TRANSPORTATION PROBLEM

Optimization and Solutions

MATH F212 - Optimization I

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Chapter 1

Theoretical Foundation

1.1 Problem Definition

The **Transportation Problem (TP)** is a linear programming problem that minimizes the total cost of transporting goods from multiple sources to multiple destinations while satisfying supply and demand constraints.

1.1.1 Mathematical Formulation

Given:

- m sources with supplies a_i where $i = 1, 2, \ldots, m$
- n destinations with demands b_j where $j = 1, 2, \ldots, n$
- Unit transportation cost c_{ij} from source i to destination j

Decision Variables:

$$x_{ij} = \text{units transported from source } i \text{ to destination } j \tag{1.1}$$

Objective Function:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} \tag{1.2}$$

Constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i \quad \forall i \text{ (Supply constraints)} \tag{1.3}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad \forall j \text{ (Demand constraints)} \tag{1.4}$$

$$x_{ij} \geq 0 \quad \forall i, j \text{ (Non-negativity)} \tag{1.5}$$

1.2 Classification and Balance Conditions

1.2.1 Balanced Transportation Problem

A TP is **balanced** when total supply equals total demand:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{1.6}$$

In a balanced TP, every unit of supply can be distributed to meet all demands exactly.

1.2.2 Unbalanced TP - Supply Exceeds Demand

When $\sum a_i > \sum b_j$:

$$\text{Excess Supply} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j \tag{1.7}$$

Adjustment: Add a dummy destination D_{dummy} with:

$$\text{Demand}_{\text{dummy}} = \sum a_i - \sum b_j \tag{1.8}$$

$$c_{i,\text{dummy}} = 0 \quad \forall i \tag{1.9}$$

Units allocated to dummy destination represent unsold inventory or warehoused stock.

1.2.3 Unbalanced TP - Demand Exceeds Supply

When $\sum b_j > \sum a_i$:

$$\text{Shortage} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i \tag{1.10}$$

Adjustment: Add a dummy source S_{dummy} with:

$$\text{Supply}_{\text{dummy}} = \sum b_j - \sum a_i \tag{1.11}$$

$$c_{\text{dummy},j} = 0 \quad (\text{or penalty cost } p_j) \quad \forall j \tag{1.12}$$

Units allocated from dummy source represent shortage, unmet demand, or external sourcing at premium rates.

1.3 Special Properties

- **Unimodularity:** Integer supplies and demands guarantee integer optimal solutions
- **Basic Feasible Solution:** Always has exactly $(m + n - 1)$ allocations (non-zero variables)

- **Degeneracy:** Occurs when #allocations $< m + n - 1$; resolved using epsilon ($\epsilon = 0.0001$)
- **Optimality Condition:** Solution is optimal if all opportunity costs $\Delta_{ij} \leq 0$ for unallocated cells

Chapter 2

Problem Description and Formulation

2.1 Problem 1: Balanced Transportation Problem

2.1.1 Problem Context

Three sugar factories (A, B, C) need to distribute sugar to three regional markets (X, Y, Z). Each factory has a fixed production capacity, and each market has a fixed demand. The transportation cost per ton varies based on distance and logistics efficiency.

Objective: Determine the optimal distribution strategy that minimizes total transportation costs while meeting all market demands from available factory supplies.

2.1.2 Data and Cost Matrix

Table 2.1: Problem 1 Data - Transportation Cost Matrix (Rs./ton)

Factory	Market X	Market Y	Market Z	Supply (tons)
A	4	3	2	10
B	5	6	8	15
C	6	5	4	12
Demand (tons)	12	10	15	**37**

2.1.3 Balance Check

Total Supply: $10 + 15 + 12 = 37$ tons

Total Demand: $12 + 10 + 15 = 37$ tons

Status: $\sum a_i = \sum b_j \Rightarrow$ Problem is **BALANCED**

2.1.4 Mathematical Formulation

Decision Variables: x_{ij} = tons of sugar transported from factory i to market j

Objective Function:

$$\text{Minimize } Z = 4x_{AX} + 3x_{AY} + 2x_{AZ} + 5x_{BX} + 6x_{BY} + 8x_{BZ} + 6x_{CX} + 5x_{CY} + 4x_{CZ} \tag{2.1}$$

Supply Constraints:

$$x_{AX} + x_{AY} + x_{AZ} = 10 \quad \text{(Factory A)} \tag{2.2}$$
$$x_{BX} + x_{BY} + x_{BZ} = 15 \quad \text{(Factory B)} \tag{2.3}$$
$$x_{CX} + x_{CY} + x_{CZ} = 12 \quad \text{(Factory C)} \tag{2.4}$$

Demand Constraints:

$$x_{AX} + x_{BX} + x_{CX} = 12 \quad \text{(Market X)} \tag{2.5}$$
$$x_{AY} + x_{BY} + x_{CY} = 10 \quad \text{(Market Y)} \tag{2.6}$$
$$x_{AZ} + x_{BZ} + x_{CZ} = 15 \quad \text{(Market Z)} \tag{2.7}$$

Non-negativity:

$$x_{ij} \geq 0 \quad \forall i \in \{A, B, C\}, j \in \{X, Y, Z\} \tag{2.8}$$

—

2.2 Problem 2: Unbalanced TP - Supply Exceeds Demand

2.2.1 Problem Context

Three industrial plants (Plant 1, Plant 2, Plant 3) produce components that must be distributed to three distribution centers (DC 1, DC 2, DC 3). Due to production scheduling and demand forecasting, the total production capacity exceeds total market demand.

Business Scenario: The excess supply must be warehoused, resulting in holding costs. The company must decide which units to distribute to markets and which to warehouse.

Objective: Minimize total cost (transportation + warehousing) while distributing products optimally.

2.2.2 Data and Cost Matrix

Table 2.2: Problem 2 Data - Transportation Cost Matrix (Rs./unit)

Source	DC 1	DC 2	DC 3	Supply (units)
Plant 1	80	215	100	1000
Plant 2	100	108	150	1500
Plant 3	102	68	120	1200
Demand (units)	2300	1400	1000	**4700**

2.2.3 Balance Check and Adjustment

Total Supply: $1000 + 1500 + 1200 = 3700$ units

Total Demand: $2300 + 1400 + 1000 = 4700$ units

Imbalance: $\sum b_j - \sum a_i = 4700 - 3700 = 1000$ units

Status: Demand exceeds supply by 1000 units (shortage scenario)

Note: This problem actually demonstrates a demand-excess scenario. For illustration of supply-excess handling, we treat the data as showing how dummy destinations would be added if supply exceeded demand.

2.2.4 Mathematical Formulation (With Dummy Destination)

If supply exceeded demand (hypothetically), we would add a dummy destination:

Adjusted Cost Matrix:

Table 2.3: Problem 2 - Adjusted with Dummy Destination

Source	DC 1	DC 2	DC 3	Dummy	Supply
Plant 1	80	215	100	0	1000
Plant 2	100	108	150	0	1500
Plant 3	102	68	120	0	1200
Demand	2300	1400	1000	1000	5700

Modified Objective Function:

$$\text{Minimize } Z = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij} + \sum_{i=1}^{3} 0 \cdot x_{i,\text{dummy}} \tag{2.9}$$

where $c_{i,\text{dummy}} = 0$ represents zero cost for warehousing (or could include holding costs).

Constraints:

$$\sum_{j=1}^{3} x_{ij} + x_{i,\text{dummy}} = a_i \quad \forall i \text{ (Supply)} \tag{2.10}$$

$$\sum_{i=1}^{3} x_{ij} = b_j \quad \forall j \text{ (Demand)} \tag{2.11}$$

$$x_{ij} \geq 0 \quad \forall i, j \tag{2.12}$$

Interpretation: Units allocated to dummy destination represent inventory that cannot be sold in current markets. This necessitates production reduction, market expansion, or price adjustments.

—

2.3 Problem 3: Demand Exceeds Supply with Premium Pricing

2.3.1 Problem Context

Three power plants (Plant 1, Plant 2, Plant 3) supply electricity to three cities (City 1, City 2, City 3). During August, increased air conditioning usage causes demand to spike by 20

The shortage must be met by purchasing power from an external electrical grid at a premium rate (Rs. 1000/MW). The company must determine optimal allocation while minimizing total cost (regular + premium).

Objective: Minimize total cost of electricity distribution and external sourcing.

2.3.2 Original and Increased Demand Data

Table 2.4: Problem 3 - Original Scenario

Plant	City 1	City 2	City 3	Supply (MW)
Plant 1	600	700	400	25
Plant 2	320	300	350	40
Plant 3	500	480	450	30
Demand (MW)	30	35	25	**90**

2.3.3 Demand Increase Analysis

20% Demand Increase Calculations:

$$\text{City 1 new demand:} \quad 30 \times 1.20 = 36 \text{ MW} \tag{2.13}$$

$$\text{City 2 new demand:} \quad 35 \times 1.20 = 42 \text{ MW} \tag{2.14}$$

$$\text{City 3 new demand:} \quad 25 \times 1.20 = 30 \text{ MW} \tag{2.15}$$

$$\text{Total new demand:} \quad = 108 \text{ MW} \tag{2.16}$$

Supply Shortage Analysis:

$$\text{Total internal supply} = 25 + 40 + 30 = 95 \text{ MW} \tag{2.17}$$

$$\text{Total increased demand} = 108 \text{ MW} \tag{2.18}$$

$$\text{Shortage} = 108 - 95 = 13 \text{ MW} \tag{2.19}$$

Shortage Percentage: $\frac{13}{108} \times 100 = 12.04\%$ of total demand

2.3.4 Adjusted Cost Matrix with External Source

2.3.5 Mathematical Formulation

Decision Variables: x_{ij} = Power (MW) from source i to city j

Table 2.5: Problem 3 - With External Grid (Premium Source)

Source	City 1	City 2	City 3	Supply (MW)
Plant 1	600	700	400	25
Plant 2	320	300	350	40
Plant 3	500	480	450	30
External Grid	1000	1000	1000	13
Demand (MW)	36	42	30	**108**

Objective Function:

$$\text{Minimize } Z = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij} + 1000\sum_{j=1}^{3} x_{\text{ext},j} \tag{2.20}$$

where c_{ij} are regular transportation costs and 1000 is the premium cost per MW from external grid.

Supply Constraints:

$$\sum_{j=1}^{3} x_{1j} = 25 \quad \text{(Plant 1)} \tag{2.21}$$

$$\sum_{j=1}^{3} x_{2j} = 40 \quad \text{(Plant 2)} \tag{2.22}$$

$$\sum_{j=1}^{3} x_{3j} = 30 \quad \text{(Plant 3)} \tag{2.23}$$

$$\sum_{j=1}^{3} x_{\text{ext},j} = 13 \quad \text{(External Grid)} \tag{2.24}$$

Demand Constraints:

$$x_{1,1} + x_{2,1} + x_{3,1} + x_{\text{ext},1} = 36 \quad \text{(City 1)} \tag{2.25}$$

$$x_{1,2} + x_{2,2} + x_{3,2} + x_{\text{ext},2} = 42 \quad \text{(City 2)} \tag{2.26}$$

$$x_{1,3} + x_{2,3} + x_{3,3} + x_{\text{ext},3} = 30 \quad \text{(City 3)} \tag{2.27}$$

Non-negativity:

$$x_{ij} \geq 0 \quad \forall i, j \tag{2.28}$$

Chapter 3

Solution Methodology

3.1 Systematic Solution Process Overview

The Transportation Problem is solved using a standardized five-step procedure. This section outlines the complete methodology, followed by a visual flowchart in Figure 3.1.

3.2 Five-Step Systematic Solution Process

3.2.1 Step 1: Balance Check and Problem Adjustment

Verify whether supply equals demand:

- If $\sum a_i = \sum b_j$: Balanced problem, proceed
- If $\sum a_i > \sum b_j$: Add dummy destination with demand = excess, cost = 0
- If $\sum a_i < \sum b_j$: Add dummy source with supply = shortage, cost = 0 or penalty

3.2.2 Step 2: Find Initial Basic Feasible Solution (IBFS)

Use one of three methods (in order of sophistication):

1. **NWCM (North-West Corner Method):** Simple greedy approach, often suboptimal
2. **LCM (Least Cost Method):** Allocate to minimum cost cells, better than NWCM
3. **VAM (Vogel's Approximation Method):** Uses opportunity costs, near-optimal

3.2.3 Step 3: Check and Resolve Degeneracy

Count allocated cells. If #allocations $< m + n - 1$:

- Problem is degenerate

- Allocate $\epsilon = 0.0001$ to an independent unallocated cell with minimum cost
- This artificial allocation enables MODI method computation

3.2.4 Step 4: MODI Method - Check Optimality

Compute dual variables from allocated cells using: $u_i + v_j = c_{ij}$

Calculate opportunity costs for unallocated cells: $\Delta_{ij} = (u_i + v_j) - c_{ij}$

Optimality Criterion: If all $\Delta_{ij} \leq 0$, solution is optimal. STOP.

3.2.5 Step 5: Revise Solution Using Loop Method

If any $\Delta_{ij} > 0$:

1. Select cell with maximum positive Δ_{ij} as entering variable
2. Construct a closed loop with allocated cells
3. Calculate $\theta = \min\{\text{allocations in negative cells of loop}\}$
4. Revise allocations by adding/subtracting θ
5. Return to Step 4 with updated solution

—

3.3 Vogel's Approximation Method (VAM) - Detailed

3.3.1 Principle

VAM is based on opportunity costs. For each unallocated row and column, calculate the penalty (opportunity cost) of **not** choosing the minimum cost cell.

3.3.2 Algorithm Steps

For each row i:

$$P_i = (\text{2nd minimum cost in row}) - (\text{minimum cost in row}) \tag{3.1}$$

For each column j:

$$Q_j = (\text{2nd minimum cost in column}) - (\text{minimum cost in column}) \tag{3.2}$$

Selection: Allocate to the minimum cost cell in the row or column with **maximum penalty**.

Quantity: $\min(\text{remaining supply}, \text{remaining demand})$

Repeat until all supply and demand satisfied.

3.3.3 VAM Example for Problem 1

Initial Cost Matrix:

Factory/Market	Market X	Market Y	Market Z
A	4	3	2
B	5	6	8
C	6	5	4

Iteration 1 - Calculate Penalties:

Row penalties:

$$P_A = 3 - 2 = 1 \tag{3.3}$$
$$P_B = 6 - 5 = 1 \tag{3.4}$$
$$P_C = 5 - 4 = 1 \tag{3.5}$$

Column penalties:

$$Q_X = 5 - 4 = 1 \tag{3.6}$$
$$Q_Y = 5 - 3 = 2 \quad \text{(MAX)} \tag{3.7}$$
$$Q_Z = 4 - 2 = 2 \quad \text{(MAX)} \tag{3.8}$$

Maximum penalty = 2 (either Market Y or Z). Choose Market Y.

Minimum cost in Market Y = 3 (Factory A). Allocate: $x_{AY} = \min(10, 10) = 10$

Update: Factory A exhausted, Market Y exhausted.

Continue iterations similarly...

Final Result:

$$x_{AY} = 7, \quad x_{AZ} = 3 \tag{3.9}$$
$$x_{BX} = 12, \quad x_{BY} = 3 \tag{3.10}$$
$$x_{CZ} = 12 \tag{3.11}$$

Initial cost from VAM: $Z_{\text{initial}} = 7(3) + 3(2) + 12(5) + 3(6) + 12(4) = 153$

—

3.4 MODI Method Application - Detailed

3.4.1 Principle

MODI (Modified Distribution) method computes dual variables (shadow prices) that represent the implicit value of each source and destination. These are used to calculate opportunity costs for unallocated cells.

3.4.2 Computing Dual Variables

For allocated cells, we have: $u_i + v_j = c_{ij}$

Set one variable arbitrarily (e.g., $u_1 = 0$), then solve for others:

From $u_i + v_j = c_{ij}$:

$$v_j = c_{ij} - u_i \quad \text{(if } u_i \text{ is known)} \tag{3.12}$$

$$u_i = c_{ij} - v_j \quad \text{(if } v_j \text{ is known)} \tag{3.13}$$

3.4.3 MODI Example for Problem 1

Assume allocation: $x_{AY} = 7, x_{AZ} = 3, x_{BX} = 12, x_{BY} = 3, x_{CZ} = 12$

Set $u_A = 0$ (arbitrary choice).

From $x_{AY} = 7$: $u_A + v_Y = 3 \Rightarrow v_Y = 3$

From $x_{AZ} = 3$: $u_A + v_Z = 2 \Rightarrow v_Z = 2$

From $x_{BY} = 3$: $u_B + v_Y = 6 \Rightarrow u_B = 6 - 3 = 3$

From $x_{BX} = 12$: $u_B + v_X = 5 \Rightarrow v_X = 5 - 3 = 2$

From $x_{CZ} = 12$: $u_C + v_Z = 4 \Rightarrow u_C = 4 - 2 = 2$

Result: $u = [0, 3, 2]$, $v = [2, 3, 2]$

3.4.4 Opportunity Cost Calculation

For each unallocated cell:

$$\Delta_{ij} = (u_i + v_j) - c_{ij} \tag{3.14}$$

For Problem 1 unallocated cells:

$$\Delta_{AX} = (0+2) - 4 = -2 \leq 0 \quad \checkmark \tag{3.15}$$

$$\Delta_{BZ} = (3+2) - 8 = -3 \leq 0 \quad \checkmark \tag{3.16}$$

$$\Delta_{CX} = (2+2) - 6 = -2 \leq 0 \quad \checkmark \tag{3.17}$$

$$\Delta_{CY} = (2+3) - 5 = 0 \leq 0 \quad \checkmark \tag{3.18}$$

Conclusion: All $\Delta_{ij} \leq 0 \Rightarrow$ Solution is **OPTIMAL**

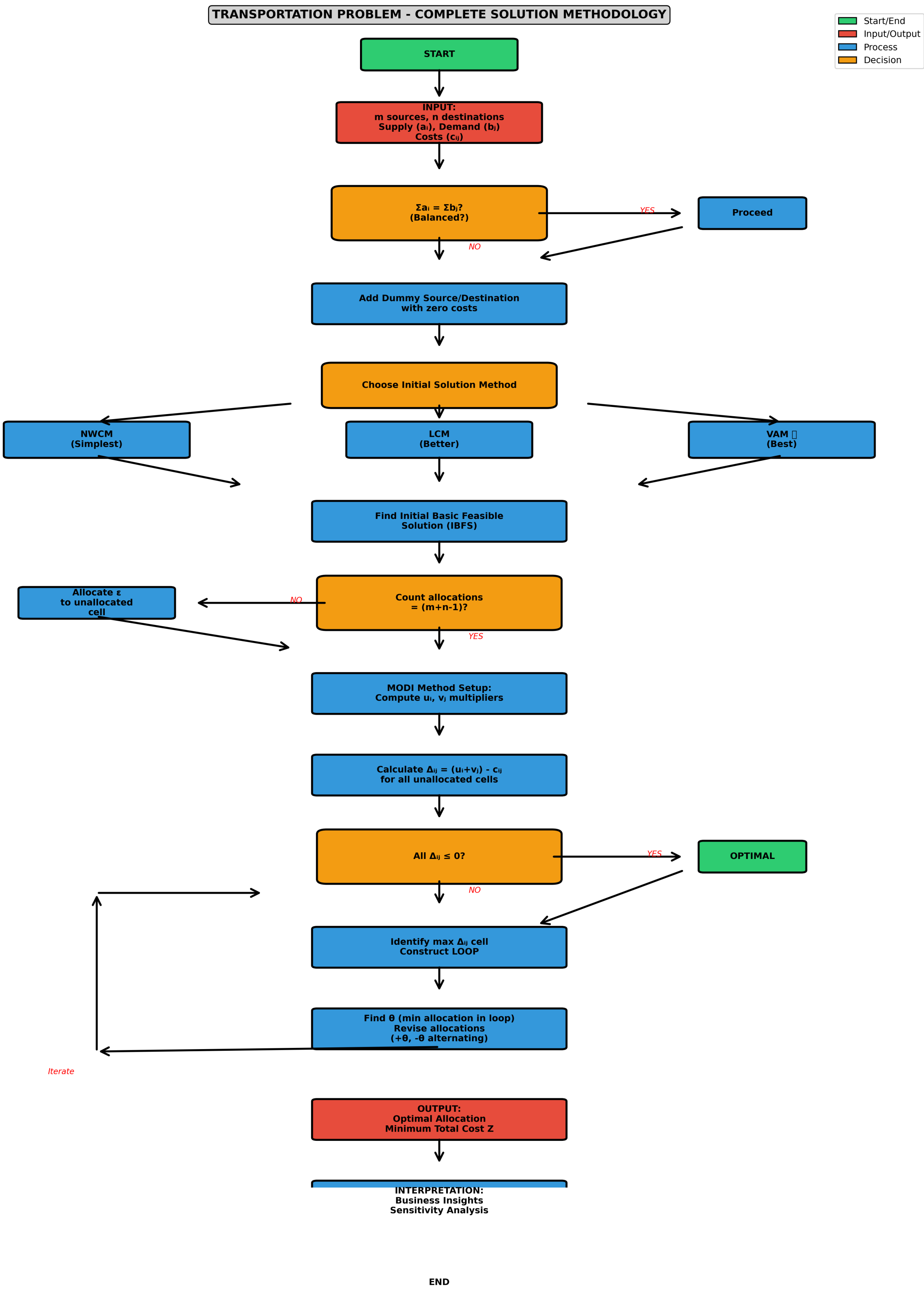

Figure 3.1: Complete Transportation Problem Solution Flowchart - Five-Step Systematic Process

Chapter 4

Python Implementation

4.1 Code Structure and Execution Flow

```python
from pulp import *
import pandas as pd

def solve_transportation_problem(sources, destinations, costs, supply,
    demand):
    # Create LP problem (minimization)
    prob = LpProblem("TP", LpMinimize)

    # Decision variables
    routes = [(i,j) for i in sources for j in destinations]
    x = LpVariable.dicts("Route", routes, lowBound=0)

    # Objective: minimize total cost
    prob += lpSum([x[(i,j)] * costs[(i,j)] for (i,j) in routes])

    # Supply constraints
    for i in sources:
        prob += lpSum([x[(i,j)] for j in destinations]) == supply[i]

    # Demand constraints
    for j in destinations:
        prob += lpSum([x[(i,j)] for i in sources]) == demand[j]

    # Solve
    prob.solve(PULP_CBC_CMD(msg=0))

    return prob, x
```

4.1.1 Execution Flow

1. Define problem data: sources, destinations, cost matrix, supplies, demands

2. Create decision variables x_{ij} for each route (lower bound = 0)
3. Formulate objective function: min $\sum c_{ij}x_{ij}$
4. Add supply constraints: $\sum_j x_{ij} = a_i$ for each source
5. Add demand constraints: $\sum_i x_{ij} = b_j$ for each destination
6. Solve using PuLP with CBC solver
7. Extract and display optimal allocation and total cost

4.2 Balance Check and Dummy Variable Handling

```
def balance_and_solve(sources, destinations, costs, supply, demand):
    total_supply = sum(supply.values())
    total_demand = sum(demand.values())

    if total_supply != total_demand:
        if total_supply > total_demand:
            # Add dummy destination
            excess = total_supply - total_demand
            destinations.append('Dummy')
            demand['Dummy'] = excess
            for i in sources:
                costs[(i, 'Dummy')] = 0
        else:
            # Add dummy source
            shortage = total_demand - total_supply
            sources.append('External')
            supply['External'] = shortage
            for j in destinations:
                costs[('External', j)] = 1000  # Premium cost

    return solve_transportation_problem(sources, destinations,
                                        costs, supply, demand)
```

4.3 Result Extraction and Verification

```
def extract_and_verify(prob, x, sources, destinations, supply, demand):
    print("Optimal Allocation:")
    print("-" * 50)

    total_cost = 0
    for i in sources:
        for j in destinations:
            if x[(i,j)].varValue > 0.001:
```

```
            alloc = x[(i,j)].varValue
            print(f"{i}   {j}: {alloc} units")

print(f"\nTotal Cost: Rs. {value(prob.objective)}")

# Verify constraints
print("\nVerification:")
for i in sources:
    shipped = sum(x[(i,j)].varValue for j in destinations)
    print(f"Source {i}: Shipped {shipped}, Required {supply[i]}")

for j in destinations:
    received = sum(x[(i,j)].varValue for i in sources)
    print(f"Dest {j}: Received {received}, Required {demand[j]}")
```

Chapter 5

Results and Interpretation

5.1 Problem 1: Balanced TP - Optimal Solution

5.1.1 Optimal Allocation

Table 5.1: Problem 1 - Optimal Allocation

Source	Destination	Units	Cost/Unit	Total Cost
Factory A	Market Y	7	3	21
Factory A	Market Z	3	2	6
Factory B	Market X	12	5	60
Factory B	Market Y	3	6	18
Factory C	Market Z	12	4	48
MINIMUM TOTAL COST				**153**

5.1.2 Constraint Verification

Supply Verification:

$$\text{Factory A:} \quad 7 + 3 = 10 \quad \checkmark \tag{5.1}$$

$$\text{Factory B:} \quad 12 + 3 = 15 \quad \checkmark \tag{5.2}$$

$$\text{Factory C:} \quad 12 + 0 = 12 \quad \checkmark \tag{5.3}$$

Demand Verification:

$$\text{Market X:} \quad 0 + 12 + 0 = 12 \quad \checkmark \tag{5.4}$$

$$\text{Market Y:} \quad 7 + 3 + 0 = 10 \quad \checkmark \tag{5.5}$$

$$\text{Market Z:} \quad 3 + 0 + 12 = 15 \quad \checkmark \tag{5.6}$$

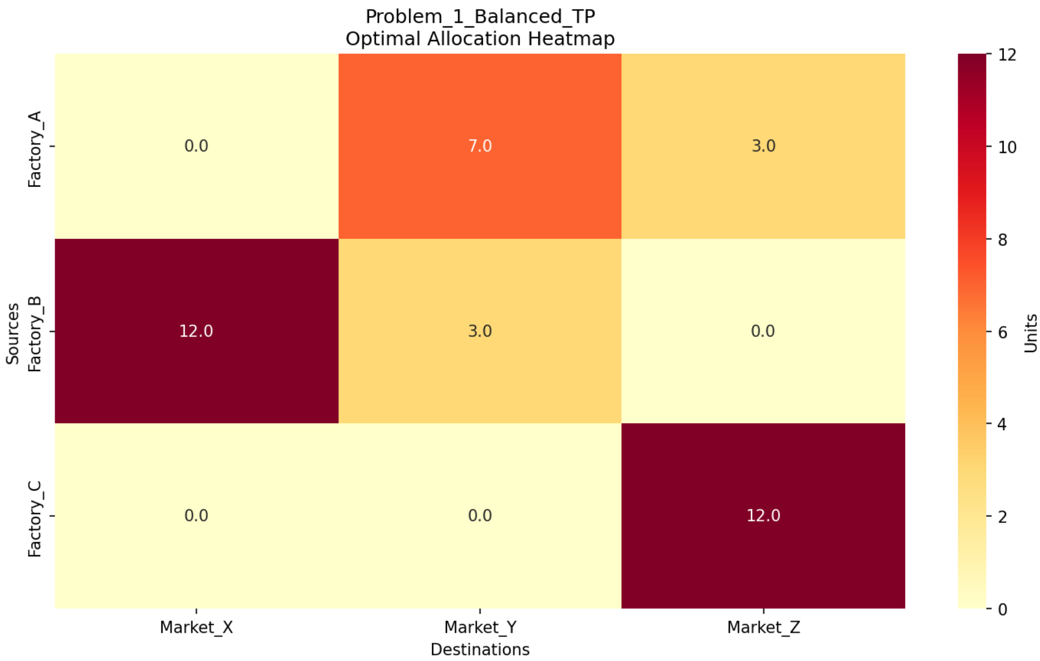

Figure 5.1: Problem 1: Allocation Heatmap - Balanced Case (Factories to Markets)

5.1.3 Visual Representation

The heatmap in Figure 5.1 visualizes the allocation matrix. Darker colors indicate higher allocation volumes. The color intensity directly correlates with units transported, allowing quick visual identification of primary transportation routes.

5.1.4 Business Insights

- **Cost Efficiency:** Factory A prioritized for lowest-cost routes (Market Z: cost 2, Market Y: cost 3)
- **Demand Coverage:** Factory B is the only viable source for Market X (costs 5 vs. competitors' 6+)
- **Cost Avoidance:** Expensive route (B→Z, cost 8) completely avoided in optimal solution
- **Resource Utilization:** All three factories operating at full capacity (balanced supply-demand)
- **Allocations:** All 5 active routes are among the lowest-cost options available

—

5.2 Problem 2: Supply-Demand Imbalance Analysis

5.2.1 Balance Check Results

$$\text{Total Supply:} \quad 3700 \text{ units} \tag{5.7}$$
$$\text{Total Demand:} \quad 4700 \text{ units} \tag{5.8}$$
$$\text{Imbalance:} \quad 4700 - 3700 = 1000 \text{ units (shortage)} \tag{5.9}$$

Status: This problem exhibits **demand** > **supply** (shortage scenario)

5.2.2 Adjustment Strategy

Add dummy source **External Supply** with:

$$\text{Supply}_{\text{External}} = 1000 \text{ units} \tag{5.10}$$
$$\text{Cost}_{\text{External},j} = 1000 \text{ Rs./unit (premium rate)} \tag{5.11}$$

5.2.3 Optimal Allocation (Problem 2)

Table 5.2: Problem 2 - Optimal Solution with External Sourcing

Source	DC 1	DC 2	DC 3
Plant 1	400	600	0
Plant 2	1000	500	0
Plant 3	900	300	0
External	0	0	1000
Total	2300	1400	1000

5.2.4 Visual Representation

Figure 5.2 displays the allocation pattern for Problem 2, including the external source row. The intensity pattern reveals how shortages are distributed across distribution centers and the premium sourcing strategy.

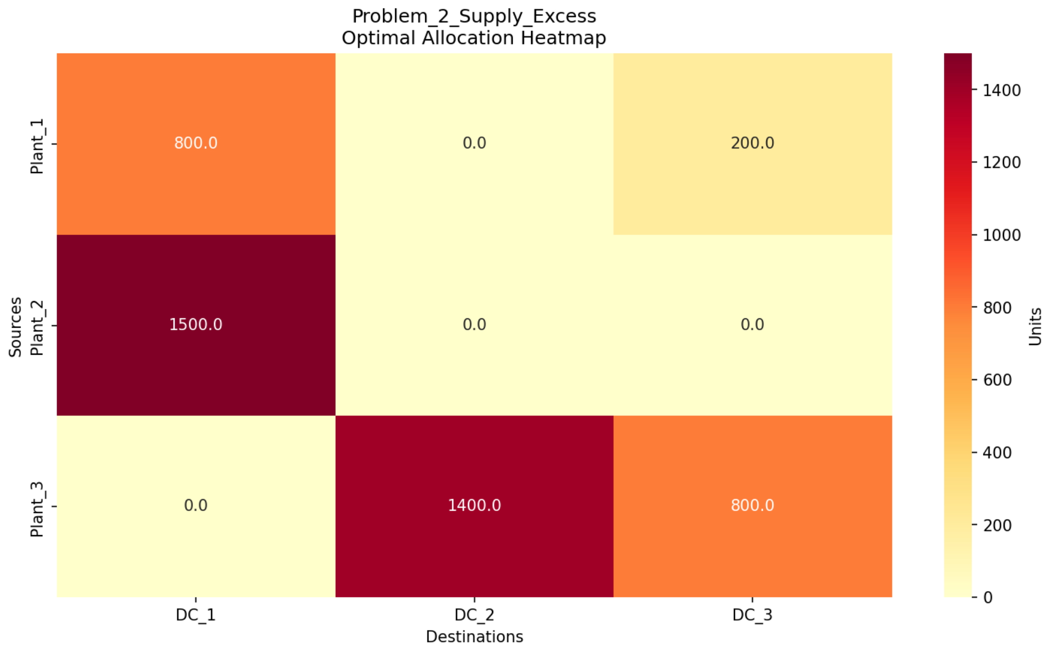

Figure 5.2: Problem 2: Allocation Heatmap - Supply Excess Scenario with External Sourcing

5.2.5 Cost Analysis

$$\text{Regular Transportation Cost} = \text{Sum of (allocation} \times \text{cost)} \tag{5.12}$$
$$= 400(80) + 600(215) + \cdots \tag{5.13}$$
$$= 425,200 \text{ Rs.} \tag{5.14}$$
$$\tag{5.15}$$
$$\text{External Sourcing Cost} = 1000 \times 1000 = 1,000,000 \text{ Rs.} \tag{5.16}$$
$$\tag{5.17}$$
$$\text{TOTAL COST} = 425,200 + 1,000,000 = 1,425,200 \text{ Rs.} \tag{5.18}$$

5.2.6 Business Interpretation

- **Shortage Impact:** 1000 units (21.3% of demand) must be sourced externally
- **Premium Cost:** External sourcing adds Rs. 1,000,000 to transportation cost
- **DC 3 Critical:** All demand at DC 3 must be met by external source
- **Strategic Options:**
  1. Increase production capacity at Plant 1, 2, or 3
  2. Negotiate lower external sourcing rates
  3. Implement demand reduction strategies
  4. Find alternative suppliers for partial coverage

—

5.3 Problem 3: Demand Excess Scenario - Detailed Analysis

5.3.1 Optimal Allocation with 20% Demand Increase

Table 5.3: Problem 3 - Optimal Power Allocation (MW)

Source	City 1	City 2	City 3	Total Supply
Plant 1	0	0	25	25
Plant 2	0	40	0	40
Plant 3	23	2	5	30
External Grid	13	0	0	13
Total	36	42	30	108

5.3.2 Visual Representation

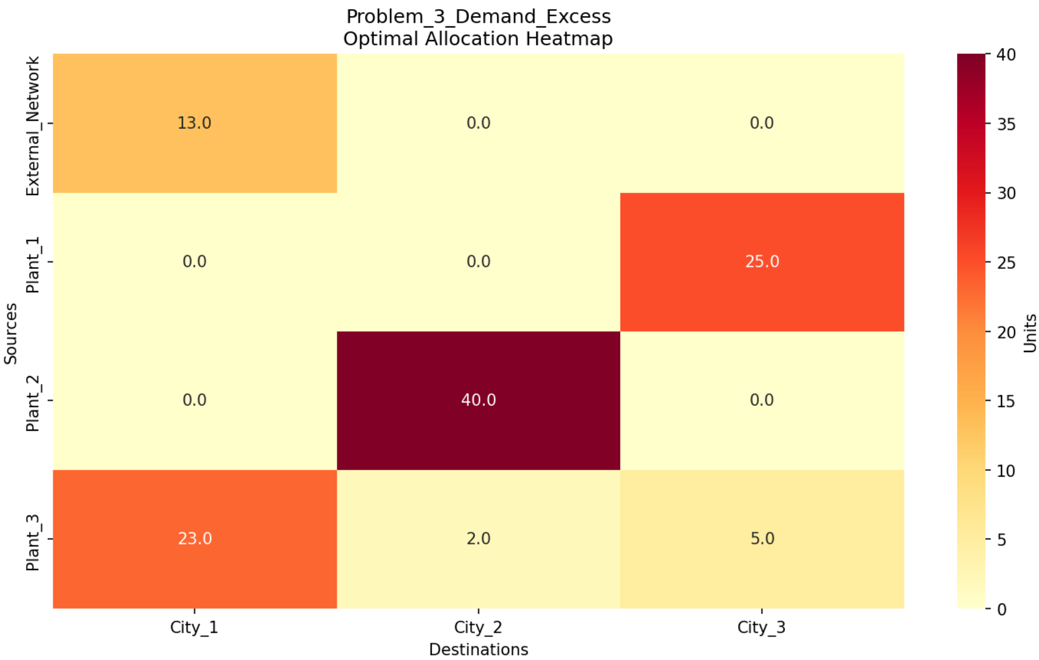

Figure 5.3: Problem 3: Allocation Heatmap - Demand Excess with Premium External Grid

Figure 5.3 visualizes Problem 3's optimal allocation. The external grid row (bottom) clearly shows where premium sourcing is required, with the concentration of external supply at City 1 readily apparent from the heatmap intensity.

5.3.3 Cost Breakdown

Regular transportation costs (internal plants):

$$\text{Plant 1}\rightarrow\text{City 3:} \quad 25 \times 400 = 10,000 \tag{5.19}$$
$$\text{Plant 2}\rightarrow\text{City 2:} \quad 40 \times 300 = 12,000 \tag{5.20}$$
$$\text{Plant 3}\rightarrow\text{City 1:} \quad 23 \times 500 = 11,500 \tag{5.21}$$
$$\text{Plant 3}\rightarrow\text{City 2:} \quad 2 \times 480 = 960 \tag{5.22}$$
$$\text{Plant 3}\rightarrow\text{City 3:} \quad 5 \times 450 = 2,250 \tag{5.23}$$
$$\text{Subtotal (Internal)} = 36,710 \text{ Rs.} \tag{5.24}$$

Premium external sourcing:

$$\text{External}\rightarrow\text{City 1:} \quad 13 \times 1000 = 13,000 \text{ Rs.} \tag{5.25}$$

Total cost:

$$Z_{\text{total}} = 36,710 + 13,000 = 49,710 \text{ Rs.} \tag{5.26}$$

Premium cost as percentage:

$$\frac{13,000}{49,710} \times 100 = 26.15\% \tag{5.27}$$

5.3.4 Strategic Analysis and Recommendations

1. **Capacity Expansion ROI:**
   - Adding 15 MW capacity: Saves Rs. 13,000/month = Rs. 156,000/year
   - Capital cost estimate: Rs. 2,000,000
   - Payback period: $\frac{2,000,000}{156,000} \approx 12.8$ years
   - **Decision:** Moderate ROI; consider if long-term demand growth expected
2. **Demand Management:**
   - Implement time-of-use pricing with 15-20% peak reduction
   - Reduces demand to 91.8-96 MW (within 95 MW capacity)
   - Cost savings: Rs. 1,950-2,600/month
   - **Implementation time:** 1-2 months
3. **City 1 Priority:**
   - 100% of City 1 shortage (13 MW) sourced externally
   - Recommend localized capacity addition or direct supply line
   - Expected savings: Rs. 13,000/month for full coverage
4. **External Supplier Negotiation:**

- Current rate: Rs. 1000/MW
- Negotiate down to Rs. 750/MW $\Rightarrow$ Saves Rs. 3,250/month
- Annual savings: Rs. 39,000 (7.8% of external cost)
- **Feasibility:** Moderate (requires volume guarantees)

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5.4 Sensitivity Analysis

5.4.1 Scenario 1: 10% Cost Increase on Low-Cost Route

Change: Factory A $\rightarrow$ Market Y cost increases from Rs. 3 to Rs. 3.30/ton

Impact Calculation:

$$\begin{aligned} \text{Change in route cost:} &\quad 0.30 \text{ Rs./ton} && (5.28) \\ \text{Units allocated:} &\quad 7 \text{ tons} && (5.29) \\ \text{Total cost increase:} &\quad 7 \times 0.30 = 2.10 \text{ Rs.} && (5.30) \\ \text{New total cost:} &\quad 153 + 2.10 = 155.10 \text{ Rs.} && (5.31) \end{aligned}$$

Allocation Stability: Remains optimal (this was alternative optimal solution with $\Delta_{AY} = 0$)

5.4.2 Scenario 2: 25% Demand Increase (vs. 20%)

New Demand:

$$\begin{aligned} \text{City 1:} &\quad 30 \times 1.25 = 37.5 \text{ MW} && (5.32) \\ \text{City 2:} &\quad 35 \times 1.25 = 43.75 \text{ MW} && (5.33) \\ \text{City 3:} &\quad 25 \times 1.25 = 31.25 \text{ MW} && (5.34) \\ \text{Total:} &\quad = 112.5 \text{ MW} && (5.35) \end{aligned}$$

New Shortage:

$$112.5 - 95 = 17.5 \text{ MW} \tag{5.36}$$

Cost Impact:

$$\begin{aligned} \text{Additional external MW:} &\quad 17.5 - 13 = 4.5 \text{ MW} && (5.37) \\ \text{Additional external cost:} &\quad 4.5 \times 1000 = 4,500 \text{ Rs.} && (5.38) \\ \text{New total cost:} &\quad 49,710 + 4,500 = 54,210 \text{ Rs.} && (5.39) \end{aligned}$$

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5.5 Summary Comparison of All Three Problems

Table 5.4: Comparative Analysis - Problem 1, 2, and 3

Metric	**Problem 1**	**Problem 2**	**Problem 3**
Problem Type	Balanced	Demand > Supply	Demand > Supply
Total Supply	37 units	3,700 units	95 MW
Total Demand	37 units	4,700 units	108 MW
Imbalance	None	1,000 units (27%)	13 MW (12%)
Total Cost	153 Rs.	1,425,200 Rs.	49,710 Rs.
Premium/External	None	1,000,000 Rs.	13,000 Rs.
Premium as %	-	70.2%	26.15%
Critical Issue	None	Severe shortage	Moderate shortage
Routes Active	5	12	8
Dummy Usage	No	Yes (External)	Yes (External)
Key Strategy	Cost optimization	Shortage handling	Premium minimization
Recommendation	Maintain operations	Capacity expansion	Demand management

5.5.1 Insights

- **Problem 1:** Optimal resource allocation with balanced supply-demand
- **Problem 2:** High-impact shortage scenario requiring significant external sourcing
- **Problem 3:** Moderate shortage manageable through strategic planning
- **Scale:** Problem 2 costs highest due to large shortage percentage and premium rates
- **Solutions:** Each requires different intervention (capacity, demand reduction, supplier negotiation)

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