Solar collector

Phys 29X Experiment 8

Student's manual

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[1]

1 Introduction

The calculations obtained in this experiment are very important. In the current model of the solar power, proposed in 1920 by Arthur Eddington (1882-1944), the powered in the Sun is generated by nuclear fusion (of hydrogen nuclei into helium).[2] The Sun is in a steady state, and this means that the power generated by the solar core is balanced by the electromagnetic radiation emitted by the Sun's photosphere. This electromagnetic radiation, called the irradiance, is what you will be measuring in this experiment; this gives an indirect way of measuring the rate of nuclear reaction at the Sun's core, if the nuclear fusion model for solar energy is indeed correct.

This model for the solar energy was once put in severe questioning when the measurements, at the Earth's surface, of electron neutrinos produced by the Sun gave approximately one third of the value predicted by the nuclear

[1] This laboratory manual is heavily based on the publication by Gil, et. al. [3], and has been modified here for teaching purposes. The data included here has been taken from this publication unless otherwise indicated.

[2] Curious fact: the model of nuclear fusion arises from the gravitational collapse of the Sun, which is kept at bay by degenerate pressure. This model in turn arises from the hypothesis that the solar system was formed from a spinning cloud of particles; this idea was initially proposed by Immanuel Kant (1724-1804), who originally studied physics and later dedicated himself to philosophy, in 1755, and it is called the 'Nebular hypothesis' [4].

fusion model for the energy of the sun. This was finally solved by a series of experiments at the Sudbury's Neutrino Observatory (SNO), in Ontario, Canada, which determined that neutrinos oscillate between their different states (electron, muon, and tau neutrinos).

2 Theory

In this experiment you will determine the solar constant, which is the rate of solar radiation reaching the Earth's orbit outside the atmosphere. For your better understanding of the following derivations, a quick overview of the apparatus and how it is used is offered here. The full information is in the Experimental Details section 3, naturally. The apparatus consists, basically, of an aluminium plate painted with black matt paint to reduce its reflectivity and a contact thermometer attached to the plate. You will be measuring the temperature changes of the plate over time as the plate is directly exposed to the Sun during different times of the day, when the Sun is at different positions in the sky.

2.1 Heat transfer unto the plate from the solar radiation

First, we start by asking ourselves how much the temperature of the plate changes per absorption of heat from the solar radiation. We start by introducing the heat capacity C of the aluminum plate; per unit mass, it is defined as the specific heat capacity c:

$$c = \frac{1}{m}\frac{đQ}{dT}, \tag{1}$$

where m is the mass of the substance heated, Q is heat and T is temperature.[3]

For a mass that remains constant during the heat transfer process, Equation 1 tells us the amount of heat transfer per 'unit' change in temperature, assuming that the initial temperature is zero for zero heat transfer at some initial time:

$$Q = cmT. \tag{2}$$

Taking the time derivative of this expression, we obtain the amount of heat transfer to the plate from the **surroundings** per unit time:

[3] The bar in $đQ$ comes from the fact that this is an inexact differential.

$$\frac{đQ}{dt} = \frac{1}{m}\frac{dT}{dt}. \tag{3}$$

Now we introduce Newton's law of cooling:[4]

$$\frac{đQ}{dt} = -hA(T - T_0), \tag{4}$$

where h is the heat transfer coefficient of the material, A is the area of contact between the material and the **surroundings** [5], and T_0 is the **stable** temperature of the surroundings. T is the **changing** temperature of the material. Further adding a term corresponding to the heat transfer from solar radiation [6], we obtain

$$\frac{đQ}{dt} = mc\frac{dT}{dt} = -hA(T - T_0) + I_{gnd}\epsilon A, \tag{5}$$

where $\epsilon = 1 - r$ is the effective emissivity of the face of the plate exposed to the sun, and r is the reflectivity of the plate. I_{gnd} is the solar irradiance at the surface of the Earth. Finally, we divide by mc and define $k \equiv \frac{hA}{mc}$:

$$\frac{dT}{dt} = -k(T - T_0) + \frac{\epsilon AI}{mc}, \tag{6}$$

The necessary values needed for the experiment are listed in Table 1, in Appendix A. Regarding the plate, DO NOT PLACE IT ON THE SCALE; the plate is too heavy for it, and by doing otherwise you will PERMANENTLY DAMAGE IT. The mass of the plate is provided in the Appendix.

In this experiment, you will cool down the plate with ice, or by placing it inside the freezer, to a temperature lower than the ambient temperature. This means that for some time $t = t_0 > 0$, $T = T_0$ and $t > t_0$, $T > T_0$. For temperatures $T \approx T_0$, the term arising from the heat exchange between the surroundings and the plate is negligible, so Equation 6 becomes:

$$\left.\frac{dT}{dt}\right|_{T\approx T_0} = \frac{\epsilon AI}{mc}, \tag{7}$$

from which you can calculate the irradiance of the Sun at the surface of the Earth by linear analysis of:

$$T(t) = \frac{\epsilon AI}{mc}t + T_i. \tag{8}$$

[4] You have probably seen this in a calculus course, but if not, here it is.

[5] The heat transfer we are considering here is by convection from the surrounding air.

[6] Newton's law of cooling only holds for very small temperature differences between the surroundings and the object for the case of heat transfer by radiation.

2.2 Absorption of light at the Earth's atmosphere

$T(t)$ in Equation 8 depends on the angle between the zenith and the line parallel to the incoming solar rays of light, indirectly though, as it is seen in what follows.[7] This is called the zenith angle. See Figure 1.

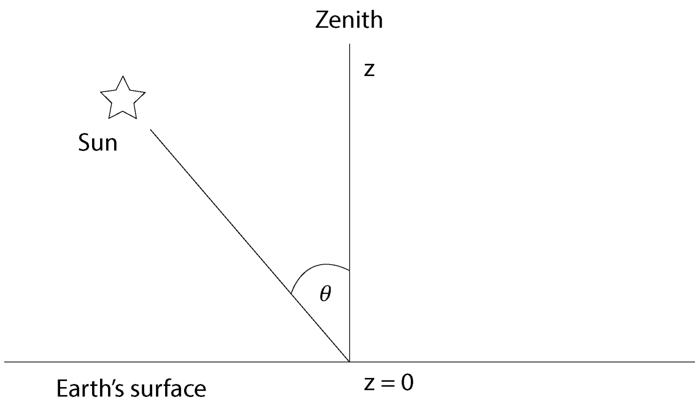

Figure 1: Zenith angle.

The experiment works for zenith angles smaller than 75 degrees. The reason is the following. Most of the solar radiation is absorbed at the lowest part of the atmosphere, the troposphere, which has a ratio of radius of curvature to thickness of about 10^3 [3]. This means that, for our purposes, the relevant layer of atmosphere can be approximated to a plane. Near the horizon, this approximation breaks because the light must travel through more atmospheric content, making any local approximation (like the plane approximation above, for instance) not valid.

In our plane model, we assume that the air density the light of the sun must traverse to reach the apparatus is only a function of the height z along the line of the zenith. We invoke the Beer-Lambert law [3, 6] for the calculation of the solar irradiance at the ground level as a function of the zenith angle:

$$dI = \frac{1}{cos(\theta)} K(z) I(z) dz, \tag{9}$$

[7]Of course the light rays are not parallel but, given the distance between the surface of the Earth and the Sun's photosphere, considering them parallel for the calculations concerning this experiment is a reasonable thing to do.

where K is the light extinction coefficient [8], I is the irradiance of the sun as a function of the height z above the ground. θ, finally, is the zenith angle. Integrating to infinity, the solar constant I_0 being given at $z \to \infty$ and defining $K_{eff} \equiv \int_{z=0}^{z=\inf} K(z)dz$, we obtain, by separation of variables:

$$\begin{aligned} \int_{\tilde{I}=I_{gnd}}^{\tilde{I}=I_0} \frac{1}{\tilde{I}} d\tilde{I} &= \frac{1}{cos(\theta)} \int_{z=0}^{z=\inf} K(z)dz \\ ln(I_{gnd}) - ln(I_0) &= -\frac{1}{cos(\theta)} K_{eff} \qquad (10) \\ ln(\frac{I}{I_0}) &= -\frac{1}{cos(\theta)} K_{eff} \end{aligned}$$

$$I_{gnd} = I_0 e^{-\mu K_{eff}}, \qquad (11)$$

where $I_{gnd}(\theta) = I_{gnd}(\mu)$ is the irradiance of the Sun at the ground and $\mu \equiv 1/cos(\theta)$. A simple exponential fit can recover the value of I_0 and K_{eff} without the need to calculate the latter independently.

3 Experimental details

For the elaboration of this experiment, you will need a sunny day with a clear sky. Choose a day that is not too windy. Because of the plane approximation of the troposphere explained above, and on which this experiment's results depend, this experiment has to be performed at the start of the term, while the sun still reaches relatively small zenith angles.[9] The experiment will take an entire day to be completed. This is the time that the data recollection will take, approximately, if you generated a data point for the irradiance of the sun at the ground level approximately every hour; as the zenith angle increases you will need to take data points at least every half an hour. It is important that you take data for a big range of angles.

Now we explain how the apparatus works and how you will be using it. The equipment consists of a cardboard box with one of its ends open and with polystyrene foam at its bottom. An aluminium plate painted with matt black paint is placed on top of the polystyrene foam. The purpose of the box is to protect the metal plate from the wind and it also provides some insulation to it, while the purpose of the polystyrene foam is to provide insulation from

[8] This coefficient depends on the particles present in the air and their amount by a function of the height above the ground. See Gil, et. al. [3] for more information.

[9] This is in part a shame, since the theory of this experiment follows that of Babinet's principle and Malus's law, naturally.

the ground, which usually gets very hot under the Sun. A thermometer is attached to the back of the metal plate. The cable of the thermometer goes through a hole in the cardboard box and the polystyrene foam. Make sure that the face of the plate to which the thermometer is attached is in contact with the polystyrene foam, opposite to the face exposed to the Sun. Attached to the cardboard box are two wooden legs designed to be able to point the surface of the plate inside the box directly into the sun at its different zenith angles. See Figure 2. Tip: place a wallet or wedge below the inclined part of the box, to avoid it falling due to the wind.

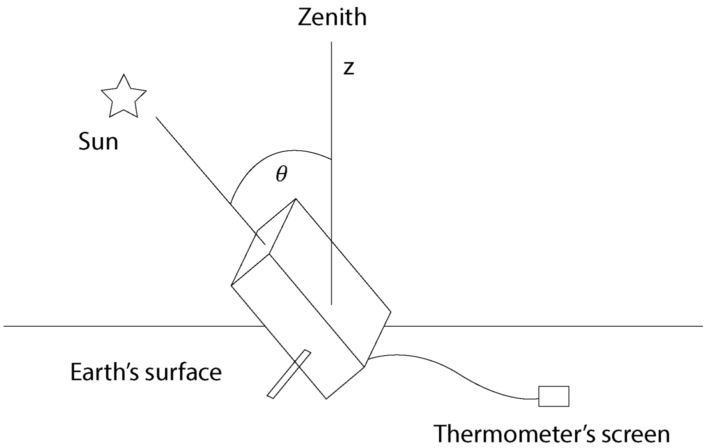

Figure 2: Experimental setup.

Before the data collection, detach the thermometer from the metal plate with the help of a screwdriver (of you do not have a screw driver, just is a coin) and measure the ambient temperature. While the thermometer's reader reaches thermal equilibrium with the surroundings, take a measurement of the zenith angle with the provided protractor; you can do this using by using basic trigonometry (show your work in the laboratory report). The correct angle will be such that the walls of the box do not project a shadow at its bottom. Doing this step before actually taking data will avoid you taking data for the same zenith angle twice. Take into account that the Sun spends more time around smaller zenith angles than at larger one. Write down the ambient temperature as measured by the thermometer in thermal equilibrium. Now attach the thermometer back to the metal plate for the collection of data. Start the data collection when the sun is a bit low, at about 70 degrees zenith angle, in the sky, then continue until it reaches its maximum height above you; finally, take more data for larger zenith angles, when the sun starts to set. The process for data collection is as follows. You will first cool the plate in ice. See Figure 3. Do not detach the thermometer from the plate and try not to get the cable wet. When the thermometer reaches its lowest temperature reading (the plate must be cooled down to a temperature below the ambient temperature) gently but quickly pull the thermometer cable while you settle the plate at the bottom of the box and point the surface of the plate directly into the Sun. To verify that the plate is indeed pointing directly to the Sun; check that there are no shadows from the box's walls upon the plate. Start taking data on the temperature change of the plate **at least every second**. A good way to do this is to take a video of the screen of the thermometer as the plate is being heated up by the solar radiation, then you can watch the video on your computer and take notes on the temperature readings for every second.[10] Wait for the thermometer to reach a temperature of at least 5 degrees above the ambient temperature. Repeat the process for different positions of the Sun in the sky as the day goes on. Make sure that you get data for a large range of zenith angles.

Figure 3: Plate cooling.

Using a linear fit analysis for Equation 8 at temperatures of the plate near the ambient temperature, you will determine values for the irradiance of the Sun at the ground level for different zenith angles. You will need to use Table 1. Then you will use equation Equation 11 to generate an exponential

[10]If you do not have access to a camera that can do this, please talk to your TA to find a way to perform the experiment while minimizing errors. One option is that you take a temperature reading live every 3 to 5 seconds, and then perform a linear fit of Equation 8 for a wider range of temperatures than the one that is best for an accurate result. It is important that you tell the TA that you did this and why you did it, as this might affect your results negatively. More on the linear fit mentioned here on the next paragraph.

fit for the data values of I_{gnd} as a function of $\mu = 1/cos(\theta)$. An extrapolation to $\mu = 0$ will give you the solar constant I_0. Hint: for the error analysis of the Equation 11, try linearizing the function by taking the natural logarithm of it.[11] Can you think of a way of determining the solar luminosity with the value you obtain for the solar constant?

References

[1] Brown, Joel. The new black: the least reflective paint ever. Bostonia (Boston's University alumni magazine). Retrieved on Aug. 13, 2020, from: http://www.bu.edu/articles/2018/jason-chase-singularity-black-paint/.

[2] Capilano University Physics Department. Physics 110/111/114/115 Labs. Appendix. Revisions: Vern Moen and Bernd Simson. Url: http://phys114115lab.capuphysics.ca/index.htm.

[3] Gil, S.; Mayochi, M.; Pellizza, L. (2006). Experimental estimation of the luminosity of the Sun. American Journal of Physics, vol. 74, 728. doi: http://doi.org/10.1119/1.2192789.

[4] Rohlf, Michael. (2020). Immanuel Kant. Stanford Encyclopaedia of Philosophy. Retrieved on Aug. 30, 2020, from: https://plato.stanford.edu/entries/kant/#LifWor.

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[6] As cited by Gil, et. al. [3]:
American Institute of Physics. (1966). American Institute of Physics Handbook. Edited by D. E. Gray. New York: McGraw Hill.
Wikipedia. Beer-Lambert law. Retrieved from: https://en.wikipedia.org/wiki/Beer-Lambert-law.

[11] A useful webpage for the determination of uncertainties when taking the logarithm and exponential of a quantity is given at the bibliography [2].

A

Quantity	Value	Unc
Aluminium specific heat [J/kg K]	910	
Typical matt black paint reflectivity	0.2	
Mass of the metal plate [kg]	0.196	0.001
Thickness of the plate [m]	0.00327	0.00001
Area of the plate [m^2]	0.022602	0.00001

Table 1

[12]

[12] The data in Table 1 has not been taken from Gil, et. al. [3]. The value of the reflectivity of matt black pain was taken from Bostonian [1], and the value of the aluminum specific heat from The Engineering ToolBox [5].