Expt. No. 7

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Experiment - 7

1. for $f = x^3 + y^2 + z^3$, evaluate ~~lapla~~ laplacian f (only expression)

Sol. $f = x^3 + y^2 + z^3$

To calculate laplacian we use the below formula

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left[3x^2\right] = 6x$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left[2y\right] = 2$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial y}\left[3z^2\right] = 6z$$

$\therefore \nabla^2 f = 6x + 2 + 6z$

Teacher's Signature

code:

```
from sympy import *
x,y,z= Symbols('x y z')
F= x**3 +y**2 + z**3
F1= diff(F,x,2)
F2= diff(F,y,2)
F3= diff(F,z,2)
Lap_F= F1+ F2+ F3
print("Required Laplacian=", Lap_F)
```

Output:

```
Required Laplacian= 6*x + 6*z + 2
```

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2. For $f = x^3y^2 + xyz^3$, evaluate laplacian f at point (1, 1, 2)

A. $f = x^3y^2 + xyz^3$

To calculate laplacian f, we use the below formula

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(3x^2y^2 + yz^3\right) = 6xy^2$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(2x^3y + xz^3\right) = 2x^3$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial z}\left(3xyz^2\right) = 6xyz$$

$$\nabla^2 f = 6xy^2 + 2x^3 + 6xyz$$

at point (1, 1, 2)

$$\nabla^2 f = 6(1)(1)^2 + 2(1)^3 + 6(1)(1)(2)$$
$$= 6 + 2 + 12$$
$$= 20$$

Teacher's Signature

Code:

```
from sympy import *
x, y, z = symbols('x y z')
f = (x**3)*(y**2) + x*y*z**3

f1 = diff(f, x, 2)
f2 = diff(f, y, 2)
f3 = diff(f, z, 2)

lap_f = f1 + f2 + f3
print(" Required laplacian =", lap_f)

point = {x: 1, y: 1, z: 2}
lap_at_point = lap_f.subs(point)
print(" Value at point (1, 1, 2) = ", lap_at_point)
```

Output:

```
Required Laplacian = 6*x*y**2 + 2*x**3 + 6*x*y*z

Value at point (1, 1, 2) = 20
```

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3) Find the matrix of linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (2x+3y, 4x-5y)$ with respect to the standard basis

Sol. $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (2x+3y, 4x-5y)$

Standard basis of $\mathbb{R}^2$ is

$$e_1 = (1,0), \quad e_2 = (0,1)$$

Applying T to each basis

$$T(e_1) = T(1,0) = [2(1)+3(0), 4(1)-5(0)]$$
$$= [2, 4]$$

$$T(e_2) = T(0,1) = [2(0)+3(1), 4(0)-5(1))$$
$$= [3, -5]$$

To form the transformation matrix,

$$[T] = [\text{Col } 1 \quad \text{Col } 2] = [T(e_1) \ T(e_2)]$$

$$\therefore [T] = \begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}$$

Teacher's Signature

Code:

```
from sympy import*
import sympy as sym
import numpy as np
K, y, x = Symbols ("K, y, x")

def T(x):
    x_new = 0
    x, y = x[0], x[1]
    x_new = np.array([2*x + 3*y, 4*x - 5*y])
    return simplify (x_new)

u1= np.array ([1,0])
u2= np.array ([0,1])
v1= np.array ([1,0])
v2= np.array ([0,1])
eq0 = np.array ([v1[0]*x, v2[0]*y])
eq1= np.array([v1[1]*x, v2[1]*y])
print(eq0, eq1)
a0 = np.array([T(u1)[0], T(u2)[0]])
a1 = np.array([T(u1)[1], T(u2)[1]])
print(a0[0]*x + a0[1]*y, 'and', a1[0]*x + a1[1]*y)
print("The matrix of linear transformation is A",
                                    Matrix([a0, a1]))
```

Output:

```
[x 0] [0 y]
2*x + 3*y  and  4*x - 5*y
The matrix of linear transformation is A  Matrix([[2,3], [4,-5]
```

4. Find the linear transformation for the matrix ([[-1, 0], [2, 0], [1, 3]]) with respect to the basis B1 = {(1, 0), (2, -1)} and
B2 = {(1, 2, 0), (0, -1, 0), (+1, -1, 1)}.

Sol. Given, $A = \begin{bmatrix} -1 & 0 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$ $u_1 = [1, 0]$
$u_2 = [2, -1]$

$v_1 = [1, 2, 0]$
$v_2 = [0, -1, 0]$
$v_3 = [1, -1, 1]$

Now, we multiply first column of A with v_1, v_2 and v_3
$\therefore T(u_1) = -1 \cdot [1, 2, 0] + 2 [0, -1, 0] + 1 \cdot [1, -1, 1]$
$= [-1, -2, 0] + [0, -2, 0] + [1, -1, 1]$
$= [0, -5, 1]$

Similarly $T(u_2) = 0 \cdot [1, 2, 0] + 0 [0, -1, 0] + 3 [1, -1, 1]$
$= [3, -3, 3]$

now, for basis B_1, we will use ~~x [~~ variables x and y with columns of u_1 and u_2
$\therefore p = x \cdot u_1[0] + y \cdot [u_2[0]] = x + 2y$
$q = x \cdot u_1[1] + y \cdot u_2[1] = 0 - y$

Multiplying $T(u_1) \cdot p$ and $T(u_2) q$, we get
$T(u_1) \cdot p = [0, -5x - 10y, x + 2y]$
$T(u_2) \cdot q = [-3y, 3y, -3y]$

Adding the above results,

$$T(u_1)\cdot p + T(u_2)\cdot q = [0, -5x - 10y, x+2y] + [-3y, 3y, -3y]$$

$$= [-3y, -5x - 7y, x - y]$$

So, the linear transformation $\cancel{T}$ is $= [-3y, -5x-7y, x-y]$

code:

```
from sympy import *
import numpy as np
a, b, x, y = Symbols ('a, b, x, y')
A = Matrix ([[-1,0], [2,0], [1,3]])
u1 = np.array([1,0])
u2 = np.array ([2, -1])
v1 = np.array ([1,2,0])
v2 = np.array ([0,-1,0])
v3 = np.array([1,-1, 1])

T0 = np.array([A[0,0] * v1 , A[1,0] * v2 , A[2,0] * v3])
T1 = np.array ([A[0,1] * v1 , A[1,1] * v2, A[2,1] * v3])

sumT0 = T0[0] + T0[1] + T0[2]
sumT1 = T1[0] + T1[1] + T1[2]
print (sumT0, sumT1)

eq = np.array ([u1[0] * x + u2[0] * y , u1[1] * x + u2[1] * y])
p, q = eq[0], eq[1]

print (p, 'and', q)

t = sumT0 * p + sumT1 * q

print ("the linear transformation T is T(x,y)=" t)
```

Output :

[0 -5 1] [3 -3 3]

x + 2 * y and -y

The linear transformation T is T (x, y) =

[-3*y -5*x - 7*y x-y]