Introduction to Quantum Mechanics

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The postulates

- Associated to any isolated physical system is a complex vector space equipped with an inner product, known as the *state space* of the system. The state of the system is completely described by its *state vector*, which is a normalized vector in the state space.
- The evolution of a **closed** quantum system is described by a unitary transformation.That is,

$$|\psi(t_1)\rangle = U(t_1, t_2)|\psi(t_2)\rangle$$

The evolution postulate can also be written as the Schrodinger equation $i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{H}\Psi(\mathbf{r}, t)$. This can be transformed back using $U(t_1, t_2) = exp(\frac{-iH(t_1-t_2)}{\hbar})$. The above expression assumes a time independent hamiltonian

The postulates – Quantum Measurements

- Quantum measurements are described by a set of measurement operators M_m which act on the state space of the system under consideration. The index m refers to the measurement outcome. The probability of measuring the outcome m given the current state is $|\psi\rangle$ is

$$p(m) = \langle\psi|M_m^* M_m|\psi\rangle$$

and after measuring m the state collapses to

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^* M_m|\psi\rangle}}$$

The measurement operators satisfy the completeness relation

$$\sum_m M_m^* M_m = I$$

The postulates – Composite Systems

- The state space of the composite system is mathematically described by the tensor product operation. If we have systems 1, 2, 3 ... with hilbert spaces $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$ ... and states $|\psi_1\rangle, |\psi_2\rangle, \ldots |\psi_n\rangle$, then

  $$\mathcal{H}_{1,2,\ldots n} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \mathcal{H}_n$$

  With the state of the composite system

  $$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots |\psi_n\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \mathcal{H}_n$$