Chaos in learning to win in Rock-Paper-Scissors

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Nash Equilibrium

Kingdom 2

Kingdom 1		Agriculture	War
	Agriculture	5->kingdom 1 5->kingdom 2	0->kingdom 1 6->kingdom 2
	War	6->kingdom 1 0->kingdom 2	1->kingdom 1 1->kingdom 2

Best for Society

Pure Strategy Nash Equilibrium

Pure Strategy: Just one move i.e. Agriculture

Mixed Strategy: A probabilistic mixture of two or more pure strategies i.e. $0.5\text{Agriculture} + 0.5\text{War}$

Nash Theorem: There is at least one mixed strategy Nash Equilibrium in a finite game

Core Assumption: All players are perfectly RATIONAL and the fact that they are rational is common knowledge.

How Rock Paper Scissors works

- Two Players: player 1 and player 2
- Each has an action set: {Rock, Paper, Scissors}

- Both players choose their action SIMULTANEOUSLY
- Scissors beats Paper, Paper beats Rock, Rock beats Scissors
- Tie if both players choose the same action

The MSNE of the symmetric RPS game is (0.33R+0.33P+0.33S, 0.33R+0.33P+0.33S).

In reality, the players are only boundedly rational and learn their strategies.

We analyse the game of asymmetric Rock-Paper-Scissors for boundedly rational players that learn their strategies over multiple games.

Payoff Matrix for the symmetric RPS game

		Player 2		
		R	**P**	**S**
Player 1	**R**	0,0	-1,1	1,-1
	P	1,-1	0,0	-1,1
	S	-1,1	1,-1	0,0

In the asymmetric game, if there is a tie (i.e. RR, PP, or SS), then player 1 gets a payoff of γ_1 and player 2 gets a payoff of γ_2.

The players start off with a random pure/mixed strategy.

After n such games, say that the strategy for player 1 is
x= (x_1, x_2, x_3) representing
x_1.R + x_2.P + x_3.S
Similarly define y for player 2.

Payoff Matrix for the asymmetric RPS game

		Player 2		
		R	**P**	**S**
Player 1	**R**	γ1,γ2	-1,1	1,-1
	P	1,-1	γ1,γ2	-1,1
	S	-1,1	1,-1	γ1,γ2

Equations governing the Dynamics

Let `(x, y) = (x1, x2, x3, y1, y2, y3)` be the frequency of `(RI , PI , SI , RII , PII , SII )` and `(f1, f2, f3, g1, g2, g3)` be the expected payoff of `(RI , PI , SI , RII , PII , SII )` with

$$\sum_{i=1}^{3} x_i = 1, \quad f_i(\mathrm{x}) = \sum_{j=1}^{3} y_j a_{ij},$$

$$\sum_{i=1}^{3} y_i = 1, \quad g_i(\mathrm{y}) = \sum_{j=1}^{3} x_j b_{ij}, \quad (i, j = 1, 2, 3)$$

Average payoff of player 1’s ith strategy when player 2 plays mixed strategy y

Frequency/ probability of jth strategy

$$\begin{cases} \dot{x}_i = x_i \sum_{j \neq i} x_j [F_{ij}(\mathrm{x}) - F_{ji}(\mathrm{x})], \\ \dot{y}_i = y_i \sum_{j \neq i} \mathbf{y}_j [G_{ij}(\mathrm{y}) - G_{ji}(\mathrm{y})], \end{cases} \quad (i, j = 1, 2, 3)$$

$$F_{ij}(\mathrm{x}) = \frac{f_i(\mathrm{x}) - f_j(\mathrm{x})}{\sum_{i=1}^{3} f_i(\mathrm{x})}, \quad G_{ij}(\mathbf{y}) = \frac{g_i(\mathrm{y}) - g_j(\mathrm{y})}{\sum_{i=1}^{3} g_i(\mathrm{y})}$$

Represents how advantageous or disadvantageous player 1’s strategy i is when compared to strategy of player 2.

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Equations

$$
\begin{cases}
\dfrac{\dot{x}_1}{x_1} = [1 + \dfrac{1}{\gamma_1} - (1 + \dfrac{3}{\gamma_1})y_1 - 2y_2] \cdot x_2 \\
\qquad +[2y_1 + (1 - \dfrac{3}{\gamma_1})y_2 + \dfrac{1}{\gamma_1} - 1] \cdot (1 - x_1), \\
\dfrac{\dot{x}_2}{x_2} = [1 - \dfrac{1}{\gamma_1} - 2y_1 + (\dfrac{3}{\gamma_1} - 1)y_2] \cdot x_1 \\
\qquad +[(1 + \dfrac{3}{\gamma_1})y_1 + 2y_2 - \dfrac{1}{\gamma_1} - 1] \cdot (1 - x_2), \\
\dfrac{\dot{y}_1}{y_1} = [1 + \dfrac{1}{\gamma_2} - (1 + \dfrac{3}{\gamma_2})x_1 - 2x_2] \cdot y_2 \\
\qquad +[2x_1 + (1 - \dfrac{3}{\gamma_2})x_2 + \dfrac{1}{\gamma_2} - 1] \cdot (1 - y_1), \\
\dfrac{\dot{y}_2}{y_2} = [1 - \dfrac{1}{\gamma_2} - 2x_1 + (\dfrac{3}{\gamma_2} - 1)x_2] \cdot y_1 \\
\qquad +[(1 + \dfrac{3}{\gamma_2})x_1 + 2x_2 - \dfrac{1}{\gamma_2} - 1] \cdot (1 - y_2).
\end{cases}
$$

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For the case of all Non-zero mixed strategy,

Lyapunov exponent for different parameters

γ_1, γ_2	LE	(0.3,0.15,0.3,0.25)	(0.5,0.25,0.5,0.15)	(0.7,0.15,0.7,0.2)
$\gamma_1 = 0.5$, $\gamma_2 = -0.5$	L1	0.275	0.2881	0.32
	L2	0.242	0.2244	0.1508
	L3	-0.2374	-0.2284	-0.142
	L4	-0.2796	-0.2851	-0.3288
$\gamma_1 = -0.1$, $\gamma_2 = 0.05$	L1	-8.9845	-12.3457	-12.3678
	L2	-18.9985	-13.5647	-13.5948
	L3	-19.9308	-21.9604	-21.9788
	L4	-39.9355	-39.873	-39.927
$\gamma_1 = 0.1$, $\gamma_2 = -0.05$	L1	0.2136	0.276	0.2595
	L2	0.1587	0.1901	0.1538
	L3	-0.1677	-0.207	-0.1806
	L4	-0.2183	-0.2709	-0.2523

- ❖ Two positive LE for $\gamma_1+\gamma_2=0$
- ❖ Four Negative LE for $\gamma_1+\gamma_2<0$
- ❖ Two positive LE for $\gamma_1+\gamma_2>0$

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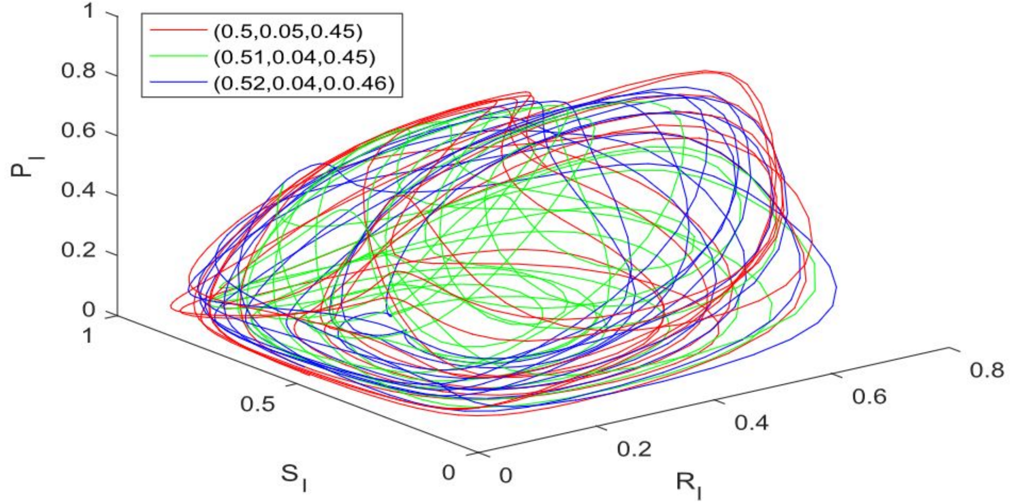
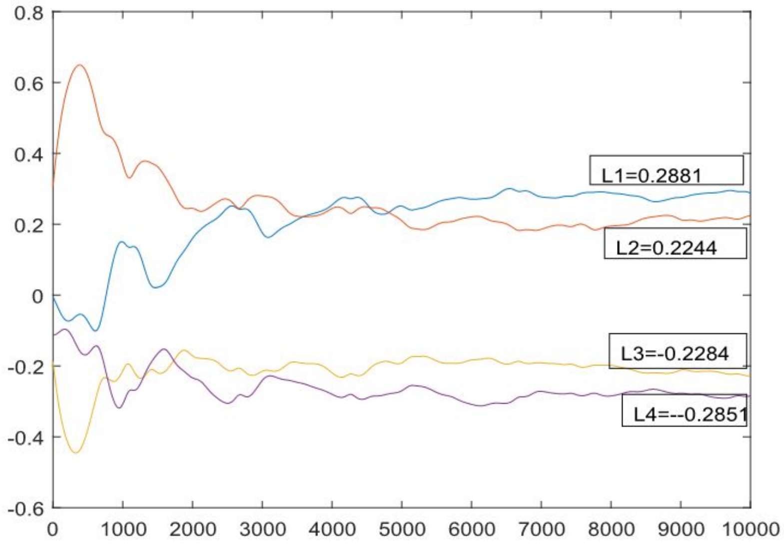
$\gamma 1+\gamma 2 = 0$



Chaotic behaviour when **γ1=0.5 and γ2=-0.5**



Lyapunov exponent of system at (0.45, 0.01, 0.45, 0.24)

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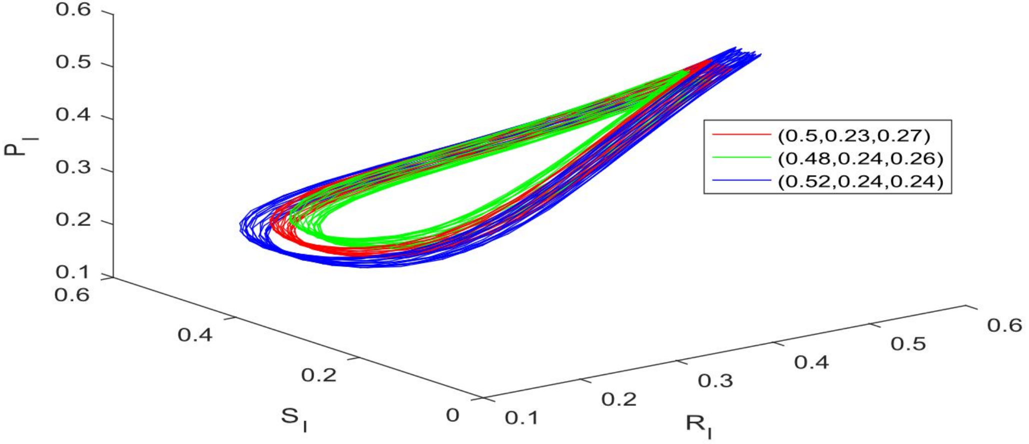
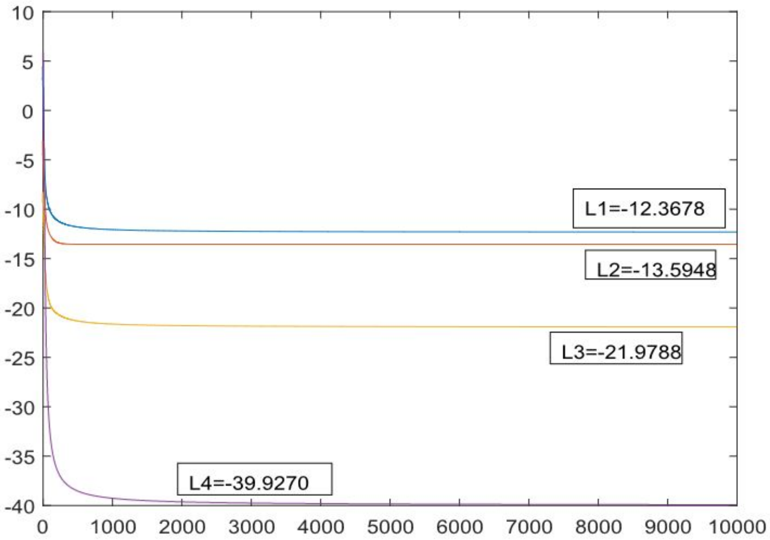
$\gamma 1+\gamma 2 < 0$



Limit cycle when **γ1=-0.1 and γ2=0.05**



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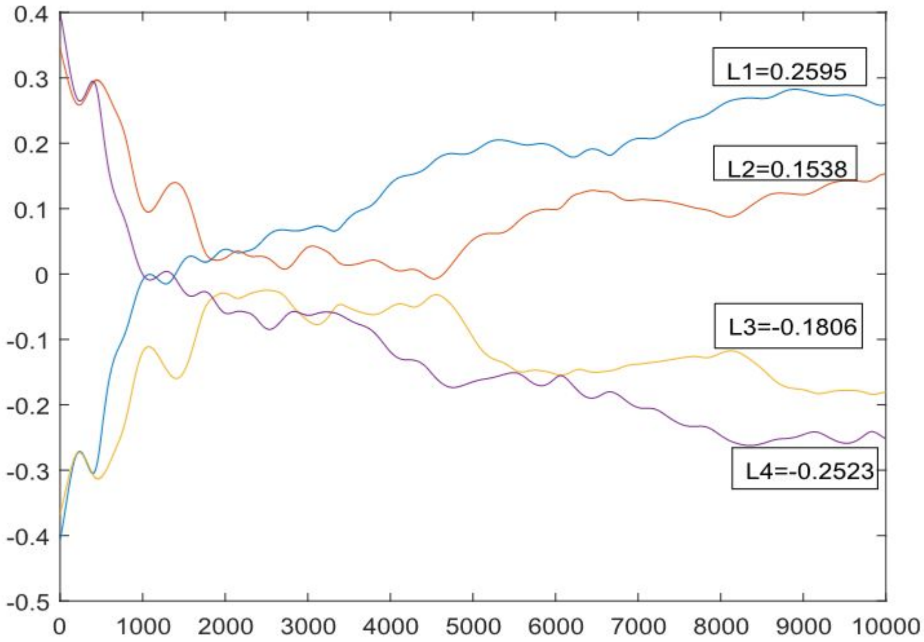
$\gamma 1+\gamma 2 > 0$



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Conclusion

- With the emergence of chaos in such a simple low-dimensional game, **RATIONALITY** may be an unrealistic assumption in the real world.
- Real world players don't seem to learn to be rational.
- The World Rock Paper Scissors society advises players to not play the Nash Equilibrium strategy.

RPS games are also found naturally in population dynamics.
Video links:

1. https://youtu.be/yoLH-nsWKtA?si=8N0yPo-mwaZl2you
2. https://www.youtube.com/watch?v=rafdHxBwIbQ