Performance modeling for Computer systems (Assignment 2)

1 Each Question is for 5 marks

1. Find the stationary distribution and the mean number of jobs in an $M/M/K/\infty$ system. Assume servers are homogenous with exponential service times with rate μ and arrivals are Poisson process with rate λ. When is the system stable? **Mor Balter pg. 258**

2. Consider an $M/M/2/\infty$ queue. Server 1 has service rate μ_1 and server 2 has service rate μ_2 where $\mu_1 < \mu_2$. Describe this markov chain, specify its Q matrix and obtain its stationary distribution. (Hint: $X(t) = \{\text{Number of jobs in the system}\}$ will not be a good descriptor because when there is only one job in the system, we cannot determine which server is serving it using this descriptor. So your descriptor should also keep track of which server is active or not) **Related: Mor Balter pg. 266**

3. Consider an $M/M/K/K$ queue. Obtain the stationary distribution for number of jobs in the system. What is the probability that arriving jobs are blocked in this system? **Mor Balter pg. 255**

4. Let $A(t)$ and $D(t)$ denote two Poisson process with rates λ and μ respectively. Consider $N(t) = max(A(t) - D(t), 0)$. Prove that $Z(t)$ is a Markov chain. Identify its Q matrix and its stationary distribution.

5. Let $S_n = \sum_{i=1}^{n} X_i$ where X_i are i.i.d and $X_i = 1$ with probability p and $X_i = -1$ with probability $1 - p$. S_n is a random walk. Model this random walk as a Markov chain. Identify conditions on the parameter p such that Markov chain is recurrent and transient. When the chain is recurrent, what is the mean recurrence time? **Mor Balter pg. 160, 163**

6. Consider an $M/M/\infty$ queue with a Poisson arrival process λ and exponential service rate μ. Let $X(t)$ denote the number of jobs receiving service at time t. Model $X(t)$ as a Markov chain and obtain its stationary distribution. Obtain the mean value $E[X]$ where X is the corresponding stationary random variable. **Mor Balter pg. 271**