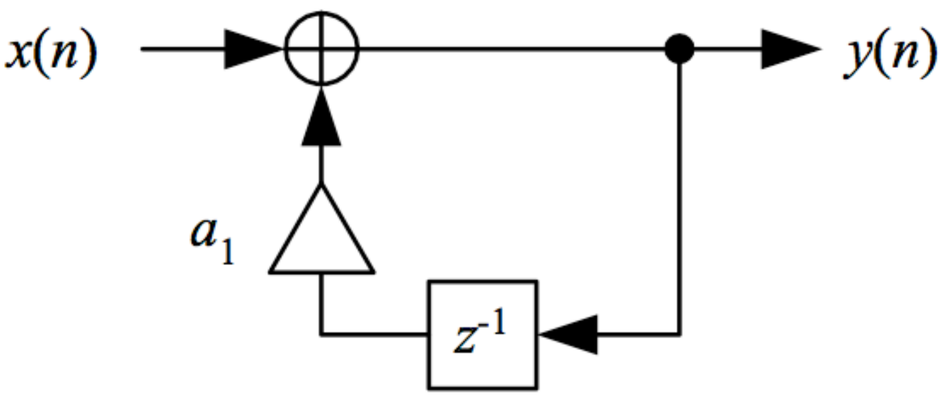
- Complex plane
- Useful in designing, analyzing and predicting system characteristics
- Poles & Zeros

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$



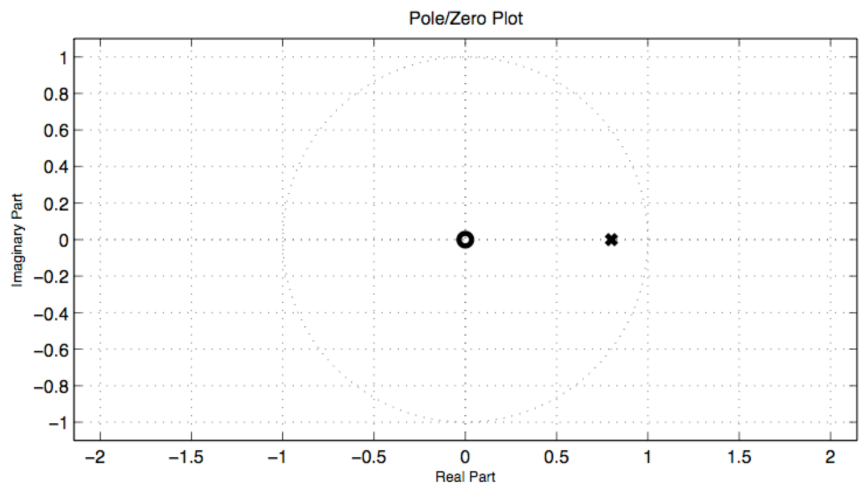
Example

- ex: **PZ plot** of First Order IIR Filter?
  - $y(n) = x(n) + a_1 y(n-1)$
  - $Y(z)/X(z) = 1/(1-a_1 z^{-1})$

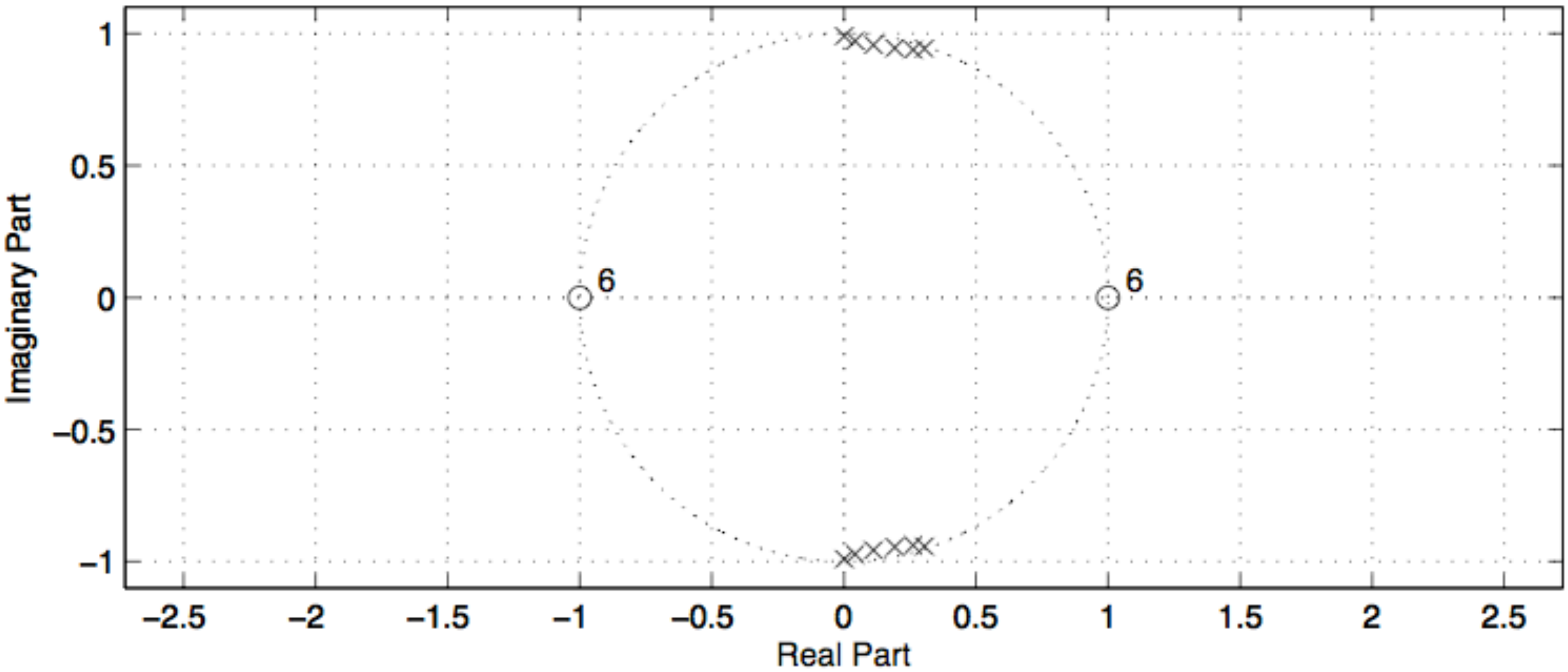


“Leaky integrator”

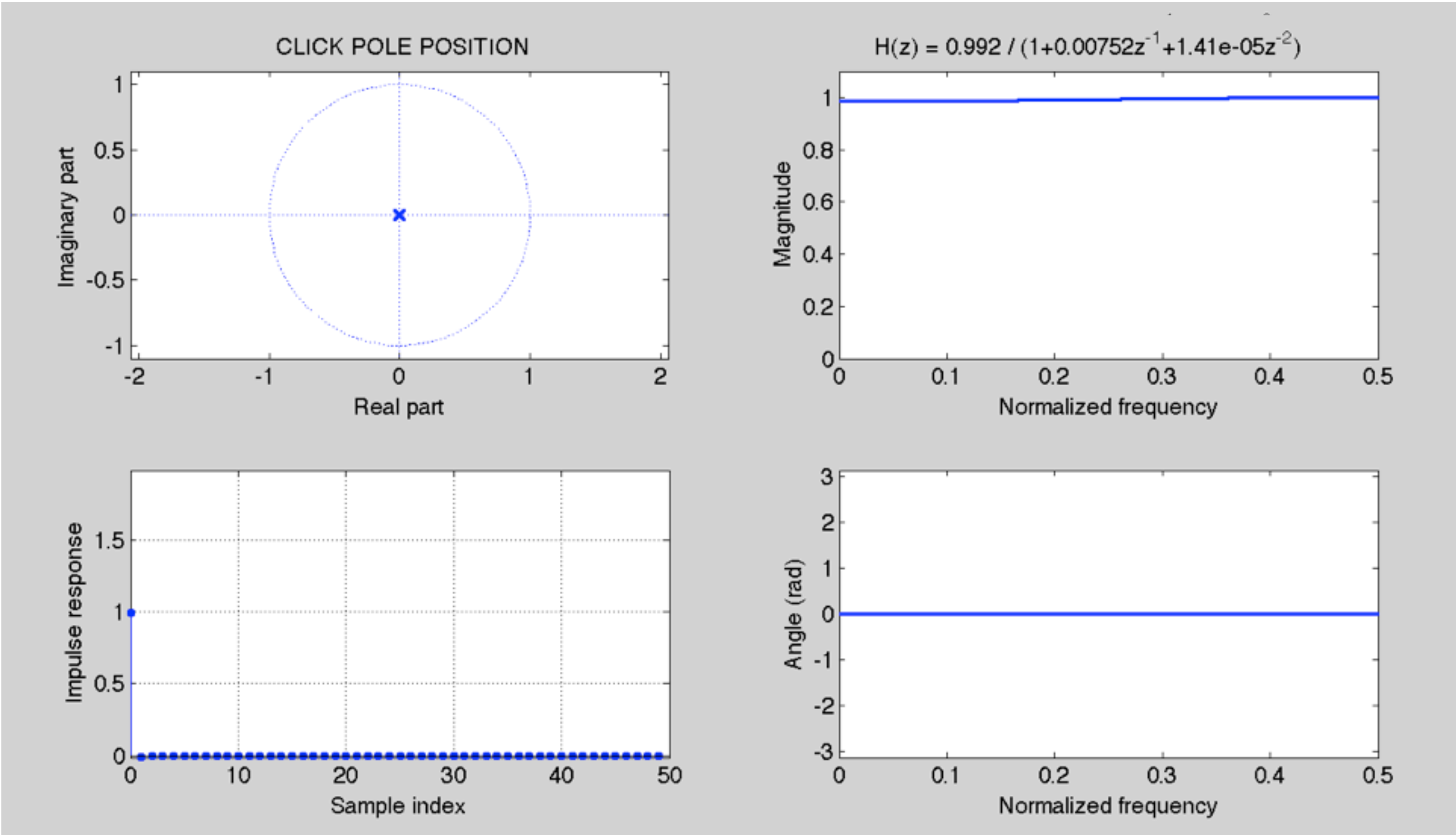
(when $0 < a_1 < 1$)



Examples



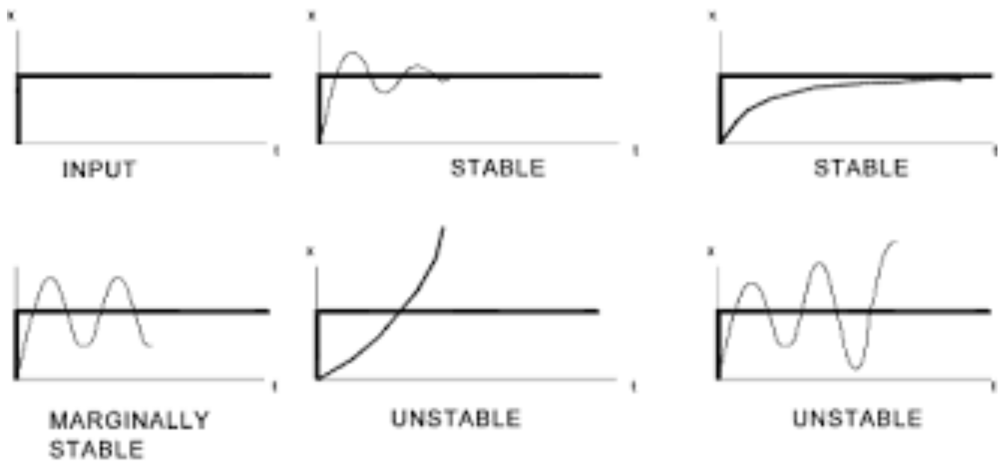
Examples


CLICK POLE POSITION
Imaginary part
Real part
H(z) = 0.992 / (1+0.00752z^-1+1.41e-05z^-2)
Magnitude
Normalized frequency
Impulse response
Sample index
Angle (rad)
Normalized frequency

Poles and Zeros

- Poles and Zeros determine how the system acts
  - what it does to the input

- Poles determines stability of the system

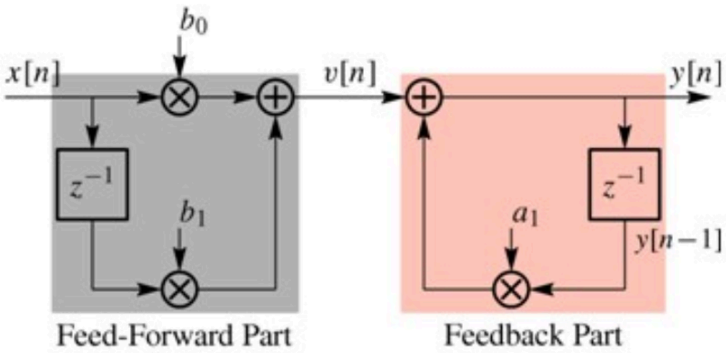


Filter Implementation

$$\boxed{y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]}$$

feedback term
recursive filter

FIR part



$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

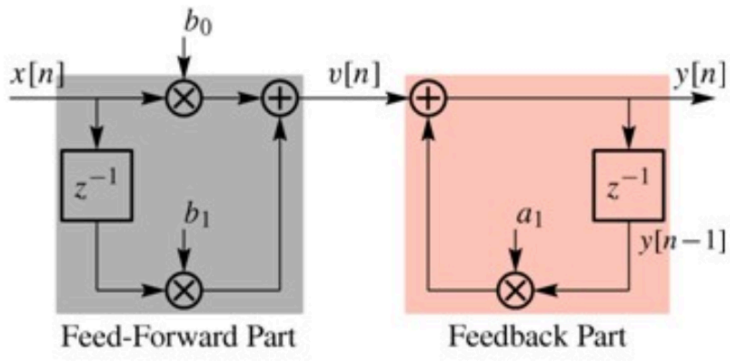
The first order case : $N = M = 1$,

```
filter One-dimensional digital filter.
   Y = filter(B,A,X) filters the data in vector X with the
   filter described by vectors A and B to create the filtered
   data Y.  The filter is a "Direct Form II Transposed"
   implementation of the standard difference equation:

   a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
                         - a(2)*y(n-1) - ... - a(na+1)*y(n-na)
```

B = [*b0 b1*]   A = [*1 -a1*]

Ex: Filter Implementation



$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$\rightarrow \quad Y(z) = a_1 z^{-1} Y(z) + b_0 X(z) + b_1 z^{-1} X(z)$$

$$\rightarrow \quad (1 - a_1 z^{-1}) Y(z) = (b_0 + b_1 z^{-1}) X(z)$$

$$\rightarrow \quad H(\mathrm{z}) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \equiv \frac{B(z)}{A(z)} \quad \begin{matrix} \rightarrow \text{FIR part} \\ \rightarrow \text{feedback part} \end{matrix}$$