Bayes Theorem

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- Doctor: “Allen: you have a very rare disease which affects 0.1% of the population in the world.”
- Allen: “How certain is it that I have the disease?”
- Doctor: “The test accurately find 99% of the people who have the disease. False alarm rate of the test is 1%.”

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- **Good news for Allen: that's not actually correct !**
- **`Thanks to Bayes Theorem !!!`**

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Bayes Theorem

Probability of observing event E given that hypothesis H is true

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- $P(H) = 0.001$
- $P(\neg H) = 0.999$
- $P(E \mid H) = 0.99$
- $P(E \mid \neg H) = 0.01$ (False alarm rate)

Chance of Allen actually having disease

- Plugging all the numbers, we get

$$P(H|E) = \frac{0.99 * 0.001}{0.99 * 0.001 + 0.01 * 0.999} = 0.09$$

- Only 9% chance that Allen having the disease !

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- 10% of the electric vehicles produced by Machine A are defective.
- 5% percent of the electric vehicles from Machine B are defective.
- If an electric vehicle is randomly chosen and found defective, what is the probability that it came from machine A?

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$$= \frac{0.1 * 0.4}{0.1 * 0.4 + 0.05 * 0.6} = \frac{4}{7}$$

Generalized Bayes Theorem

Let $E_1, E_2, \ldots, E_n$ be (pairwise) mutually exclusive events such that $E_1 \cup E_2 \cdots \cup E_n = S$, where S denotes the sample space. Let F be an event such that $P(F) \neq 0$, Then

$$P(E_1 \mid F) = \frac{P(F \mid E_1)P(E_1)}{P(F \mid E_1)P(E_1) + P(F \mid E_2)P(E_2) + \cdots + P(F \mid E_n)P(E_n)}$$

Questions?

Bayes Classifier

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- As the name suggests, it is based on Bayes Theorem.
- Lets see …

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	the	red	dog	cat	eats	food
1. the red dog	1	1	1	0	0	0
2. cat eats dog	0	0	1	1	1	0
3. dog eats food	0	0	1	0	1	1
4. red cat eats	0	1	0	1	1	0

- $\mathscr{X}$ could be a space of vectors in $\mathbb{R}^d$. Elements of these vectors represent various features of an object. For example, in disease prediction, these feature values can be temperature, blood pressure, oxygen level, heart rate etc.

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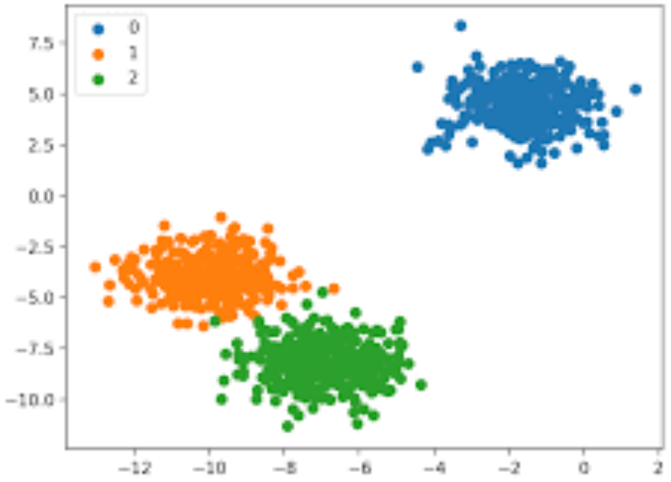

- The goal is to find a classifier which is inspired by Bayes Theorem.

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$$\hat{y} = \mathrm{arg}max_{i \in \{1,\ldots,k\}} P(C = i \mid \mathbf{x}) = \mathrm{arg}max_{i \in \{1,\ldots,K\}} P(C = i)p(\mathbf{x} \mid C = i)$$

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- This can be a tedious task.
- Naive Bayes makes life easy !

Naive Bayes Assumption

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- It assumes that given the class label, features are independent to each other. In other words

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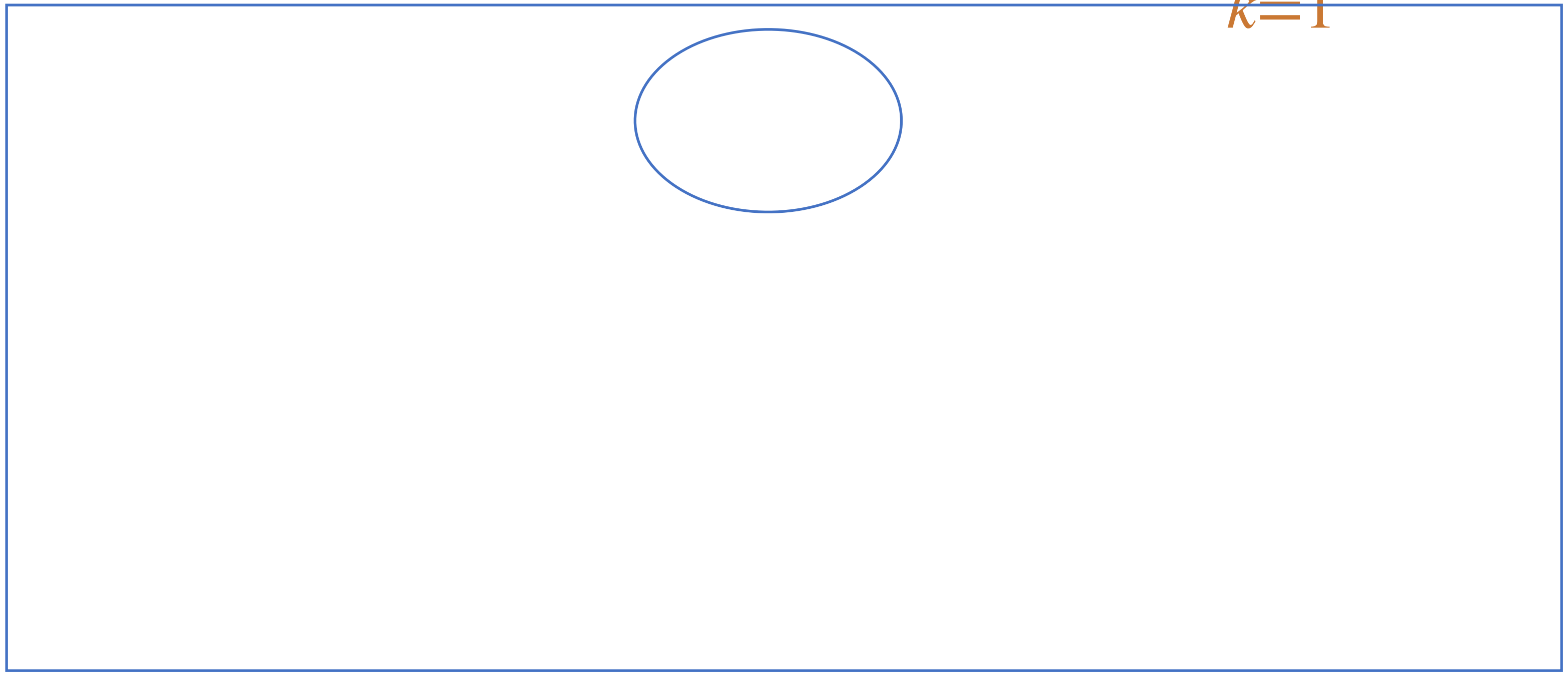

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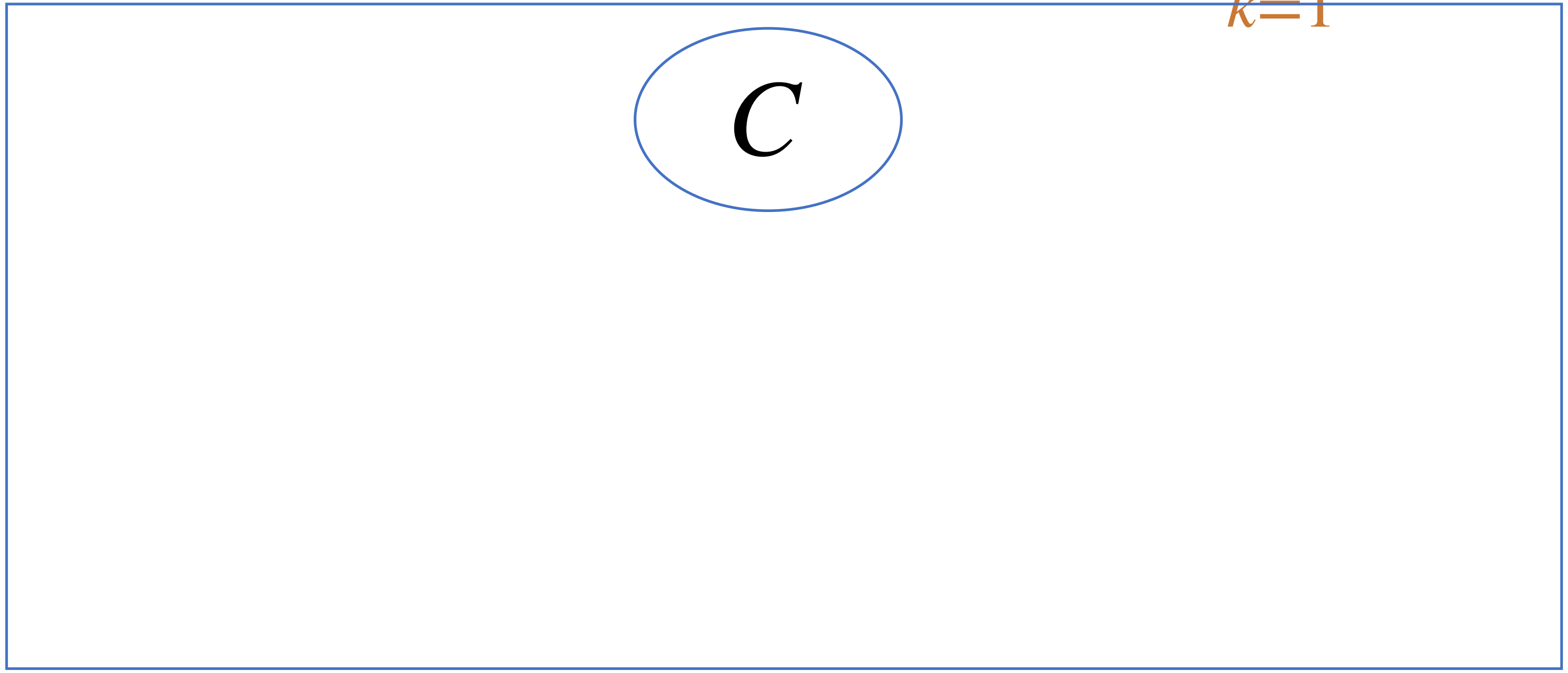

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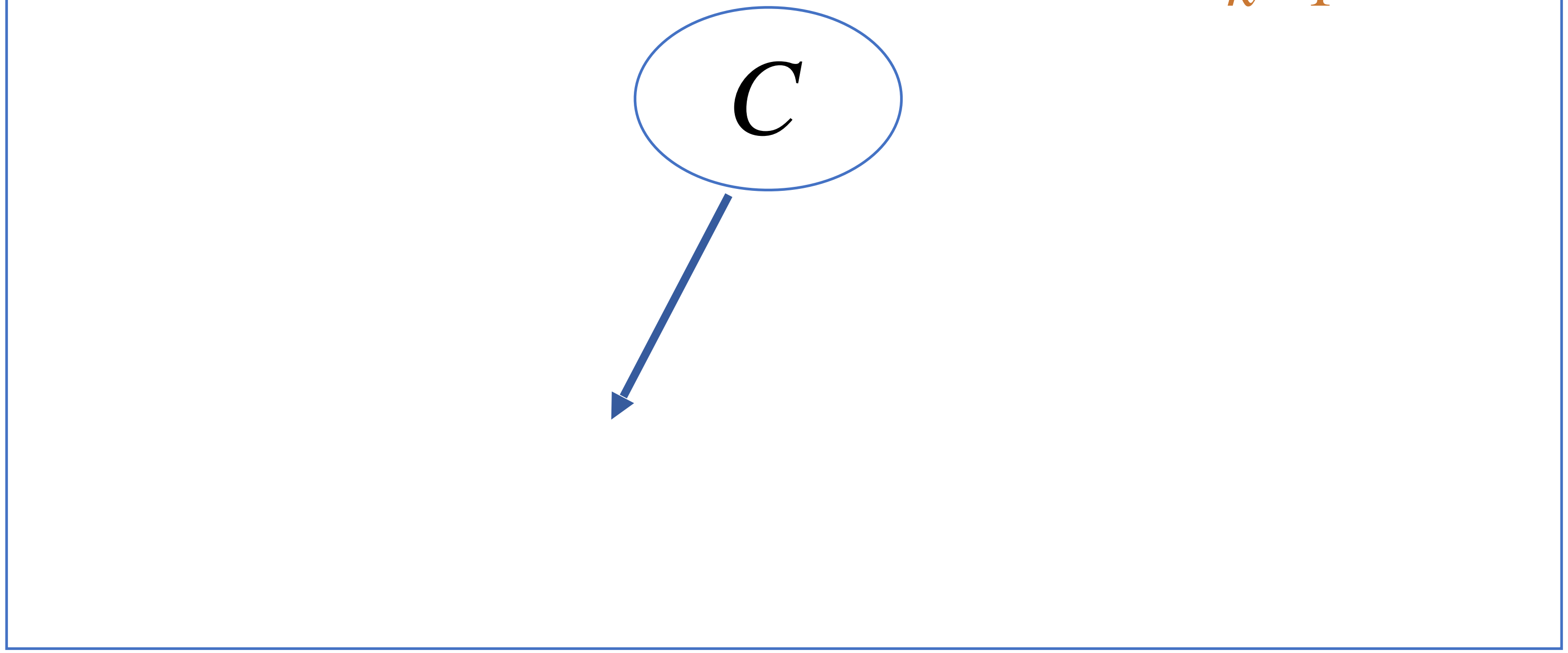

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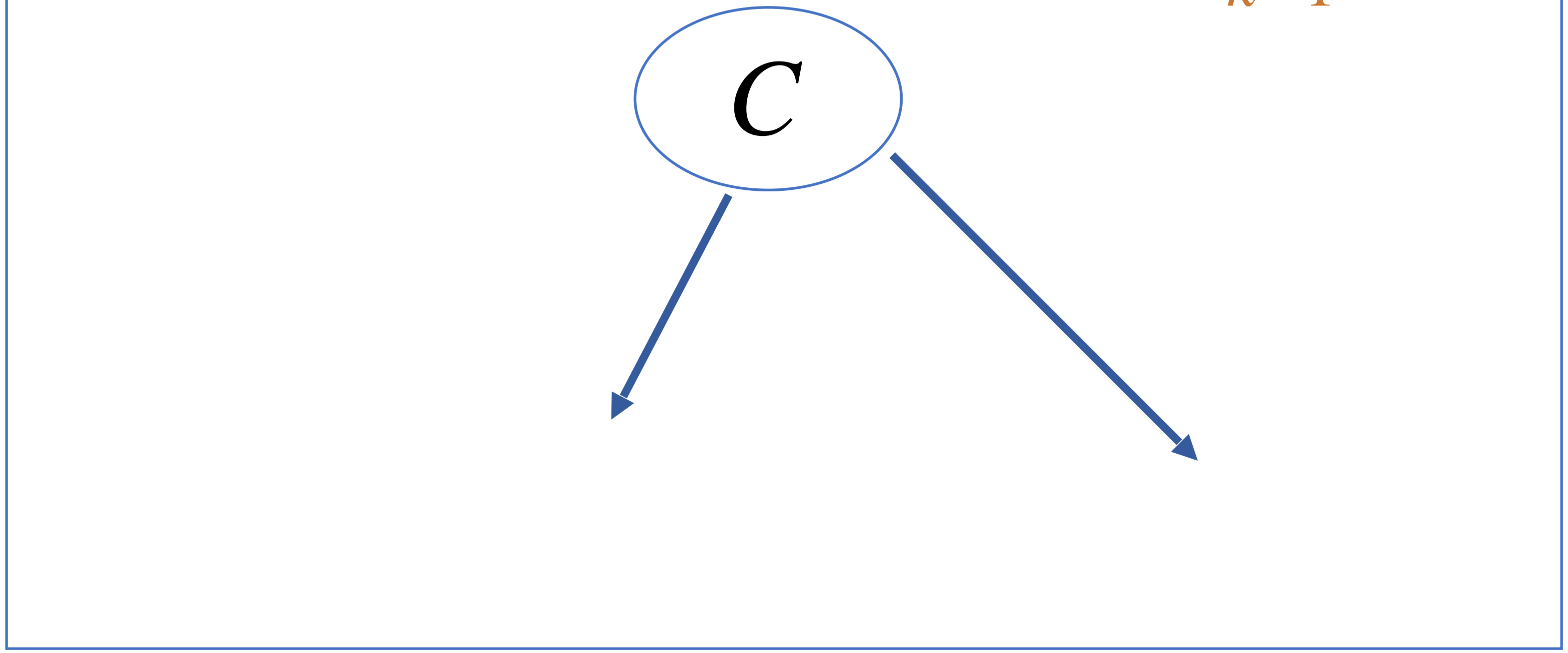

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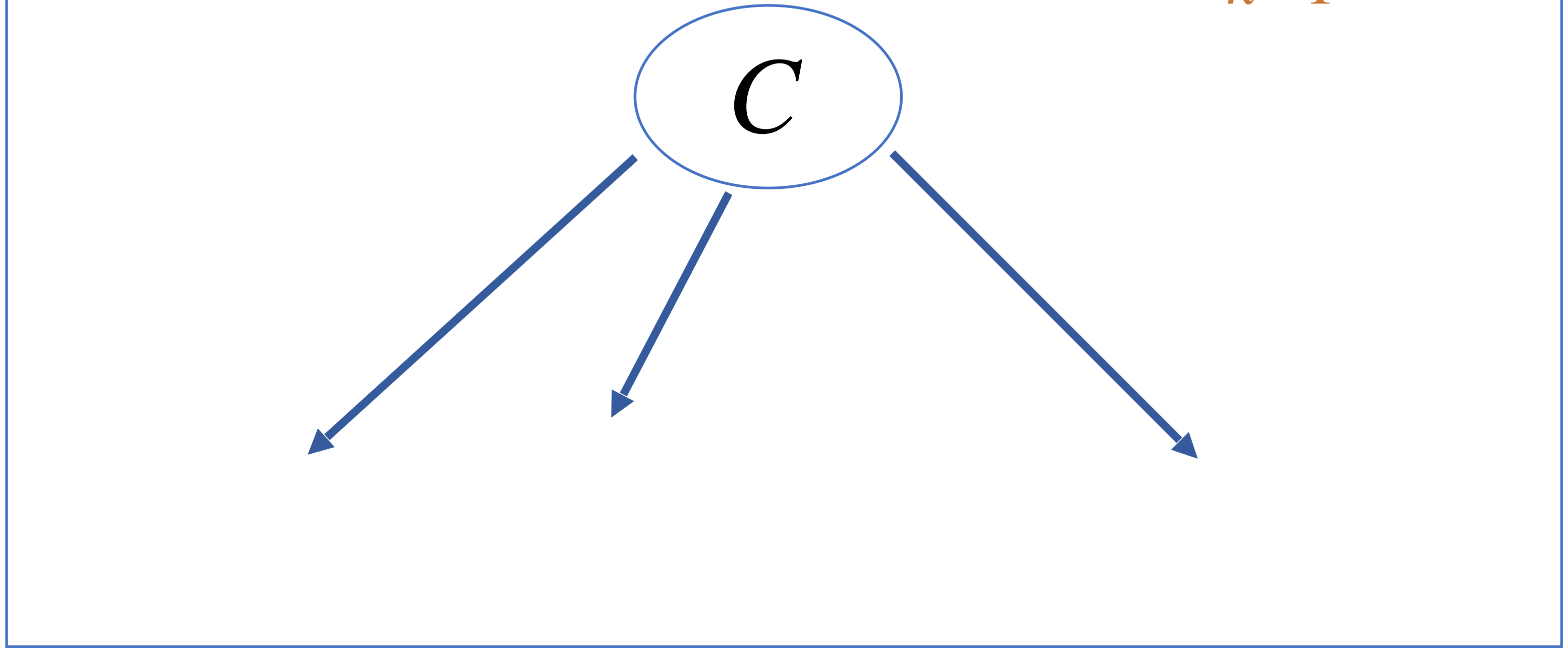

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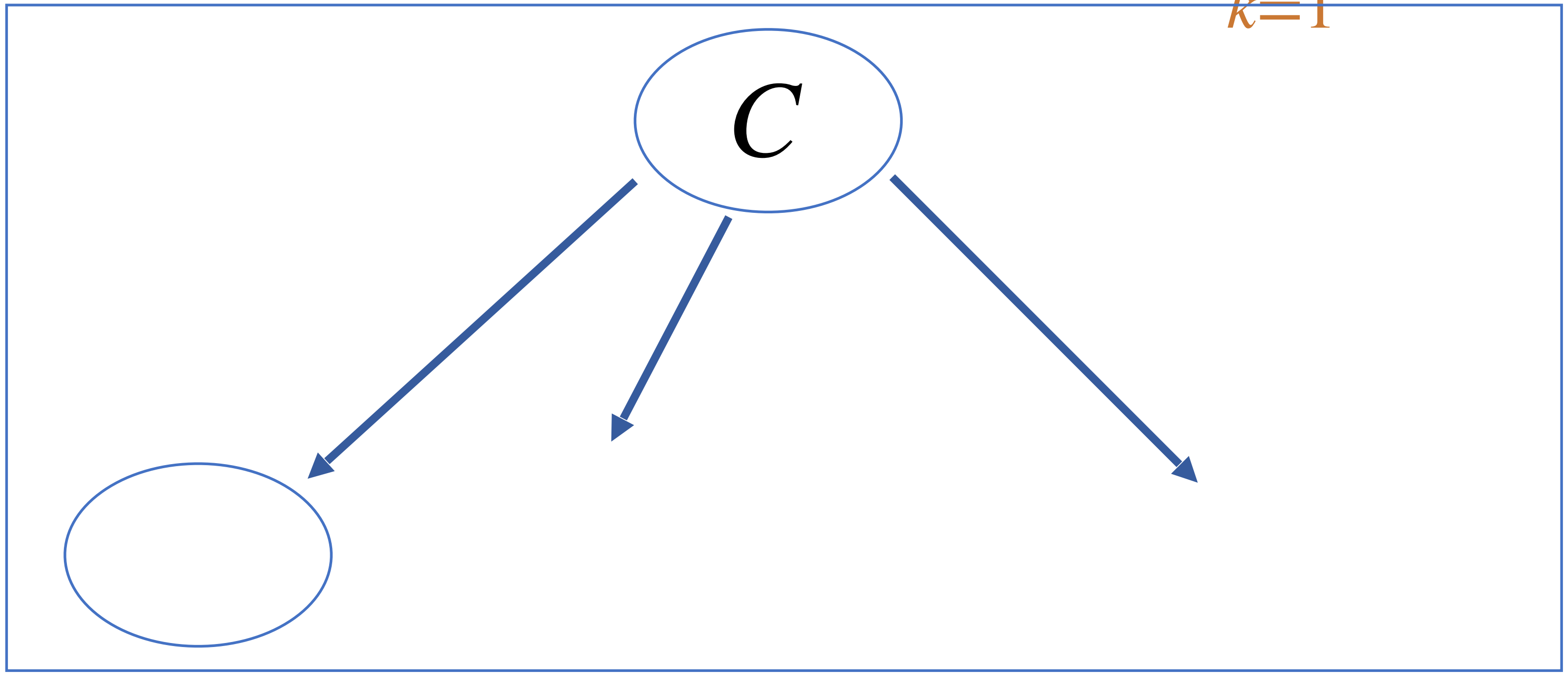

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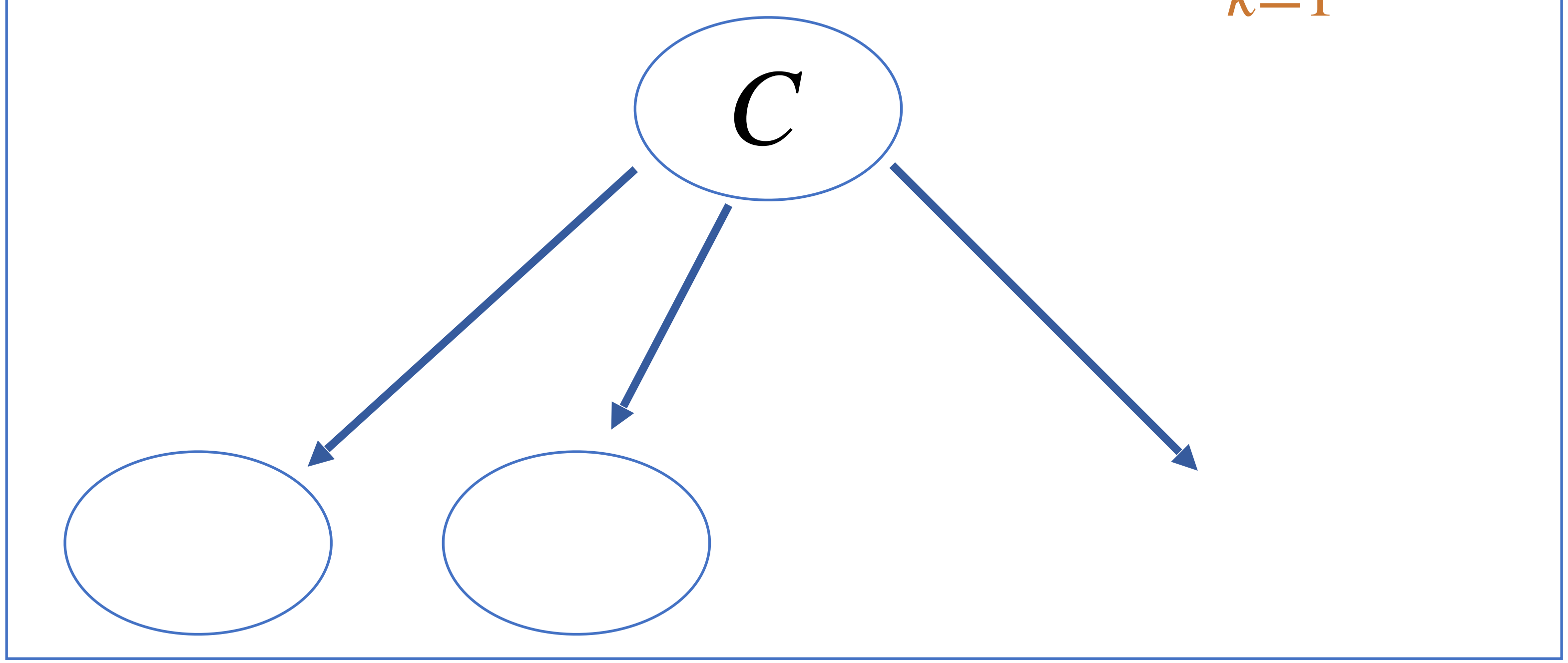

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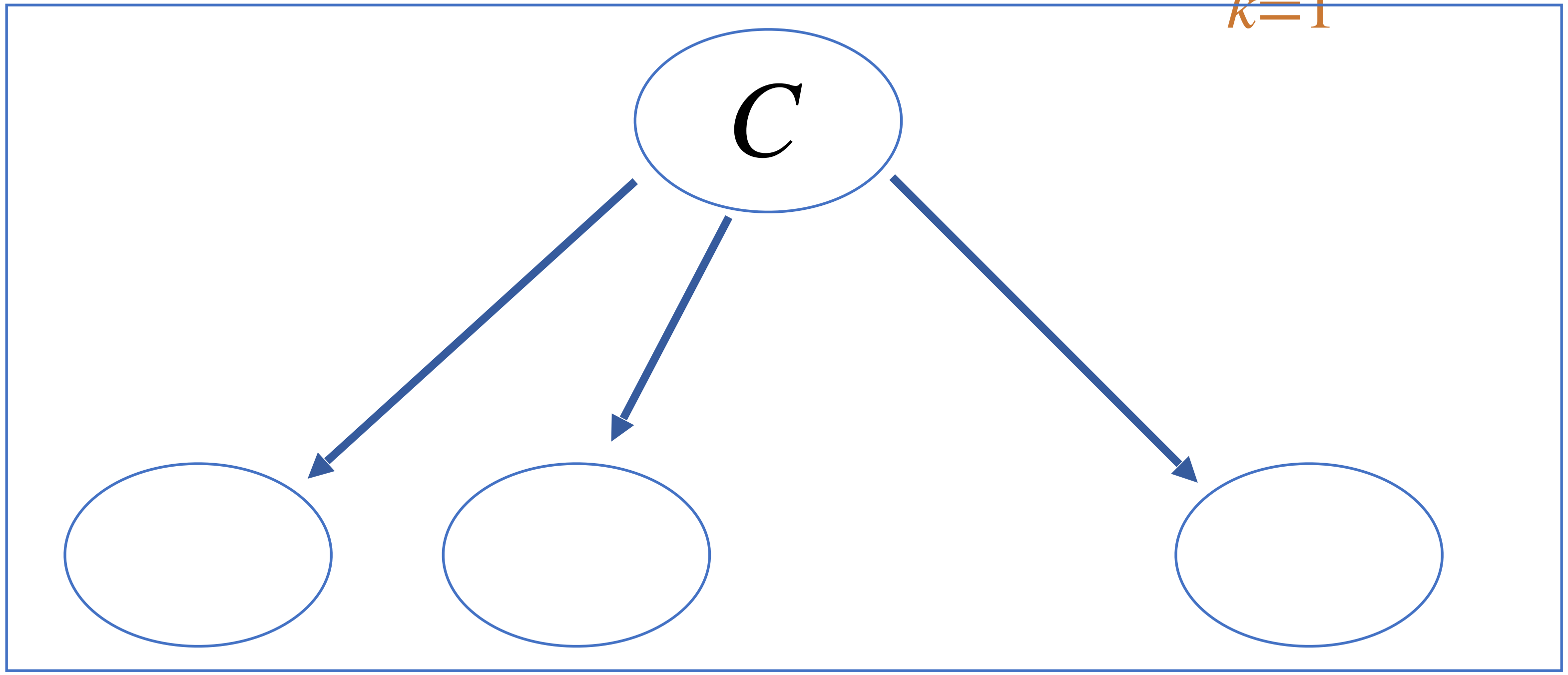

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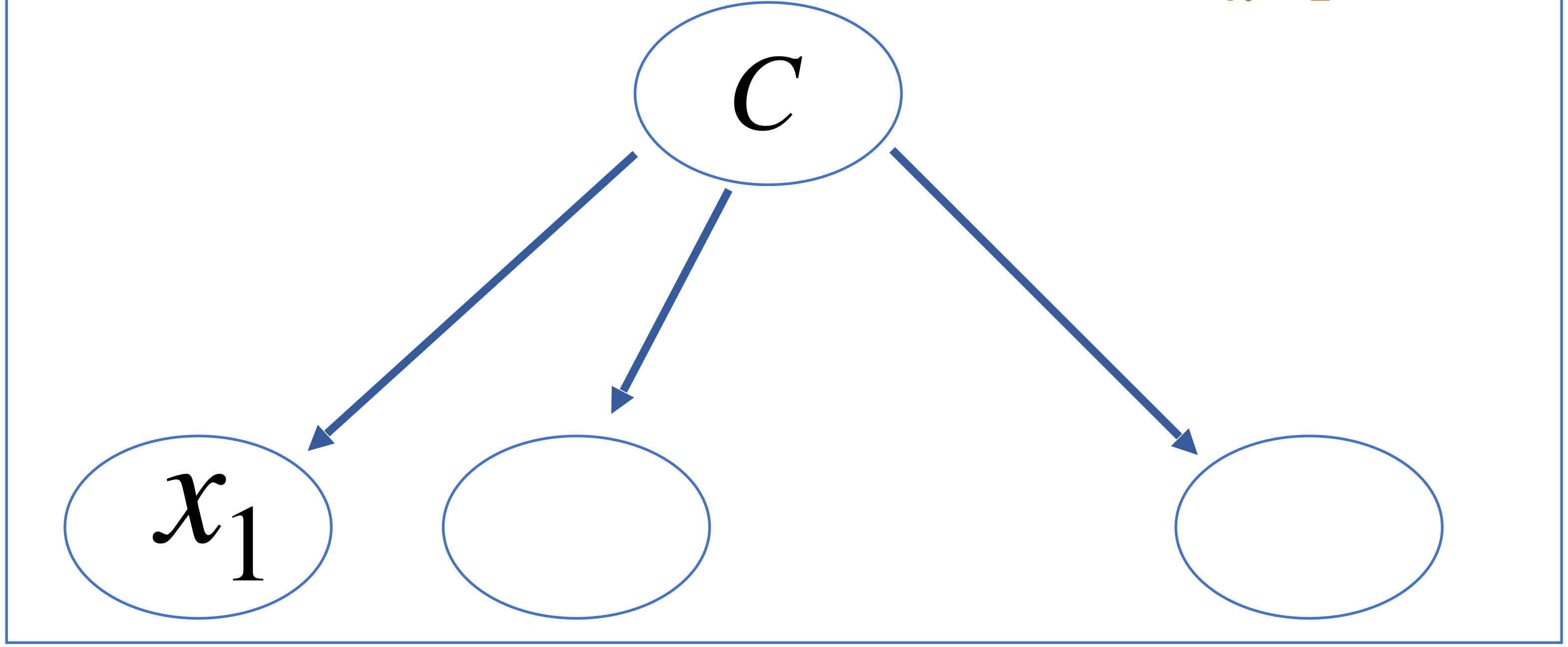

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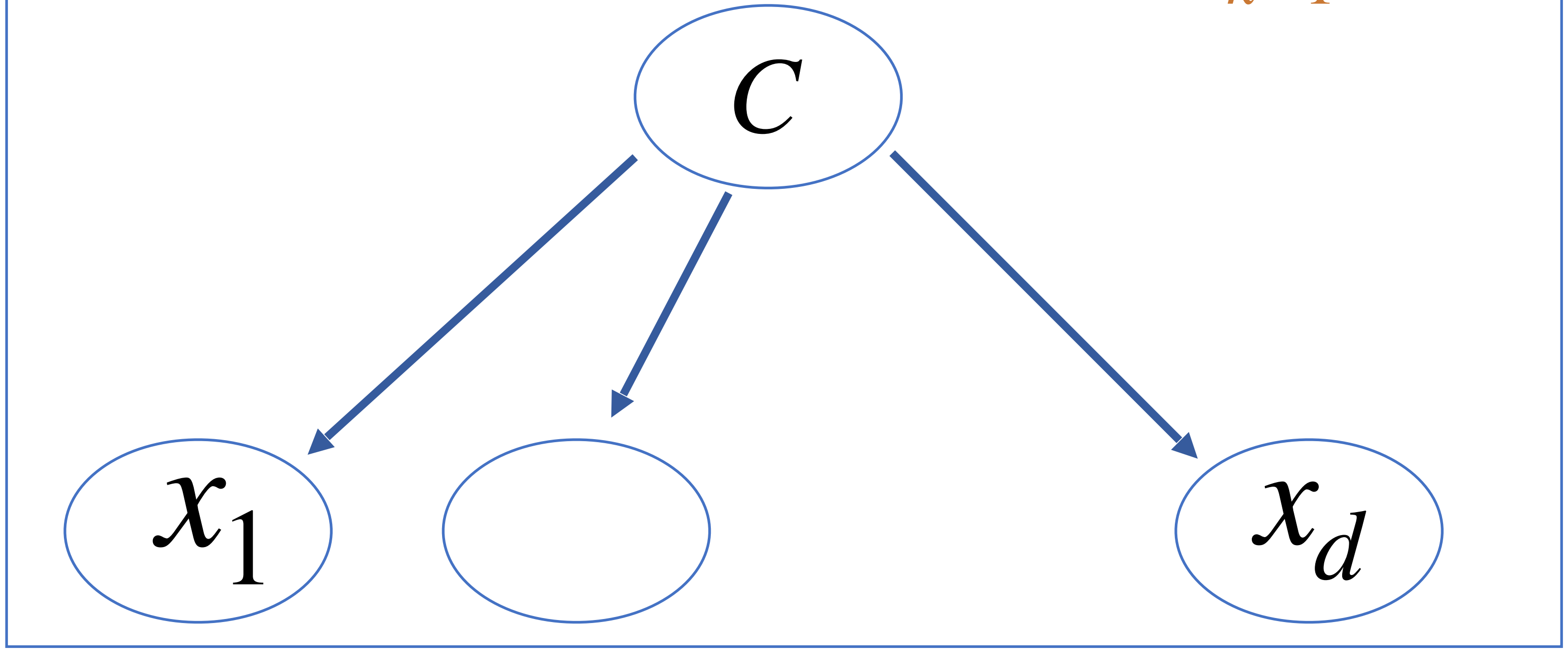

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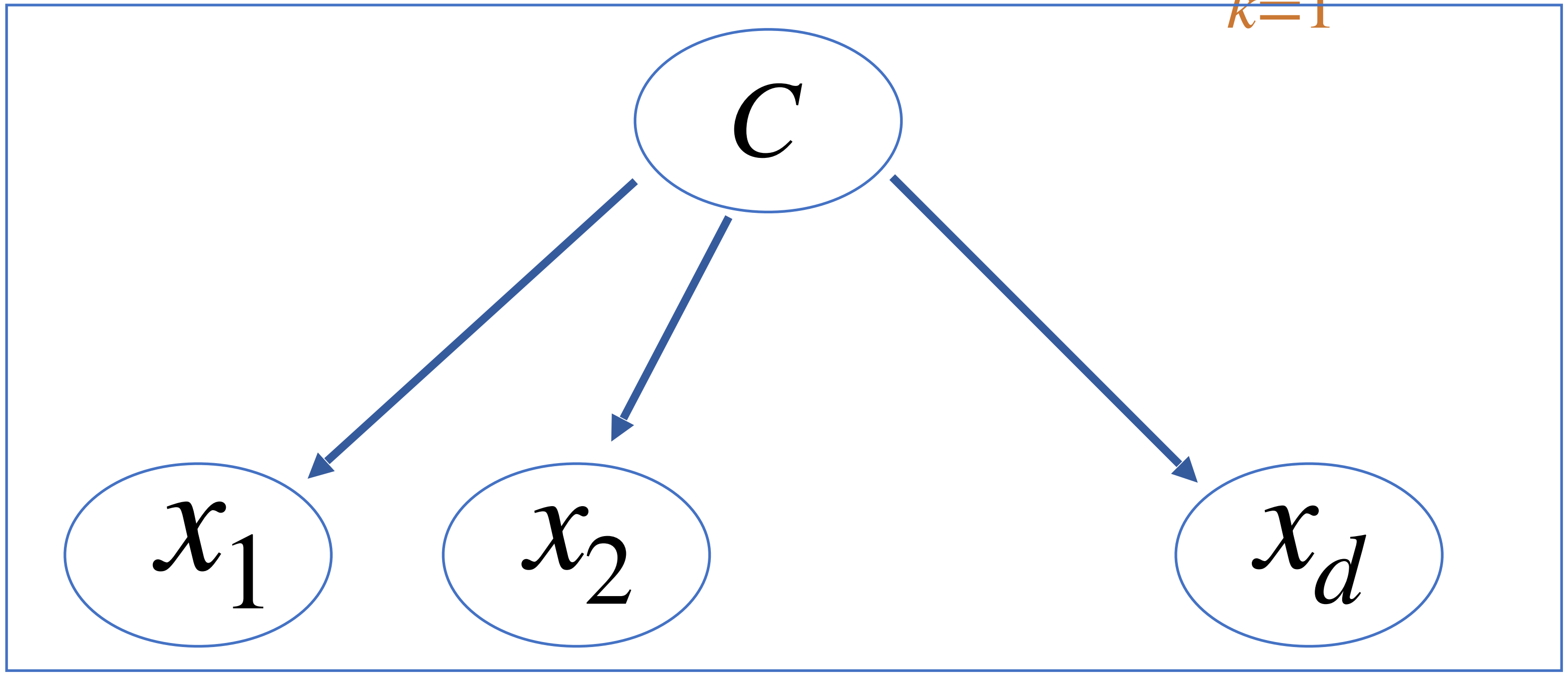

Naive Bayes Classifier

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- **Naive Bayes Classifiers predicts class label $\hat{y}$ as follows:**

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- Thus, it only requires the to estimate the distribution of each feature given the class label.
- This not only makes the overall model simple, also reduces the computations required.

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- When assumption of independence holds, a Naive Bayes classifier performs better compare to other models like logistic regression and you need less training data.

Cons:

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- **Zero Frequency Issue:** If categorical variable has a category in test data set, which was not observed in training data set, then model will assign a zero probability and will be unable to make a prediction. This is often known as "Zero Frequency".
- **Independence Assumption:** Another limitation of Naive Bayes is the assumption of independent predictors. In real life, it is almost impossible that we get a set of predictors which are completely independent.

Questions

Examples of Naive Bayes Classifier

Naive Bayes Classifier Example 1:
Predicting whether to play given the weather conditions

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Predicting whether to play given the weather conditions

Weather Data Record of 14 Days

Day	Outlook	Humidity	Wind	Play
1	Sunny	High	Week	No
2	Sunny	Normal	Strong	Yes
3	Overcast	High	Week	Yes
4	Rain	Normal	Strong	No
5	Rain	Normal	Week	No
6	Sunny	High	Strong	Yes
7	Overcast	Normal	Strong	Yes
8	Sunny	High	Week	Yes
9	Overcast	High	Strong	Yes
10	Rain	Normal	Week	No
11	Overcast	High	Strong	No
12	Rain	High	Week	No
13	Sunny	Normal	Week	No
14	Overcast	High	Strong	Yes

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- **Step 1: Find the posterior probabilities as follows:**

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$$P(Play = Yes \mid Outlook, Humidity, Wind) = \frac{P(Play = Yes)P(Outlook, Humidity, Wind \mid Play = Yes)}{P(Outlook, Humidity, Wind)}$$

$$= \frac{P(Play = Yes)P(Outlook \mid Play = Yes)P(Humidity \mid Play = Yes)P(Wind \mid Play = Yes)}{P(Outlook, Humidity, Wind)}$$

$$P(Play = No \mid Outlook, Humidity, Wind) = \frac{P(Play = No)P(Outlook, Humidity, Wind \mid Play = No)}{P(Outlook, Humidity, Wind)}$$

$$= \frac{P(Play = No)P(Outlook \mid Play = No)P(Humidity \mid Play = No)P(Wind \mid Play = No)}{P(Outlook, Humidity, Wind)}$$

Classification Rule:

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- **Decide to Play if**

$$P(Play = Yes \mid Outlook, Humidity, Wind) > P(Play = No \mid Outlook, Humidity, Wind)$$

OR

$$P(Play = Yes)P(Outlook \mid Play = Yes)P(Humidity \mid Play = Yes)P(Wind \mid Play = Yes)$$
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- **Decide Not to Play if**

$$P(Play = No \mid Outlook, Humidity, Wind) > P(Play = Yes \mid Outlook, Humidity, Wind)$$

OR

$$P(Play = No)P(Outlook \mid Play = No)P(Humidity \mid Play = No)P(Wind \mid Play = No)$$
$$> P(Play = Yes)P(Outlook \mid Play = Yes)P(Humidity \mid Play = Yes)P(Wind \mid Play = Yes)$$

Quantities defining the classifier

$$P(Play = Yes) = 0.5$$
$$P(Play = No) = 0.5$$
$$P(Outlook \mid Play = Yes)$$
$$P(Outlook \mid Play = No)$$
$$P(Humidity \mid Play = Yes)$$
$$P(Humidity \mid Play = No)$$
$$P(Wind \mid Play = Yes)$$
$$P(Wind \mid Play = No)$$

Likelihoods of Outlook

Outlook	$P(Outlook \mid Play = Yes)$	$P(Outlook \mid Play = No)$
Sunny	$P(Outlook = Sunny \mid Play = Yes) = \frac{3}{7}$	$P(Outlook = Sunny \mid Play = No) = \frac{2}{7}$
Overcast	$P(Outlook = Overcast \mid Play = Yes) = \frac{4}{7}$	$P(Outlook = Overcast \mid Play = No) = \frac{1}{7}$
Rain	$P(Outlook = Rain \mid Play = Yes) = 0$	$P(Outlook = Rain \mid Play = No) = \frac{4}{7}$

Likelihoods for Humidity

Humidity	$P(Humidity \mid Play = Yes)$	$P(Humidity \mid Play = No)$
High	$P(Humidity = High \mid Play = Yes) = \frac{5}{7}$	$P(Humidity = High \mid Play = No) = \frac{3}{7}$
Normal	$P(Humidity = Normal \mid Play = Yes) = \frac{2}{7}$	$P(Humidity = High \mid Play = No) = \frac{4}{7}$

Likelihoods of Wind

Wind	$P(Wind \mid Play = Yes)$	$P(Wind \mid Play = No)$
Week	$P(Wind = Week \mid Play = Yes) = \frac{2}{7}$	$P(Wind = Week \mid Play = No) = \frac{5}{7}$
Strong	$P(Wind = Strong \mid Play = Yes) = \frac{5}{7}$	$P(Wind = Strong \mid Play = No) = \frac{2}{7}$

Classification for a Test Example

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- Let on a given day, we observe the following: Outlook=Sunny, Humidity=High, Wind=Strong.
- What is the prediction of Naive Bayes Classifier (Play or Not to Play) ?

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- **Let on a given day, we observe the following: Outlook=Sunny, Humidity=High, Wind=Strong.**
- **What is the prediction of Naive Bayes Classifier (Play or Not to Play) ?**

$$P(Play = Yes)P(Outlook = Sunny \mid Play = Yes)P(Humidity = High \mid Play = Yes)P(Wind = Strong \mid Play = Yes)$$

$$= 0.5 * \frac{3}{7} * \frac{5}{7} * \frac{5}{7}$$

$$= 0.11$$

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Classification for a Test Example

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$$P(Play = Yes)P(Outlook = Sunny|Play = Yes)P(Humidity = High|Play = Yes)P(Wind = Strong|Play = Yes)$$
$$= 0.5*\frac{3}{7}*\frac{5}{7}*\frac{5}{7}$$
$$= 0.11$$

$$P(Play = No)P(Outlook = Sunny|Play = No)P(Humidity = High|Play = No)P(Wind = Strong|Play = No)$$
$$= 0.5*\frac{2}{7}*\frac{3}{7}*\frac{2}{7}$$
$$= 0.02$$

- Thus, the classifier predicts to Play

Naive Bayes Classifier Example 2:
Continuous features with Gaussian Distribution

Class Conditional Distribution of Each Feature is Gaussian

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- **Consider binary classification problem.**
- **Let** $p(x_k \mid C = i) = \mathcal{N}(\mu_{i,k}, \sigma_k^2), i = 1,2; k = 1 \ldots d$

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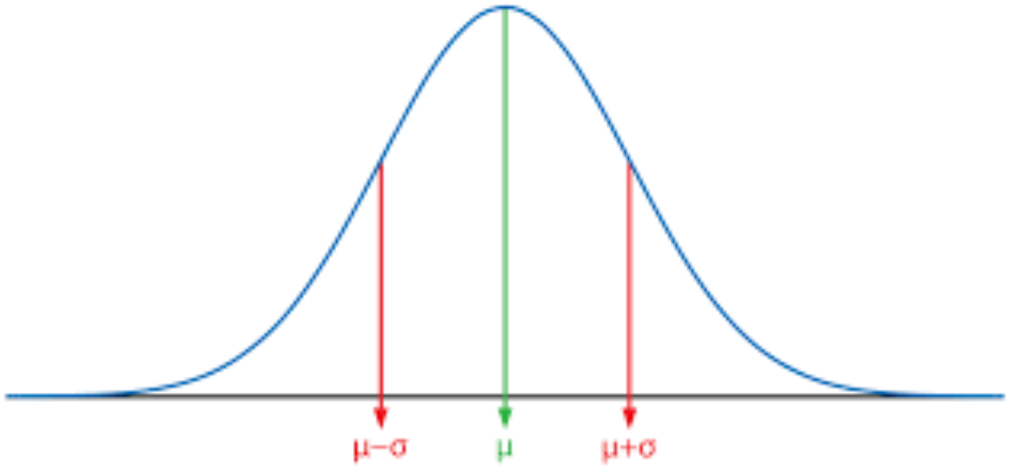

Naive Bayes Classifier

Naive Bayes Classifier

- **For a new example $\mathbf{x}$, the Naive Bayes Classifier predicts class 1 if**

$$P(C=1)\prod_{k=1}^{d} p(x_k \mid C=1) > P(C=2)\prod_{k=1}^{d} p(x_k \mid C=2)$$

$$\Rightarrow \log\left(\prod_{k=1}^{d} \frac{p(x_k \mid C=1)}{p(x_k \mid C=2)}\right) > \log\left(\frac{P(C=2)}{P(C=1)}\right)$$

$$\Rightarrow \sum_{k=1}^{d} \frac{1}{\sigma_k^2}\left[(x_k-\mu_{2,k})^2-(x_k-\mu_{1,k})^2\right] > \log(P(C=2)) - \log(P(C=1))$$

-

$$\Rightarrow \sum_{k=1}^{d} \frac{2\left(\mu_{1,k}-\mu_{2,k}\right)}{\sigma_k^2} x_k > \log(P(C=2)) - \log(P(C=1)) + \sum_{k=1}^{d} \frac{\mu_{1,k}^2-\mu_{2,k}^2}{\sigma_k^2}$$

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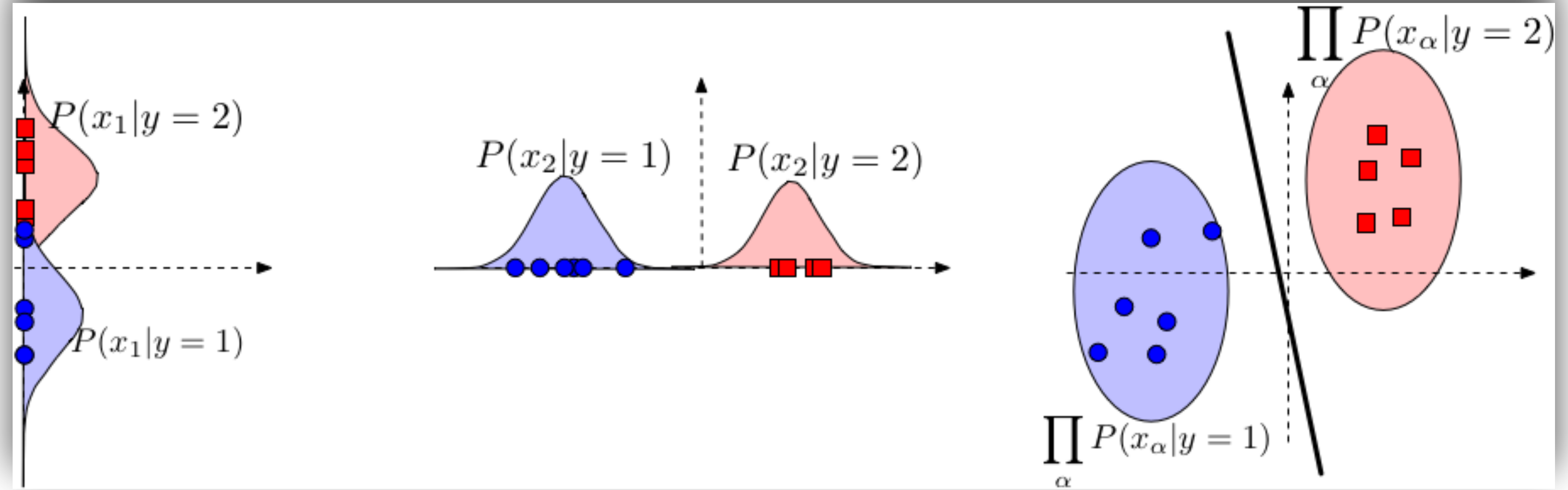
- Naive Bayes Classifier turns out to be Linear

Gaussian Naive Bayes Classifier



Other Applications

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- NEWS Classification: Classify the news type (e.g., sports, political, national, international, finance, stock market, cinema, educations etc.) given the news content.
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- Medical Diagnosis
- Many more ….

scikit-learn Usage

```
>>> from sklearn.datasets import load_iris
>>> from sklearn.model_selection import train_test_split
>>> from sklearn.naive_bayes import GaussianNB
>>> X, y = load_iris(return_X_y=True)
>>> X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.5,
random_state=0)
>>> gnb = GaussianNB()
>>> y_pred = gnb.fit(X_train, y_train).predict(X_test)
```

The documentation contains lot of useful details and explanations

https://scikit-learn.org/stable/modules/naive_bayes.html

Questions?