Performance modeling for Computer systems (Assignement 1)

5 mks

1. Consider $Y = \sum_{i=1}^{N} X_i$ where X_i are iid random variables and N is an independent non negative integer valued random variable. Let $M_X(t)$ and $M_N(t)$ denote the moment generating functions for X and N respectively. Obtain the MGF $M_Y(t)$ in terms of that of X and N.

2. Obtain the second moment of Poisson distribution and Geometric distribution.

3. Consider an $M/G/\infty$ system. Find the distribution (pmf or cdf) of $X(t)$ where $X(t)$ denotes the number of jobs jobs present in the system at time t and where the service time distribution follows 1) Exponential distribution with parameter μ 2) Uniform distribution over interval $[0, 1]$.

4. Show that the Binomial process has independent and stationary increments.

5. Consider a sequence $\{X_i\}$ of i.id exponential random variables with parameter λ.. Define $S_n = \sum_{i=1}^{n} X_i$. Derive the pdf of S_n and obtain its mean and variance.

6. Prove the splitting property of the Poisson process.

7. Prove the merging property of the Poisson process.

8. For a Poisson process $N(t)$ derive the expression for $P(N(s) = k | N(t) = n)$ where $s < t$.