

Bubble chamber

Phys 294/5 Experiment 4

Instructor's manual

August 10, 2020

University of Alberta, Department of Physics

1 Theory

This experiment looks at the conservation of energy and momentum in relativistic elastic and inelastic collisions between particles. The studied collision is that of a pion π^- and a proton p^+ . A π^- has a negative charge of magnitude equal to e , the charge of a proton ($-e$ is the charge of the electron). The negative pions are passed through a chamber filled with liquid hydrogen, so that they interact with the hydrogen protons. Moving charged particles, such as the negative pions and protons, leave tracks in this kind of chambers; the traces are bubbles formed along the path of the charged particle. Neutral particles do not leave any trace. The chamber itself is exposed to a uniform magnetic field ¹, which means that a charged particle **moving** through the chamber experiences the Lorentz's force (Equation ¹ ²):

$$\vec{F} = q\vec{v} \times \vec{B}, \tag{1}$$

where q is the charge of the moving particle, v , its speed, and B is the external magnetic field.

Lorentz's force says that a charged particle passing through a magnetic field is subject to a force component that is perpendicular to its direction

¹This means that the direction of the magnetic field and its magnitude are the same all over the space of consideration, in this case, the bubble chamber.

²To understand this formula you just have to look at it. Its derivation is simply experimental observation.

of motion, making it experience a centripetal acceleration. It is important to note that the Lorentz's force only applies to moving particles, and this is based on experimental observation only. The collisions you will be looking at are of a moving negative pion and a proton at rest (we will come back to this later).

For this kind of experiments, one works with momentum, since it is always a conserved quantity. There is a way of expressing the information in Equation 1 in terms of momentum. For the sake of simplicity, and for the purposes of this experiment, we assume that the direction of the magnetic field is always perpendicular to the direction of motion of the particles.³ Because we are working with charged particles moving at relativistic speeds, i.e. at very high speeds, comparable in size to the speed of light in vacuum (and nothing can move faster than the speed of light in vacuum), this assumption is approximately valid.

It happens that the Lorentz's force, as expressed in Equation 1, is valid in the relativistic limit, and for a particle moving approximately perpendicular to the external magnetic field, magnitude of the Lorentz's force is simply given by Equation 2:

$$F = qvB. \quad (2)$$

Therefore, a charged particle passing through the chamber moves approximately along the circumference of a circle, whose radius depends on the momentum of the particle. One just needs to recall the formula for the centripetal force of a particle experiencing uniform circular motion (Equation 3):

$$F_c = pv/R, \quad (3)$$

where R is the radius of the circle along whose circumference the charge particle is moving. Equation Equations 2 & 3, one finds Equation 4, for the magnitude of the momentum of the moving charged particle:

$$p = qBR. \quad (4)$$

There is a very important formula in special relativity for expressing the energy of a massive particle moving at relativistic speeds. This formula is

³This is not too far fetched if the charged particles are moving fast enough to make the radius of the circle along whose circumference they move very large. Then in a small enough segment of the circumference, lines traced from the center of the circle to the position of the charged particle in the segment of the circumference are approximately parallel to each other. If these lines represented a magnetic vector field, then one could consider the magnetic vector field to be uniform.

given by Equation 5. You do not need to know its derivation at this point, but you will get to that in later courses throughout the Physics program. The following, however, should give you some useful information.

$$E^2 = m^2 c^4 + p^2 c^2, \quad (5)$$

where E is the energy of the moving charged particle, m is its mass, p , its momentum, and c is the magnitude of the speed of light.

In special relativity, one works with 4-vectors, i.e. vectors with four components. The fourth component comes from the fact that time is not absolute at all, and it can be “stretched” or “shrunk”. The expression for the 4-momentum is the following:

$$p_\mu = (E/c, \vec{p}) = (E/c, p_x, p_y, p_z). \quad (6)$$

The scalar product for 4-vectors is the square of its first component **minus** the square of the three-dimensional scalar product of its last three components. This is:

$$p_\mu^2 = E^2/c^2 - p^2 = E^2/c^2 - (p_x^2 + p_y^2 + p_z^2). \quad (7)$$

Now there is an important property, and this is that the scalar product of a 4-momentum is always equal to $p_\mu^2 = m^2 c^2$.⁴ Then you can see that we recover Equation 5.

2 Experimental details and procedure

Going back to the collision between the incoming π^- and the p^+ at rest. There are three possible outcomes:

- 1) $\pi^- + p^+ \rightarrow \pi^- + p^+$,
- 2) a) $\pi^- + p^+ \rightarrow \pi^- + p^+ + \pi^0$,
- b) $\pi^- + p^+ \rightarrow \pi^- + p^+ + n$,

where π^0 is a neutral pion, and n is a neutron.

In interaction 1, the only particles present are the ones that leave traces; you would expect three visible traces, from the incoming negative pion and the outgoing negative pion and proton. In interaction 2a and 2b, there is a negative outgoing particle, a neutral pion and a neutron, respectively; you

⁴For a good explanation of how this is obtained, see Griffiths’s Introduction to elementary particles, Section 3.3.

would expect three visible traces, from the incoming negative pion and the outgoing negative pion and proton, and one invisible trace, from the outgoing neutral pion or neutron, whichever is the case.

In order to identify which collision is being observed, start by looking at the traces in the bubble chamber pictures and identifying whether the momentum is conserved by the particles leaving a trace alone. If momentum is conserved by the particles leaving a trace alone, then you are looking at collision 1. Otherwise, you are looking at either collision 2a or 2b. The incoming pion has a momentum of $921 \text{ MeV}/c \pm 3\%$, and in order to determine the momenta of the outgoing negative pion and proton leaving a trace, you will use plastic templates that have lines that follow the circumference of circles of different radii. Then you can calculate the momentum by using Equation 4 (we will get to how the magnetic field is determined shortly). You will require momentum to be conserved, and then require the energy to be conserve. In case of collisions 2a and 2b, you will determine which one occurred by trying out the masses of the outgoing neutral particles. For the purposes of this experiment, the particles are assumed to move approximately in a plane, i.e. in two dimensions. The mass that gives energy conservation indicates which interaction occurred, 2a or 2b. The summary of the procedure is the following:

1. Choose three photographs from the bubble chamber from the provided set. Indicate which photographs you chose (they are labeled). These are from a real experiment conducted at the Brookhaven Laboratory. Try to perform the following steps for at least two of the photographs that you picked. Do the complete analysis for one photograph first, so that you can improve your experimental procedure when looking at the second photograph (indicate which improvements you made, if any). Provide pictures of all the traces that you draw (more about this in the next steps).

Photograph used: 16B.

Size of the photograph: $18.94 \text{ cm} \times 28 \text{ cm}$ (ratio: 23:34).

2. Using the plastic templates with the lines following the circumference of circles with different radii, match a circumference in the template with the lines left by the incoming negative pion in the photograph. Write down the radii that you find match the lines, then take the average and use that radius for the rest of the experiment. Try not to use a line that branches into two, i.e. the lines of the incoming pion that did collide with a resting proton; this line is too short. You will need to perform this step for each photograph that you chose.

3. The momentum of the incoming negative pions is $921 \text{ MeV}/c \pm 3\%$. From this value and the value of the radius you determined in Step 2, you can estimate, with Equation 4 the magnitude of the magnetic field that would produce such a trace left by the incoming pion with this momentum.

The radius was found to be (in our scaling): $R_1 = 2.6 \pm 0.1$.

From here on, the subscripts 1, 2, 3, 4 refer to the incoming negative pion, the outgoing negative pion, the outgoing proton, the outgoing neutral charges, respectively. The necessary calculations are shown in the attached Excel document. Using Equation 4, the fact that the momentum of the incoming negative pion is $p_1 = 921 \text{ MeV}/c \pm 3\% = 4.9 \pm 0.1 \times 10^{-19} \text{ kg m/s}$ and the curvature with radius $R_1 = 2.6 \pm 0.1 \text{ m}$ matched this momentum (in our scaling), one finds that a magnetic field needed to produce such a momentum is $B = 1.18 \text{ T}$.

4. Look at a line that branches in two. Using the plastic templates, match the traces left by the outgoing negative pion and proton to get values for the radii of the curvatures they follow. Then using these radii and the value you estimated for the magnetic field, determine their momenta. Pay close attention to the orientation of the curvatures when assigning the momenta to the outgoing particles: negatively charged particles' tracks are bent towards a particular direction, and positively charged particles' tracks are bent in the opposite direction to the ones of the negatively charge particles'.

Figure 1 is a scan of the traces drawn to analyse the momentum curves

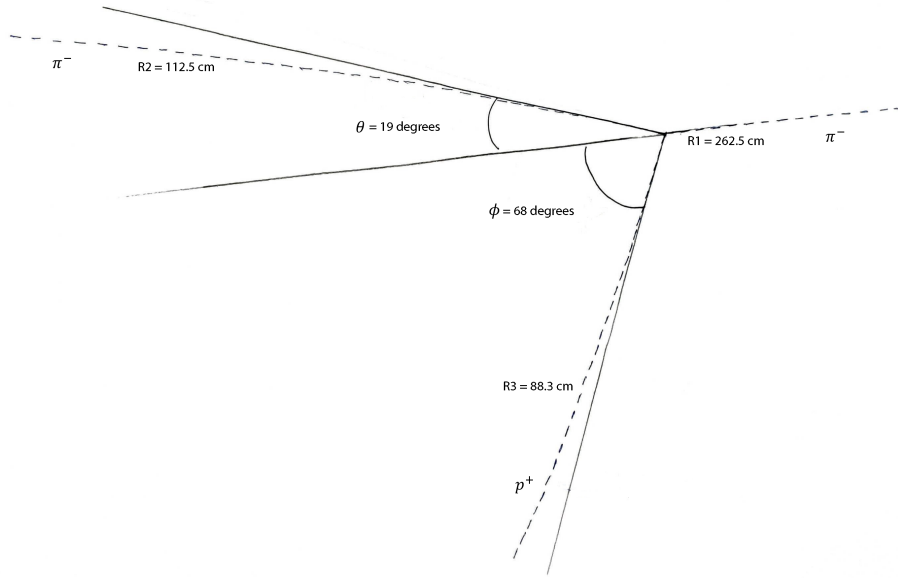


Figure 1

5. Now calculate the x and y components of the three momenta you have determined. To do this, draw tangent lines to the three trajectories you are looking at the point of collision. Use the line tangent to the trajectory of the incoming negative pion as your x axis. This means that, for the incoming negative pion, $p = (p_x, 0, 0)$. Using trigonometry, calculate the x and y components of the momenta of the outgoing negative pion and proton. See Figure 1.
6. Check whether the momentum is conserved in the x and y directions with the momenta corresponding to the three traces alone. If it is conserved, then you are looking at collision 1. If it is not conserved, you are looking at collision 2a or 2b; in this case, by requiring conservation of momentum in x and y , you can determine the magnitude of the momentum of the outgoing neutral particle.

One can find the momentum of the outgoing negative pion and proton, again with Equation 4. These are $p_2 = 1.8 \pm 0.4 \times 10^{-19} \text{ kg m/s}$, for a radius $R_2 = 1.12 \pm 0.05 \text{ m}$, and $p_3 = 1.4 \pm 0.3 \times 10^{-19} \text{ kg m/s}$, for a radius $R_3 = 0.88 \pm 0.03 \text{ m}$. Taking the tangent line to the trace left by the incoming negative pion as the x axis, from Figure 1 one can calculate the x and y components of the momenta of the outgoing

| Energy | Value [$\times 10^{-10}$ J] | Unc [$\times 10^{-10}$ J] |
|-------------------------------|------------------------------|----------------------------|
| $E_1 = E_{initial}$ | 1.49 | 0.01 |
| E_2 | 0.7 | 0.1 |
| E_3 | 0.5 | 0.1 |
| E_4 | 0.8 | 0.1 |
| $E_2 + E_3 + E_4 = E_{final}$ | 2.0 | 0.2 |

Table 2

negative pion and proton, and then require conservation of momentum. In the photograph analyses, the observed collision was one with a neutral outgoing particle. Table 1 summarizes this information.

| Momentum | Value [$\times 10^{-19}$ kg m/s] | Unc [$\times 10^{-19}$ kg m/s] |
|-----------------------|-----------------------------------|---------------------------------|
| p_1, p_{1x}, p_{1y} | 4.9, 4.9, 0 | 0.1, 0.1, 0 |
| p_2, p_{2x}, p_{2y} | 2.1, 2.0, 6.9 | 0.5, 0.4, 0.1 |
| p_3, p_{3x}, p_{3y} | 1.7, 6.3, -1.5 | 0.4, 0.1, 0.3 |
| p_4, p_{4x}, p_{4y} | 2.4, 2.3, 8.6 | 0.5, 0.5, 0.4 |

Table 1

- Now that you have all the momenta you need, determine the energies of the all the particles by using Equation 5. For collision 1, check that energy is conserved. For collisions 2a and 2b, require energy conservation and determine which of the two occurred by trying out the values of the masses of the outgoing neutral particles in the calculations.

Knowing the four momenta of the particles in question, the incoming negative pion and the outgoing negative pion, proton, and neutral pion, one ca calculate the energies and check if energy is conserved. The results are summarized in Table 2. The initial and final energies do not agree within error for neither case, 2a or 2b. However, the discrepancy between the results obtained using the mass of the neutral pion and mass of the neutron are big enough to conclude that the observed collision is 2a.

- Discuss what the results tell you. Do the initial energy and final energy agree within error? If not, do they agree in some degree? Can you still reach some conclusions? What could be causing the disagreement? How would you change the experiment, as described in this manual, to obtain better results? (These questions are only guidelines. The best thing to do is to think of your own questions, try to explore the

results. If you follow the procedure correctly, and did the calculations properly, there should not be a correct or incorrect answer at this point, so do not “make” the initial and final energies match within error. The important part here is to think and explore).

The disagreement between the initial and final energies could be due to the fact that the dimensions of the photograph used are not big enough, making the tracing of the lines tangent to the path following by the particles less accurate (this leads to an underestimating of uncertainties). In order to fix this, one could make the dimensions of the photograph bigger, always keeping the ratio between its height and width the same; one could also consider the size factor of the photograph in the uncertainties of the measurements.

3 Notes

- Table 3 gives the values of (rest) masses of the particles in question.

| Particle | Charge | Rest mass [MeV/ c^2] |
|----------------|----------|----------------------------|
| e^- | $-e$ | 0.51099 |
| π^-, π^+ | $-e, +e$ | 139.5669 |
| π^0 | 0 | 134.9626 |
| p^+ | $+e$ | 938.272 |
| n | 0 | 939.566 |

Table 3: Table of rest masses. Taken from the University of Alberta [3].

- For constants, units and uncertainties, access the webpage of “The fundamental physical constants from NIST (National Institute of Standards and Technology)”.
- If you are more interested in particle physics, you can download the Particle Physics Booklet from the Particle Data Group, for free.

References

- [1] Griffiths, D. (2017). Introduction to electrodynamics. 4th. ed. Cambridge University Press.

- [2] Griffiths, D. (2008). Introduction to elementary particles. 2nd., revised ed. Wiley-VCH.
- [3] Physics Laboratory Manual: Phys 292/294/295/297. University of Alberta, Department of Physics. (2006). Coordinator: David Leonard.