MDL Assigment 1

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- LCD/CSD

1. a) Let p be the proposition "It rains." and q be the proposition "Raju carries an umbrella"

∴ To prove: $((p \to q) \land q) \to p$: Ans

p	q	$p \to q$	$(p\to q)\land q$	Ans $= ((p\to q)\land q)\to p$
0	0	1	0	1
0	1	1	1	0
1	0	0	0	1
1	1	1	1	1

Here, 0 represents F and 1 represents T.

As we can see from the truth table, the expression 'Ans' is not a Tautology, therefore the given argument is not valid.

b) Let
p: "The weather is warm."
q: "The sky is clear."
r: "We go swimming."
s: "We go boating."

We are given the following argument:

$(p \land q) \to (r \lor s)$ {Let A}
$\neg(\neg r \to \neg q)$ {Let B}

$\therefore\ p \lor s$ {Let C}

~~We are to prove $(A \land B) \to C$ is a tautology.~~

We need to determine whether $(A \wedge B) \rightarrow C$ is a Tautology or not.

p	q	r	s	$p \wedge q$	$r \vee s$	A	$\neg r \rightarrow \neg q$	B	C	$A \wedge B$	$(A \wedge B) \rightarrow C$
0	0	0	0	0	0	1	1	0	0	0	1
0	0	0	1	0	1	1	1	0	1	0	1
0	0	1	0	0	1	1	1	0	0	0	1
0	0	1	1	0	1	1	1	0	1	0	1
0	1	0	0	0	0	1	0	1	0	1	0
0	1	0	1	0	1	1	0	1	1	1	1
0	1	1	0	0	1	1	1	0	0	0	1
0	1	1	1	0	1	1	1	0	1	0	1
1	0	0	0	0	0	1	1	0	1	0	1
1	0	0	1	0	1	1	1	0	1	0	1
1	0	1	0	0	1	1	1	0	1	0	1
1	0	1	1	0	1	1	1	0	1	0	1
1	1	0	0	1	0	0	0	1	1	0	1
1	1	0	1	1	1	1	0	1	1	1	1
1	1	1	0	1	1	1	1	0	1	0	1
1	1	1	1	1	1	1	1	0	1	0	1

As it can be seen, the proposition $(A \wedge B) \rightarrow C$ is not a tautology, therefore we can say the argument presented is invalid.

2. a) $\neg(A \vee X)$

$\Rightarrow \neg(T \vee F)$

$\Rightarrow \neg(T)$

$\Rightarrow F$

b) $A \vee (X \wedge Y)$

$\Rightarrow T \vee (F \wedge F)$

$\Rightarrow T$

c) $A \wedge (X \vee (B \wedge Y))$

$\Rightarrow T \wedge (F \vee (T \wedge F))$

$\Rightarrow T \wedge (F \vee F)$

$\Rightarrow T \wedge F \Rightarrow F$

d) $[(A \wedge X) \vee \neg B] \wedge \neg[(A \wedge X) \vee \neg B]$

$\Rightarrow F \quad \{ p \wedge \neg p = F \}$

e) $(\neg A \vee X) \wedge (P \wedge Q)$

$\Rightarrow (\neg T \vee F) \wedge (P \wedge Q)$

$\Rightarrow (F) \wedge (P \wedge Q)$

$\Rightarrow F.$

d) $p \to q$ is T whenever q is T or p is F.

$\therefore [\underbrace{X \wedge Y \to A}_{T}] \to [\underbrace{X \to (Y \to A)}_{F}]$

$\Rightarrow T \to T \Rightarrow T.$

3. a) formal proof:

1. $P \to \neg Q$
2. $\neg Q \to R$

$\Rightarrow$ 3. $P \to R$ [1 & 2 : Hypothetical syllogism]

Method of Resolution:

1. $P \to \neg Q \equiv \neg P \vee \neg Q$
2. $\neg Q \to R \equiv Q \vee R$

$\Rightarrow$ 3. $\neg P \vee R \equiv P \to R$ [Resolution]

b) The statement tells us that with no premise, the ~~state~~ proposition $((P \vee Q) \wedge \neg P) \rightarrow Q$ is true.

For it to be true, the proposition must be a tautology.

$\therefore$ It suffices to prove $[(P \vee Q) \wedge \neg P] \rightarrow Q$ is a tautology.

Formal method:

$$[(P \vee Q) \wedge \neg P] \rightarrow Q$$

$$\Rightarrow \neg [(P \vee Q) \wedge \neg P] \vee Q.$$

$$\Rightarrow \cancel{\neg P \wedge \neg Q}\ \neg (P \vee Q) \vee P \vee Q$$

$$\Rightarrow \neg (P \vee Q) \vee (P \vee Q) \Rightarrow T \quad \{\neg P \vee P = T\}$$

Method of Resolution:

Given,

1. $P \vee Q$
2. $\neg P$

$\Rightarrow$ 3. $\neg P \vee F$ $\quad \{A \vee F = A\}$

$\Rightarrow$ 4. $Q \vee F$ $\quad \{1 \,\&\, 3:$ Resolution$\}$

$\Rightarrow$ 5. Q $\quad \{A \vee F = A\}$

$\therefore$

$$\begin{array}{l} P \vee Q \\ \neg P \\ \hline \therefore Q \end{array}$$

is a valid argument.

$\therefore$ $[(P \vee Q) \wedge \neg P] \rightarrow Q$ is a tautology.