Orbit Propagation

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1 Introduction

The Orbit Propagator has the responsibility to provide the current estimated position and velocity of the satellite as we can't use GPS for all times due to its higher power requirements and its heating effects. Orbit Propagator takes the input values (Initial Position and Velocity vectors) from GPS and propagates them until next update from GPS. Outputs from ephemeris models such as IGRF, WMM and Earth reflectivity for albedo are all dependant on the position of the satellite as well as relative position of the sun as well. Thus, an accurate and verified satellite navigation system is a necessity for the computation of reference frame vectors such as sun and magnetic field vectors. Moreover, considering that the payload has high over- all pointing requirements, there is a high demand for accurate satellite position and velocity information as it affords greater room for inaccuracy in body frame vector formation as well as to the entire attitude determination and estimation system. The aim of this document is to record the development of Team Anant's Orbit Propagator.

2 Gravitational Force

We started working on Orbit Propagator by applying different force models one by one, staring with the simplest one that is the simple Newton's law of gravitation. We started by considering a simple two body system, the only force acting on the bodies will be F = $\frac{GM_1M_2}{r^2}$. Initially when we applied this equation and compared the data with that obtained from GMAT we were getting quite high error (around 40km over a day) then expected, in order to resolve that we used the values of the constants (Instead of using Gravitational constant we used Gravitational parameter as it is more accurate) of higher accuracy, then the error reduced significantly to 10^{-7} - 10^{-8} order which was in acceptable range of error.

3 J2 Perturbations

Then we started working on applying J2 perturbations or the second degree gravitational forces due to Earth's oblateness. The J2 perturbations changes the Longitude of the ascending node (Ω) of the orbit of a satellite. Many Walking Orbits like Sun-synchronous orbit (SSO) use these forces to have a desired change in Ω. Our satellite is supposed to be in an SSO which means that it will while completing one revolution around the Sun, the orbit will also have completed its one revolution, the following image best describes it.

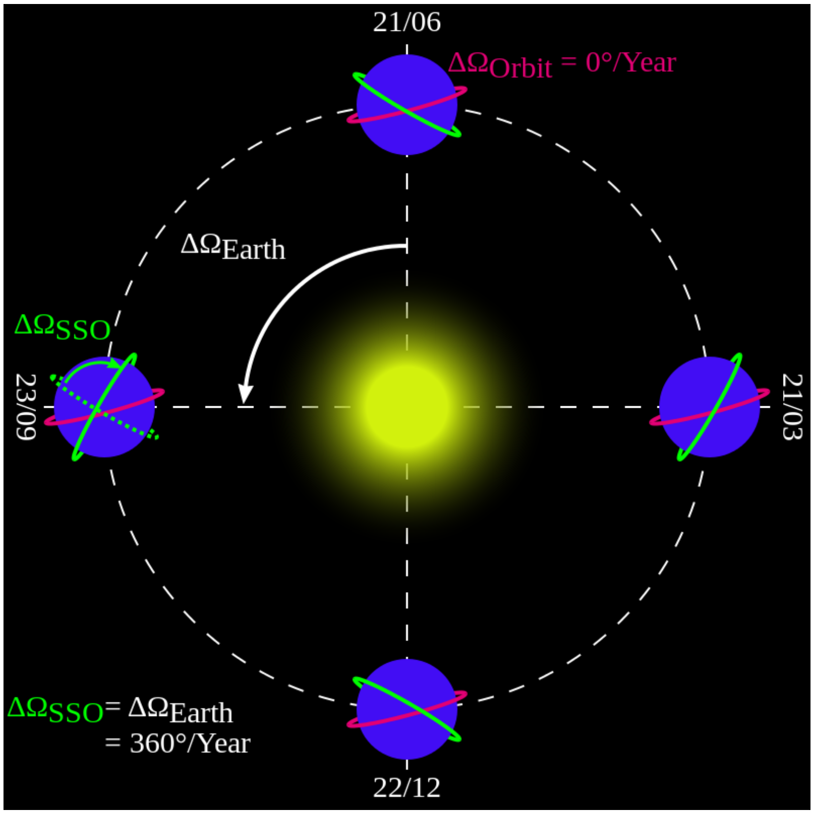

Figure 1: SSO Orbit

3.1 Calculations

Following equations are used to calculate the accelerations caused by J2 perturbations:

$a_x = \frac{\mu \cdot J_2 \cdot R_e^2}{2} \cdot (15 \cdot \frac{xz^2}{r^7} - 3 \cdot \frac{x}{r^5})$

$a_y = \frac{\mu \cdot J_2 \cdot R_e^2}{2} \cdot (15 \cdot \frac{yz^2}{r^7} - 3 \cdot \frac{y}{r^5})$

$a_z = \frac{\mu \cdot J_2 \cdot R_e^2}{2} \cdot (15 \cdot \frac{z^3}{r^7} - 9 \cdot \frac{z}{r^5})$

Using these equations we first tried to apply it for an Equatorial Orbit for which z = 0 which will cause these perturbing accelerations along Z axis to be 0 but when we compared the data with that obtained from GMAT we got an error of about 2 km for one day. Something was wrong as it was not supposed to happen as for a circular equatorial orbit, logically speaking will effectively reduce to a 2 body case which we solved earlier. In order to figure out what was wrong we started looking from the basic which is explained in upcoming section.

3.2 Frame Selection

There are various perturbing forces acting on the satellite like Gravitational force, Aerodynamic Drag force due to Earth's Atmosphere, Solar Drag force and Second degree - Zeroth order Gravitational perturbing forces due to Earth's Oblateness. It is easier to account for these force models in an inertial frames. Initially we took Earth's ECI (Earth Centered Inertial) frame as our reference frame. In an ECI frame, the Z axis is along the Earth's Rotational axis, the X axis is along the Vernal Equinox and the Y axis is calculated as Z×X.

When we looked into the problem discussed in 3.1, we found out that the Earth's Rotational Axis is not steady thus ECI frame is not completely inertial, that was the only possible explanation, then after a bit of research we came to a conclusion to use J2000 equatorial frame as our reference frame. J2000 equatorial frame is also an ECI frame defined on 12:00 pm 1st January 2000. Basically GMAT returns the values in its ECI frame which was J2000 and we were comparing it with current ECI frame, that was the conclusion we reached. Now the problem was how to convert our current ECI frame to J2000 frame, as we need to account for the change of Earth's Rotational axis over a period of roughly 20 years.

3.3 Frame Conversion

For the ease of calculations the change in Earth's Rotational axis can be summed up as the resultant of three different types of motion which are Axial Precession, Earth's Nutation and Polar Motion.

3.3.1 Precession

Axial Precession is a gravity-induced, slow, and continuous change in the orientation of an Earth's rotational axis. It is mainly caused by the uneven gravitational forces of Sun and the Moon due to Earth's oblateness. It causes gradual shift in the orientation of Earth's axis of rotation in a cycle of approximately 26,000 years. Out of the three types of motions, Axial Precession affects the most. The following figure best describes this phenomenon. The complex

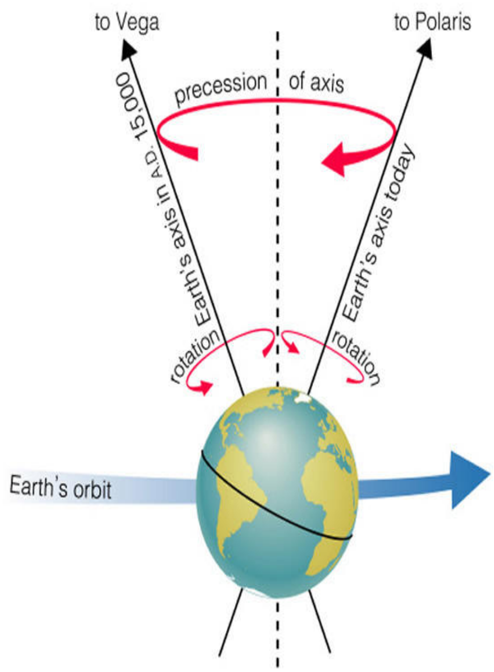

Figure 2: Axial Precession

motion of general precession can be specified by three angles ζ, z, and θ. The precession matrix (D) transforms coordinates from the CIS (mean inertial system of epoch) or ECI System to the mean inertial system of date i.e 1st January 2000. The coordinate axes for the CIS (mean inertial system of epoch) are designated in following figure as X_1, Y_1, and Z_1. The X_1-axis is a vector in the plane of the mean celestial equator of epoch pointing toward the mean vernal equinox of epoch ($\bar{\gamma_0}$). The positive end of the X_1-axis is upwards, perpendicular to the page as shown in the following figure. The Z_1axis is a vector perpendicular to the plane of the mean celestial equator of epoch, positive toward the north celestial pole ($\bar{P_0}$). The Y1-axis is in the plane of the mean celestial equator of epoch, completing a right-handed orthogonal coordinate system (90 degrees east of X_1). The epoch is J2000.0, which is at noon on 1 January in the year 2000.
Precession actually consists of three rotations through three angles:
- A positive rotation about the Z1-axis through the angle (90-ζ)
- A positive rotation about the Xi-axis through the angle (θ)
- A negative rotation about the Z2-axis through the angle (90+z).

The effective Precession matrix (D) is the product of these three rotations and is given by:

$$D = \begin{bmatrix} cosz \cdot cos\theta \cdot cos\zeta - sinz \cdot sin\zeta & -cosz \cdot cos\theta \cdot sin\zeta - sinz \cdot cos\zeta & -cosz \cdot sin\theta \\ sinz \cdot cos\theta \cdot cos\zeta + cosz \cdot sin\zeta & -sinz \cdot cos\theta \cdot sin\zeta + cosz \cdot cos\zeta & -sinz \cdot sin\theta \\ sin\theta \cdot cos\zeta & -sin\theta \cdot sin\zeta & cos\theta \end{bmatrix}$$

where z, θ and ζ are calculated as follows (The following angles are calculated in arc seconds):
$\zeta = 2306.2181 \cdot \mathrm{T} + 0.30188 \cdot \mathrm{T}^2 + 0.017988 \cdot \mathrm{T}^3$
$\mathrm{z} = 2306.2181 \cdot \mathrm{T} + 1.09468 \cdot \mathrm{T}^2 + 0.018203 \cdot \mathrm{T}^3$
$\theta = 2004.3109 \cdot \mathrm{T} + 0.42665 \cdot \mathrm{T}^2 + 0.041833 \cdot \mathrm{T}^3$
Here 'T' is the Julian centuries from the epoch that is 1st January 2000.
$\therefore \mathrm{T} = \frac{JulianDate - 2451545}{36525}$

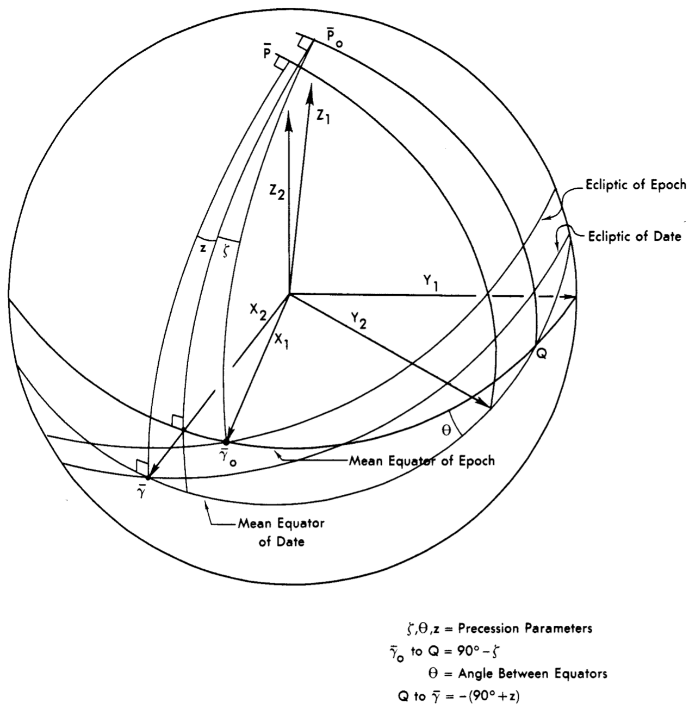

Figure 3: Axial Precession Calculations

3.3.2 Nutation

Nutation is caused principally by the gravitational forces of the Moon and Sun acting upon the non-spherical figure of the Earth. Precession is the effect of these forces averaged over a very long period of time, and a time-varying moment of inertia, and has a timescale of about 26,000 years, on the other hand Nutation occurs because the forces are not constant, and vary as the Earth revolves around the Sun, and the Moon revolves around the Earth. Basically, there are also torques from other planets that cause planetary precession which contributes to about 2% of the total precession. Because periodic variations in the torques from the sun and the moon, the wobbling (nutation) comes into place.
In other words Nutation is the motion caused by the change in inclination angle of the Earth's Rotational axis. The following figure best describes it.

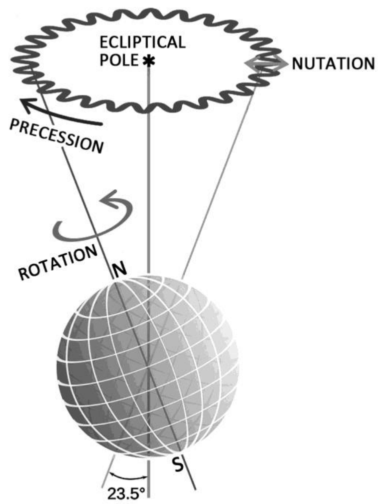

Figure 4: Earth's Nutation

The astronomic nutation matrix (C) transforms coordinates from the mean inertial system of date to the true inertial system of date. This is resolved into nutation-induced corrections known as nutation in ecliptic longitude ($\Delta\psi$) and nutation in obliquity ($\Delta\epsilon$). The X_2, Y_2, Z_2 axes are rotated to X_3, Y_3 and Z_3 axes respectively.
Nutation can be expressed as the sum of three rotations which are as follows:
- A positive rotation along X_2-axis through the angle $\bar{\epsilon}$.
- A negative rotation about Z_2'-axis through the angle of $\Delta\psi$.
- A negative rotation about X_3-axis through the angle of ϵ. The effective Nutation matrix is a product of these three rotations and is given by:

$$C = \begin{bmatrix} cos\Delta\psi & -sin\Delta\psi \cdot cos\bar{\epsilon} & -sin\Delta\psi \cdot sin\bar{\epsilon} \\ cos\epsilon \cdot sin\Delta\psi & cos\epsilon \cdot cos\Delta\psi \cdot cos\bar{\epsilon} + sin\epsilon \cdot sin\bar{\epsilon} & cos\epsilon \cdot cos\Delta\psi \cdot sin\bar{\epsilon} - sin\epsilon \cdot cos\bar{\epsilon} \\ sin\epsilon \cdot sin\Delta\psi & sin\epsilon \cdot cos\Delta\psi \cdot cos\bar{\epsilon} - cos\epsilon \cdot sin\bar{\epsilon} & sin\epsilon \cdot cos\Delta\psi \cdot sin\bar{\epsilon} + cos\epsilon \cdot cos\bar{\epsilon} \end{bmatrix}$$

where ϵ, $\Delta\psi$ and $\bar{\epsilon}$ are calculated as follows:
Here 'T' is the Julian centuries from the epoch that is 1st January 2000.
$\therefore$ T = $\frac{JulianDate-2451545}{36525}$ $\epsilon_0 = 23^o26'21.448''$
$\bar{\epsilon}$ = Mean Obliquity of the Ecliptic = ϵ_0 - 46.8150 $\cdot$ T - 0.00059 $\cdot$ T^2 + 0.001813 $\cdot$ T^3 arc seconds
$\Delta\psi$ = Nutation in Longitude = $\sum_{i=1}^{106} \Delta\psi_i$
Here $\Delta\psi_i = (A_i + B_i \cdot T) \cdot \sin(a_{1i} \cdot l + a_{2i} \cdot l' + a_{3i} \cdot F + a_{4i} \cdot D + a_{5i} \cdot \Omega)$
l = Mean Anomaly of Moon = 485866.733 + (1325^γ + 715922.633)$\cdot$T + 31.310$\cdot T^2$ + 0.064$\cdot T^3$ arc seconds
l' = Mean Anomaly of Sun = 1287099.804 + (99^γ + 1292581.244)$\cdot$T - 0.577$\cdot T^2$ - 0.012$\cdot T^3$ arc seconds
F = Mean Longitude of Moon - Ω = 335778.877 + (1342^γ + 295263.137)$\cdot$T - 13.257$\cdot T^2$ + 0.011$\cdot T^3$ arc seconds
D = Mean Elongation of Moon from Sun = 1072261.307 + (1236^γ + 1105601.328)$\cdot$T - 6.891$\cdot T^2$ + 0.019$\cdot T^3$ arc seconds
Ω = Longitude of Ascending Node of Lunar Mean Orbit on Ecliptic Measured From Mean Equinox of Date = 450160.280 - (5^γ + 482890.539)$\cdot$T + 7.455$\cdot T^2$ + 0.008$\cdot T^3$ arc seconds
1^γ = 129600 arc seconds
$\Delta\epsilon$ = Nutation in Obliquity = $\sum_{i=1}^{106} \Delta\epsilon_i$
Here $\Delta\epsilon_i = (C_i + D_i \cdot T) \cdot \cos(a_{1i} \cdot l + a_{2i} \cdot l' + a_{3i} \cdot F + a_{4i} \cdot D + a_{5i} \cdot \Omega)$
ϵ = True Obliquity of Ecliptic = $\bar{\epsilon} + \Delta\epsilon$
The values of A_i, B_i, C_i, D_i, a_{1i}, a_{2i}, a_{3i} , a_{4i} and a_{5i} are obtained from the table attached in appendix.

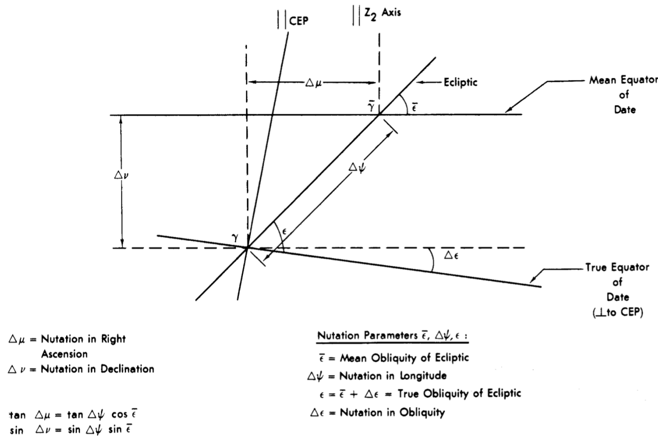

Figure 5: Earth's Nutation Calculations

3.3.3 Polar Motion

Polar motion of the Earth is the motion of the Earth's rotational axis relative to its crust. This is measured with respect to a reference frame in which the solid Earth is fixed. Polar Motion is partly due to motions in the Earth's core and mantle, and partly to the redistribution of water mass as the Greenland ice sheet melts, and to isostatic rebound, i.e. the slow rise of land that was formerly burdened with ice sheets or glaciers. Since about 2000, there is a dramatic eastward shift in drift direction is attributed to the global scale mass transport between the oceans and the continents. Major earthquakes cause abrupt polar motion by altering the volume distribution of the Earth's solid mass. These shifts are quite small in magnitude relative to the long-term core/mantle and isostatic rebound components of polar motion. Due to these various factors and limited understanding of Earth's composition we currently don't have any force model to account for it. The following image tries to show the direction of motion of Earth's Rotational poles prior and after 2000. After applying these matrices we reduced the error along the Z axis into an acceptable range. Then we tried some more highly eccentric orbits as test runs and still got acceptable errors when compared with GMAT.

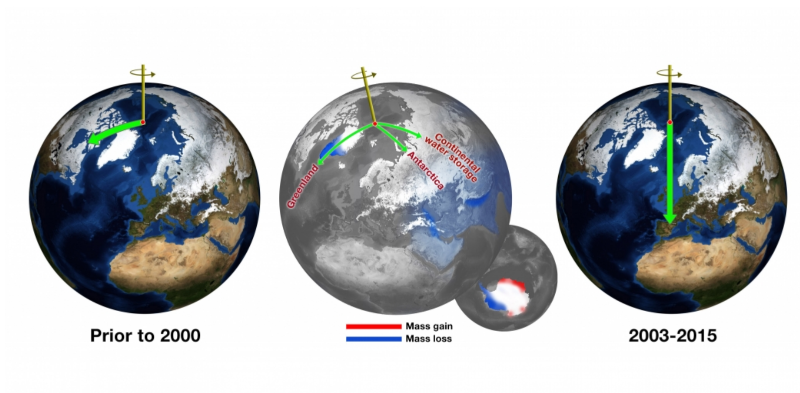

Figure 6: Polar Motion

4 Aerodynamic Drag

Based on the altitude, the orbits are classified into 4 categories - Low Earth Orbits (below 2000km), Medium Earth Orbits (2000km to 35000km), Geosynchronous Orbits (35786km) and High Earth Orbits (above 35786km). The following image shows different orbital altitudes.
The cyan colored portion represents the LEO altitudes.
The yellow colored portion represents the MEO altitudes.
The green dotted line is the altitude at which the GPS satellites are orbiting the earth.
The black dotted line is the Geosynchronous altitude and beyond that lies the HEO altitudes.

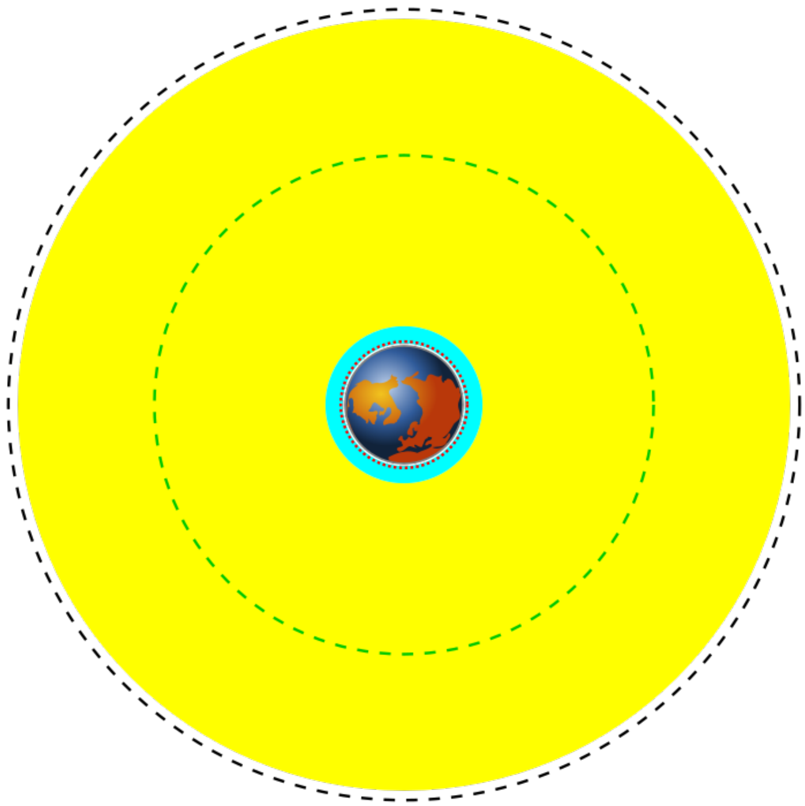

Figure 7: Different types of Orbits

In Low Earth Orbits the affect of Aerodynamic drag forces is significantly high as compared to other categories, and since our satellite will be orbiting in a Low Earth Orbit we need to account for that. The drag force is given by:

$$F_D = \frac{1}{2}\rho\nu^2 C_D A$$

where the variables are:
F_D = Drag force
ρ = Atmospheric density
C_D = Drag coefficient
A = Projected area along the velocity vector or the cross-sectional area
Generally for the satellites in Low Earth Orbits the value of C_D is taken to be 2.2, it is also the default value in GMAT's drag model, we have also assumed it to be 2.2 for the sake of simplicity. Although the more accurate value can experimentally determined in a wind tunnel.
The value of A is calculated by the dot product of the area vector and the unit vector along the velocity vector, summing this up for 3 sides of the cuboid (because maximum of three faces of a cuboid can contribute to the cross-sectional are of the satellite).
For the calculation of atmospheric density there are various atmospheric models which helps us to calculate its value, depending upon the order of accuracy we need and the computational power available we can decide which model

to use. Jacchia Roberts Atmospheric model is used by GMAT and it is one of the most accurate models currently available. It calculates the atmospheric density from the exo-space temperature around the satellite which can be calculated from sun vector and albedo effects, since these two are the only major sources of heat. One of the main disadvantage of this atmospheric model is that it requires high computational power since on board we only have limited computational power, we used a simpler one. We have used exponentially decaying model atmosphere which uses only a single equation that is

$$\rho = \rho_0 \cdot exp(-\frac{h - h_0}{H})$$

where the values of ρ_0, h, h_0 and H are obtained from the table in appendix. It is important to note that in this atmospheric model it is assumed that the density of the Earth is spherically symmetric in 3D space. Now we have our drag force which we divide it by the mass of the satellite to obtain the effective retardation caused by the aerodynamic drag.

5 Solar Radiation Pressure

Solar radiation pressure (SRP) is another non-conservative force acting on space- craft. It is dominated by drag for spacecraft in low-Earth orbit, but SRP will generally outweigh drag in higher altitude orbits (≤800 km). Like drag, SRP can be characterized using either a simple or high fidelity model depending upon the level of accuracy needed as well as a priori knowledge of the spacecraft. The mechanism by which SRP affects the orbit of a spacecraft is through momentum exchange between the spacecraft and photons incident on the spacecraft. Because of this, SRP is a fundamentally different perturbation from that of drag. Whereas drag acts throughout the entire orbit, SRP only contributes as times when the spacecraft is not in the shadow of the Earth or another body. The most general equation of Solar drag force is given by:

$$F_{SRP} = -P_{\odot} c_{SRP} S e_{sat\odot}$$

here $P_{\odot}$ = Solar Radiation Pressure = $\frac{S_{\odot}}{cr^2_{sat\odot}}$ where $S_{\odot}$ is the solar constant whose value is around 1362 Wm^{-2} which is the flux density of energy radiated by sun at a distance of 1AU, c is the speed of light and r is the distance from sun. It keeps fluctuating based on 11 year solar cycle, these fluctuations are very difficult to predict.
c_{SRP} = Solar drag coefficient
S = Sun facing area
$e_{sat\odot}$ = unit vector directed from satellite to the center of the sun
The value of c_{SRP} = 1 means that it is 100% transparent and c_{SRP} = 2 means it is reflecting 100% of the light falling on it, generally for satellites its value is assumed to be 1.8.
The value of sun facing area is computed as the summation of the dot product of the area vector of sun facing sides of the satellite and the unit sun vector.
The value of $e_{sat\odot}$ is computed by simple vector algebra, we know the vector from Earth to satellite (initial position vector), Sun to Earth (depending on the date) then we can find out a vector from satellite to sun.
The above mentioned model is the most basic model of Solar drag. Following equation is a bt complex and more accurate Solar Drag model, we are planning on using it but it needs values of certain constants which can only be determined by performing small experiments which is why it is currently on hold.

$$F_{SRP} = -P_{\odot} \sum_{i=1}^{N} S_i cos\theta^i_{SRP} [2(\frac{R^i_{diff}}{3} + R^i_{spec} cos\theta^i_{SRP}) n^i_I + (1 - R^i_{spec}) e_{sat\odot}]$$

It is important to note that the summation is only carried out for plates with $\cos\theta^i_{SRP} \geq 0$. The variables are as follows:
θ^i_{SRP} = Inclination of the i^{th} plate to the spacecraft-to-Sun vector
S_i = Area of the plate
R^i_{diff} = Diffuse Reflectivity
R^i_{spec} = Specular Reflectivity
$n^i_I = A^T\, n^i_B$ where A is the attitude matrix of the satellite and n^i_B is the outward unit normal vector of the plate in body frame. The rest variables have the usual meaning. Specular Reflectivity is the ratio of the incident power to the normal reflected power and the Diffuse reflectivity is the ratio of the incident power to the scattered reflected power, it can be computed by a simple experiment explained in this link.

6 Appendix

1980 IAU Theory of Nutation

-Series for Nutation in Longitude ($\Delta\psi$) and Obliquity ($\Delta\varepsilon$) (Mean Equator and Equinox of Date)

i	a_1	a_2	a_3	a_4	a_5	A	B	C	D
1	0	0	0	0	1	-171996	-174.2	92025	8.9
2	0	0	0	0	2	2062	0.2	-895	0.5
3	-2	0	2	0	1	46	0.0	-24	0.0
4	2	0	-2	0	0	11	0.0	0	0.0
5	-2	0	2	0	2	-3	0.0	1	0.0
6	1	-1	0	-1	0	-3	0.0	0	0.0
7	0	-2	2	-2	1	-2	0.0	1	0.0
8	2	0	-2	0	1	1	0.0	0	0.0
9	0	0	2	-2	2	-13187	-1.6	5736	-3.1
10	0	1	0	0	0	1426	-3.4	54	-0.1
11	0	1	2	-2	2	-517	1.2	224	-0.6
12	0	-1	2	-2	2	217	-0.5	-95	0.3
13	0	0	2	-2	1	129	0.1	-70	0.0
14	2	0	0	-2	0	48	0.0	1	0.0
15	0	0	2	-2	0	-22	0.0	0	0.0
16	0	2	0	0	0	17	-0.1	0	0.0
17	0	1	0	0	1	-15	0.0	9	0.0
18	0	2	2	-2	2	-16	0.1	7	0.0
19	0	-1	0	0	1	-12	0.0	6	0.0
20	-2	0	0	2	1	-6	0.0	3	0.0
21	0	-1	2	-2	1	-5	0.0	3	0.0
22	2	0	0	-2	1	4	0.0	-2	0.0
23	0	1	2	-2	1	4	0.0	-2	0.0
24	1	0	0	-1	0	-4	0.0	0	0.0
25	2	1	0	-2	0	1	0.0	0	0.0
26	0	0	-2	2	1	1	0.0	0	0.0
27	0	1	-2	2	0	-1	0.0	0	0.0
28	0	1	0	0	2	1	0.0	0	0.0
29	-1	0	0	1	1	1	0.0	0	0.0
30	0	1	2	-2	0	-1	0.0	0	0.0
31	0	0	2	0	2	-2274	-0.2	977	-0.5
32	1	0	0	0	0	712	0.1	-7	0.0
33	0	0	2	0	1	-386	-0.4	200	0.0
34	1	0	2	0	2	-301	0.0	129	-0.1
35	1	0	0	-2	0	-158	0.0	-1	0.0
36	-1	0	2	0	2	123	0.0	-53	0.0
37	0	0	0	2	0	63	0.0	-2	0.0
38	1	0	0	0	1	63	0.1	-33	0.0
39	-1	0	0	0	1	-58	-0.1	32	0.0
40	-1	0	2	2	2	-59	0.0	26	0.0
41	1	0	2	0	1	-51	0.0	27	0.0
42	0	0	2	2	2	-38	0.0	16	0.0
43	2	0	0	0	0	29	0.0	-1	0.0
44	1	0	2	-2	2	29	0.0	-12	0.0
45	2	0	2	0	2	-31	0.0	13	0.0
46	0	0	2	0	0	26	0.0	-1	0.0
47	-1	0	2	0	1	21	0.0	-10	0.0
48	-1	0	0	2	1	16	0.0	-8	0.0
49	1	0	0	-2	1	-13	0.0	7	0.0
50	-1	0	2	2	1	-10	0.0	5	0.0
51	1	1	0	-2	0	-7	0.0	0	0.0
52	0	1	2	0	2	7	0.0	-3	0.0
53	0	-1	2	0	2	-7	0.0	3	0.0
54	1	0	2	2	2	-8	0.0	3	0.0
55	1	0	0	2	0	6	0.0	0	0.0
56	2	0	2	-2	2	6	0.0	-3	0.0
57	0	0	0	2	1	-6	0.0	3	0.0
58	0	0	2	2	1	-7	0.0	3	0.0
59	1	0	2	-2	1	6	0.0	-3	0.0
60	0	0	0	-2	1	-5	0.0	3	0.0
61	1	-1	0	0	0	5	0.0	0	0.0
62	2	0	2	0	1	-5	0.0	3	0.0
63	0	1	0	-2	0	-4	0.0	0	0.0
64	1	0	-2	0	0	4	0.0	0	0.0
65	0	0	0	1	0	-4	0.0	0	0.0
66	1	1	0	0	0	-3	0.0	0	0.0
67	1	0	2	0	0	3	0.0	0	0.0
68	1	-1	2	0	2	-3	0.0	1	0.0
69	-1	-1	2	2	2	-3	0.0	1	0.0
70	-2	0	0	0	1	-2	0.0	1	0.0
71	3	0	2	0	2	-3	0.0	1	0.0
72	0	-1	2	2	2	-3	0.0	1	0.0
73	1	1	2	0	2	2	0.0	-1	0.0
74	-1	0	2	-2	1	-2	0.0	1	0.0
75	2	0	0	0	1	2	0.0	-1	0.0
76	1	0	0	0	2	-2	0.0	1	0.0
77	3	0	0	0	0	2	0.0	0	0.0
78	0	0	2	1	2	2	0.0	-1	0.0
79	-1	0	0	0	2	1	0.0	-1	0.0
80	1	0	0	-4	0	-1	0.0	0	0.0
81	-2	0	2	2	2	1	0.0	-1	0.0
82	-1	0	2	4	2	-2	0.0	1	0.0
83	2	0	0	-4	0	-1	0.0	0	0.0
84	1	1	2	-2	2	1	0.0	-1	0.0
85	1	0	2	2	1	-1	0.0	1	0.0
86	-2	0	2	4	2	-1	0.0	1	0.0
87	-1	0	4	0	2	1	0.0	0	0.0
88	1	-1	0	-2	0	1	0.0	0	0.0
89	2	0	2	-2	1	1	0.0	-1	0.0
90	2	0	2	2	2	-1	0.0	0	0.0
91	1	0	0	2	1	-1	0.0	0	0.0
92	0	0	4	-2	2	1	0.0	0	0.0
93	3	0	2	-2	2	1	0.0	0	0.0
94	1	0	2	-2	0	-1	0.0	0	0.0
95	0	1	2	0	1	1	0.0	0	0.0
96	-1	-1	0	2	1	1	0.0	0	0.0
97	0	0	-2	0	1	-1	0.0	0	0.0
98	0	0	2	-1	2	-1	0.0	0	0.0
99	0	1	0	2	0	-1	0.0	0	0.0
100	1	0	-2	-2	0	-1	0.0	0	0.0
101	0	-1	2	0	1	-1	0.0	0	0.0
102	1	1	0	-2	1	-1	0.0	0	0.0
103	1	0	-2	2	0	-1	0.0	0	0.0
104	2	0	0	2	0	1	0.0	0	0.0
105	0	0	2	4	2	-1	0.0	0	0.0
106	0	1	0	1	0	1	0.0	0	0.0

Units: A = C = 0.0001"; B = D = 0.0001" Per Julian Century (T from Epoch J2000.0)

Figure 8: Constants for computing Nutation Matrix

h (km)	h_0 (km)	ρ_0 (kg/m^3)	H (km)
0–25	0	1.225	8.44
25–30	25	3.899×10^{-2}	6.49
30–35	30	1.774×10^{-2}	6.75
35–40	35	8.279×10^{-3}	7.07
40–45	40	3.972×10^{-3}	7.47
45–50	45	1.995×10^{-3}	7.83
50–55	50	1.057×10^{-3}	7.95
55–60	55	5.821×10^{-4}	7.73
60–65	60	3.206×10^{-4}	7.29
65–70	65	1.718×10^{-4}	6.81
70–75	70	8.770×10^{-5}	6.33
75–80	75	4.178×10^{-5}	6.00
80–85	80	1.905×10^{-5}	5.70
85–90	85	8.337×10^{-6}	5.41
90–95	90	3.396×10^{-6}	5.38
95–100	95	1.343×10^{-6}	5.74
100–110	100	5.297×10^{-7}	6.15
110–120	110	9.661×10^{-8}	8.06
120–130	120	2.438×10^{-8}	11.6
130–140	130	8.484×10^{-9}	16.1
140–150	140	3.845×10^{-9}	20.6
150–160	150	2.070×10^{-9}	24.6
160–180	160	1.224×10^{-9}	26.3
180–200	180	5.464×10^{-10}	33.2
200–250	200	2.789×10^{-10}	38.5
250–300	250	7.248×10^{-11}	46.9
300–350	300	2.418×10^{-11}	52.5
350–400	350	9.158×10^{-12}	56.4
400–450	400	3.725×10^{-12}	59.4
450–500	450	1.585×10^{-12}	62.2
500–600	500	6.967×10^{-13}	65.8
600–700	600	1.454×10^{-13}	79.0
700–800	700	3.614×10^{-14}	109.0
800–900	800	1.170×10^{-14}	164.0
900–1,000	900	5.245×10^{-15}	225.0
>1,000	1,000	3.019×10^{-15}	268.0

Figure 9: Constants for computing Atmospheric Density