Geometric Arguments in the Generation of Simulated Real Decays

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For simplicity of argument, the following calculations on the generation of decay tracks are performed in the “standard looking” Cartesian space. In the application of the results of these calculations, however, the Cartesian space is rotated 90° in the x axis such that the coordinates x, y, and z become x, $-z$, y, respectively, in the coding application.

The Cartesian Space

The following calculations are for the cases when the incoming particle's (the assumption is that the trajectory of this particle is perpendicular to the back walls of the box, i.e, it is originally traveling in the positive y direction, and that it decays at the origin of the Cartesian space in Figure 1) particle's decay products hit the back walls of the boxes, the left and right walls, and the upper and bottom walls. Of course, the code takes into consideration the cases when a mixture of these three could happen as the decay products travel through the boxes' walls.

(i) Back Wall

We define the opening angle between the two decay tracks, θ, the rotation angle of the plane made by the decay tracks, ϕ, and the distance between the point of decay (the origin in the coordinate system) and the point of intersection with a wall, $r_1 = r_2$. We look at a single track; its sibling will follow a similar case but at $-\theta$.
From Figure 2 one can see that

$$\begin{aligned} x_1 &= r_1 \sin(\theta) \\ y_1 &= 2.40 \\ z_1 &= 0 \\ r_1 &= y_1 / \cos(\theta) \end{aligned} \tag{1}$$

$$\begin{aligned} x_2 &= x_1 \cos(\phi) \\ y_2 &= y_1 \\ z_2 &= x_1 \sin(\phi) \\ r_2 &= r_1. \end{aligned} \tag{2}$$

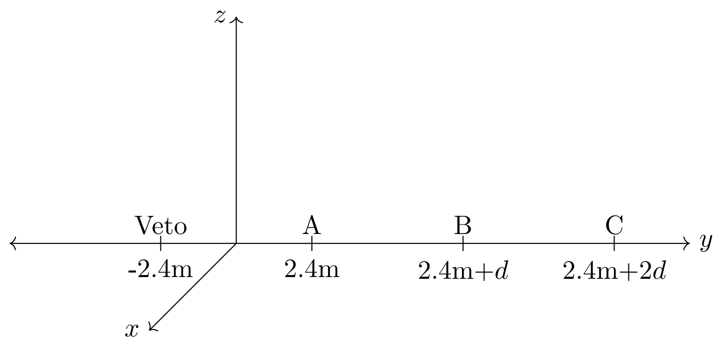

Figure 1: Cartesian space showing the positions of the back walls of the boxes A, B, and C, and the position of the veto wall. d is the separation distance between the boxes' back walls; this parameter can be modified in the code.

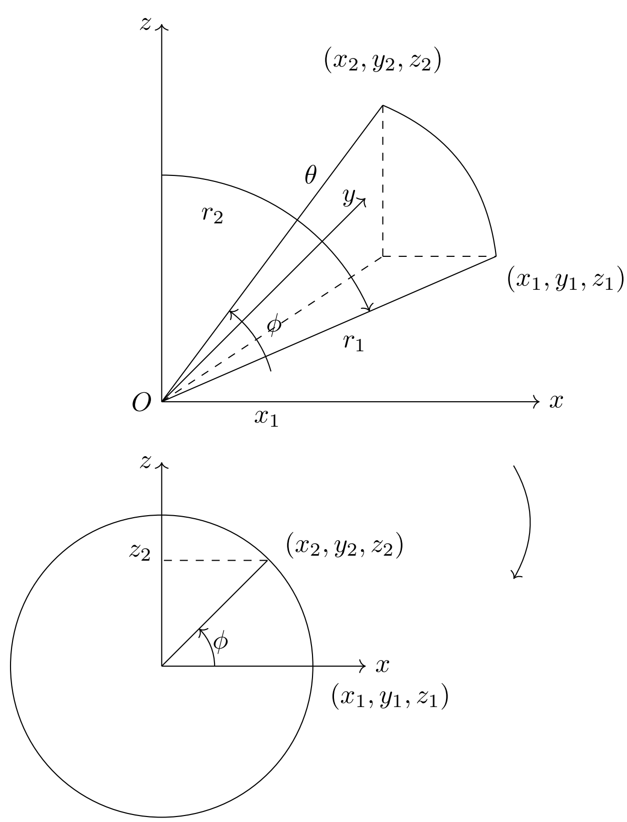

Figure 2: Decay track hitting the back wall of a detection box at (x_2, y_2, z_2).

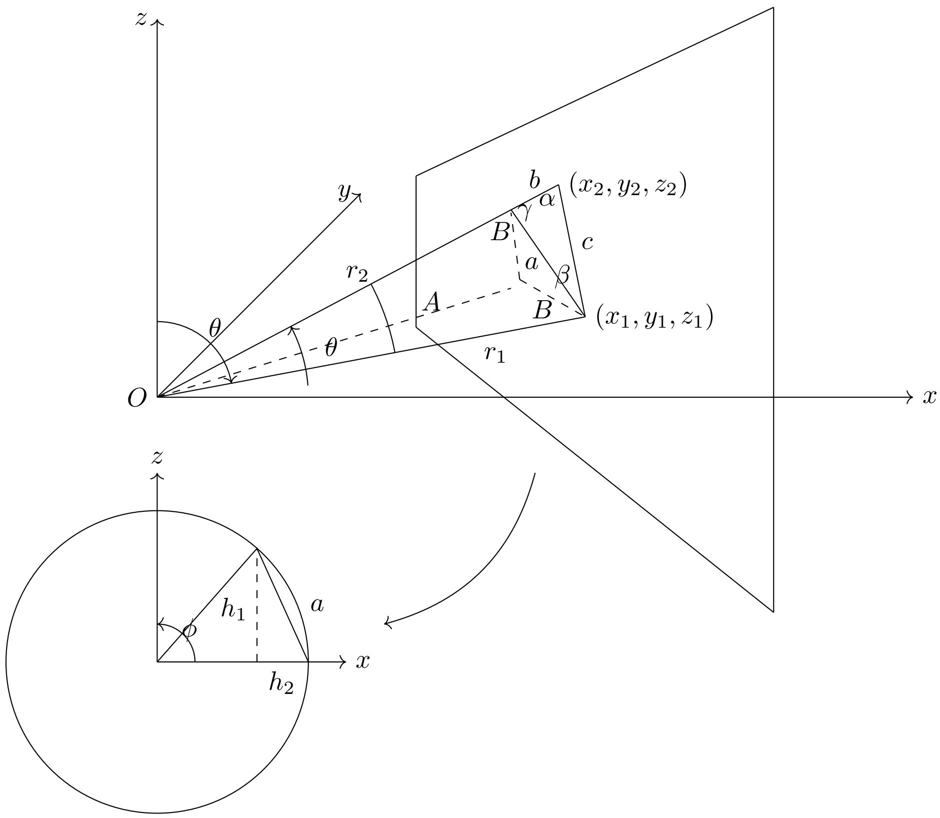

Figure 3: Decay track hitting the right wall of a detection box at (x_2, y_2, z_2). The y axis points into the page.

(ii) Left (or Right) Wall

We see from Figure 3 that

$$\begin{aligned} x_1 &= 1.50 \\ y_1 &= r_1 \cos(\theta) \\ z_1 &= 0 \\ r_1 &= x_1/\sin(\theta). \end{aligned} \tag{3}$$

In order to find (x_2, y_2, z_2), we make note of the following, again, from Figure 3 (Figure 4 is the triangular pyramid within Figure 3):

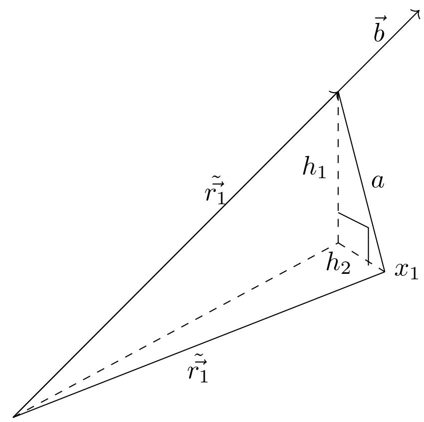

Figure 4

$$\begin{aligned}
a^2 &= 2x_1^2 - 2x_1^2\cos(\phi) \text{ (cosine law)} \\
a &= x_1\sqrt{2\left(1-\cos(\phi)\right)} \\
\text{But } a^2 &= 2r_1^2 - 2r_1^2\cos(A) \\
\frac{a^2}{2r_1^2} &= 1-\cos(A) \\
\Rightarrow A &= \arccos\left(1-\frac{a^2}{2r_1^2}\right) \qquad (4) \\
h_1 &= x_1\sin(\phi) \\
h_2 &= x_1\left(1-\cos(\phi)\right) \\
\Rightarrow B &= \frac{180}{2} - \frac{A}{2} = \frac{\pi}{2} - \frac{A}{2} \\
\gamma &= 180 - B = \pi - B.
\end{aligned}$$

$$\begin{aligned}
\tilde{x}_1 &= x_1 - h_2 \\
\tilde{y}_1 &= y_1 \\
\tilde{z}_1 &= h_1 \\
\vec{r}_2 &= \tilde{\vec{r}}_1 + \left(\frac{\tilde{\vec{r}}_1}{r_1}\right)t \qquad (5) \\
\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} &= \begin{pmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{pmatrix} + \begin{pmatrix} \tilde{x}_1/r_1 \\ \tilde{y}_1/r_1 \\ \tilde{z}_1/r_1 \end{pmatrix} t
\end{aligned}$$

Then

$$
\begin{aligned}
x_2 &= \tilde{x}_1 + \frac{\tilde{x}_1}{r_1} t = x_1 \\
y_2 &= \tilde{y}_1 + \frac{\tilde{y}_1}{r_1} t = y_1 \left(1 + \frac{t}{r_1} \right) \\
z_2 &= \tilde{z}_1 + \frac{\tilde{z}_1}{r_1} = h_1 \left(1 + \frac{t}{r_1} \right)
\end{aligned} \tag{6}
$$

So

$$x_2 = x_1 - h_2 + \frac{x_1 - h_2}{r_1} t = x_1$$

$$\Rightarrow t = h_2 \left(\frac{r_1}{x_1 - h_2} \right) \tag{7}$$

(iii) Top (or Bottom) Wall

From Figures 5 & 6 we see that

$$
\begin{aligned}
x_1 &= 0 \\
y_1 &= r_1 \sin(\theta) \\
z_1 &= 1.50 \\
r_1 &= z_1 / \cos(\theta) \\
\tilde{\phi} &= \frac{\pi}{2} - \phi
\end{aligned} \tag{8}
$$

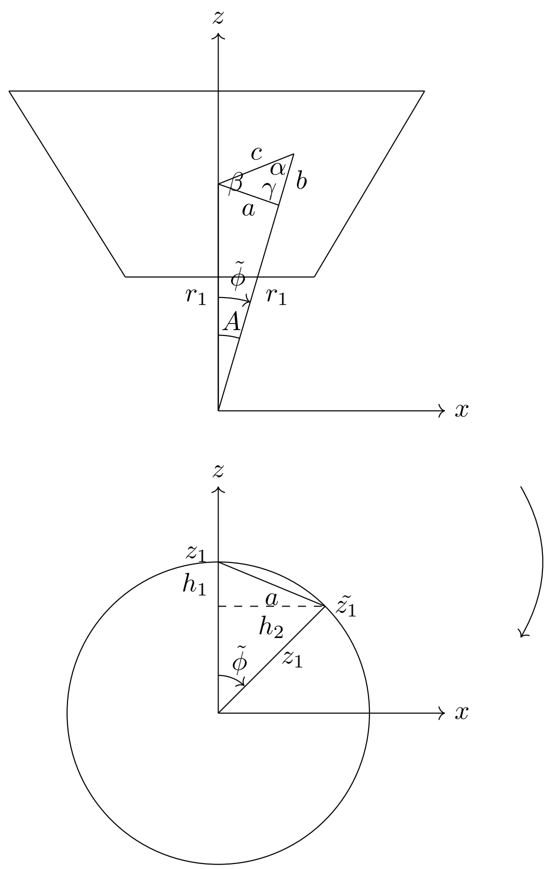

Figure 5: Decay track hitting the top wall of a detection box. The y axis points into the page.

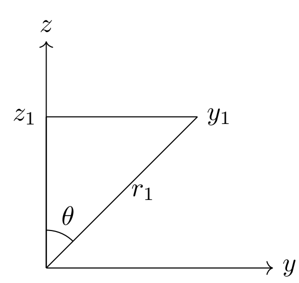

Figure 6

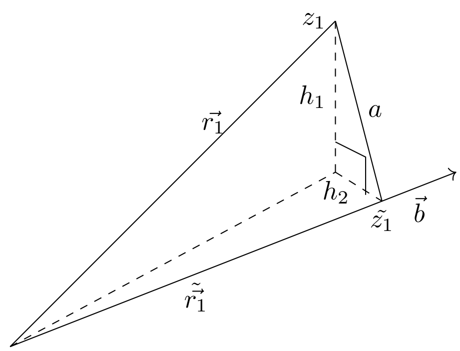

Figure 7: The inner angles (not equal to $\pi/2$) in the $a - z_1 - \tilde{z}_1$ triangle are C.

Also

$$\begin{aligned}
h_1 &= z_1\left(1-\cos(\tilde{\phi})\right)\\
h_2 &= z_1\sin(\tilde{\phi})\\
a^2 &= 2z_1^2 - 2z_1^2\cos(\tilde{\phi})\\
a &= z_1\sqrt{2\left(1-\cos(\tilde{\phi})\right)}\\
\text{But } a^2 &= 2r_1^2 - 2r_1^2\cos(A)\\
\Rightarrow A &= \arccos(1-\frac{a^2}{2r_1^2})\\
B &= \frac{\pi}{2} - \frac{A}{2}\\
\gamma &= \pi - B
\end{aligned} \tag{9}$$

We also see from Figure 7 that

$$\begin{aligned}
C &= \frac{\pi}{4}\\
h &= a\cos(C)\\
\tilde{x}_1 &= h_2\\
\tilde{y}_1 &= y_1\\
\tilde{z}_1 &= z_1 - h_1
\end{aligned} \tag{10}$$

And

$$\begin{aligned}
\vec{r_2} &= \tilde{\vec{r_1}} + \frac{\tilde{\vec{r_1}}}{r_1}t\\
\begin{pmatrix} x_2\\ y_2\\ z_2 \end{pmatrix} &= \begin{pmatrix} \tilde{x}_1\\ \tilde{y}_1\\ \tilde{z}_1 \end{pmatrix} + \begin{pmatrix} \tilde{x}_1/r_1\\ \tilde{y}_1/r_1\\ \tilde{z}_1/r_1 \end{pmatrix} t
\end{aligned} \tag{11}$$

So

$$z_2 = (z_1 - h_1) + \frac{z_1 - h_1}{r_1} t = z_1$$

$$\Rightarrow t = h_1 \left(\frac{r_1}{z_1 - h_1} \right) \tag{12}$$

And

$$\begin{aligned} x_2 &= h_2 \left(1 + \frac{t}{r_1} \right) \\ y_2 &= y_1 \left(1 + \frac{t}{r_1} \right) \end{aligned} \tag{13}$$