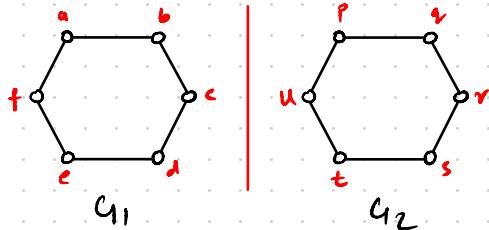


d. 2. Isomorphic Pairs:



WL-Test

Iteration 1:

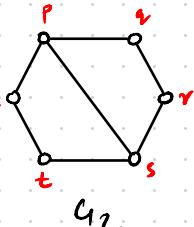
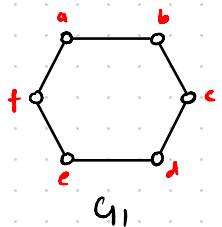
Graph 1			Graph 2		
Node	hash	colour	Node	hash	colour
a	111	3	p	111	3
b	111	3	q	111	3
c	111	3	r	111	3
d	111	3	s	111	3
e	111	3	t	111	3
f	111	3	u	111	3

Iteration 2:

Graph 1			Graph 2		
Node	hash	colour	Node	hash	colour
a	333	9	p	333	9
b	333	9	q	333	9
c	333	9	r	333	9
d	333	9	s	333	9
e	333	9	t	333	9
f	333	9	u	333	9

Since colours have not changed from iteration 1 & 2,
the test has converged and we get the graphs to be isomorphic.

Non-Isomorphic Pairs :



WL-Test

Iteration 1 :

Graph 1			Graph 2		
Node	hash	colour	Node	hash	colour
a	111	3	p	1111	4
b	111	3	q	111	3
c	111	3	r	111	3
d	111	3	s	1111	4
e	111	3	t	111	3
f	111	3	u	111	3

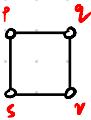
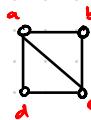
Iteration 2 :

Graph 1			Graph 2		
Node	hash	colour	Node	hash	colour
a	333	9	p	3344	14
b	333	9	q	334	10
c	333	9	r	334	10
d	333	9	s	3344	14
e	333	9	t	334	10
f	333	9	u	334	14

Since colours have not changed from iteration 1 & 2,
the test has converged and we get the graphs to be isomorphic.

Q. 3. a)

1.



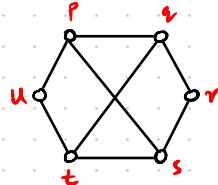
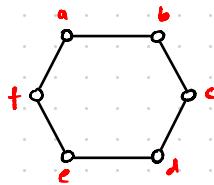
$$\text{Mean: } a, b, c, d = 1, 1, 1, 1 \\ p, q, r, s = 1, 1, 1, 1$$

$$\text{Max: } a, b, c, d = 1, 1, 1, 1 \\ p, q, r, s = 1, 1, 1, 1$$

$$\text{Sum: } a, b, c, d = 3, 2, 3, 2$$

$$p, q, r, s = 2, 2, 2, 2$$

2.



$$\text{Mean: } a, b, c, d, e, f = 1, 1, 1, 1, 1, 1$$

$$p, q, r, s, t, u = 1, 1, 1, 1, 1, 1$$

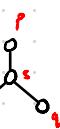
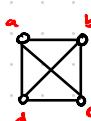
$$\text{Max: } a, b, c, d, e, f = 1, 1, 1, 1, 1, 1$$

$$p, q, r, s, t, u = 1, 1, 1, 1, 1, 1$$

$$\text{Sum: } a, b, c, d, e, f = 2, 2, 2, 2, 2, 2$$

$$p, q, r, s, t, u = 3, 3, 2, 3, 3, 2$$

3.



$$\text{Mean: } a, b, c, d = 1, 1, 1, 1$$

$$p, q, r, s = 1, 1, 1, 1$$

$$\text{Max: } a, b, c, d = 1, 1, 1, 1$$

$$p, q, r, s = 1, 1, 1, 1$$

WL-Test

Graph 1

Node	Embedding
a	1111
b	1111
c	11111
d	1111

Graph 2

Node	Embedding
p	111
q	1111
r	1111
s	1111

∴ The graphs are non-isomorphic

WL-Test

Graph 1

Node	Embedding
a	1111
b	1111
c	111
d	1111
e	1111
f	1111

Graph 2

Node	Embedding
p	11111
q	11111
r	11111
s	11111
t	11111
u	11111

∴ The graphs are non-isomorphic.

WL-Test

Graph 1

Node	Embedding
a	11111
b	11111
c	11111
d	11111

Graph 2

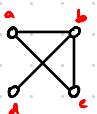
Node	Embedding
p	11
q	11
r	11
s	11111

∴ The graphs are non-isomorphic

Sum: $a, b, c, d = 3, 3, 3, 3$

$p, q, r, s = 1, 1, 1, 3$

4.



Mean: $a, b, c, d = 1, 1, 1, 1$

$p, q, r, s = 1, 1, 1, 1$

Max: $a, b, c, d = 1, 1, 1, 1$

$p, q, r, s = 1, 1, 1, 1$

Sum: $a, b, c, d = 2, 3, 2, 1$

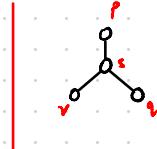
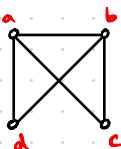
$p, q, r, s = 1, 2, 2, 1$

WL-Test

Graph 1		Graph 2	
Node	Embedding	Node	Embedding
a	111	p	11
b	1111	q	111
c	111	r	111
d	11	s	11

\therefore The graphs are non-isomorphic

5.



Mean: $a, b, c, d = 1, 1, 1, 1$

$p, q, r, s = 1, 1, 1, 1$

Max: $a, b, c, d = 1, 1, 1, 1$

$p, q, r, s = 1, 1, 1, 1$

Sum: $a, b, c, d = 3, 3, 2, 2$

$p, q, r, s = 1, 1, 1, 3$

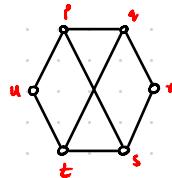
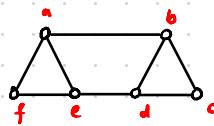
WL-Test

Graph 1		Graph 2	
Node	Embedding	Node	Embedding
a	1111	p	11
b	1111	q	11
c	111	r	11
d	111	s	1111

\therefore The graphs are non-isomorphic

b) GIN fails to differentiate

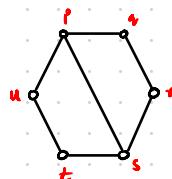
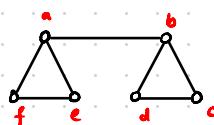
1)



$$\text{Sum: } a, b, c, d, e, f = 3, 3, 2, 3, 3, 2$$

$$p, q, r, s, t, u = 3, 3, 2, 3, 3, 2$$

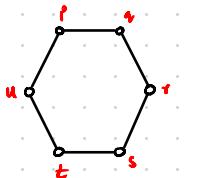
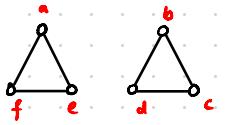
2)



$$\text{Sum: } a, b, c, d, e, f = 3, 3, 2, 2, 2, 2$$

$$p, q, r, s, t, u = 3, 2, 2, 3, 2, 2$$

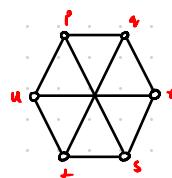
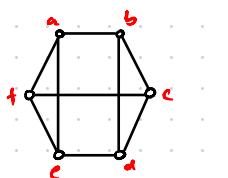
3)



$$\text{Sum: } a, b, c, d, e, f = 2, 2, 2, 2, 2, 2$$

$$p, q, r, s, t, u = 2, 2, 2, 2, 2, 2$$

4)



$$\text{Sum: } a, b, c, d, e, f = 3, 3, 3, 3, 3, 3$$

$$p, q, r, s, t, u = 3, 3, 3, 3, 3, 3$$

WL-Test

Graph 1

Node Embedding

a	1111
b	1111
c	1111
d	1111
e	1111
f	1111

Graph 2

Node Embedding

p	1111
q	1111
r	1111
s	1111
t	1111
u	1111

∴ WL-Test fails

WL-Test

Graph 1

Node Embedding

a	1111
b	1111
c	1111
d	1111
e	1111
f	1111

Graph 2

Node Embedding

p	1111
q	1111
r	1111
s	1111
t	1111
u	1111

∴ WL-Test fails

WL-Test

Graph 1

Node Embedding

a	111
b	111
c	111
d	111
e	111
f	111

Graph 2

Node Embedding

p	111
q	111
r	111
s	111
t	111
u	111

∴ WL-Test fails

WL-Test

Graph 1

Node Embedding

a	1111
b	1111
c	1111
d	1111
e	1111
f	1111

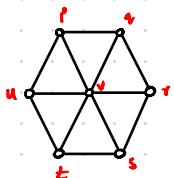
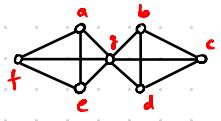
Graph 2

Node Embedding

p	1111
q	1111
r	1111
s	1111
t	1111
u	1111

∴ WL-Test fails

5)



Sum: $a, b, c, d, e, f, g = 3, 3, 3, 3, 3, 3, 6$

$P, Q, R, S, T, U, V = 3, 3, 3, 3, 3, 3, 6$

WL-Test

Graph 1

Node Embedding

Node	Embedding
a	
b	
c	
d	
e	
f	
g	

Graph 2

Node Embedding

Node	Embedding
P	
Q	
R	
S	
T	
U	
V	

∴ WL-Test Fails