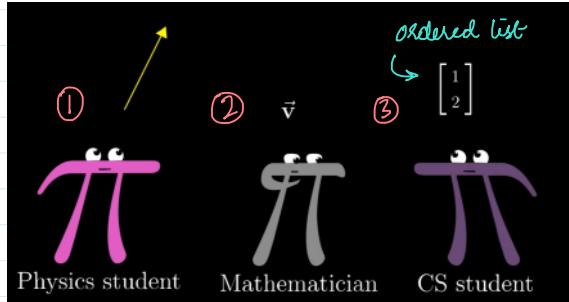


Vectors | Chapter 1, Essence of linear algebra + Math for ML book

↳ what are scalars? real numbers or elements in the field.

↳ what are vectors? 3 perspectives



↳ coordinate system & vectors rooted in origin.

↳ nD vector as a point in nD space

↳ vector addition

Δ law (11th law)

↳ multiplying vector by scalar → scaling

↳ components of a vector

$$a\hat{i} + b\hat{j}$$

Linear combinations, span, and basis vectors | Chapter 2, Essence of linear algebra

↳ think about the coordinates as scalars. (Basis aka unit vectors)

& then the addition/subtraction/multiplication

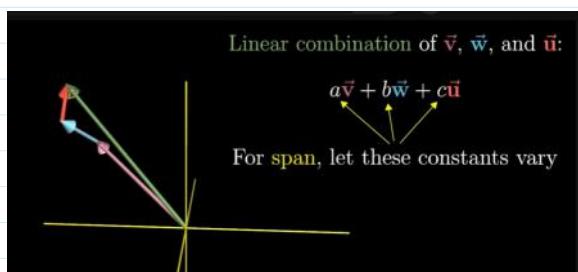
↳ special vectors \hat{i}, \hat{j} are basis in 2D vector space

↳ why linear combination? Reaching all possible vectors with pairs of 2D vectors is called span of those vectors.



→ think of collection of vectors as points in the vector space

↳ span of 1 vector → line
2 LI vectors → plane
3 LI vectors → hyperplane



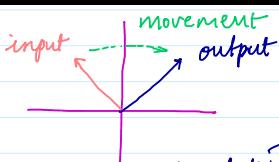
→ 3 linearly independent (LI) vectors span 3D space

→ we need n LI vectors to span nD space

Linear transformations and matrices | Chapter 3

Matrices as Linear transformations

Listen up folks, this one is particularly important



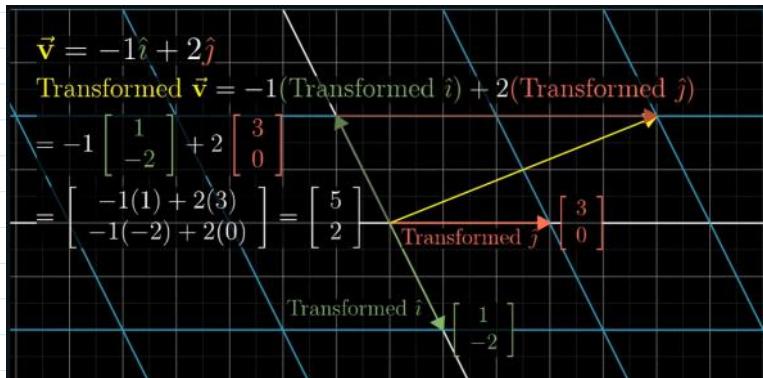
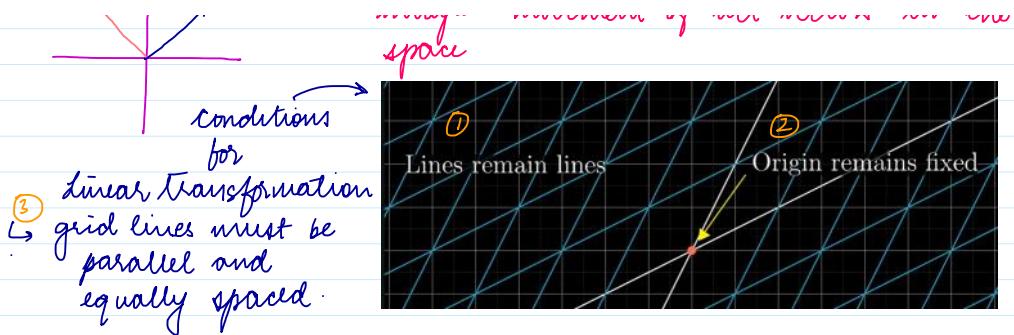
↳ matrix is best understood by visualization.

↳ matrix vector multiplication or linear transformation

vector input → function → vector output

↳ imagine movement of all vectors in the space





→ to understand the transformation, follow the basis vectors. How their positions change? where do they land?

→ plugging these transformed values of \hat{i} & \hat{j} in the original vector equation will give us the output vector.

$i \rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix}$	$j \rightarrow \begin{bmatrix} 3 \\ 0 \end{bmatrix}$
$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow x \begin{bmatrix} 1 \\ -2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$	

"2x2 Matrix"

$$\begin{bmatrix} (3) & (2) \\ (-2) & (1) \end{bmatrix}$$

Where \hat{i} lands Where \hat{j} lands

"2x2 Matrix"

$$\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = 5 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + 7 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

→ transformed unit vectors

→ scaled version of basis vectors

→ linear combination

→ output vector

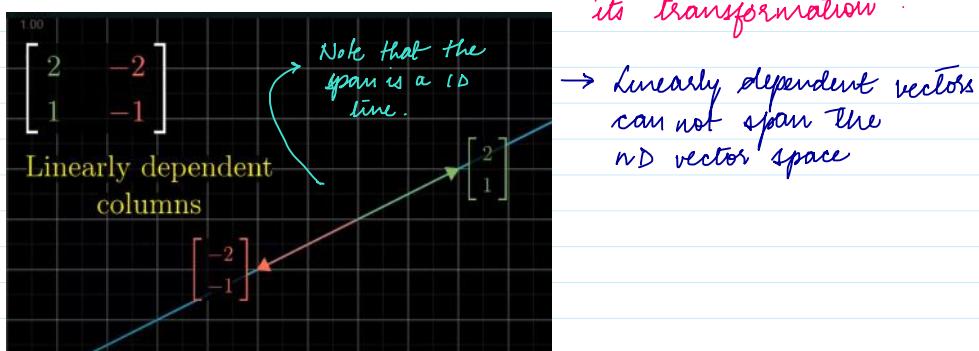
→ generalization

Where all the intuition is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x \underbrace{\begin{bmatrix} a \\ c \end{bmatrix}}_{\text{Where all the intuition is}} + y \underbrace{\begin{bmatrix} b \\ d \end{bmatrix}}_{\text{Where all the intuition is}} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

eg: rotation by 90°
shear

→ we can also start from a matrix & deduce its transformation



Essence of Linear Algebra

- Chapter 1: Vectors, what even are they?
- Chapter 2: Linear combinations, span and bases
- Chapter 3: Matrices as linear transformations
- Chapter 4: Matrix multiplication as composition
- Chapter 5: The determinant
- Chapter 6: Inverse matrices, column space and null space
- Chapter 7: Dot products and cross products
- Chapter 8: Change of basis
- Chapter 9: Eigenvectors and eigenvalues
- Chapter 10: Abstract vector spaces

All we learnt in video 4
is the underlying concept
of following videos.

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Composition}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}} \begin{bmatrix} x \\ y \end{bmatrix}$$

applying
rotation &
then shear
(read from
right to left)
similar to functions
 $f(g(x))$
② ①

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Composition}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}}$$

where \hat{i} goes →
mat multⁿ is just composition
of transformations and get a
new transformation.

Note, matrix multⁿ is associative

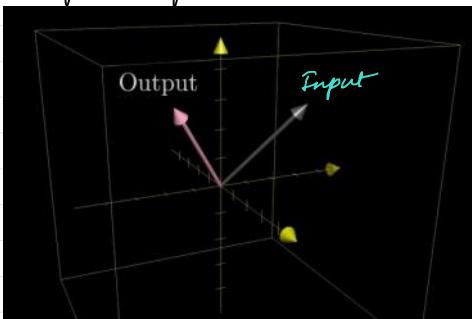
$$M_1(M_2 M_3) = (M_1 M_2) M_3 \Rightarrow \text{consider as applying transf } n \text{ one after the other}$$

$$\underbrace{\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}}_{M_1} = \underbrace{\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}}_{\text{Composition}}$$

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

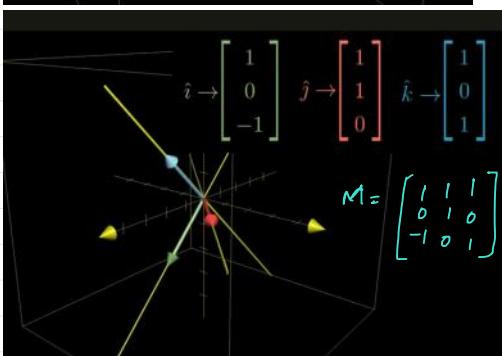
Three-dimensional linear transformations | Chapter 5

↳ in previous two videos we saw
linear transformations in 2D
they easily translate to higher Ds



→ as discussed
earlier, we
see the
transfⁿ of
unit vectors
now there are
3 (i, j, k)

$$\underbrace{\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix}}_{\text{Second transformation}} \underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}}_{\text{First transformation}}$$

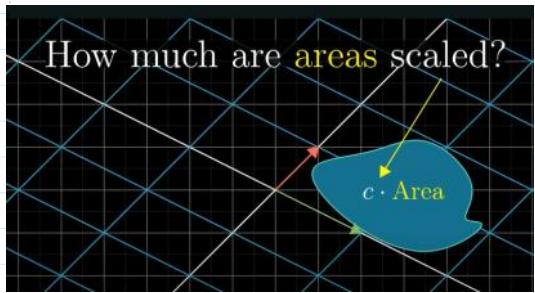


$$\underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}}_{\text{Transformation}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{Input vector}} = \underbrace{x \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} + y \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + z \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}}_{\text{Output vector}}$$

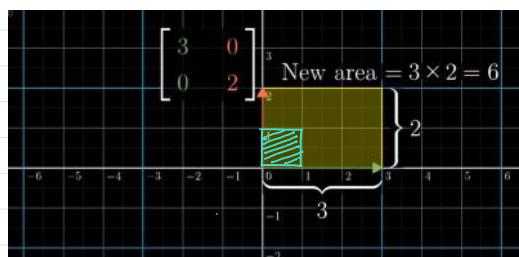
↳ composition of transfⁿ is also
the same as seen.

The determinant | Chapter 6

↳ transformations generally stretch or squeeze the space.



↳ Note: since the grid lines are parallel and equispaced, the transf applied to one square is translated to all the squares in the space



↳ the scaling factor of the area is determinant.
(linear transf) ↗ matrix

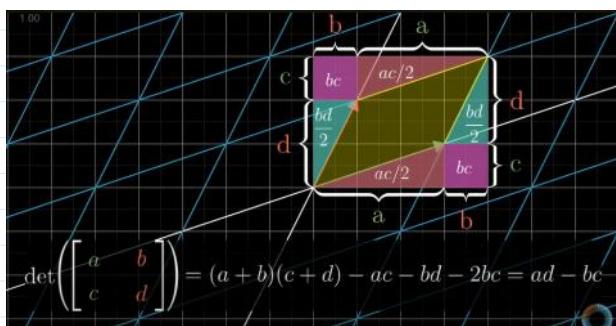
→ unit square scaled by a factor of 6.

→ if scaling factor is -ve it rotates / changes orientation of the space.

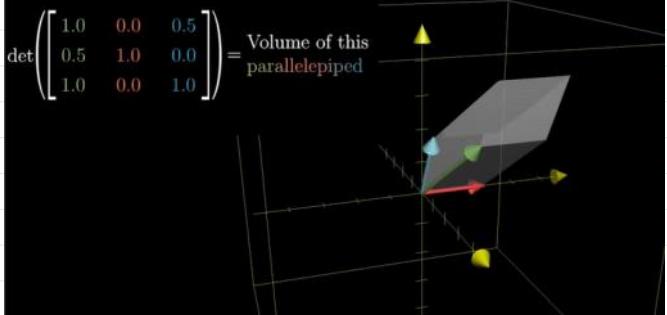
↳ In 3D we get volume instead of area (how?)

Note: when we rotate the space \hat{i} & \hat{j} come closer and at some point overlap. Here the det is 0. And if we go further, it becomes -ve.

↳ right hand rule can tell the orientation.



$$\det \begin{pmatrix} 1.0 & 0.0 & 0.5 \\ 0.5 & 1.0 & 0.0 \\ 1.0 & 0.0 & 1.0 \end{pmatrix} = \text{Volume of this parallelepiped}$$



Inverse matrices, column space and null space |

Chapter 7

↳ linear algebra has its use mainly because it helps to solve linear system of equations

we are looking for the vector x which when transformed by A , it lands on b .

$$A\vec{x} = \vec{b}$$

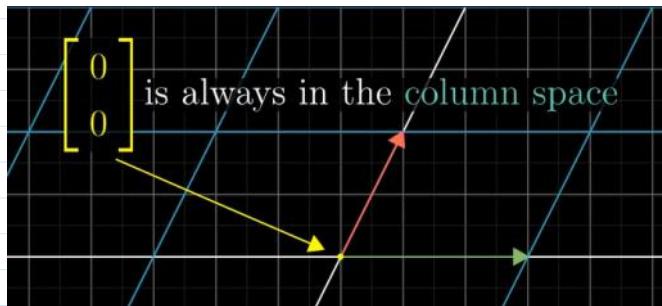
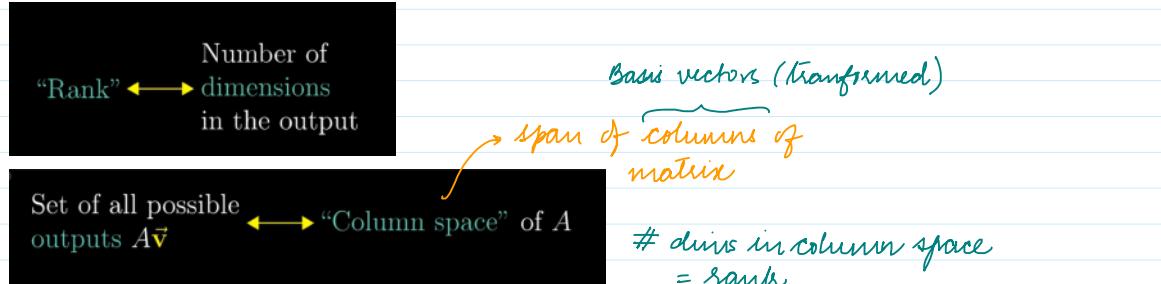
↳ any transf that undoes the effect of a transf is called inverse of that transf.

↳ the solution depends on the $\det(A)$ does it ↓ dims or leave as is.

↳ when # variables = # equations then probably we have unique solution

↳ only when $\det(A) \neq 0$ we have A^{-1} as $\det(A) = 0$ squeezes the space to a

↪ only when $\det(A) \neq 0$ we have A^{-1}
 as $\det(A) = 0$ squeezes the space to a
 lower dim. → we can not undo
 to go to higher dim. At least not by
 using a function as it can take single i/p
 and single o/p at a time
 zero area (2D) or zero volume (3D)



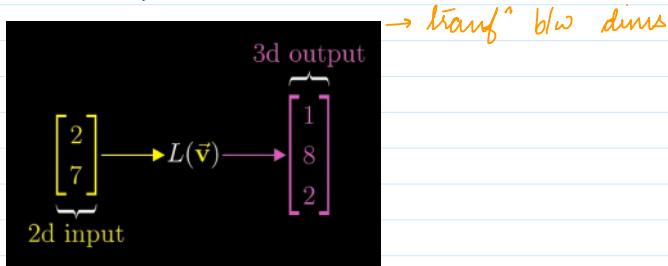
Under the light of linear transformations
 Inverse matrices
 Column space
 Rank
 Null space

↪ in case of full rank,
 only 0 vector itself can
 land on origin.

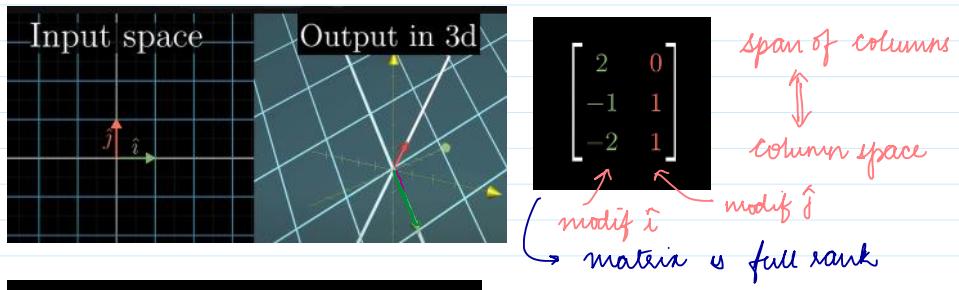
↪ but when rank is not full,
 dim is reduced set of
 many vectors can pass the origin
 This set is called null space
 or kernel of the matrix

Non-square matrices as transformations between dimensions | Chapter 8

↪ so far we have discussed linear transf^r
 in 2D, 3D.



↪ for non square transf,
 again think about
 the unit vectors,
 where do they land after
 the "transf" is applied?

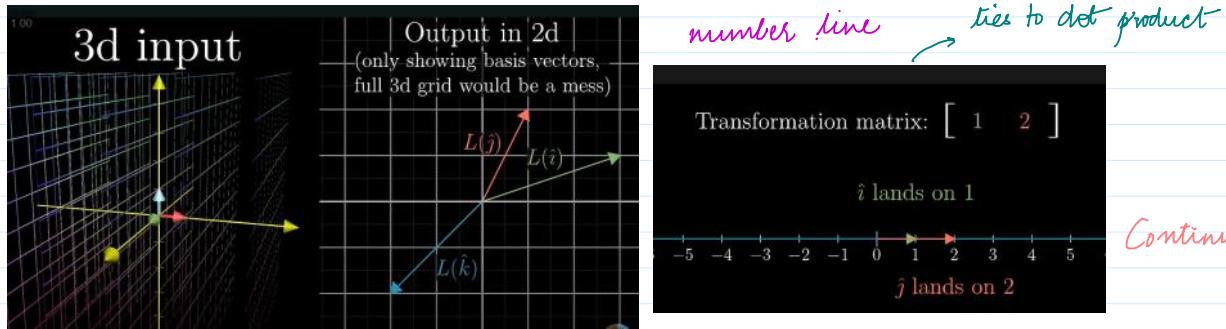


3 basis vectors

$$\left[\begin{array}{ccc} 3 & 1 & 4 \\ 1 & 5 & 9 \end{array} \right]$$

} 2 coordinates for each landing spots

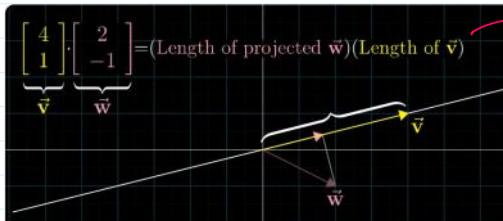
\rightarrow "transf" from 3D to 2D
similarly, we can think
of 2D to 1D transf



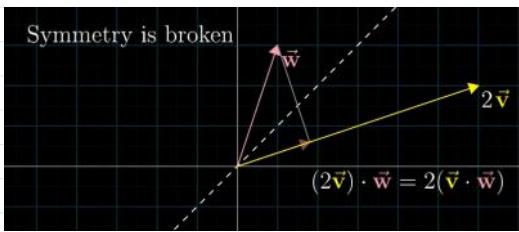
Continued in page 2 ...

Dot products and duality | Chapter 9

↪ must understand transfⁿ first

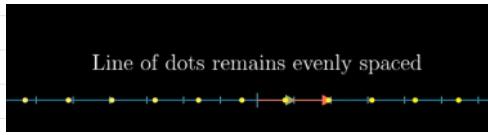


either of the vectors can be projected & multiplied in any order

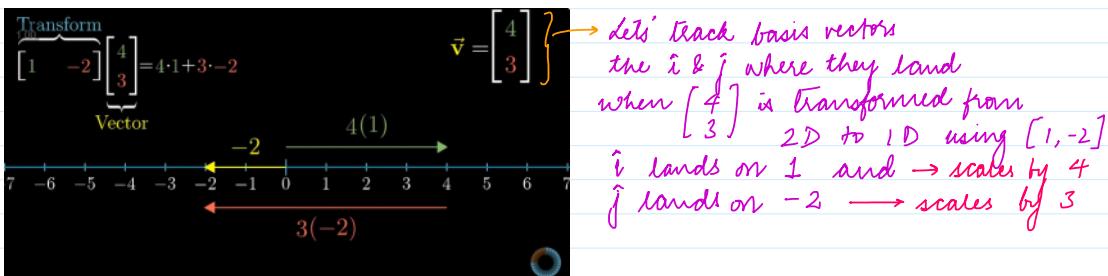


→ think about the case when \vec{v} & \vec{w} are of same length here symmetry holds but on scaling either vector the respective projection is also scaled by the same factor.

linear transfⁿ from nD to 1D



if transfⁿ is not linear, then the dots will be unevenly spaced

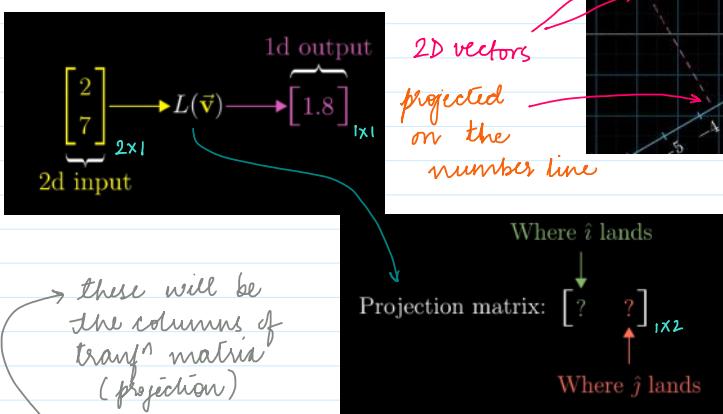


Let's track basis vectors the \hat{i} & \hat{j} where they land when $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ is transformed from 2D to 1D using $[1, -2]$
 \hat{i} lands on 1 and → scales by 4
 \hat{j} lands on -2 → scales by 3

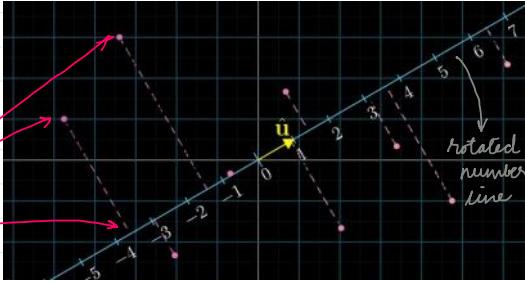
$$\begin{bmatrix} 2 & 7 \\ 1 & -2 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 7 \\ 1 \\ -2 \end{bmatrix}$$

1 × 2 matrices \longleftrightarrow 2d vectors

what does this association mean geometrically?
↪ linear transfⁿ that takes numbers to vectors & vice versa

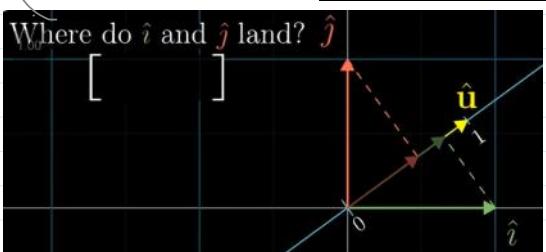


2D vectors projected on the number line



these will be the columns of transfⁿ matrix (projection)

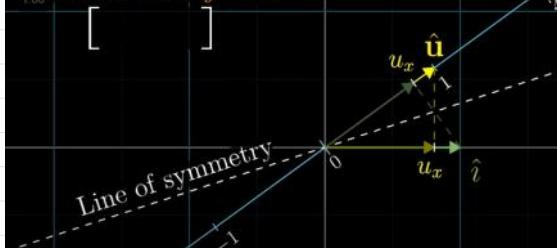
Where \hat{i} lands
Projection matrix: $\begin{bmatrix} ? & ? \end{bmatrix}_{1 \times 2}$
Where \hat{j} lands



because of symmetry, note that \hat{i} projection on \hat{i} ie u_2 is same as



Where do \hat{i} and \hat{j} land?



note that \hat{i}^0
 \hat{i} projection on \hat{i} ie u_x
 is same as
 \hat{u} projection on \hat{i} ie u_x
 similarly for \hat{j} we get u_y
 projection matrix = $[u_x \ u_y]$

$$\begin{array}{l} \hookrightarrow \text{we see the relation } [u_x \ u_y] \begin{bmatrix} x \\ y \end{bmatrix} = u_x \cdot x + u_y \cdot y \\ \text{Matrix-vector product} \\ \Downarrow \\ \text{Dot product} \end{array}$$

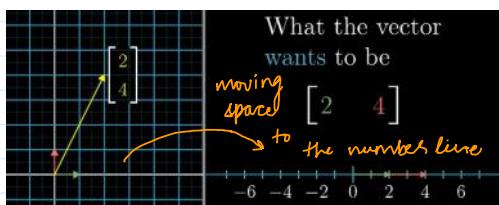
$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = u_x \cdot x + u_y \cdot y$$

→ for non-unit vectors.

this is concept of 'duality'. Anytime we have a 2d to 1d linear transf there exists a unique vector such that applying the "transf" is same as taking dot product with that vector.

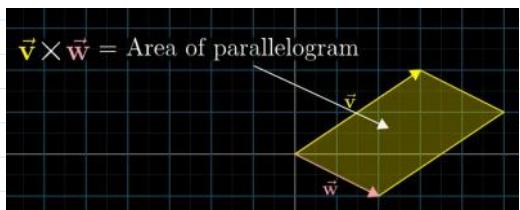
To sum up, dot product is tool to know

- direction of vectors w.r.t one another
- how the vector transforms (how it is & what it wants to be)



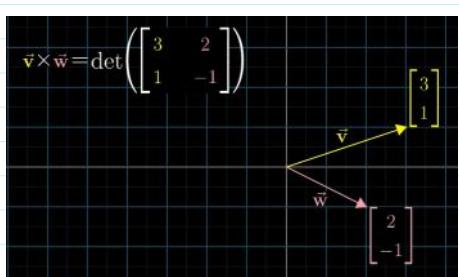
→ transf' on arrows in space
 vs
 moving the space

Cross products | Chapter 10



→ order of basis
 vector defines orientation

→ when \vec{v} & \vec{w} is swapped,
 the area is -ve.



→ recall that determinant (linear transf)
 measures how the area of unit square changes.
 → thus when columns are interchanged,
 the sign of area also changes
 (det)

$$(3\vec{v}) \times \vec{w} = 3(\vec{v} \times \vec{w})$$

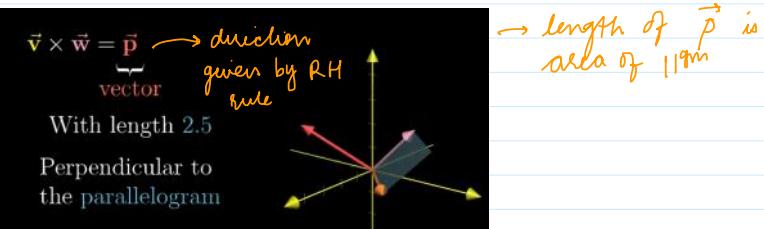
→ when vectors are ⊥ area is max^m

↪ we use right hand rule to find direction of the result (cross product)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \det \left(\begin{bmatrix} \hat{i} & v_1 & w_1 \\ \hat{j} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix} \right)$$

$$\hat{i} \underbrace{(v_2 w_3 - v_3 w_2)}_{\text{Some number}} + \hat{j} \underbrace{(v_3 w_1 - v_1 w_3)}_{\text{Some number}} + \hat{k} \underbrace{(v_1 w_2 - v_2 w_1)}_{\text{Some number}}$$

$$\underbrace{\hat{i}(v_2 w_3 - v_3 w_2)}_{\text{Some number}} + \underbrace{\hat{j}(v_3 w_1 - v_1 w_3)}_{\text{Some number}} + \underbrace{\hat{k}(v_1 w_2 - v_2 w_1)}_{\text{Some number}}$$

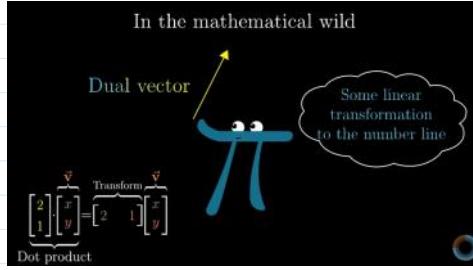


→ length of \vec{p} is area of 119m

Cross products in the light of linear transformations |

Chapter 11

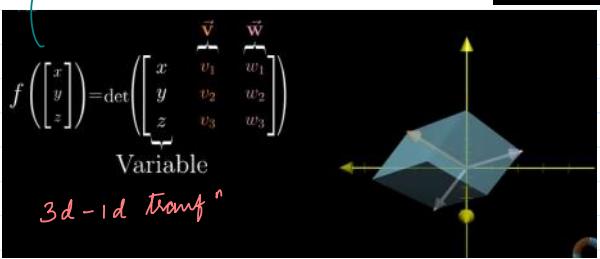
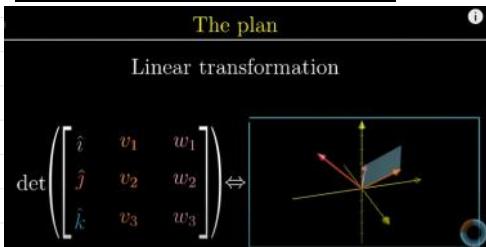
Recall



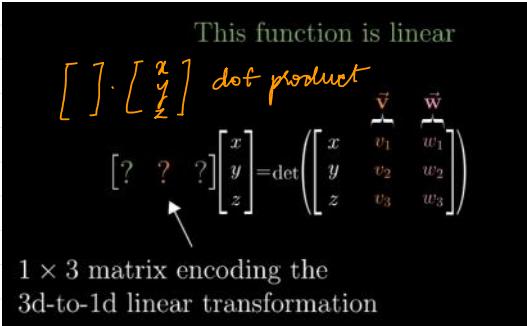
- The plan
1. Define a 3d-to-1d linear transformation in terms of \vec{v} and \vec{w}
 2. Find its dual vector
 3. Show that this dual is $\vec{v} \times \vec{w}$

Recall the 2d cross product & compare with 3d cross prod.

→ consider this function

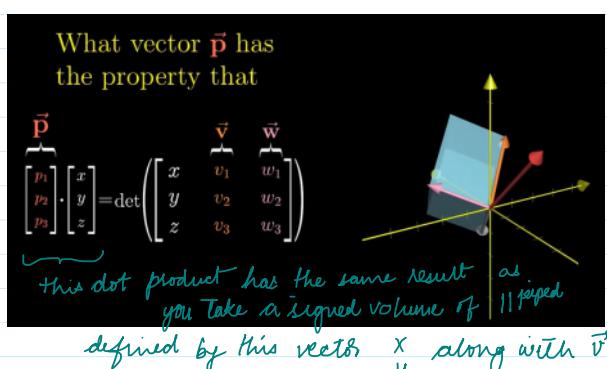


→ compare the coefficients to get p_1, p_2, p_3
the x, y, z are comparable to $\vec{i}, \vec{j}, \vec{k}$ as we saw earlier.



$$\begin{bmatrix} \vec{p} \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \det \begin{pmatrix} x & v_1 & w_1 \\ y & v_2 & w_2 \\ z & v_3 & w_3 \end{pmatrix}$$

$$x(p_2 \cdot w_3 - p_3 \cdot w_2) + y(p_3 \cdot v_1 - p_1 \cdot v_3) + z(p_1 \cdot v_2 - p_2 \cdot v_1)$$



if we choose the appropriate direction of \vec{p} the cases will line up where the area of 119m is -ve.
(the orientation is given by RH rule)

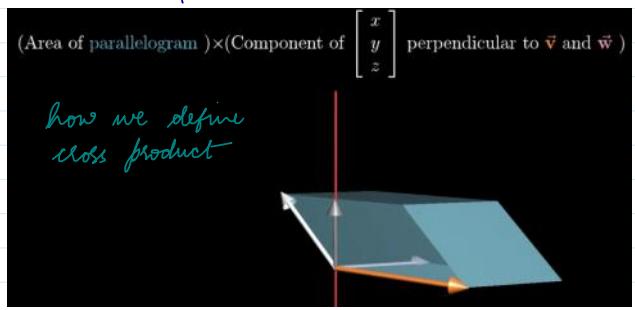
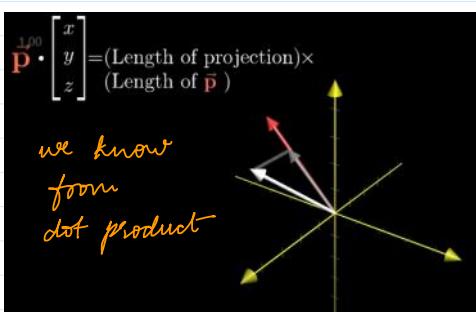
(Area of parallelogram) × (Component of $\begin{bmatrix} x \\ y \end{bmatrix}$ perpendicular to \vec{v} and \vec{w})

$\int_{-1}^1 \begin{bmatrix} x \\ y \end{bmatrix}$

this is the same thing as taking a dot product of \vec{x} and

Σ

✓ this is the same thing as taking a dot product

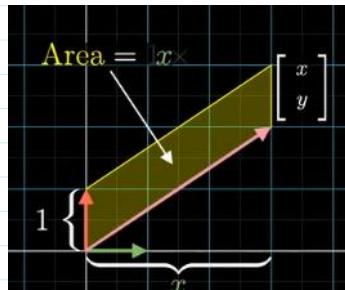
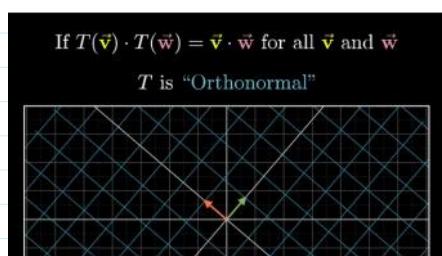
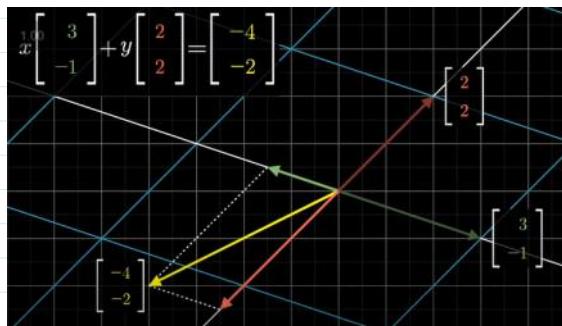
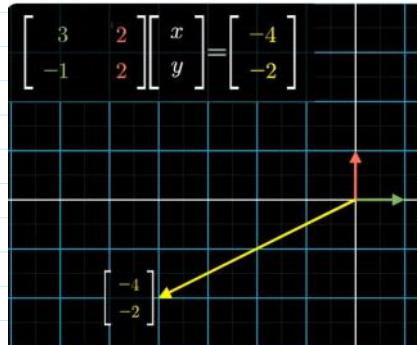


a vector (\vec{p}) that is \perp to \vec{v} & \vec{w} and length is equal to the area of $\parallel gm$.

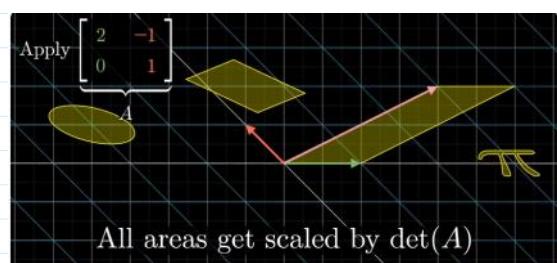
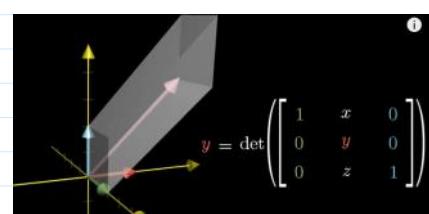
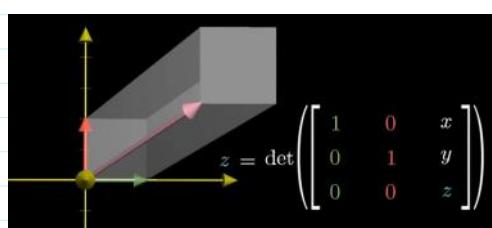
Cramer's rule, explained geometrically | Chapter 12

“Cramer’s rule”

$$x = \frac{\det\begin{bmatrix} 7 & 2 & 3 \\ -8 & 0 & 2 \\ 3 & 6 & -9 \end{bmatrix}}{\det\begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 2 \\ -4 & 6 & -9 \end{bmatrix}} \quad y = \frac{\det\begin{bmatrix} -4 & 7 & 3 \\ -1 & -8 & 2 \\ -4 & 3 & -9 \end{bmatrix}}{\det\begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 2 \\ -4 & 6 & -9 \end{bmatrix}} \quad z = \frac{\det\begin{bmatrix} -4 & 2 & 7 \\ -1 & 0 & -8 \\ -4 & 6 & 3 \end{bmatrix}}{\det\begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 2 \\ -4 & 6 & -9 \end{bmatrix}}$$



To preserve the properties of resultant of dot product of two vectors, the transf must be Orthonormal.



Similarly,

$$x = \det\left(\begin{bmatrix} x & 0 & 0 \\ y & 1 & 0 \\ z & 0 & 1 \end{bmatrix}\right)$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Area = $\det(A)x$

$$y = \frac{\text{Area}}{\det(A)} = \frac{\det\begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}}{\det\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Area = $\det(A)x$

$$x = \frac{\text{Area}}{\det(A)} = \frac{\det\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}}{\det\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}}$$

Chambers rule

$$3x + 2y - 7z = 4$$

$$1x + 2y - 4z = 2$$

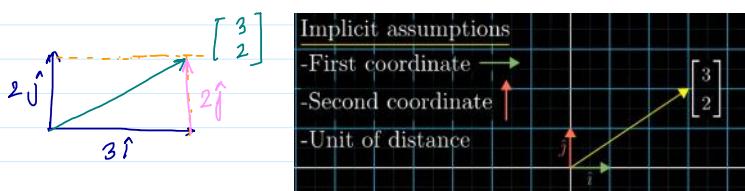
$$4x + 0y + 1z = 5$$

$$\begin{bmatrix} 3 & 2 & -7 \\ 1 & 2 & -4 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

Mystery input

→ take for example the z coordinate of input as the volume spanned by \hat{i}, \hat{j} and the input vector
what happens to this volume after the transfo. What ways can you find the volume?

Change of basis | Chapter 13



why do we care about change of basis?

we see this in eigen values & eigen vectors

↳ any way to translate bw set of numbers & vectors is called coordinate system

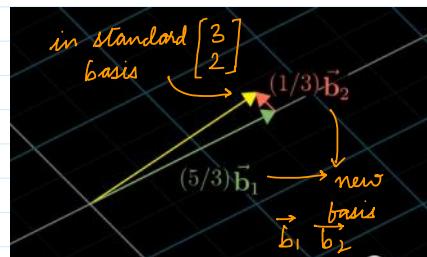
"Coordinate system"

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \Rightarrow$$

"Basis vectors"

$$\begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix}$$

→ we now introduce the idea of using different standard vectors.



$$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

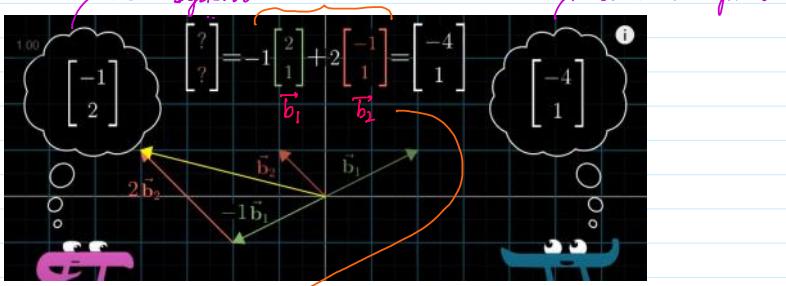
and

$$\vec{b}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\vec{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

in new system $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

matrix multipⁿ



$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \text{vector in } \hat{i}, \hat{j} \text{ system} \end{bmatrix}$$

Inverse change of basis Same vector

$$\underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{\text{transf } ^n \text{ vector in new system}} \underbrace{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}_{\text{vector in } i^j \text{ system}} = \underbrace{\begin{bmatrix} ? \\ ? \end{bmatrix}}_{\text{vector in } i^j \text{ system}}$$

Vector in her coordinates

new \downarrow *old*

$$A \begin{bmatrix} x_j \\ y_j \end{bmatrix} = \begin{bmatrix} x_o \\ y_o \end{bmatrix}$$

change of basis matrix

Same vector in our coordinates

90° rotation

Follow our choice of basis vectors

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Record using our coordinates

Inverse change of basis matrix

Same vector in her language

$$\underbrace{\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix}}_{\text{Written in our language}} \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}_{\text{Written in our language}} = \underbrace{\begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix}}_{\text{Written in our language}}$$

How to translate a matrix

$$S = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Transformation matrix in our language}} \underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{\text{vector in our language}}$$

transformed vector in new language

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1} S$$

Transformation matrix in her language

$$\underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{T_{\text{new}}}^{-1} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Written in our language}} \underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{T_{\text{new}}} = \begin{bmatrix} 1/3 & -2/3 \\ 5/3 & -1/3 \end{bmatrix}$$

$\vec{v}_{\text{new}} = T_{\text{new}} \vec{v}$

a vector in new language

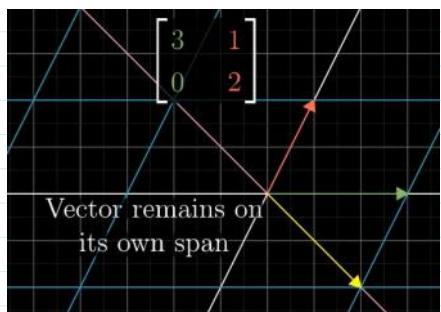
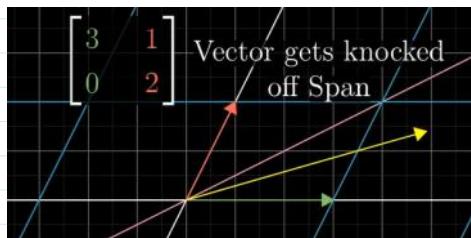
transformed vector in new language

Eigenvalues and eigenvectors | Chapter 14

Eigenvalues and Eigenvectors

$$\det \left(\begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix} \right) = 0$$

Why are we doing this?
What does this actually mean?



Many prerequisites

Linear transformations

Determinants

The "determinant" of a transformation $\det \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix} = 6$

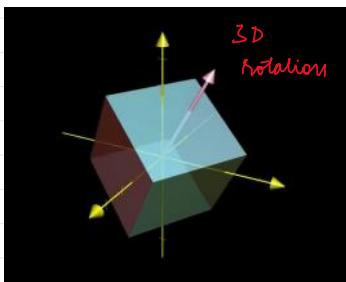
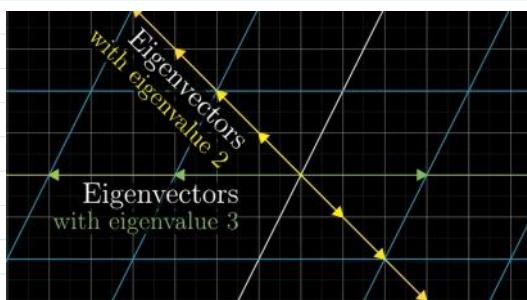
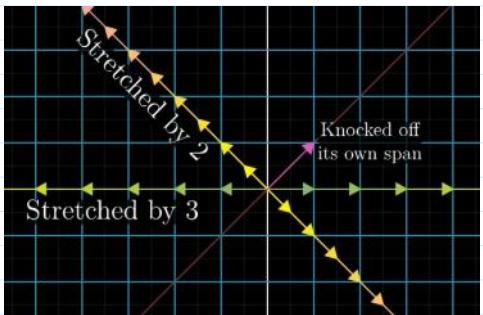
Linear systems

$$\begin{array}{l} 2x + 5y + 3z = -3 \\ 4x + 0y + 8z = 0 \\ 1x + 3y + 0z = 2 \end{array} \rightarrow \begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

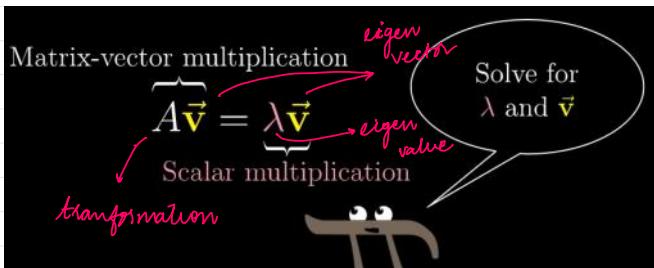
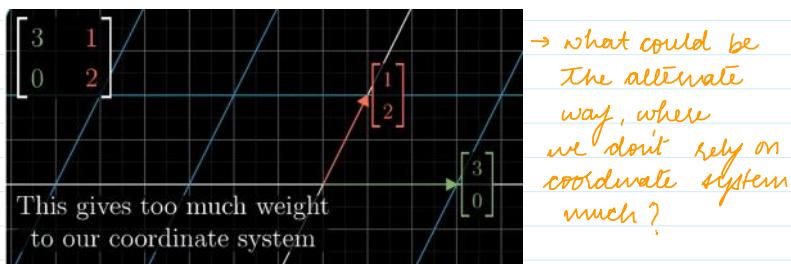
Change of basis

The concept itself is easy.
the concept of eigen value/vectors is hard to understand due to gap in prerequisites

→ a transformation such that the span of vectors is preserved



for 3D rotation,
eigen vector is
the axis of rotation
& eigen value is
1 since there is
no stretch or squeeze
in case of rotations



LHS is matrix vector multipⁿ and
RHS is scalar vector multipⁿ. multiplⁿ by matrix
How are they equal? $\lambda \leftrightarrow A$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

columns of matrix is λ times = $\lambda(I)$
unit vector

so, $A\vec{v} = (\lambda I)\vec{v}$ now both LHS & RHS

$$(A - \lambda I)\vec{v} = \vec{0}$$

↑ we want non zero \vec{v} (non trivial solution)

that means $\det(A - \lambda I)$ must be zero

↪ (only then product of a matrix & non zero vector is zero)

this means the transfⁿ squishes the space to lower dim.
that corresponds to zero det.

so, we need to find such a λ .

this vector v is eigen vector staying in its own span after transformation A

to a line
ie area = 0

$$\det \begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix} = (-\lambda)(-\lambda) - (1)(1) = \lambda^2 + 1 = 0$$

$\lambda = i$ or $\lambda = -i$

→ reason why solution has no eigen values

$$\det \begin{pmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{pmatrix} = (1-\lambda)(1-\lambda) = 0$$

$\lambda = 1$

Eigenvectors with eigenvalue 1

→ consider shear it has only one eigen value all eigenvectors have this eigen value

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Scale everything by 2

→ a single eigen value can have more than a line full of eigenvectors

→ the eigen value is 2 but every vector in the plane is eigenvector

Eigen basis, imagine basis vectors to be the eigen vectors
think of a diagonal matrix

"Diagonal matrix"

$$\begin{bmatrix} -5 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

→ each column is eigen basis
and the diagonal entries are eigen values

→ a set of basis vectors which are also eigenvectors are called eigen basis

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3^4 x \\ 2^4 y \end{bmatrix}$$

→ $\begin{bmatrix} 3^5 x \\ 2^5 y \end{bmatrix}$ can go to any number of repeated multiplication

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

Use eigenvectors as basis

$$100 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{\text{Change of basis matrix}} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

Use eigenvectors as basis

→ guaranteed to be diagonal as the basis vectors get scaled during "transf"

A quick trick for computing eigenvalues | Chapter 15

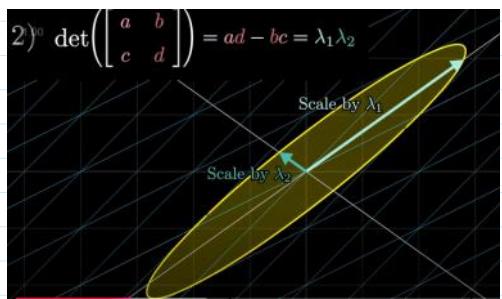
Find the eigenvalues of $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$

$$\det \begin{pmatrix} 3-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = (3-\lambda)(1-\lambda) - (1)(4) = (3-4\lambda+\lambda^2) - 4 = \underbrace{\lambda^2 - 4\lambda - 1}_{\text{Characteristic polynomial of } \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}}$$

→ Roots of this eqⁿ are the eigen values

→ area scaled by

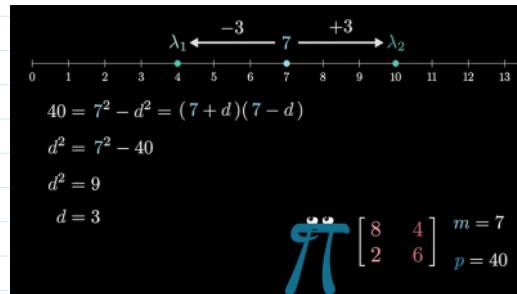
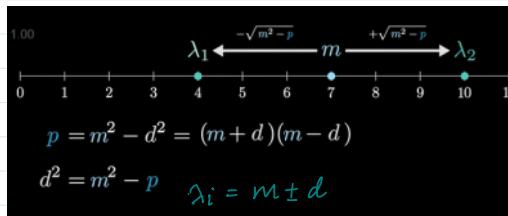
Characteristic polynomial of $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$



\rightarrow area scaled by
 $\lambda_1 \lambda_2$ so
product of eigen
values = $\det(A)$

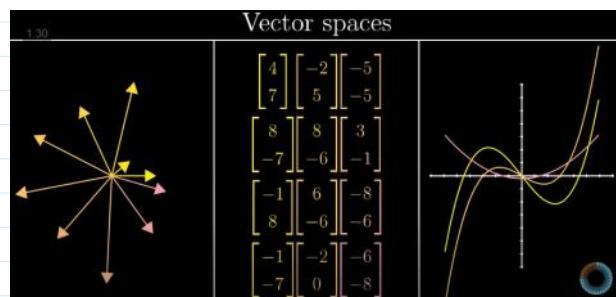
1) $\frac{1}{2} \text{tr} \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \frac{a+d}{2} = \frac{\lambda_1 + \lambda_2}{2} = m$ (mean)

2) $\det \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc = \lambda_1 \lambda_2 = p$ (product)



Abstract vector spaces | Chapter 16

Linear algebra concepts	Alternate names when applied to functions
Linear transformations	Linear operators
Dot products	Inner products
Eigenvectors	Eigenfunctions



Rules for vectors addition and scaling

1. $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
2. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
3. There is a vector $\mathbf{0}$ such that $\mathbf{0} + \vec{v} = \vec{v}$ for all \vec{v}
4. For every vector \vec{v} there is a vector $-\vec{v}$ so that $\vec{v} + (-\vec{v}) = \mathbf{0}$
5. $a(b\vec{v}) = (ab)\vec{v}$
6. $1\vec{v} = \vec{v}$
7. $a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$
8. $(a+b)\vec{v} = a\vec{v} + b\vec{v}$

“Axioms”