Properties of commutator

i) $[A, B] = -[B, A]$

ii) $[\alpha A_1 + \beta A_2, B] = \alpha [A_1, B] + \beta [A_2, B]$

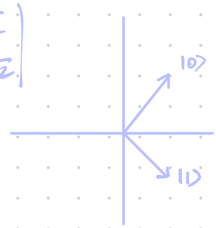
iii) $[A, BC] = [A, B]C + B[A, C]$

iv) $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$

$A^\dagger$ = transpose + conjugate

$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\qquad H = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$



$|+\rangle\langle+| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$|-\rangle\langle-| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$e^{A+B} = 1 + \frac{(A+B)}{1!} + \frac{(A+B)^2}{2!} + \frac{(A+B)^3}{3!} + \dots$

$e^A e^B = (1 + A + \frac{A^2}{2!} + \dots)(1 + B + \frac{B^2}{2!} + \dots)$

$\langle i | x | j \rangle$

$f(A) =$

$AB \quad BA$

$|\psi\rangle \to \lambda$

λ_1, λ_2

$\langle \lambda_1 | Q | \lambda_2 \rangle^\dagger$

$AB|\psi\rangle = BA|\psi\rangle$

$= B\lambda|\psi\rangle$

$A(B|\psi\rangle) = \lambda B|\psi\rangle$

$B|\psi\rangle = \mu|\psi\rangle$

$|\psi\rangle$

$= \langle \lambda_2 | Q | \lambda_1 \rangle$

$\lambda_1 \langle \lambda_2 | \lambda_1 \rangle$

$= \lambda_2 \langle \lambda_2 | \lambda_1 \rangle$

$(\lambda_2 - \lambda_1)\langle \lambda_1 | \lambda_2 \rangle = 0$

$\boxed{\langle \lambda_2 | \lambda_1 \rangle = 0}$

$\lambda \to$

$(A|\psi\rangle)^\dagger = \langle \psi | A^\dagger$

$\langle \psi | \lambda^* = \langle \psi | A^\dagger$

$\lambda^* \langle \psi | \psi \rangle = \langle \psi | A | \psi \rangle$

$\lambda^* = \lambda \langle \psi | \psi \rangle$

$\mathrm{Tr}(\overset{A}{\langle \psi|}\underbrace{\hat{O}|\psi\rangle}^{B})$

$\mathrm{Tr}(\hat{O}|\psi\rangle\langle\psi|)$

$\mathrm{Tr}(\hat{O}\rho)$

$\underset{1\times n}{} \; \underset{n\times n}{\hat{O}} \; |\psi\rangle_{n\times 1} \to 1$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x \qquad \sigma_y \qquad \sigma_z$$

$\det(\sigma_1) = 1$

σ_2

$0 - (i)(-i)$

i^2

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\begin{vmatrix} \lambda-1 & 1 & 1 \\ 1 & \lambda-1 & 1 \\ 1 & 1 & \lambda-1 \end{vmatrix}$$

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1p} \\ 1 & x_{21} & x_{22} & \cdots & x_{2p} \\ 1 & x_{21} & x_{32} & \cdots & x_{3p} \\ \vdots & & & & \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}_{n \times (p+1)}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1} \qquad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{n \times 1}$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}_{(p+1) \times 1} \qquad Y = X\beta + \varepsilon$$

$$L(\beta) = \varepsilon^T \varepsilon = (Y - X\beta)^T (Y - X\beta)$$

$1 \times p+1$

$$\frac{\partial}{\partial \beta} L(\beta) = 0$$

$$\overset{1 \times n \times n \times (p+1)}{-2 Y^T X \beta}$$

$$\frac{\partial}{\partial \beta} (Y - X\beta)^T (Y - X\beta) = \frac{\partial}{\partial \beta} \left(Y^T Y - Y^T X \beta - \beta^T X^T Y + \beta^T X^T X \beta \right)$$

$$= -2 Y^T X + \beta^T (X^T X + X^T X)$$

$$= -2 Y^T X + 2 \beta^T X^T X = 0$$

$$\beta^T X^T X = Y^T X$$

$$X^T X \beta = X^T Y$$

$$\beta = \left[(X^T X)^{-1} X^T \right] Y$$