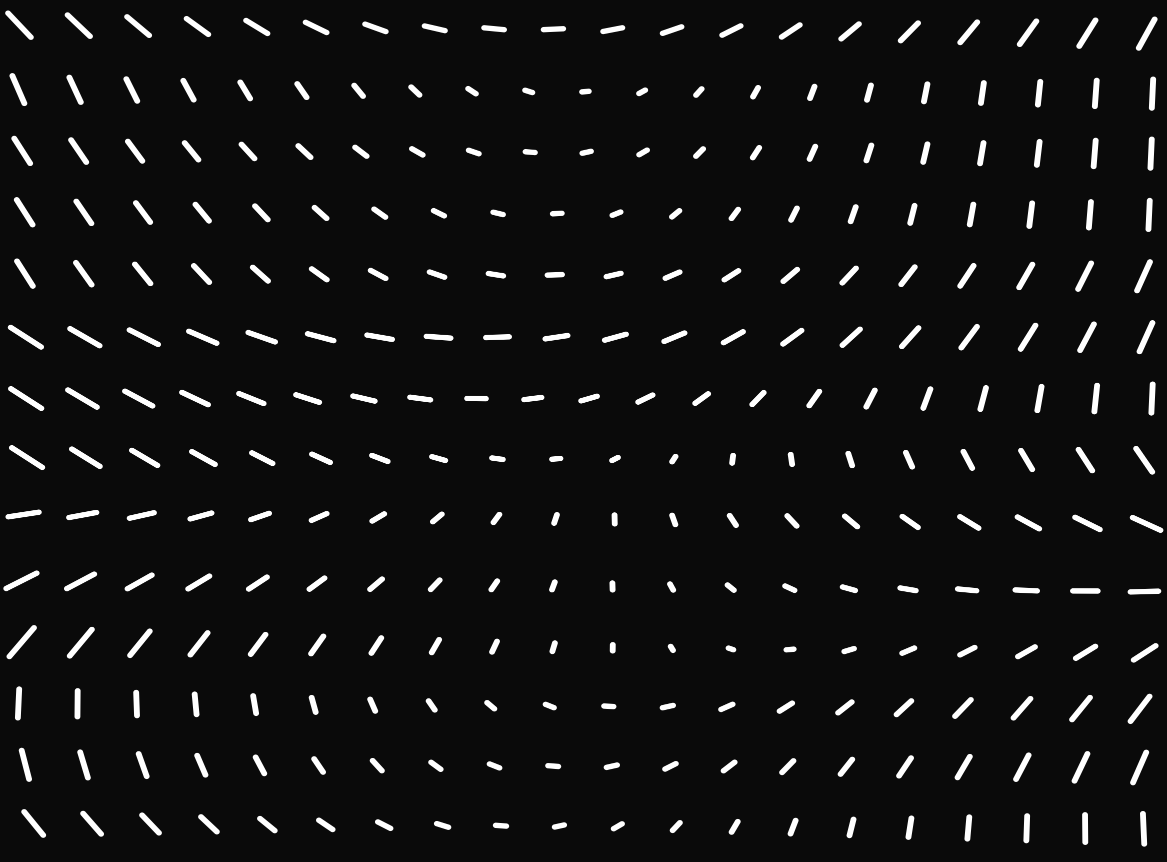


$P_1, P_2, \dots, P_n$

$1\ 2\ 3\ 4 \dots (n-1), n \rightarrow P_1 \quad 0$

$1\ 2\ 3\ 4 \dots (n), (n-1) \rightarrow P_2 \quad 1$

$1\ 2\ 3\ 4 \dots (n), (n-2), (n-1) \longrightarrow P_3 \quad 2$

$1\ 2\ 3\ 4 \dots (n), (n-1), (n-2) \longrightarrow P_4 \quad 3$

$1\ 2\ 3\ 4 \dots (n), (n-3), (n-2), (n-1) \rightarrow P_5 \rightarrow 3$

$(n), (n-3), (n-1), (n-2) \rightarrow P_6 \rightarrow 4$

$(n), (n-2), (n-1), (n-3) \rightarrow P_6 \rightarrow$

$E(V) =$

$$V_i = \begin{cases} 1 & \text{if present afterprocess} \\ 0 & \text{o/w} \end{cases}$$

$$Pr(V_i = 1) = \frac{1}{d}.$$

$$Pr(V_i = 0) = 1 - \frac{1}{d}$$

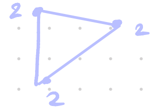
$$E(V_i) = \frac{1}{d}$$

$E(V)$ = Expected no. of vertices after process

$$= \sum_{i=1}^{n} E(V_i) = \sum_{i=1}^{n} \frac{1}{d} = \frac{n}{d}.$$

$n(1 - \frac{1}{d}) \rightarrow$ Expected no. of vertices deleted

A vertex was connected to d edges in expectation



$\sum ND = 2E$

$\sum ND = nd$

$E(ND) = d$

$nd \times (1-\frac{1}{d})$ edges deleted

$\frac{nd}{2} - nd(1-\frac{1}{d})$

$nd[\frac{1}{2} - 1 + \frac{1}{d}]$

$nd[\frac{1}{d} - \frac{1}{2}]$

wrong

2×3

$\frac{2E}{N} =$

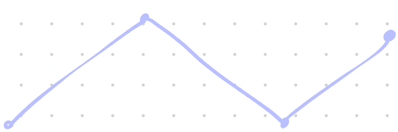
A edge survives if and only if two of its vertices survive.

$\therefore$ Expected no. of edges to survive $= \frac{nd}{2}\cdot\left(\frac{1}{d}\right)^2$

$= \frac{n}{2d}$

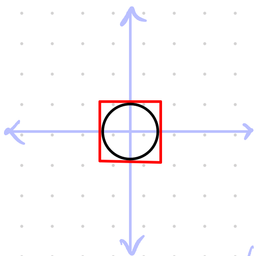
Algorithm to get Independent set

Let m_p be the no. of edges after



Theorem: Let $G = (V, E)$ be a graph on n vertices with $nd/2$ edges. Then G has an independent set with atleast $n/2d$ vertices.

$$\frac{nd}{2} \geq \frac{n}{2} \qquad \frac{n^2}{4 \cdot \frac{nd}{2}} = \frac{n^2}{4m}$$



$$e^{-\frac{\cancel{m\pi}}{4} \cdot \frac{16 \cdot \varepsilon^2}{\cancel{(m\pi)^2}}}$$

$$e^{-\frac{4\varepsilon^2}{m\pi}}$$

$$E(z) = \frac{\pi}{4}$$

$$E(W) = \frac{m\pi}{4}$$

$$Pr\left(\left|W - \frac{m\pi}{4}\right| \geq \varepsilon\right) \leq e^{-\frac{m\pi}{4} \cdot \left(\frac{4\varepsilon}{m\pi}\right)^2}$$

$$E(z) = \frac{\pi}{4}$$

$$E(z^2) = \frac{\pi}{4}$$

$$Pr\left(\left|\frac{4W}{m} - \pi\right| \geq \varepsilon\right) = Pr\left(\left|\frac{4W}{m} - \pi\right| \geq \overset{\delta}{\frac{\varepsilon}{\pi}} \cdot \pi\right)$$

$$\leq 2e^{-\pi \cdot \left(\frac{\varepsilon}{\pi}\right)^2 / 3}$$

$$= 2e^{-\varepsilon^2/3\pi}$$

standard deviation σ of z will be

$$\sqrt{E(z^2) - (E(z))^2} = \sqrt{\frac{\pi}{4} - \left(\frac{\pi}{4}\right)^2} =$$

$$\Rightarrow \quad \sqrt{\left(\frac{\pi}{4}\right)\left(1-\frac{\pi}{4}\right)} = \sigma =$$

$$Var\left(\frac{Z_1 + Z_2 + \cdots + Z_m}{m}\right) = \frac{1}{m^2} \cdot m \left(\frac{\pi}{4}\right)\left(1-\frac{\pi}{4}\right)$$

$$= \frac{\left(\frac{\pi}{4}\right)\left(1-\frac{\pi}{4}\right)}{m} = \varepsilon^2$$

$$m = \frac{4\left(\frac{\pi}{4}\right)\left(1-\frac{\pi}{4}\right)}{\varepsilon^2}$$

0.001

for our estimation to be 0.001-accurate, we would need approximately 2,696,766 trials.

Alternatetively, if we run our experiment 200,000 times, our estimate would be 0.003670 accurate.

$$Pr(X > u) < \frac{E(X)}{u} = 1$$

$$Pr(X < u) > 0$$

$x_1 + x_2 \equiv 0 \mod 3$

$x_1 \quad x_2$

(0, 0) ✓

(0, 1) ✗

(0, 2) ✗

(1, 0) ✗

(1, 1) ✗

(1, 2) ✓

(2, 0) ✗

(2, 1) ✓

(2, 2) ✗

$n = 3$ $\quad g$

$p = 2$

$f_0(x) = 2x$

$f_0(x) \equiv 0 \mod 2$

$f_1(x) \equiv x \mod 2 \qquad f_1(x) = x$

$h_0(x) = g(f_0(x)) \qquad x, y = 1, 2$

$h_1(x) = g(f_1(x))$

$h_0(x) = g(2x) \equiv 2x \mod 3$

$h_1(x) = g(x) \equiv x \mod 3$

$H = \{h_0, h_1\}$

$(x_1 \vee \bar{x}_2) \wedge (x_2 \vee x_3) \wedge (\bar{x}_3 \vee x_1)$

$y_1 + (1 - y_2) \geq z_1$

$Pr(X =$

$e_1, e_2, \ldots e_m$

$X_i \to 1$ if e_i is bet^n A & B

$$E(X_i) = 1 \cdot \left[Pr\left([u_i = A \,\&\, v_i = B] \vee [u_i = B \,\&\, v_i = A]\right)\right]$$
$$+ 0 \cdot \left[Pr\left[(u_i = A \,\&\, v_i = A) \vee (u_i = B \,\&\, v_i = B)\right]\right]$$
$$= 1 \cdot \left[\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}\right]$$
$$= \frac{1}{2}$$

$$E(X) = E\left[\sum_{i=1}^{m} X_i\right] = \sum_{i=1}^{m} E[X_i] = \sum_{i=1}^{m} \frac{1}{2} = \frac{m}{2}$$

$$Pr(X \geq \frac{m}{2}) = p = \sum_{i=m/2}^{m} Pr(X = i)$$

$$1 - p = \sum_{i=0}^{m/2 - 1} Pr(X = i)$$

$$E(X) = \sum_{i=0}^{m} i \cdot Pr(X=i) = m/2$$

$$= \sum_{i=0}^{m/2-1} i \cdot Pr(X=i) + \sum_{i=m/2}^{m} i \cdot Pr(X=i)$$

$$\le \left(\frac{m}{2}-1\right)(1-p) + m p$$

$$\frac{m}{2} \le p\left[m - \frac{m}{2} + 1\right] + \frac{m}{2} - 1$$

$$p\left(\frac{m}{2}+1\right) \ge 1$$

$$p \ge \frac{1}{\frac{m}{2}+1}$$

$\left(\frac{m}{2}+1\right) \to \frac{1}{p}$, in expectation, the number of trails to get

To find a cut with at least $m/2$ edges, simply sample $m/2 + 1$ random assignments of vertices to A & B.

$$E(X \mid x_1) = E(X)$$

$$E(X \mid x_1, \dots, x_k) = \frac{1}{2} E(X \mid x_1, \dots, x_k, x_{k+1} = A) + \frac{1}{2} E(X \mid x_1, x_2, \dots, x_{k+1} = B)$$

$$\leq E(X \mid x_1, x_2, \dots, x_{k+1})$$

To calculate conditioned expectation,

1) calculate the # edges contributing to the cut betn A & B by the vertices that are already placed. Call it λ.

2) The condition expectation is just the $\lambda + \frac{1}{2} \cdot$ # remaing vertices.

GU RO T HHO ANDO DE BE RA PO ME N TO
グロトッホ アンド デベラポメント

クナル

ホテル

ハジメマッテ

エデワンス アルゴリダム

トピクス イン デプ ラニング

ライナ
エレン
ミカサ
ガビ
メリタ

父 母
母 父

アイウエオ
マミムメモ
ルヒフヘホ
カキクケコ
サシスセソ
ラリルレロ
ナニヌネノ
タチツテト
ワ ヲ
ヤ ヨ

7 o'clock
七 日寺

読みます

いま 二日寺ごろです

いま 二日寺ごろです。

山口 中山 田中

中山

私 東京

東京はにすんでいますお？

に

東京はしうしんですか?

Are you from Tokyo?

ハコ あれー

私はしけんです

ドキドキ

東京 東 日本

リトライイチアル

京都 京 都

クラブ

都 都

a e i o u

あ

京都

Let $X = \sum_{i=1}^{m} X_i$, $\mu = E(X)$

and $p_i = Pr(X_i = 1)$

$$Pr(|X-\mu| \geq \delta\mu) \leq 2e^{-\mu\delta^2/3}$$

we want to bound the probability of the following

$$Pr\left(\left| \frac{\sum X_i}{m} - \mu \right| \geq \varepsilon\mu \right)$$

$Pr(X_i = 1) = \mu$

$E(X_i) = \mu$

$\therefore E(X) = \mu m$

$$Pr(|\overset{X}{\sum X_i} - \mu m| \geq \varepsilon m\mu) \leq 2e^{-\varepsilon^2 m\mu/3} \leq \delta$$

we want

$$\delta \geq 2e^{-\varepsilon^2 m\mu/3}$$

$$\log\left(\frac{\delta}{2}\right) \geq \frac{-\varepsilon^2 m\mu}{3}$$

$$3\log\left(\frac{2}{\delta}\right) \leq \varepsilon^2 m\mu$$

$$m \geq 3\log\left(\frac{2}{\delta}\right) \cdot \frac{1}{\varepsilon^2\mu}$$

$$f((x+y) \bmod n) = [F(x)+F(y)] \bmod m$$

x	$F(x)$
1	$f(1)$
2	$f(2)$
3	$f(3)$
$\vdots$	$\vdots$
$n-1$	$f(n-1)$

Pl state $\longrightarrow X$

$$-\ell_1 \quad \cdots \quad -2 \quad\quad -1 \quad\quad 0 \quad\quad 1 \quad\quad 2 \quad\quad 3 \quad \cdots \quad \ell_2$$

$$X = x_1 + \cdots + x_n$$

$$p(|X|^2 = i^2)$$