11.08.2023

# Statistical Methods in AI (CS7.403)

Lecture-5: k-Nearest Neighbours

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https://ravika.github.io

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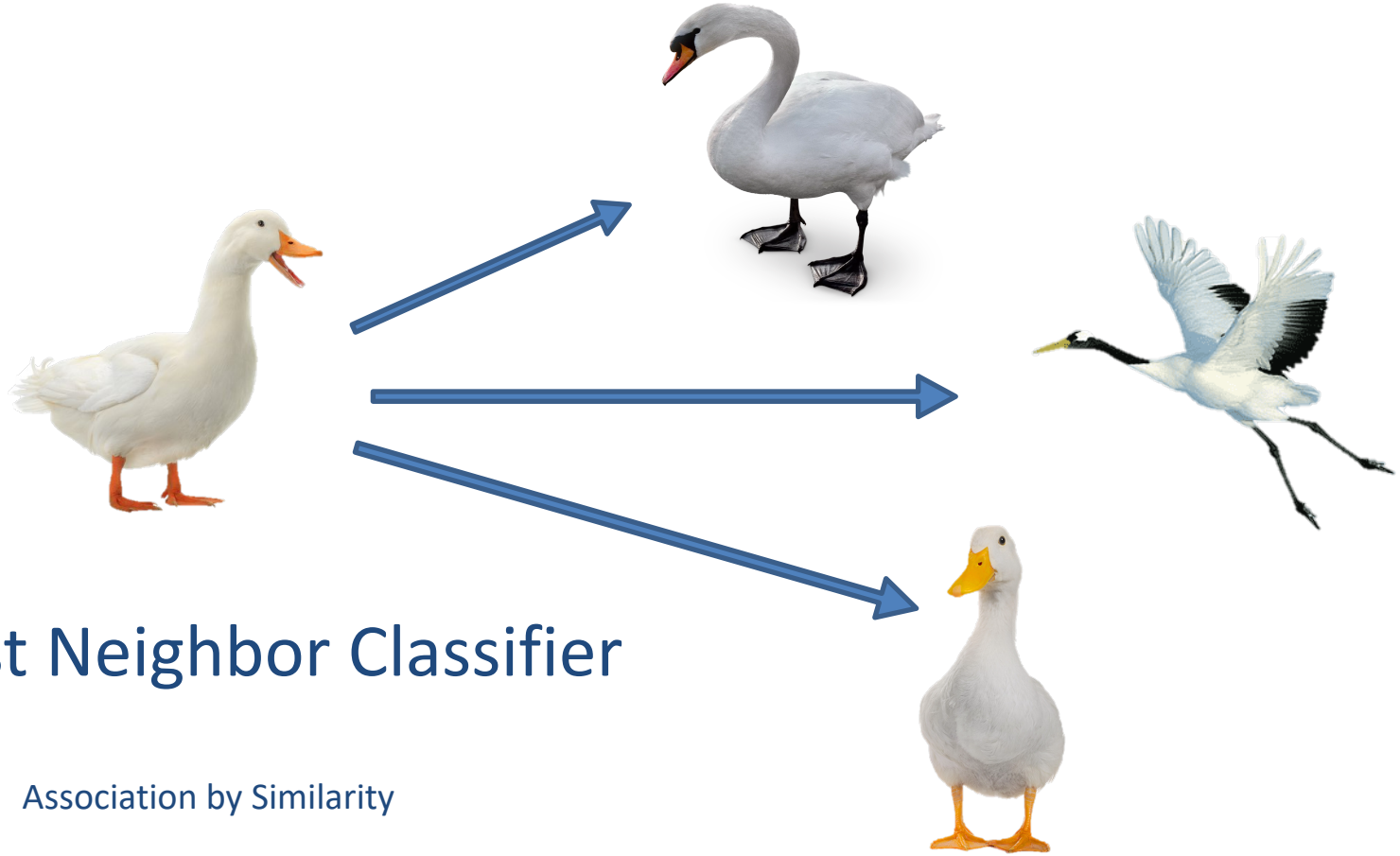
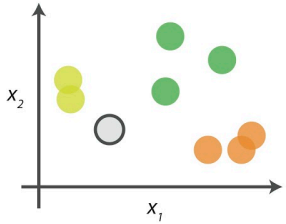


# Nearest Neighbor Classifier

Association by Similarity

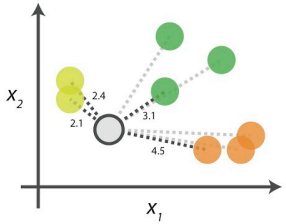
# k-NN algorithm in pictures

## 0. Look at the data



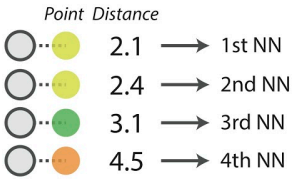

Say you want to classify the grey point into a class. Here, there are three potential classes - lime green, green and orange.

## 1. Calculate distances



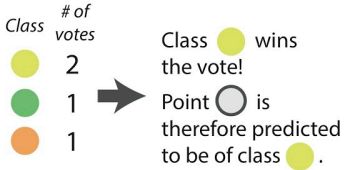

Start by calculating the distances between the grey point and all other points.

## 2. Find neighbours



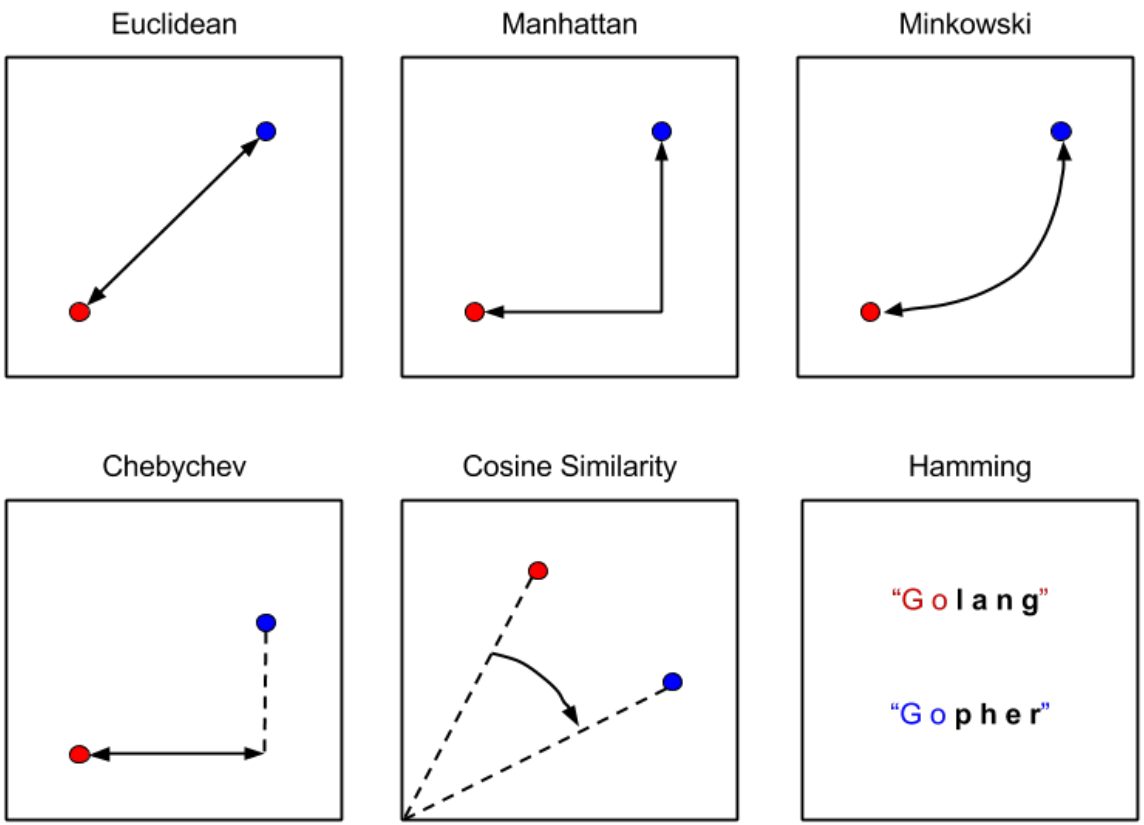

Next, find the nearest neighbours by ranking points by increasing distance. The nearest neighbours (NNs) of the grey point are the ones closest in dataspace.

## 3. Vote on labels



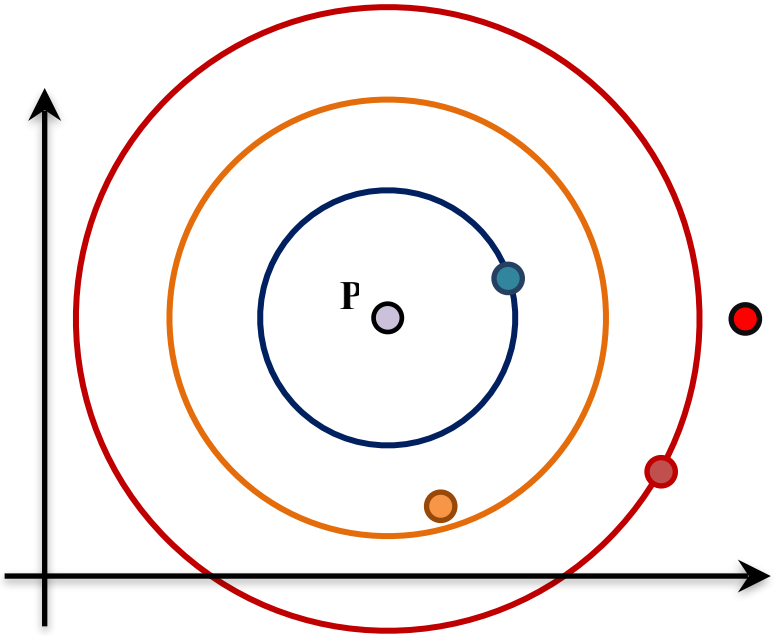

Vote on the predicted class labels based on the classes of the k nearest neighbours. Here, the labels were predicted based on the k=3 nearest neighbours.

# Distance measures



# Distance measure can affect k-NN classification

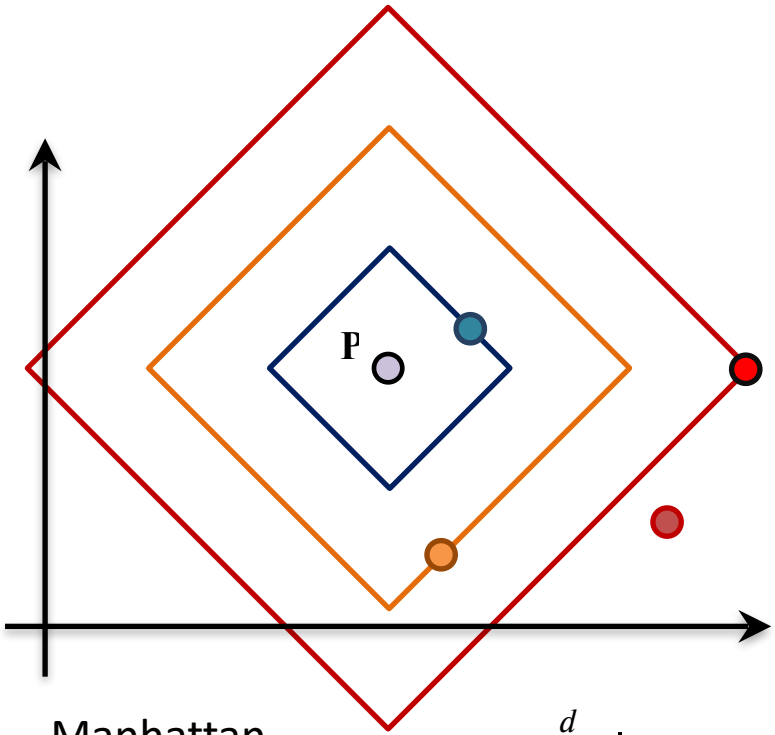
Iso-surfaces




Euclidean

$$d(P,Q)^2 = \sum_{i=1}^{d} (p_i - q_i)^2$$

Iso-surfaces

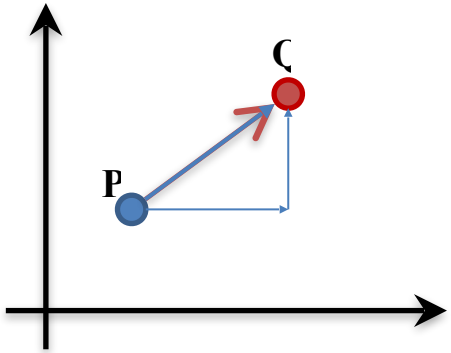



Manhattan

$$d(P,Q) = \sum_{i=1}^{d} \left| (p_i - q_i) \right|$$

# Minkowski Dist.

$$d(P,Q)^r = \sum_{i=1}^{d} \left| p_i - q_i \right|^r$$



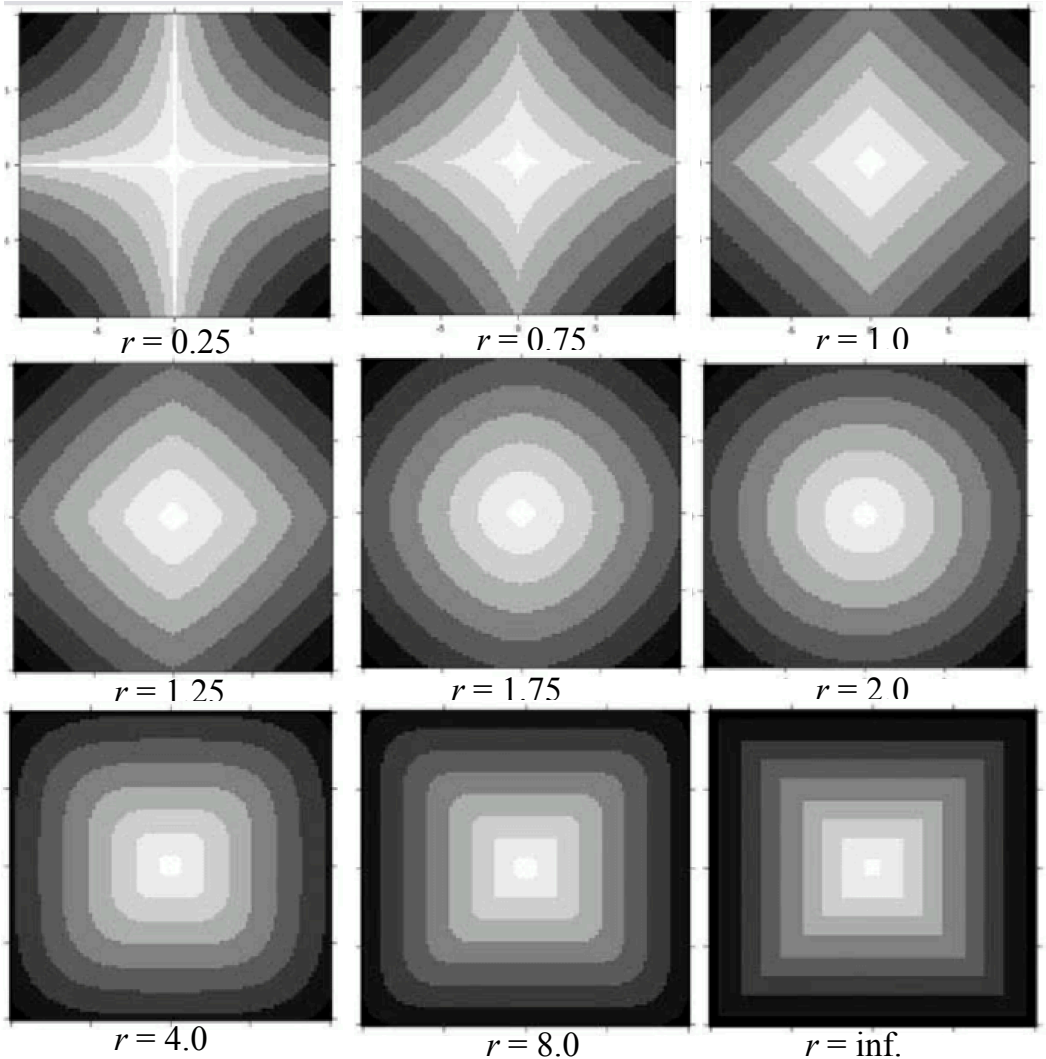
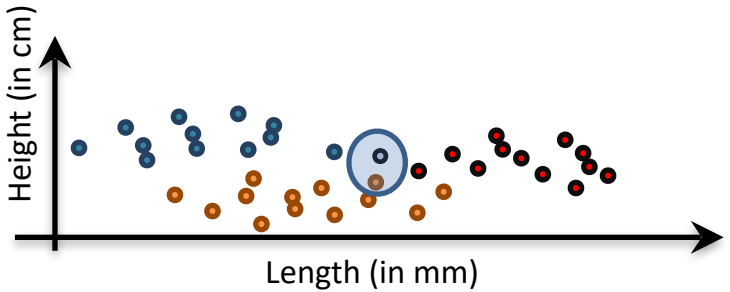
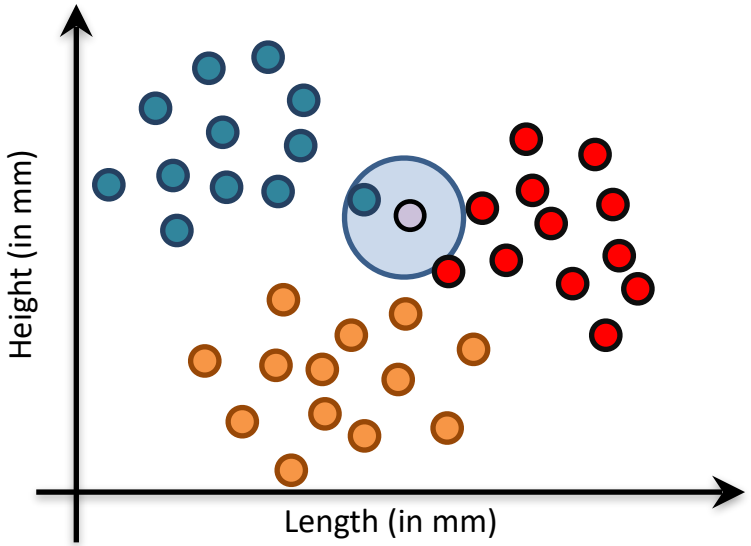




Fig. by Lu et al., "The Minkowski Approach for Choosing the Distance Metric in Geographically Weighted Regression"

# Note: Feature Normalization

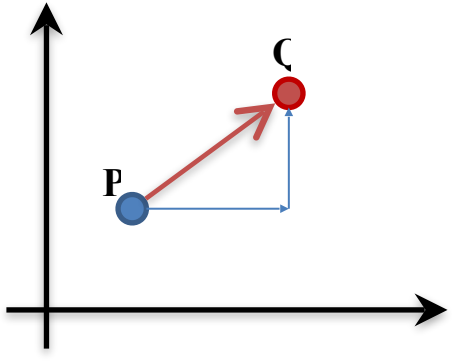
- If different features have different variances
  - Some features will dominate distance computation
  - Normalization can reduce this feature bias

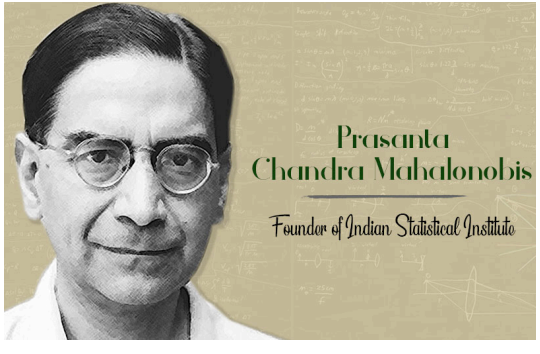


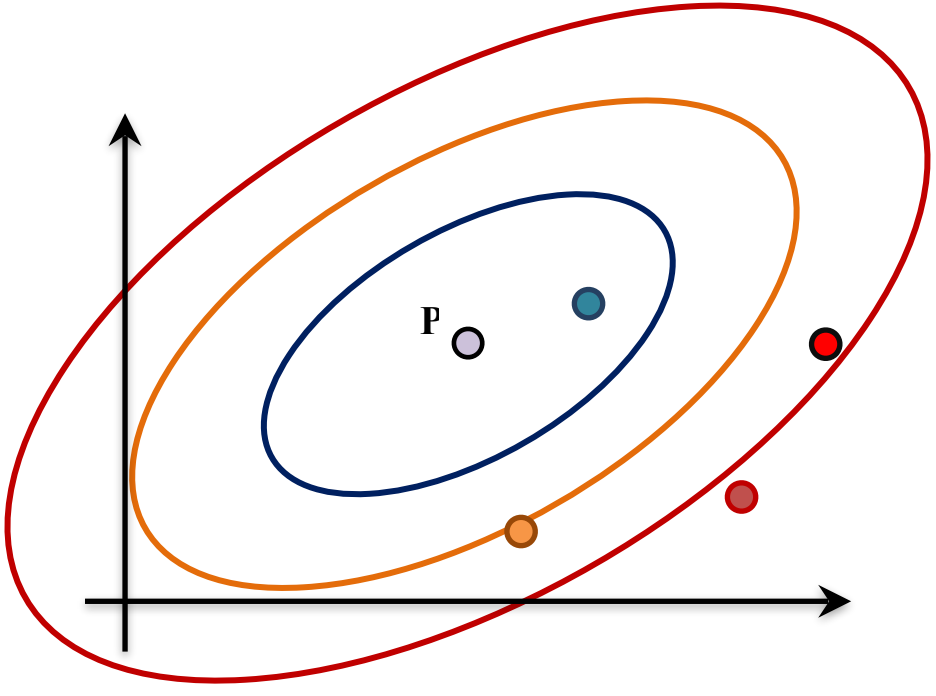
# Mahalanobis Distance

$$d(P,Q)^2 = (P-Q)^T \mathbf{S}^{-1}(P-Q)$$




Iso-surfaces

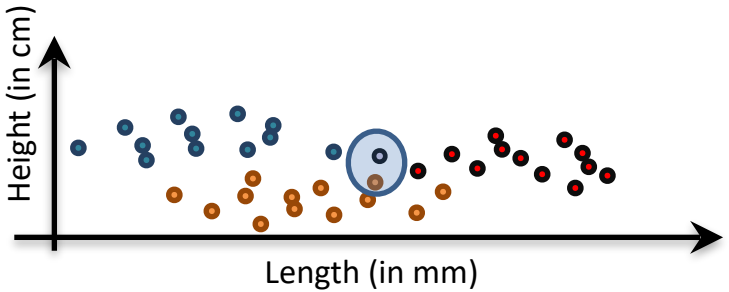


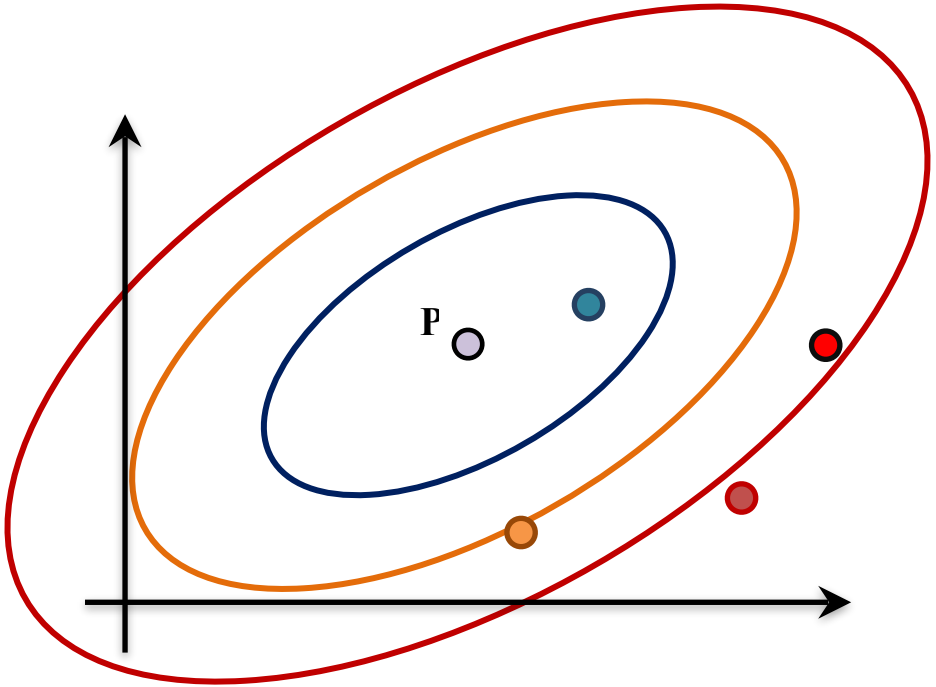

https://www.getbengal.com/details/the-indian-statistical-institute-in-kolkata-was-founded-by-the-father-of-indian-statistics-pc--mahalanobis-getbengal-story

# Mahalanobis Distance

Iso-surfaces

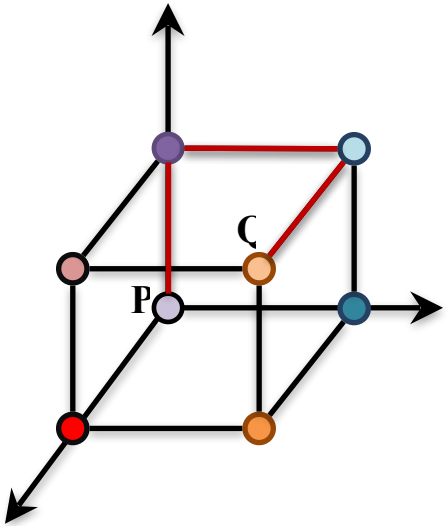
$$d(P,Q)^2 = (P-Q)^T \mathbf{S}^{-1}(P-Q)$$

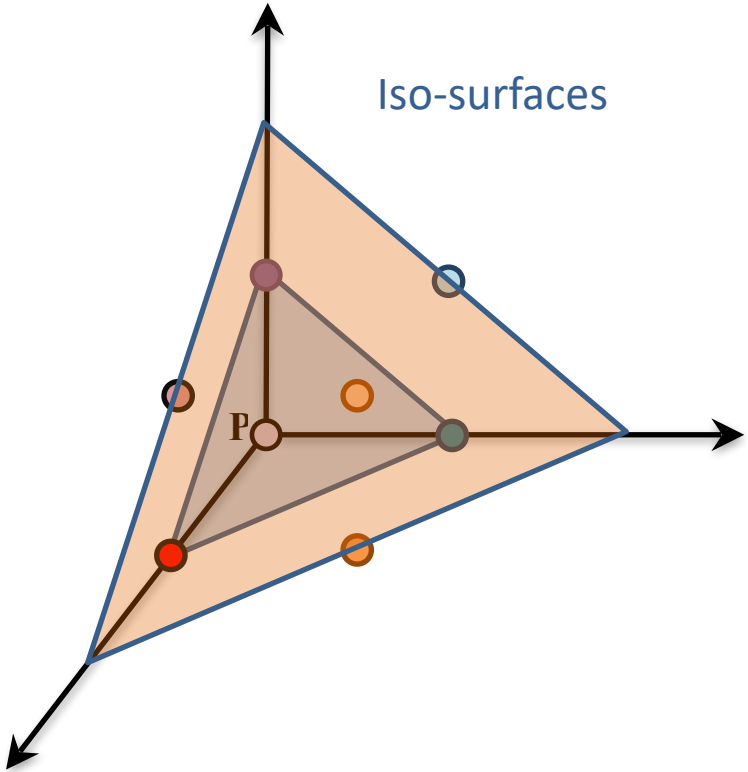


# Hamming Distance

$$d(P,Q) = \sum_{i=1}^{d} \mathrm{I}\left(p_i \neq q_i\right)$$



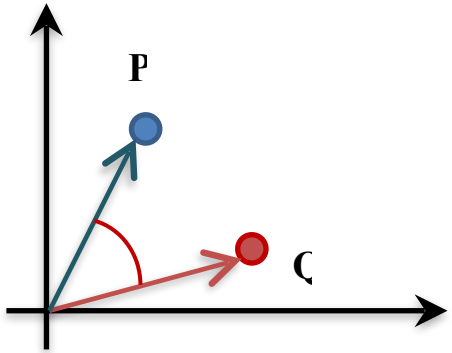




| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 |

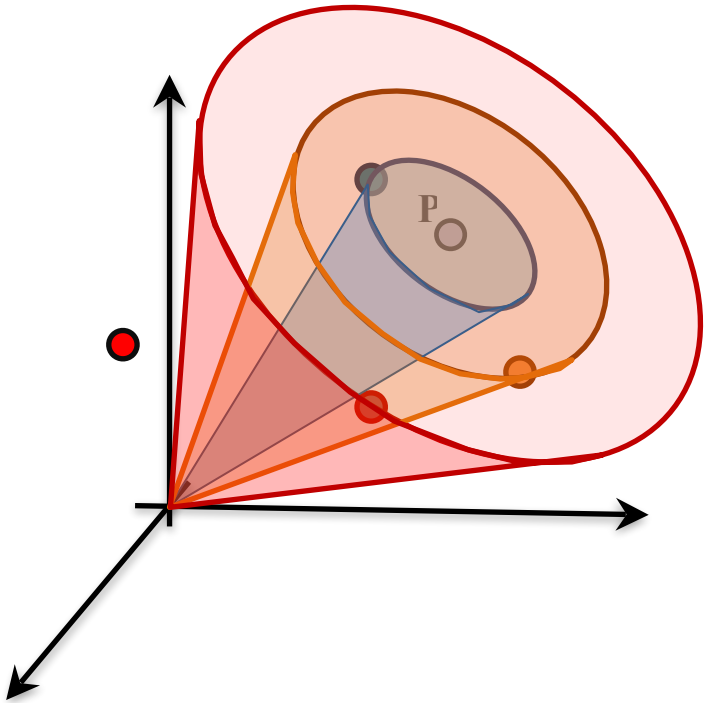
# Cosine Distance

$$s(P,Q) = cos\theta = \frac{\mathbf{P} \cdot \mathbf{Q}}{\|\mathbf{P}\| \|\mathbf{Q}\|}$$

$$d(P,Q) = 1 - s(P,Q)$$



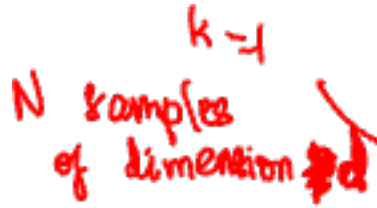

Iso-surfaces



# Summary of Distances

- Distance Metrics decide Neighborhoods
- Need not be a ”metric”
  - Jaccard Distance
  - Edit Distance
- Feature vectors need not be of same length
- Selection of metric depends on the nature of feature vector

# Complexity of k-NN




| | 1-NN | k-NN |
|---|---|---|
| • Training | | |
| – Time: | $O(1)$ | $O(1)$ |
| – Space: | $O(Nd)$ | $O(Nd)$ |
| • Testing | | |
| – Time: | ~~$O(Nd+N)$~~ $O(Nd)$ | $O(Nd+N\log N)$ |
| – Space: | $O(Nd)$ | $O(Nd)$ |

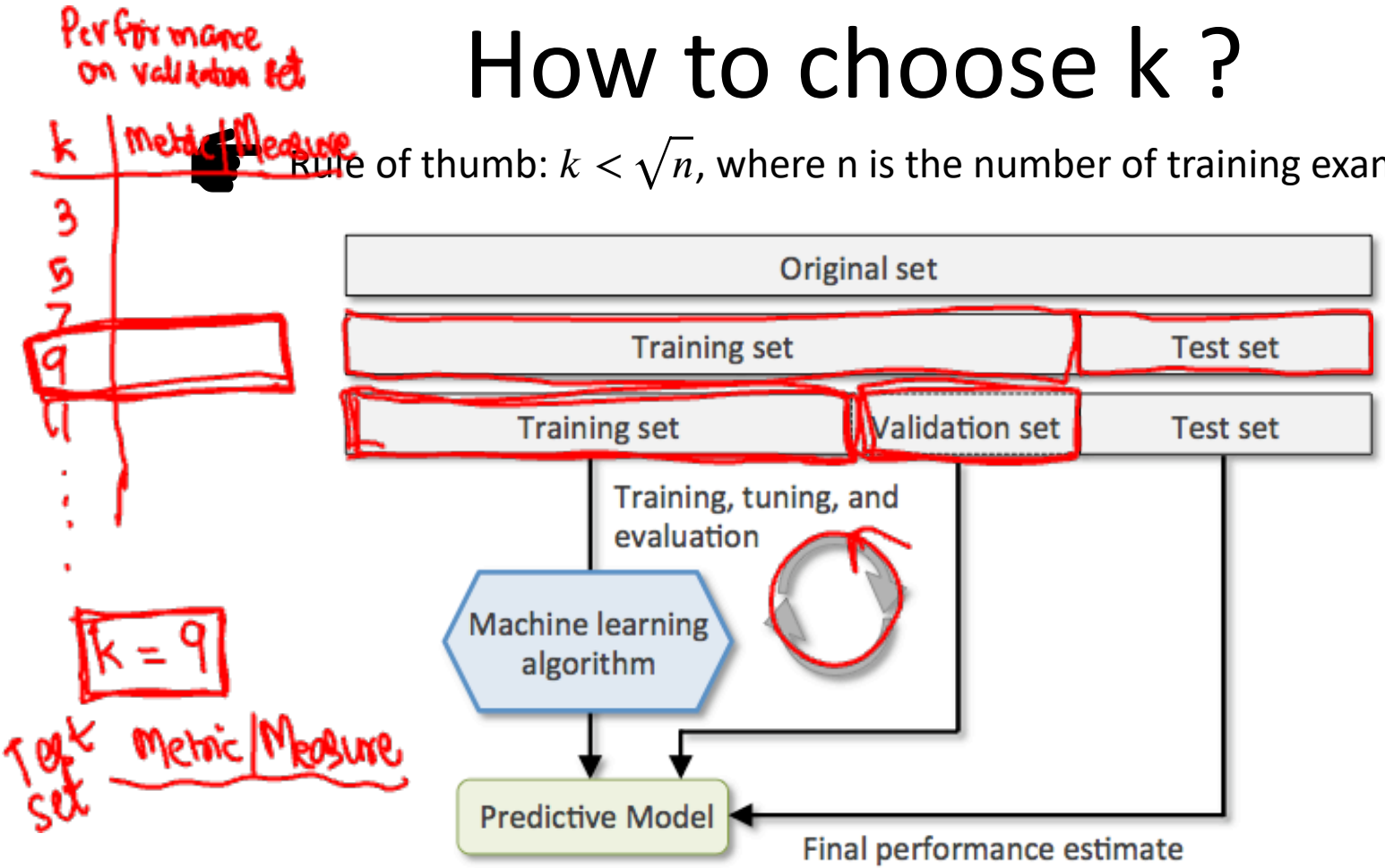
$O(Nd)$ : $1\ 2\ 3\ \ldots\ k\ \ldots\ N$ — $O(k)$ | $\log k(N-k)$

$(N-k)\log k + k\log k \rightarrow O(N\log k)$ assuming $k \ll N$

# How to choose k in k-NN?

# How to choose k ?

Rule of thumb: $k < \sqrt{n}$, where n is the number of training examples



# Properties and Issues with k-NN

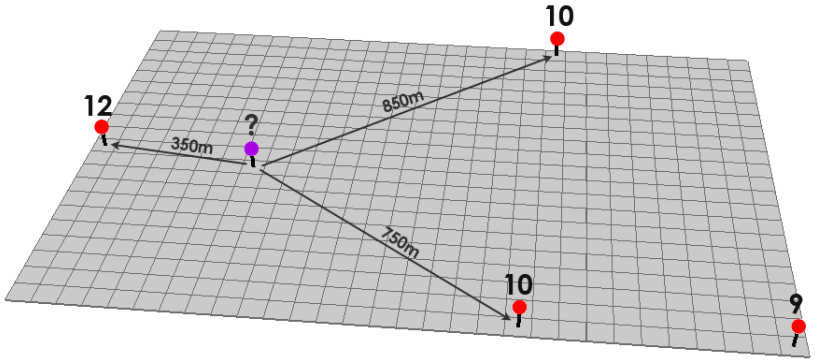
- Non-parametric
- ‘Lazy’ learner
- Simple baseline (after 0-effort baselines)
- GOOD
  - No training
  - Learns highly non-linear decision boundaries
- BAD
  - Need to keep all training points around
  - Curse of dimensionality ! (suggested #dims < 20)

# Improving the k-NN Classifier

Faster, Leaner, Meaner

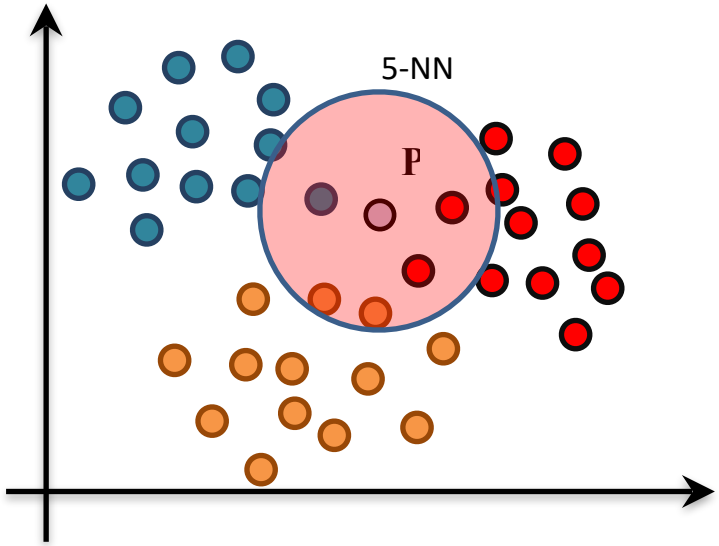
# Weighted k-NN

- Helps in case of class skew
- Helps in case of even k (breaking ties)



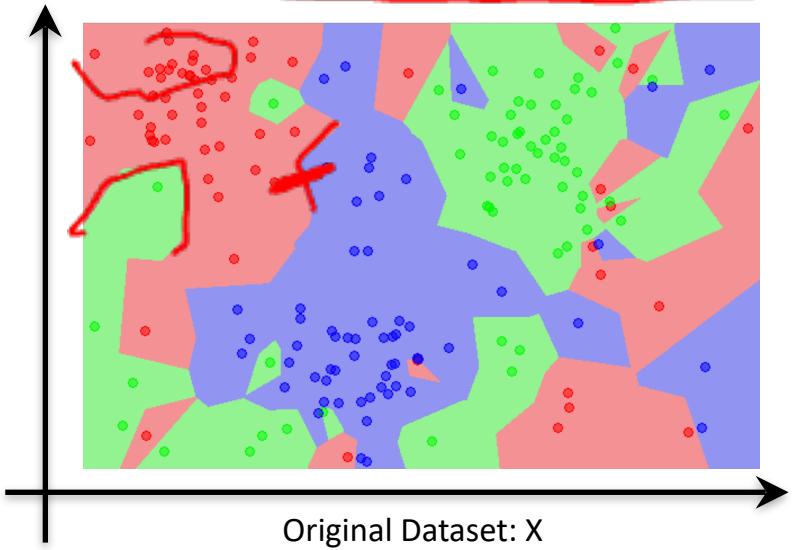
# Weighted k-NN

- If there is a tie in majority labels, one can do weighted voting
  - Samples are weighted by inverse of distance to the point p
  - e.g., $w_i = \frac{1}{1 + d(p, x_i)}$
  - Example:
    - Blue:
      - Distance: 1
      - Weight: 0.5
    - Orange:
      - Distances: 1.5, 1.6
      - Weight: 0.4 + 0.38 = 0.78
    - Red:
      - Distances: 1.1, 1.3
      - Weight: 0.48 + 0.43 = 0.91
  - ***Label(p)*** = **Red**

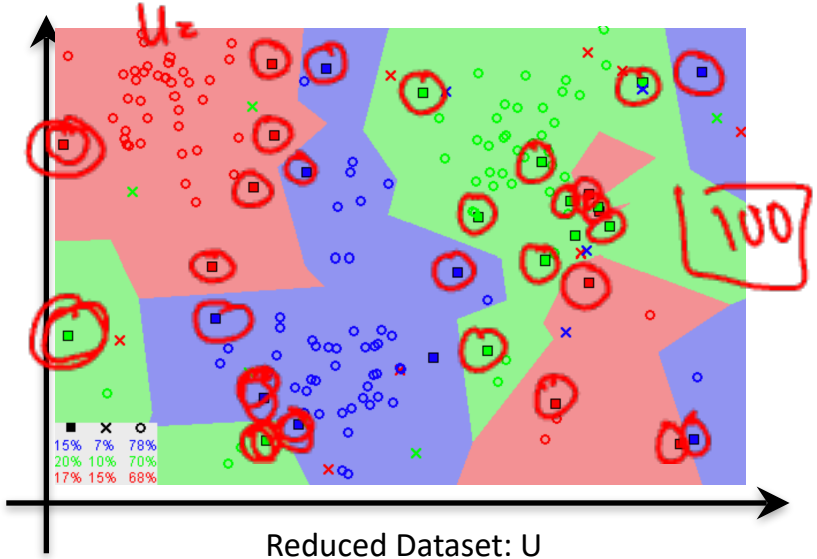


# Data Reduction: Condensed NN

- Given a training set X, and the condensed set U ={},
  1. Choose an x whose nearest prototype in U has a different label than x.
  2. Move x from X to U
  3. Repeat until no more prototypes are added to U.
- Use U instead of X for classification.



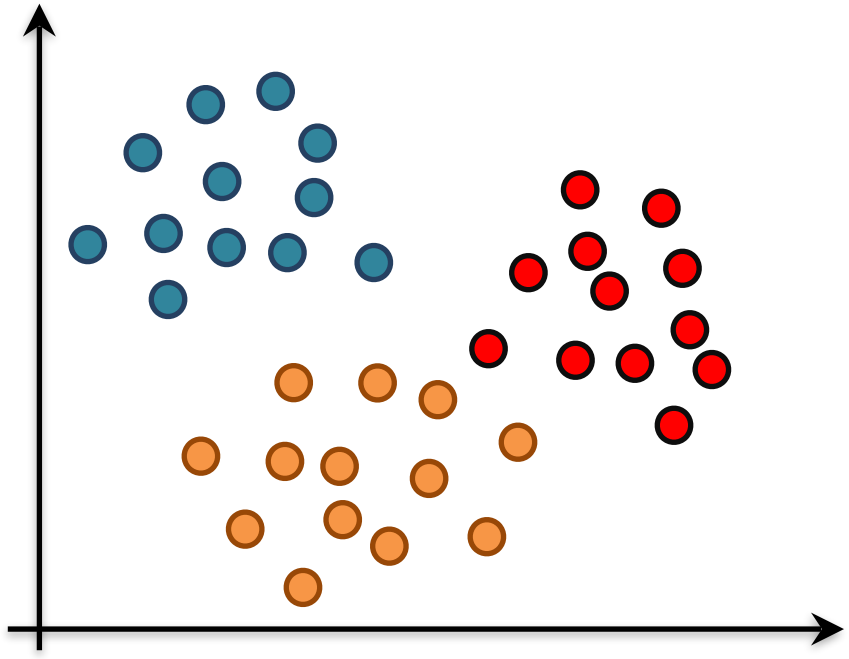
Original Dataset: X

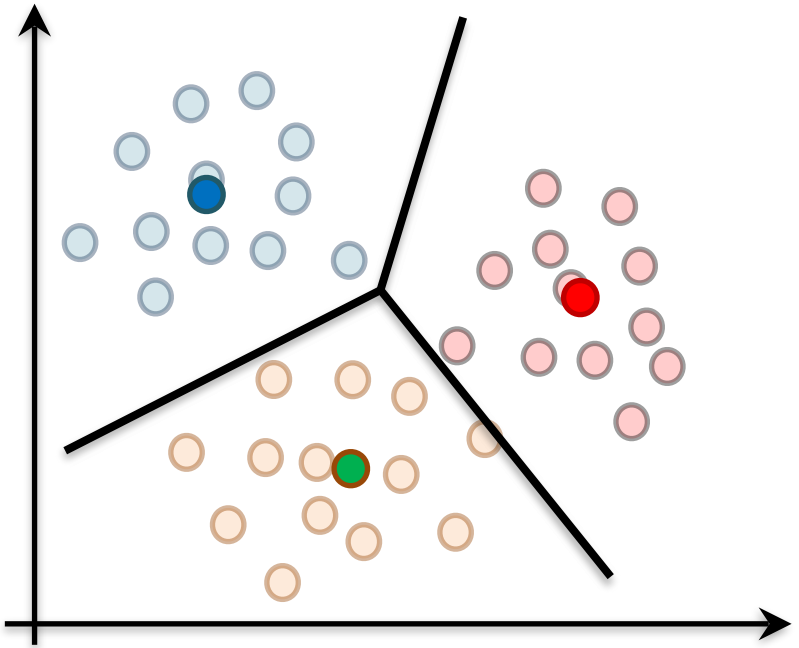



Reduced Dataset: U

# Nearest Mean Classifier

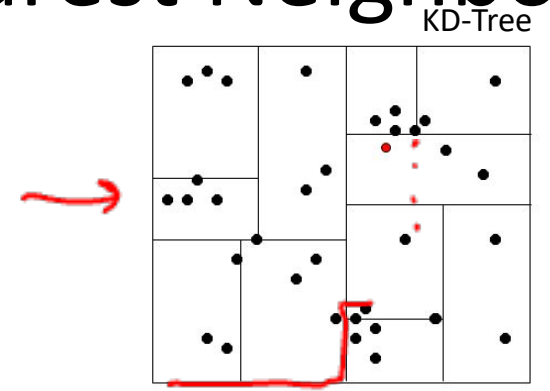
- Represent each class by a single prototype; Its Mean

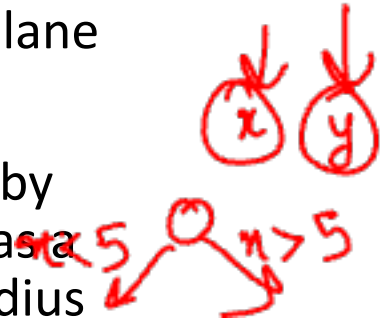


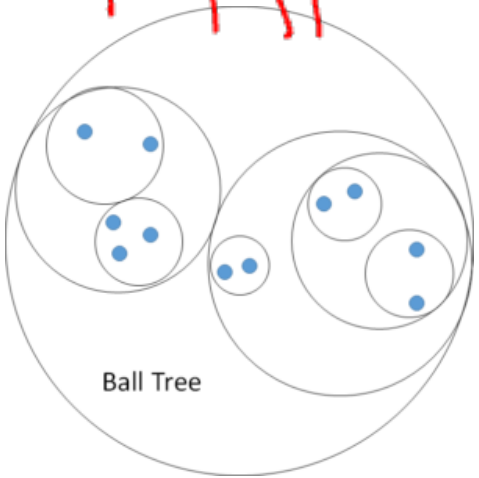


# Fast / Approximate Nearest Neighbor

- Quick search for NN with possible errors
- KD Tree
  - Binary space-partitioning trees with axis-parallel splits. Each node is a hyperplane
- Ball Tree
  - Samples are grouped by spheres. Each node has a specific center and radius



# scikit-learn Usage

```
>>> from sklearn.neighbors import NearestNeighbors
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> nbrs = NearestNeighbors(n_neighbors=2, algorithm='ball_tree').fit(X)
>>> distances, indices = nbrs.kneighbors(X)
```

The documentation contains lot of useful details and explanations

https://scikit-learn.org/stable/modules/neighbors.html

# Some use cases for k-NN

- Decent performance when lots of data

- Yann LeCunn – MNIST Digit Recognition
  - Handwritten digits
  - 28x28 pixel images: $d = 784$
  - 60,000 training samples
  - 10,000 test samples
- Nearest neighbour is competitive

| | Test Error Rate (%) |
|---|---|
| Linear classifier (1-layer NN) | 12.0 |
| K-nearest-neighbors, Euclidean | 5.0 |
| K-nearest-neighbors, Euclidean, deskewed | 2.4 |
| K-NN, Tangent Distance, 16x16 | 1.1 |
| K-NN, shape context matching | 0.67 |
| 1000 RBF + linear classifier | 3.6 |
| SVM deg 4 polynomial | 1.1 |
| 2-layer NN, 300 hidden units | 4.7 |
| 2-layer NN, 300 HU, [deskewing] | 1.6 |
| LeNet-5, [distortions] | 0.8 |
| Boosted LeNet-4, [distortions] | 0.7 |

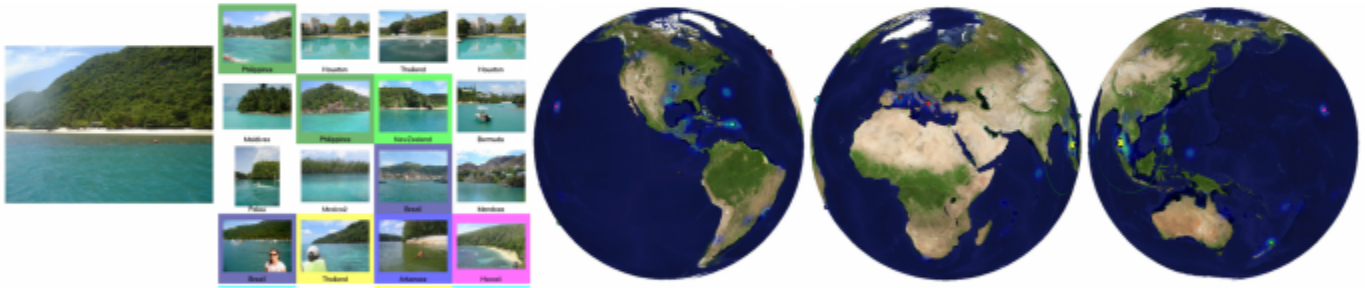
# Some use cases for k-NN

- Problem: Where (e.g., which country or GPS location) was this picture taken?

[Paper: James Hays, Alexei A. Efros. im2gps: estimating geographic information from a single image. CVPR'08. Project page: http://graphics.cs.cmu.edu/projects/im2gps/]

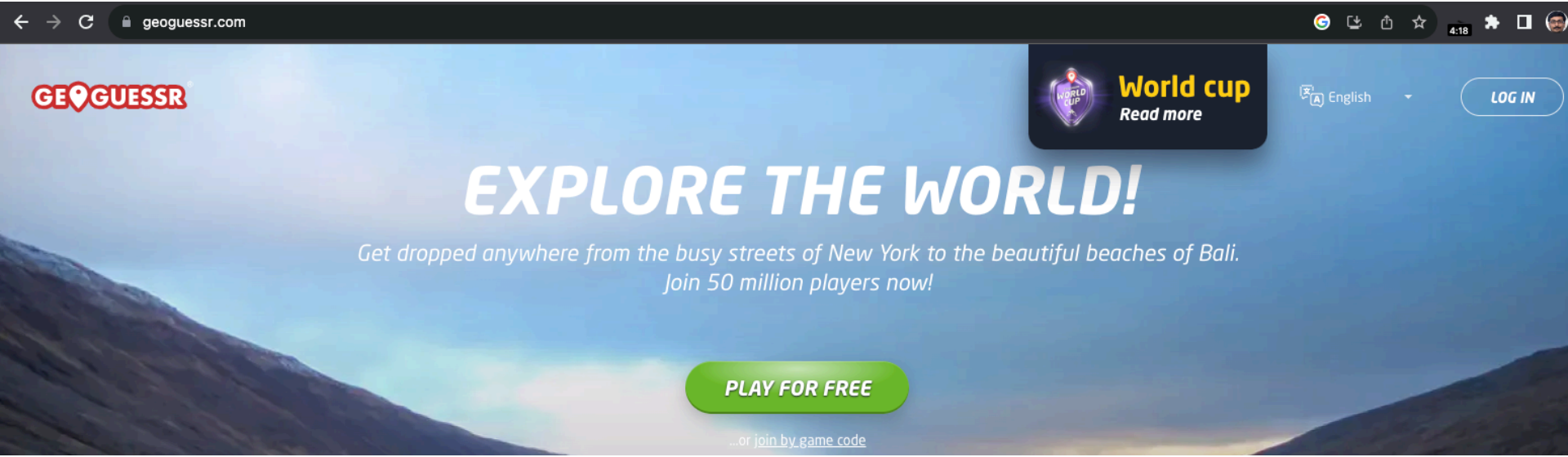
# Some use cases for k-NN

- Problem: Where (eg, which country or GPS location) was this picture taken?
  - Get 6M images from Flickr with gps info (dense sampling across world)
  - Represent each image with meaningful features
  - Do kNN (large $k$ better, they use $k = 120$)!



[Paper: James Hays, Alexei A. Efros. im2gps: estimating geographic information from a single image. CVPR'08. Project page: http://graphics.cs.cmu.edu/projects/im2gps/]

# GeoGuessr


geoguessr.com
GEOGUESSR
World cup
Read more
English
LOG IN
EXPLORE THE WORLD!
Get dropped anywhere from the busy streets of New York to the beautiful beaches of Bali.
Join 50 million players now!
PLAY FOR FREE
...or join by game code

# WHO WOULD WIN?

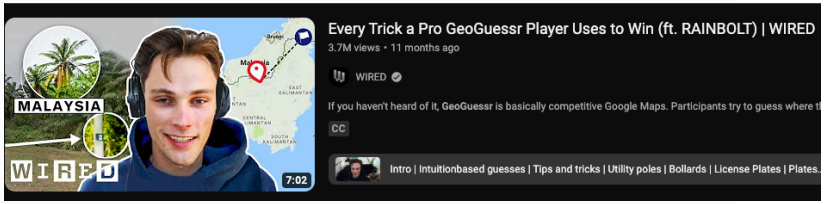
graphics.cs.cmu.edu/projects/im2gps/

IM2GPS: estimating geographic information from a single image

People

James Hays

Alexei Efros




https://www.youtube.com/watch?v=0p5Eb4OSZCs


The Return of the
KNN

# ACL 2023 (#1 NLP conference)

**"Low-Resource" Text Classification: A Parameter-Free Classification Method with Compressors**

**Zhiying Jiang**[1,2], **Matthew Y.R. Yang**[1], **Mikhail Tsirlin**[1], **Raphael Tang**[1], **Yiqin Dai**[2] and **Jimmy Lin**[1]

[1] University of Waterloo [2] AFAIK

{zhiying.jiang, m259yang, mtsirlin, r33tang}@uwaterloo.ca
quinn@afaik.io jimmylin@uwaterloo.ca

https://aclanthology.org/2023.findings-acl.426.pdf

# Intuition

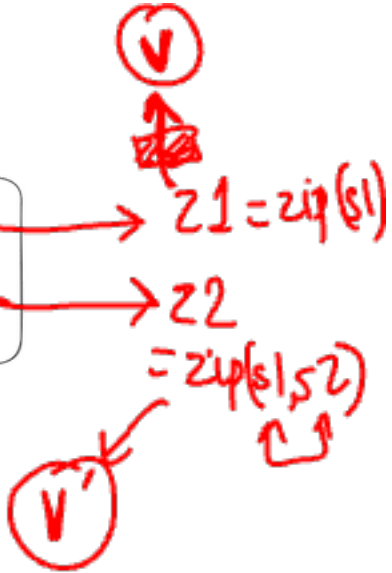
s1. The sun sets over the calm ocean
s2. A peaceful evening as the sun sets over the sea
s3. Jazz music fills the air as the sun rises in the morning

Bag of words

| word | s1 | s2 | s3 |
|---|---|---|---|
| the | 2 | 2 | 3 |
| sun | 1 | 1 | 1 |
| sets | 1 | 1 | 0 |
| over | 1 | 1 | 0 |
| calm | 1 | 0 | 0 |
| ocean | 1 | 0 | 0 |
| a | 0 | 1 | 0 |
| peaceful | 0 | 1 | 0 |
| evening | 0 | 1 | 0 |
| as | 0 | 1 | 1 |
| sea | 0 | 1 | 0 |
| jazz | 0 | 0 | 1 |
| music | 0 | 0 | 1 |
| fills | 0 | 0 | 1 |
| air | 0 | 0 | 1 |
| rises | 0 | 0 | 1 |
| morning | 0 | 0 | 1 |

The sun sets over the calm ocean
A peaceful evening as the sun sets over the sea

Common words/phrases b/w s1 and 2 that LZ77 will compress





https://twitter.com/abhi9u/status/1688150392738385920

```
To classify a sample t:
1 compress t using gzip
2 for each sample s in the training dataset:
  2.1 compress s using gzip
  2.2 compute distance between gzip(s) and gzip(t)
3 find k-nearest neighbours for t based on distances
computed in 2.2
4 pick the majority class as the target label from the k
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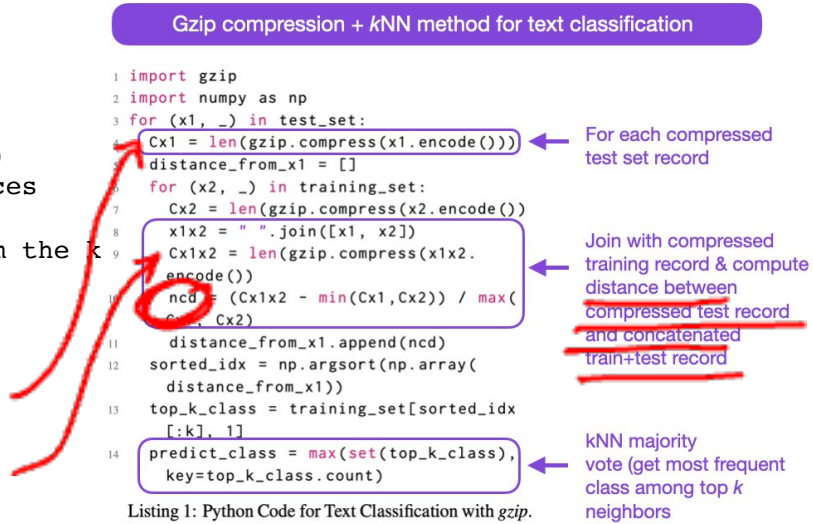

Listing 1: Python Code for Text Classification with *gzip*.

https://codeconfessions.substack.com/p/decoding-the-acl-paper-gzip-and-knn
https://pbs.twimg.com/media/F09sVTxWAAEjEZE?format=jpg&name=large

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**Gzip compression + *k*NN method for text classification**

```python
import gzip
import numpy as np
for (x1, _) in test_set:
    Cx1 = len(gzip.compress(x1.encode()))
    distance_from_x1 = []
    for (x2, _) in training_set:
        Cx2 = len(gzip.compress(x2.encode())
        x1x2 = " ".join([x1, x2])
        Cx1x2 = len(gzip.compress(x1x2.
        encode())
        ncd = (Cx1x2 - min(Cx1,Cx2)) / max(
        Cx1, Cx2)
        distance_from_x1.append(ncd)
    sorted_idx = np.argsort(np.array(
        distance_from_x1))
    top_k_class = training_set[sorted_idx
        [:k], 1]
    predict_class = max(set(top_k_class),
        key=top_k_class.count)
```

For each compressed test set record

Join with compressed training record & compute distance between compressed test record and concatenated train+test record

kNN majority vote (get most frequent class among top *k* neighbors

Listing 1: Python Code for Text Classification with *gzip*.

**Classification accuracy across different test datasets (higher is better)**

Red: outperformed by gzip method

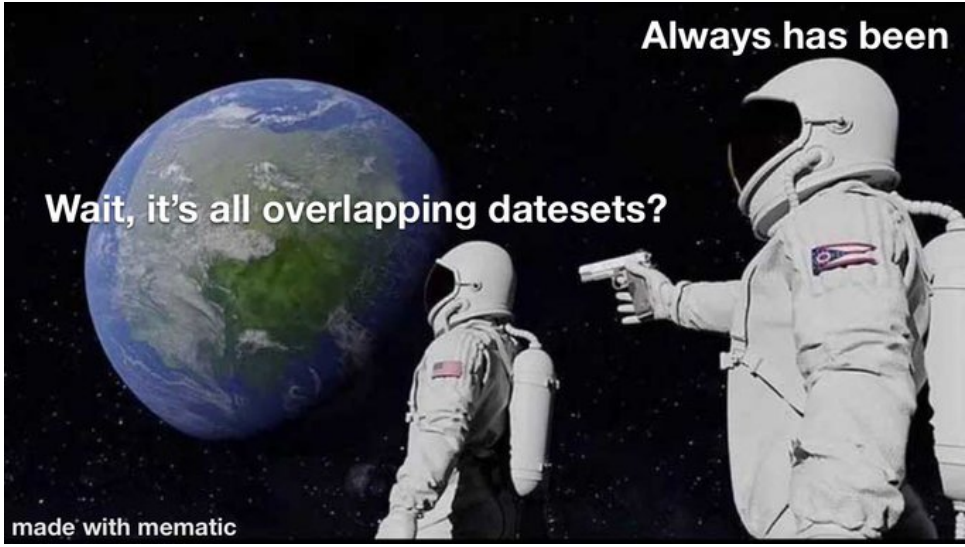
| Model | Pre-training | Training | AGNews | DBpedia | YahooAnswers | 20News | Ohsumed | R8 | R52 |
|---|---|---|---|---|---|---|---|---|---|
| TFIDF+LR | ✗ | ✓ | 0.898 | 0.982 | 0.715 | 0.827 | 0.549 | 0.949 | 0.874 |
| LSTM | ✗ | ✓ | 0.861 | 0.985 | 0.708 | 0.657 | 0.411 | 0.937 | 0.855 |
| Bi-LSTM+Attn | ✗ | ✓ | 0.917 | 0.986 | 0.732 | 0.667 | 0.481 | 0.943 | 0.886 |
| HAN | ✗ | ✓ | 0.896 | 0.986 | 0.745 | 0.646 | 0.462 | 0.960 | 0.914 |
| charCNN | ✗ | ✓ | 0.914 | 0.986 | 0.712 | 0.401 | 0.269 | 0.823 | 0.724 |
| textCNN | ✗ | ✓ | 0.817 | 0.981 | 0.728 | 0.751 | 0.570 | 0.951 | 0.895 |
| RCNN | ✗ | ✓ | 0.912 | 0.984 | 0.702 | 0.716 | 0.472 | 0.810 | 0.773 |
| VDCNN | ✗ | ✓ | 0.913 | 0.987 | 0.734 | 0.491 | 0.237 | 0.858 | 0.750 |
| fastText | ✗ | ✓ | 0.911 | 0.978 | 0.702 | 0.690 | 0.218 | 0.827 | 0.571 |
| BERT | ✓ | ✓ | 0.944 | 0.992 | 0.768 | 0.868 | 0.741 | 0.982 | 0.960 |
| W2V | ✓ | ✗ | 0.892 | 0.961 | 0.689 | 0.460 | 0.284 | 0.930 | 0.856 |
| SentBERT | ✓ | ✗ | 0.940 | 0.937 | 0.782 | 0.778 | 0.719 | 0.947 | 0.910 |
| TextLength | ✗ | ✗ | 0.275 | 0.093 | 0.105 | 0.053 | 0.090 | 0.455 | 0.362 |
| *gzip* (ours) | ✗ | ✗ | 0.937 | 0.970 | 0.638 | 0.685 | 0.521 | 0.954 | 0.896 |

Proposed gzip method

https://codeconfessions.substack.com/p/decoding-the-acl-paper-gzip-and-knn
https://pbs.twimg.com/media/F09sVTxWAAEjEZE?format=jpg&name=large

Complex problems have simple, easy to understand, wrong answers.

Erik Spiekermann


https://twitter.com/yanndubs/status/1681325900170776578

https://twitter.com/predict_addict/status/1681562362569019393


```
%%capture --no-stdout

from data import *
dataloaders = dict(DengueFilipino=load_filipino,
                   KirundiNews=load_kirnews,
                   KinyarwandaNews=load_kinnews,
                   SwahiliNews=load_swahili)

for data_name, loader in dataloaders.items():
    train, test = loader();
    overlap = 1 - len(set(test) - set(train)) / len(set(test))
    print(data_name, f"train<->test overlap: {overlap * 100:.1f}%")
```

```
DengueFilipino train<->test overlap: 100.0%
KirundiNews train<->test overlap: 90.4%
KinyarwandaNews train<->test overlap: 23.8%
SwahiliNews train<->test overlap: 0.5%
```

**Lucas Beyer** @giffmana · Jul 18

Looks like the gzip paper I was enthusiastic about ov[illegible]ted its scores because of a bug in the code: it did top-2 knn instead of k=2.

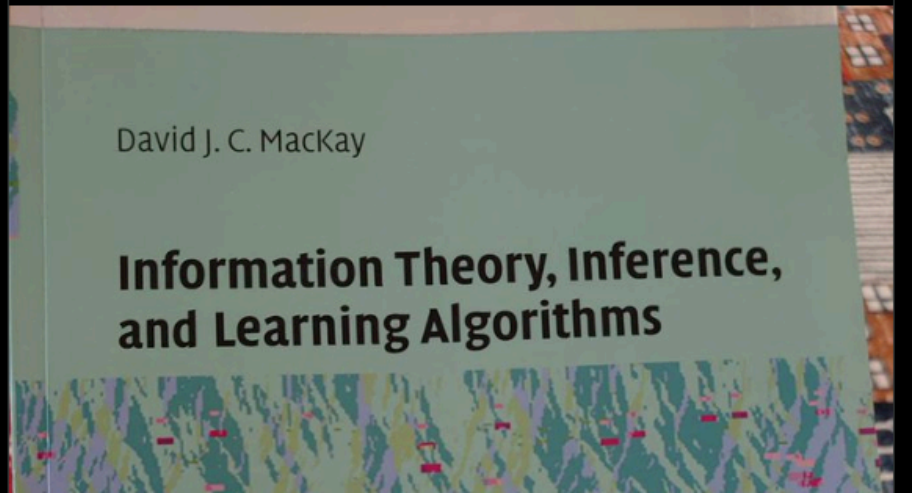
We should remember this as (yet another) a strong case for testing in ml code.

I still like that it put a new idea in my toolbox. twitter.com/amilios/status...


https://twitter.com/yanndubs/status/1681325900170776578

**Paul Snively**
@paul_snively

Should be much better known that it is. Makes explicit that compression == learning, up to and including that axioms in a logical system are compressed information, and the rules of inference are the "decompression" algorithm.

> 
> **Frank Nielsen** @FrnkNlsn · Aug 12
> A very unique textbook "Information theory, inference and learning algorithms" by Sir MacKay combining Information Theory with Machine Learning.
>
> Nicely written!
>
> Book PDF freely available at:
> 👉inference.org.uk/itila/book.html
>
> 
> 

# Related topics