Gaussian statistics: random error and basic statistics

Phys 294/5 Experiment 1

Instructor's manual

June 24, 2020

University of Alberta, Department of Physics

1 Introduction

Contrary to the naïve expectation, the experiments in physics typically involve not only the measurements of various quantitative parameters of nature. In almost all the situations the experimentalist has also to present an argument showing how confident he or she is about the numeric values obtained. Among other things, this confidence in the validity of the presented numeric data strongly depends on the accuracy of the measurement procedure. As a simple example, it is impractical to measure a mass of a feather using the scale from the truck weighing station, which is hardly sensitive to the weight less than a few pounds. Another challenge has to be met when the scientist tries to compare the results of her experiment with the data from another experiments, or with the theoretical predictions. Since the conditions of the measurement almost always vary from an experiment to an experiment, and since they are also different from the idealized situation of the theoretical model, the compared values most likely will not match each other exactly. The task is then to figure out how important the factors creating this discrepancy are. If these factors are stable (do not change from measurement to measurement) and noticeable, they are called systematic errors. If, on the contrary, these factors are variable, they are called random, or statistical, errors. An important fact is that the uncertainty due to the random errors can be reduced by increasing the number of measurements to average over the random variations.

The main purpose of this experiment is to introduce you to methods of dealing with the uncertainties of the experiment. The basic procedures to correctly estimate the uncertainty in the knowledge of the measured value (the error of the measurement) include:

- the correct treatment of the random errors and systematic errors of the experiment (Taylor [1], Chapters 4 and 5);
- the rounding off the insignificant digits in the directly measured and calculated quantities (Taylor [1], Chapter 2, and the Reference Guide).

2 Goals

1. Understand basic statistical measures of uncertainty
2. Learn when standard deviation vs. standard deviation of the mean is appropriate
3. Distinguish between systematic and random errors
4. Learn one method for estimating the random errors
5. Learn how to estimate systematic errors
6. Test for a statistically significant difference from an expected value
7. To measure a time with accuracy better than half a percent

3 Preliminary discussion

Before the lab, you are asked to read and understand the theoretical material for this lab (Taylor [1]). Before the experiment starts, your group needs to decide which information will be relevant to your experiment. Discuss what you will do in the lab and what preliminary knowledge is required for successful completion of each step. Think hard about organizing your work in an efficient way. What measurements will you need to make? Go through your lab manual with a highlighter, then make a checklist of the needed measurements. What tables or spreadsheets will you need to make to organize the calculations data? How should you use Excel to expedite your calculations and unit conversions (when necessary)? What tables will you need to summarize your analysis and conclusions from the data? This lab will have more explicit reminders about tables than future labs, but you should be thinking about this organization of data taking, data reduction, and summarization in every lab.

3.1 Questions for the preliminary discussion

You should write your own answer to the following questions in your lab book, but leave space to change it after discussion. If you do change your answer, say why.

1. Calculators and computers typically return results with as many digits as possible, including digits well beyond our measurement uncertainty. What procedure will you follow to systematically get rid of these significant digits?

2. What tables will you need to record your input data? What tables will you need to summarize your results? Scan the lab and write down your tables.

4 Experimental details

4.1 Random uncertainties

In this experiment we ask you to perform a simple repetitive measurement in order to investigate random and systematic errors. The idea is that we will compare the time interval of a stand-alone digital clock (either one about the house or a wristwatch) with a more precise instrument (a timer on your cellphone, for example). If you do not have access to such a device you can use the You-Tube movie of a digital clock footnoted below [1]. We want to perform a test to see if the time scale of the digital clock is correct or not—that is, whether using the large digital clock would cause systematic errors were we to use it to measure time.

Although the timers have very accurate time scales, we need to use rather imprecise hand-eye coordination to operate them. The systematic error of the timers is small, and guaranteed by the manufacturer. But you will have to measure the random error from your hand-eye coordination, since at the start you don't know it. We will do it by repeating measurements of the same time interval on the large digital clock, and using the variation of the

[1] https://www.youtube.com/watch?v=Lsq0FiXjGHg

measurements to calculate the random error in a single time measurement. We will then use the fact that by repeatedly measuring time intervals, we can decrease the uncertainty of our estimate of the wall clock counting rate. So we are using an understanding of random errors to measure an effect (clock count rate) which might be a systematic error if used in another experiment without correcting for it.

4.2 Time measurements

At the front of the room is a large digital clock. Assume that it counts at a constant rate, but do not assume that the rate is one count per second. The clock will count from 1 to 20, blank out for some unspecified length of time, and then begin counting again. To measure the clock's count rate, you can use the google play app "Millisecond Stopwatch & Timer" [2] whose systematic error is less than 0.001 seconds for time intervals of ten seconds or less. However, you will be relying on hand-eye coordination, which means your measurements will have random uncertainties. Your reaction time is unavoidably variable. You may also be systematically underestimating or overestimating the total time.

- Observe the clock. Write down whether you believe the clock is counting reasonably close to one count per second. also, before doing any measurements, guess how much your reaction time would vary from one measurement to the next (this is your initial guess for your random error).

  - *0.050 seconds (initial guess)*

- Now choose a counting interval at least 10 counts long, and time 25 of them. Avoid starting a timing interval on the first count. Use the first three or more counts to develop a tempo with which to synchronize your start. Write down in your notebook the initial and final clock count you use to define the interval.

[2] https://play.google.com/store/apps/details?id=myApp.schre.stopwatch&hl=en_US

Measurement no.	Measured time interval [s]
1	10.062
2	10.128
3	9.853
4	9.921
5	10.038
6	10.040
7	9.964
8	10.024
9	10.035
10	9.970
11	10.072
12	9.978
13	10.005
14	9.987
15	9.996
16	10.020
17	10.016
18	10.012
19	10.004
20	9.968
21	9.987
22	9.988
23	10.001
24	9.954
25	10.083

This experiment can be done on your own. If you are working with a partner online (via skype or zoom, etc.) one person should time while the other records the data on the data sheet belonging to the person timing. To avoid an unconscious skewing of data, the person timing should not look at the data sheet until all 25 measurements have been recorded. This is essential; otherwise, you will introduce a bias into your measuring procedure! Make a few practice runs before taking data. Exchange places with your partner, and time 25 more counting intervals. Each person will have a data sheet with 25 timings recorded on it. Assuming you have prepared well for the lab, you should both be able to analyze your own data. In any case, you should provide your own answers to the questions at the end.

4.3 Data analysis

Plot your data first! Then if it is wildly non-Gaussian, make another trial before sinking a lot of analysis time. Use Excel to make a histogram from your twenty-five measurements. The x axis represents the time measured T, and the y axis the number of measurements falling in the k-th time bin. The data should be mainly peaked at a center value, and roughly symmetrical. You can consult with your instructor to see whether to take more data.

- *Did get a somewhat symmetrical bell curve for 25 measurements. Method: took a measurement of the time y pressing the "lap" button in the application every 10-count interval in the video. Stopping the counter and resetting is not necessary. The uncertainty in starting the stopwatch is expected to be the same as the uncertainty in pressing the "lap" button, which, effectively, becomes the "start" button for the next measurement.*

To analyze your data, you will calculate 3 quantities, the mean, the standard deviation, and the standard deviation of the mean value. If the formulas don't make sense to you, check Taylor [1], Chapter 4 again. The average or "mean" time per interval, $\overline{T}$ (pronounced "T bar"), is:

$$\overline{T} = \frac{1}{N}\sum_{i=1}^{N} T_i, \tag{1}$$

where the $\sum$ stands for summation, T_i represents the i-th single measurement of the time per interval, and N is the number of measurements. This formula directs you to add up the N values of T. The deviation of the N values of T around $\overline{T}$ is given by:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(T-\overline{T})^2}. \tag{2}$$

These two quantities, T and σ, are characteristics of the distribution of your time measurements: the "center" (mean) and the "width" (standard deviation) of the distribution. As you make more measurements, your estimates should stabilize and converge to constant "true" values, though since each individual measurement you make varies randomly about the true values, your estimates will also vary, though less with more data available.

Finally, you can estimate the standard deviation of the mean (also called the standard error). It is related to the standard deviation of the individual values:

$$\sigma_m = \frac{\sigma}{\sqrt{N}} \tag{3}$$

The standard deviation of the mean σ_m, is the best estimate of the uncertainty in the measurement of the mean. Note that, unlike the standard deviation, this uncertainty can be made arbitrarily small by taking a sufficiently large number of measurements.

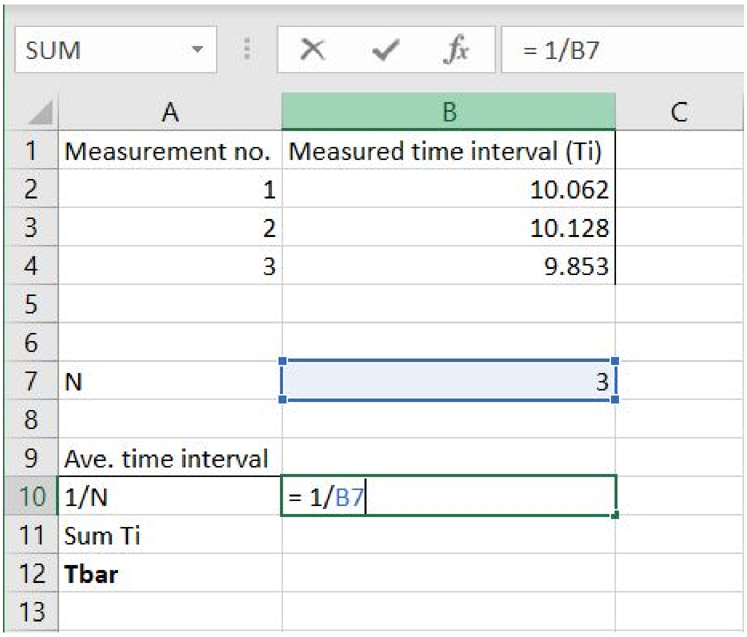
To demonstrate that you understand them, write out the calculations of these three quantities explicitly (say with Excel but not using Excel statistical formulas [you can use SUM(), for example, but NOT AVERAGE(), STDEV(), and the like])) for the first 3 measurements ($N = 3$), as if these were the only data you had taken. Repeat the 3 measurement calculations using Excel. You may then use the computer for the full 25 values. Distinguish in your hand writeup which item is hand calculated and which is from the output of Excel Functions Statistics (How could you check that you are looking at the right entry? Hint: try $N = 3$ first before $N = 25$). You will need the values of σ and σ_m in what follows. What is the label of the output in Excel that gives you σ and which gives you σ_m?

- *For the calculations performed by hand, work with as much significant digits as you can (Excel does this automatically), and then express the final quantity with the appropriate amount of significant digits.*

  Using Excel (by hand):
  $N = 3$*:* $\overline{T}$*: 10.014 s, std. dev.: 0.14356 s*
  $N = 5$*,* $\overline{T}$*: 10.000 s, std. dev.: 0.11127 s*
  $N = 10$*,* $\overline{T}$*: 10.003 s, std. dev.: 0.078171 s*
  $N = 25$*,* $\overline{T}$*: 10.004 s, std. dev.: 0.054661 s*

  With built-in functions (AVERAGE(rowi:rowf), STDEV(rowi:rowf)):
  $N = 3$*:* $\overline{T}$*: 10.014 s, std. dev.: 0.14356 s*
  $N = 5$*,* $\overline{T}$*: 10.000 s, std. dev.: 0.11127 s*
  $N = 10$*,* $\overline{T}$*: 10.003 s, std. dev.: 0.078171 s*
  $N = 25$*,* $\overline{T}$*: 10.004 s, std. dev.: 0.054661 s*

  The calculations for $N = 3$ *are shown in the following images: By hand:* $1/N$*:*

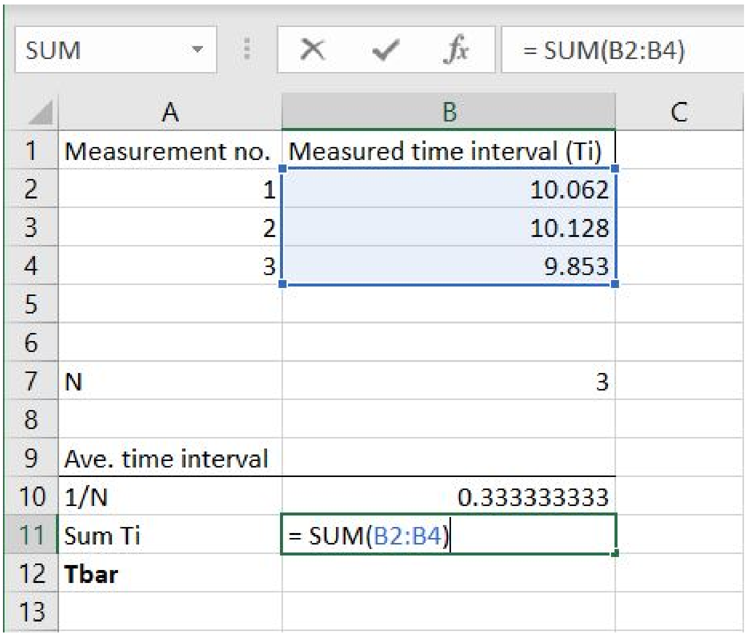


SUM = 1/B7

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	= 1/B7	
11	Sum Ti		
12	**Tbar**		
13			

Figure 1

$\sum T_i$:

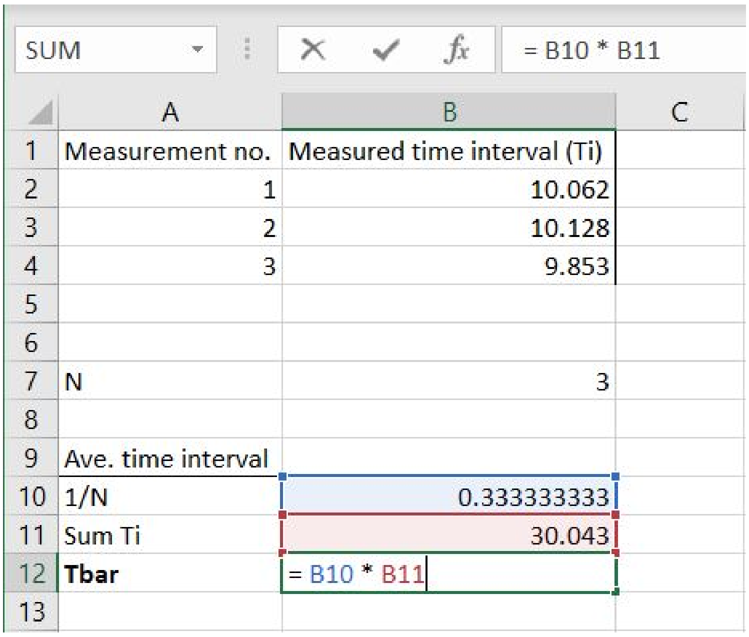


SUM = SUM(B2:B4)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	= SUM(B2:B4)	
12	**Tbar**		
13			

Figure 2

$\overline{T}$:

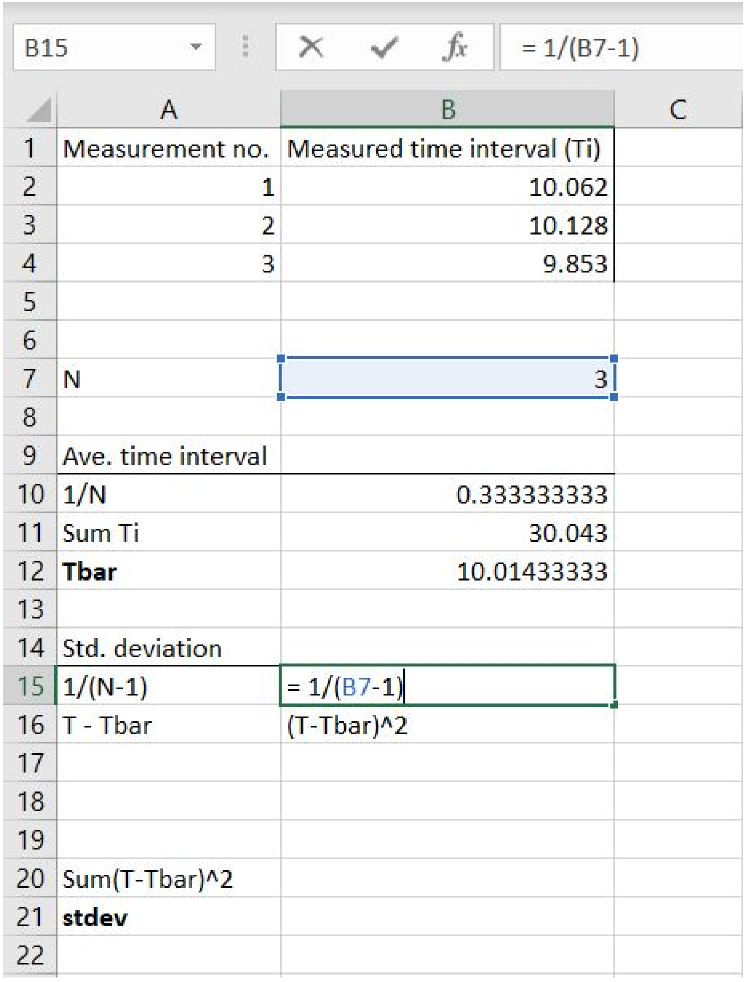


SUM | = B10 * B11

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	= B10 * B11	
13			

Figure 3

$1/(N-1)$:



B15 = 1/(B7-1)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	10.01433333	
13			
14	Std. deviation		
15	1/(N-1)	= 1/(B7-1)	
16	T - Tbar	(T-Tbar)^2	
17			
18			
19			
20	Sum(T-Tbar)^2		
21	**stdev**		
22			

Figure 4

$T_i - \overline{T}$:

It is important to place the dollar signs in the cell number of $\overline{T}$; this will be \$B\$12\$ in the following figure. This will make sure that $\overline{T}$ is the same for each $T_i - \overline{T}$ calculation. In order to do this, highlight the cell number of $\overline{T}$ and press F4. Then press ENTER.

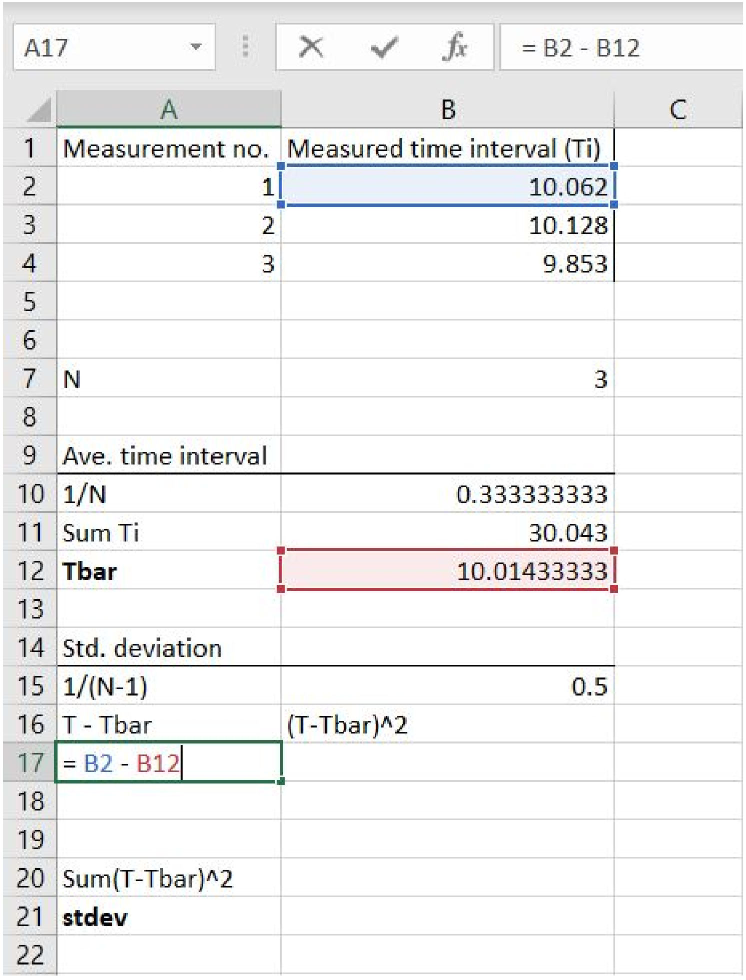


Figure 5

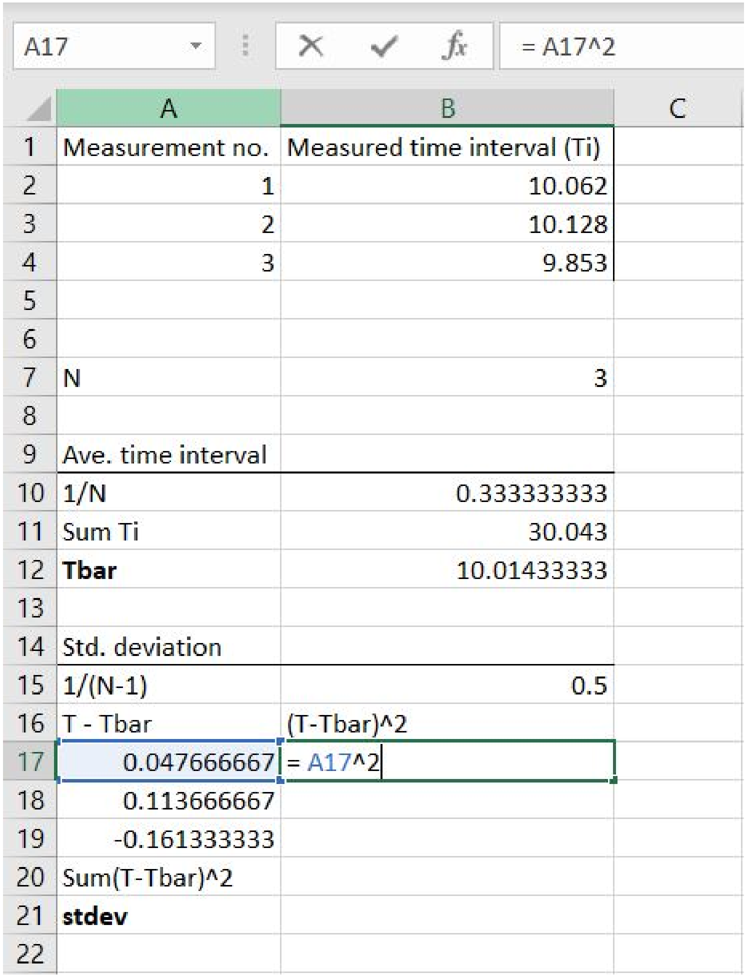
Refering to the figure above, to calculate the rest of the $T_i - \overline{T}$ *values, click on the green little square at the lower right corner of the cell A17, and drag it down two steps (for* $N = 5$ *you drag it down four steps, and so on).*

A17 = B2 - B12

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	10.01433333	
13			
14	Std. deviation		
15	1/(N-1)	0.5	
16	T - Tbar	(T-Tbar)^2	
17	0.047666667		
18	0.113666667		
19	-0.161333333		
20	Sum(T-Tbar)^2		
21	**stdev**		
22			

Figure 6

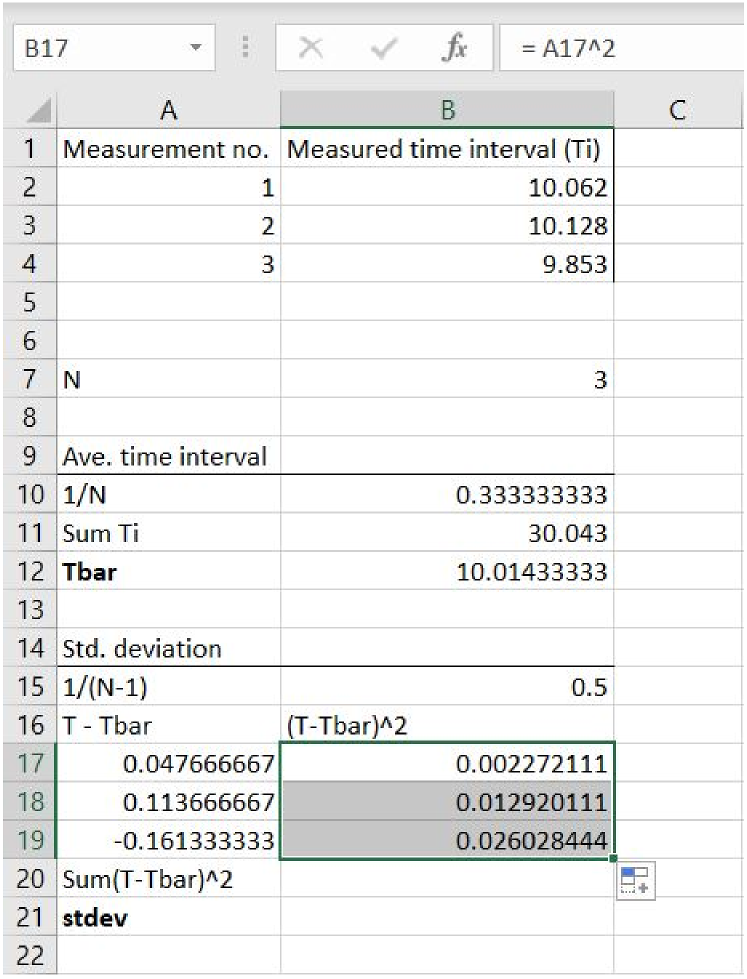
$(T_i - \overline{T})^2$:



A17 = A17^2

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	10.01433333	
13			
14	Std. deviation		
15	1/(N-1)	0.5	
16	T - Tbar	(T-Tbar)^2	
17	0.047666667	= A17^2	
18	0.113666667		
19	-0.161333333		
20	Sum(T-Tbar)^2		
21	**stdev**		
22			

Figure 7

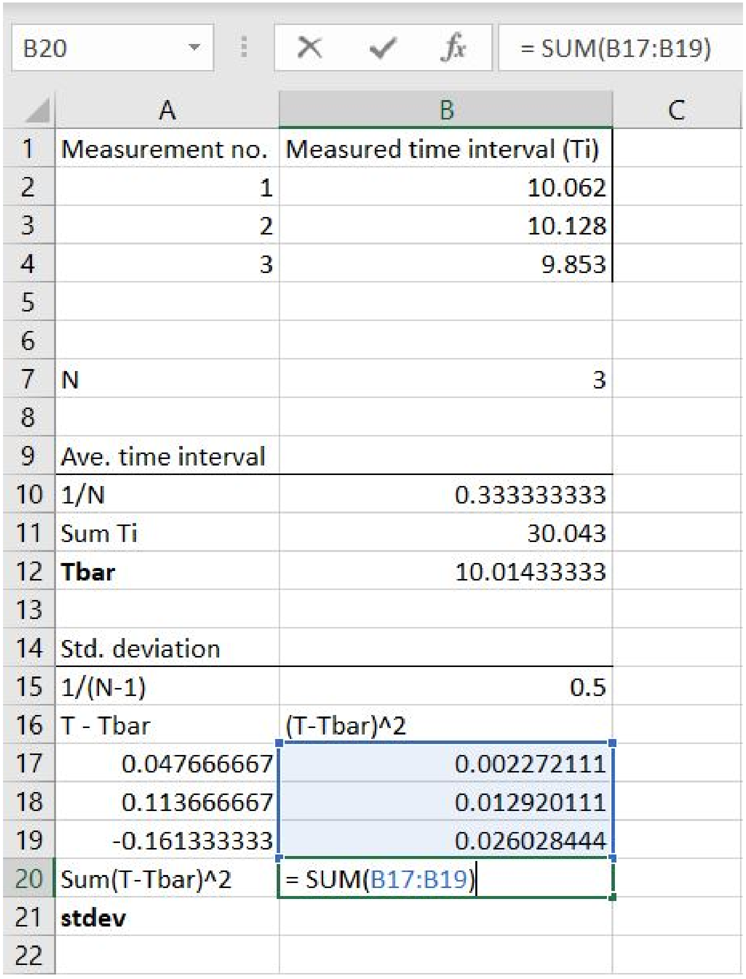


B17 | = A17^2

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	10.01433333	
13			
14	Std. deviation		
15	1/(N-1)	0.5	
16	T - Tbar	(T-Tbar)^2	
17	0.047666667	0.002272111	
18	0.113666667	0.012920111	
19	-0.161333333	0.026028444	
20	Sum(T-Tbar)^2		
21	**stdev**		
22			

Figure 8

$\sum(T_i - \overline{T})^2$:

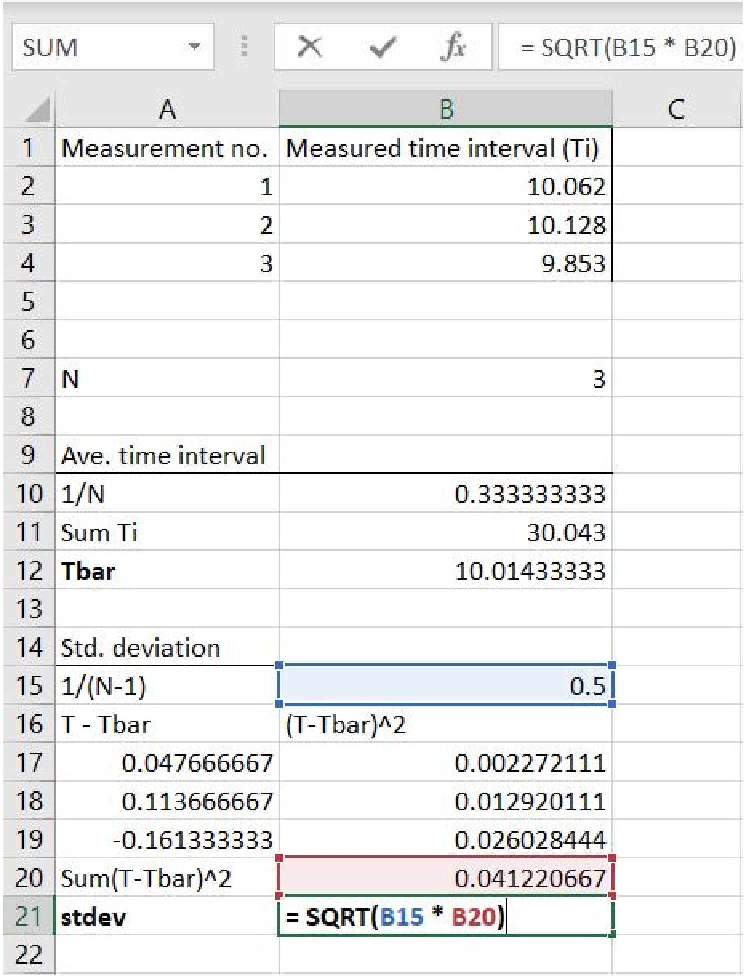


B20 | = SUM(B17:B19)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	10.01433333	
13			
14	Std. deviation		
15	1/(N-1)	0.5	
16	T - Tbar	(T-Tbar)^2	
17	0.047666667	0.002272111	
18	0.113666667	0.012920111	
19	-0.161333333	0.026028444	
20	Sum(T-Tbar)^2	= SUM(B17:B19)	
21	**stdev**		
22			

Figure 9

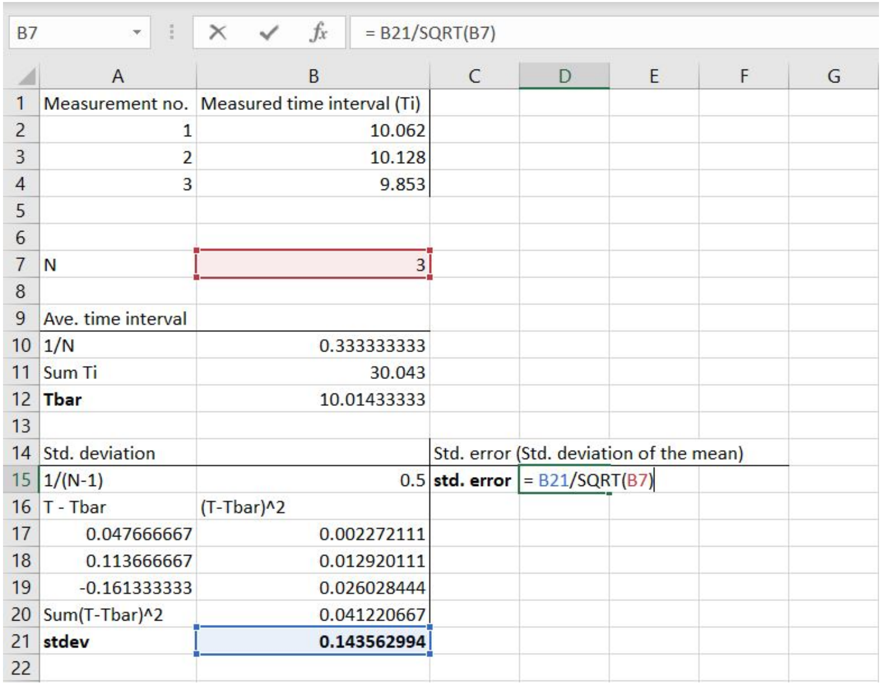
σ:


SUM = SQRT(B15 * B20)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6			
7	N	3	
8			
9	Ave. time interval		
10	1/N	0.333333333	
11	Sum Ti	30.043	
12	**Tbar**	10.01433333	
13			
14	Std. deviation		
15	1/(N-1)	0.5	
16	T - Tbar	(T-Tbar)^2	
17	0.047666667	0.002272111	
18	0.113666667	0.012920111	
19	-0.161333333	0.026028444	
20	Sum(T-Tbar)^2	0.041220667	
21	**stdev**	**= SQRT(B15 * B20)**	
22			

Figure 10

σ_m:



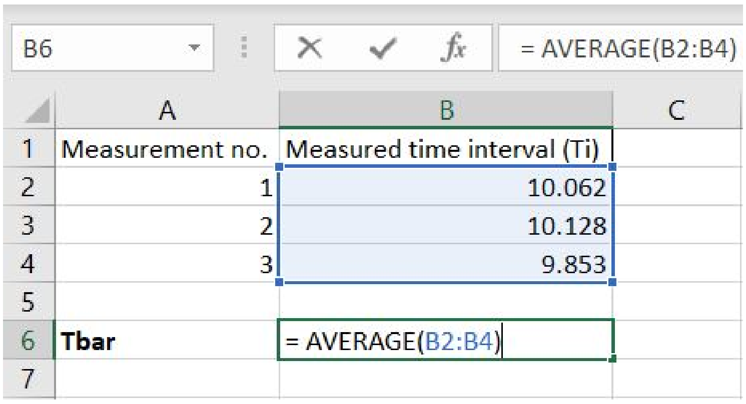
B7 = B21/SQRT(B7)

	A	B	C	D	E	F	G
1	Measurement no.	Measured time interval (Ti)					
2	1	10.062					
3	2	10.128					
4	3	9.853					
5							
6							
7	N	3					
8							
9	Ave. time interval						
10	1/N	0.333333333					
11	Sum Ti	30.043					
12	**Tbar**	10.01433333					
13							
14	Std. deviation		Std. error (Std. deviation of the mean)				
15	1/(N-1)	0.5	**std. error**	= B21/SQRT(B7)			
16	T - Tbar	(T-Tbar)^2					
17	0.047666667	0.002272111					
18	0.113666667	0.012920111					
19	-0.161333333	0.026028444					
20	Sum(T-Tbar)^2	0.041220667					
21	**stdev**	**0.143562994**					
22							

Figure 11

Using built-in functions:

$\overline{T}$:

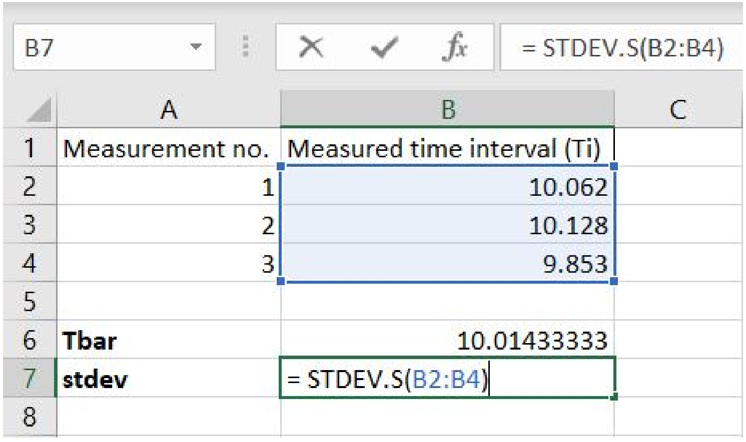


B6 = AVERAGE(B2:B4)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6	**Tbar**	= AVERAGE(B2:B4)	
7			

Figure 12

σ:

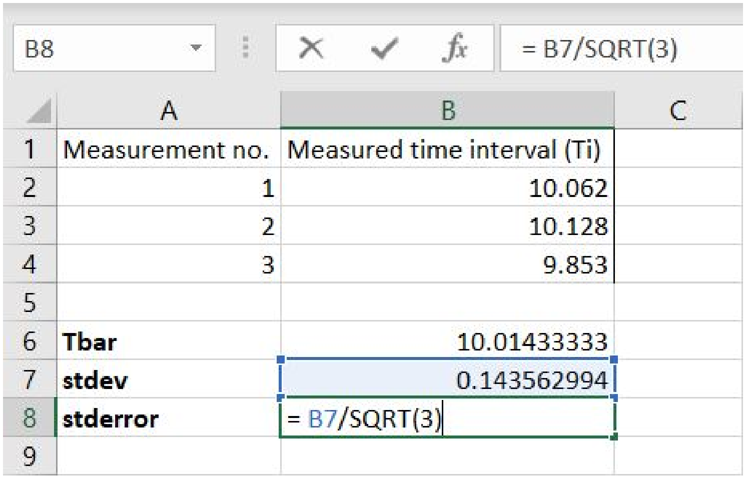


B7 | = STDEV.S(B2:B4)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6	**Tbar**	10.01433333	
7	**stdev**	= STDEV.S(B2:B4)	
8			

Figure 13

σ_m:



B8 | = B7/SQRT(3)

	A	B	C
1	Measurement no.	Measured time interval (Ti)	
2	1	10.062	
3	2	10.128	
4	3	9.853	
5			
6	**Tbar**	10.01433333	
7	**stdev**	0.143562994	
8	**stderror**	= B7/SQRT(3)	
9			

Figure 14

- Predict how T, σ, and σ_m vary with N, the number of measurements involved in their calculation. Then use Excel to calculate them for the first $N = 3, 5, 10$, and 25 (all) of your measurements, again imagining that you had only the first 3, 5, 10, or 25 data points. Then comment whether your predictions for the changes as N gets larger are approximately correct. In particular, why do σ and σ_m behave differently?
  - *Prediction:*
    - *$\overline{T}$ will get closer to 10*

    * σ *will become smaller (as approximately the power of -1/2 [from the* $(N-1)^{-1/2}$ *term])*
    * σ_m *will also become smaller (as approximately the power of -1 [from the term* $(N-1)^{-1/2} * N^{-1/2}$*])*
  - *Generate the plots:*
    * $\overline{T}$*: the prediction did not quite happen. The values of* $\overline{T}$ *kept "oscillating" between 10 (see attached Excel document)*
    * σ *and* σ_m *followed the prediction (see attached Excel document)*
  - *Generating the plots (example with* σ_m*:*
    1) *Select 'Insert' > 'Scatter'*

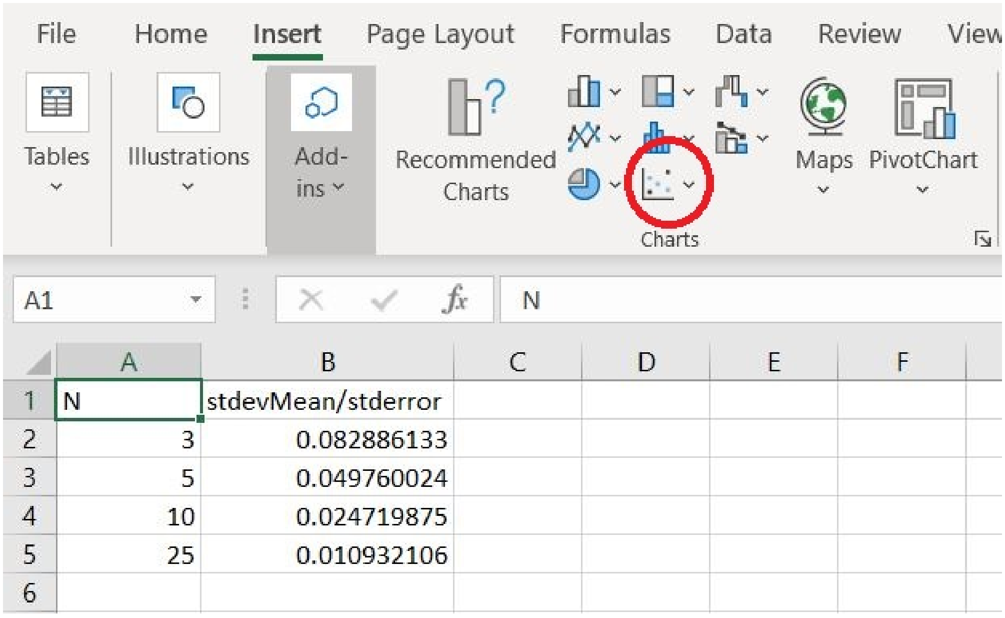


Figure 15

    2) *Right click on the graph and select 'Select Data'*
    3) *Select 'Edit' in the 'Legend Entries (Series)' window*

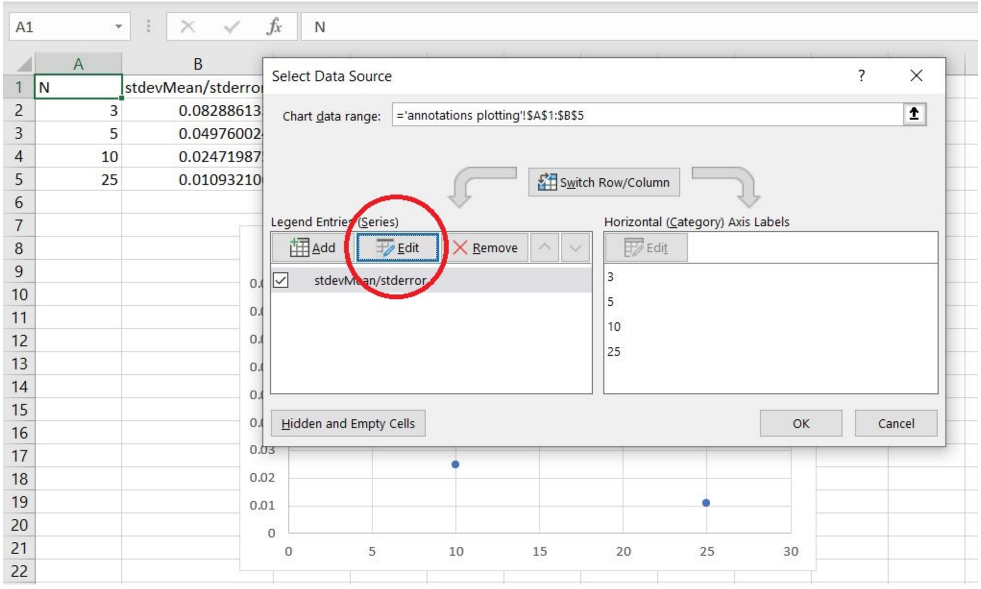


Figure 16

4) a. *Give a name to the data series*

b. *Select the x values. In this case, the values of N, which you can insert in the 'Series X values' box by selecting the cells with the different values of N. To do this, empty the box, click on the first cell, then add a comma, click on the second cell, add a comma, and so on*

c. *Select the y values. In this case, the values σ_m, which you can insert in the 'Series Y values' box by selecting the cells with the different values of σ_m. There are two different ways you can do this:*
1) If you do not have data arrange in columns, as shown through the walk-through figures: empty the box, click on the first cell, then add a comma, click on the second cell, add a comma, and so on
2) If you have the data arrange in columns: empty the box, click in the box, then highlight the data

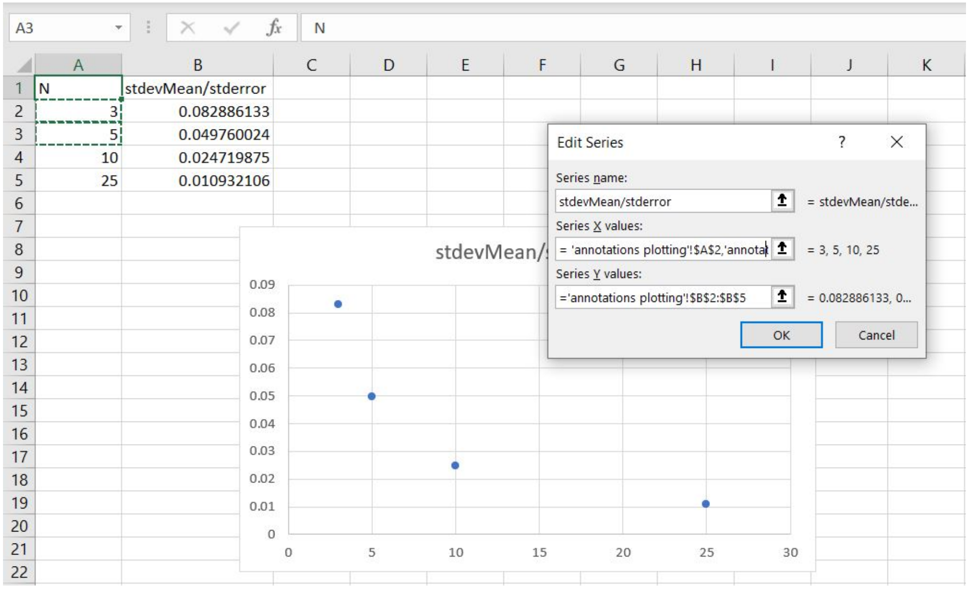


Figure 17

5) *Select 'OK'*
6) *Left click on the data point in the graph and select 'Add Trend-line'*
7) *Select 'Power' and tick 'Display Equation on chart'*

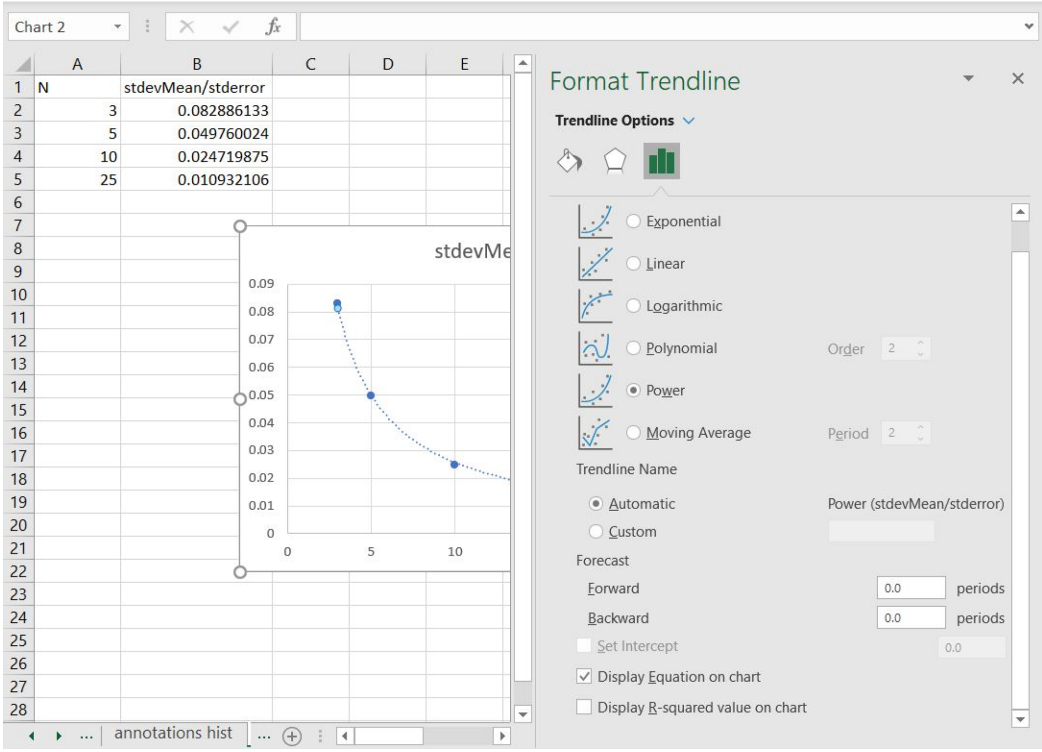


Figure 18

- Compare your $N = 25$ value of σ with our previous guess of the variability of your reaction time.

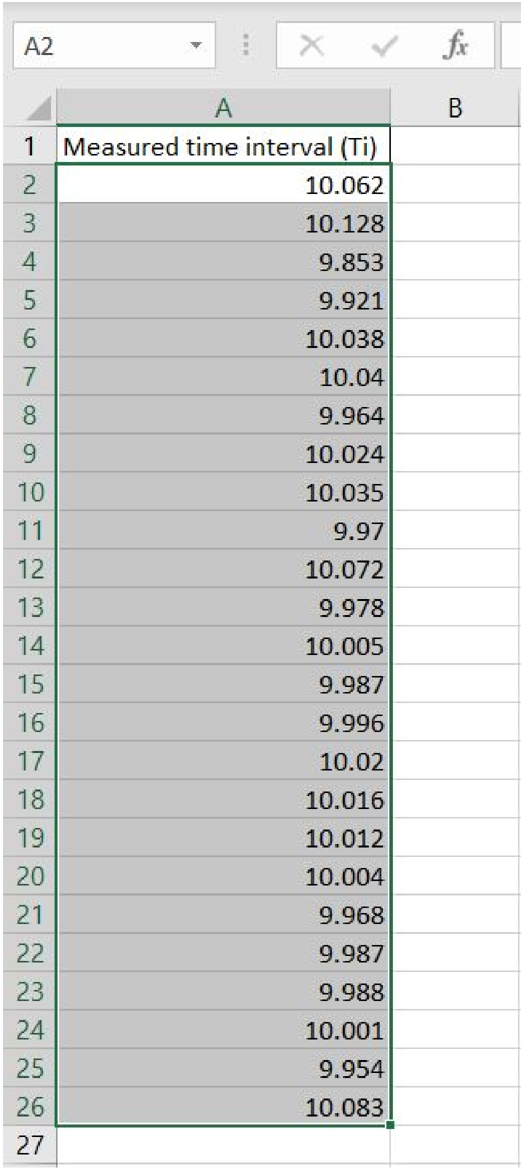
  - *Guess: 0.04 seconds*
  - *σ: 0.054661 seconds*

    Values close to each other. Correct order of magnitude

- Use Excel to make your final histogram from your twenty-five measurements. Adjust settings so the bin width to a "round" value about ≈ 0.04. Show the calculation in your report. How can you set the bin width with Excel? (See the attached Excel document for the histogram.)

  Creating a histogram:

  1) *Highlight the data (click on the uppermost cell, then press CTRL + ALT + DOWN)*



	A	B
1	Measured time interval (Ti)	
2	10.062	
3	10.128	
4	9.853	
5	9.921	
6	10.038	
7	10.04	
8	9.964	
9	10.024	
10	10.035	
11	9.97	
12	10.072	
13	9.978	
14	10.005	
15	9.987	
16	9.996	
17	10.02	
18	10.016	
19	10.012	
20	10.004	
21	9.968	
22	9.987	
23	9.988	
24	10.001	
25	9.954	
26	10.083	
27		

Figure 19

2) *Select 'Insert' > 'Histogram'*

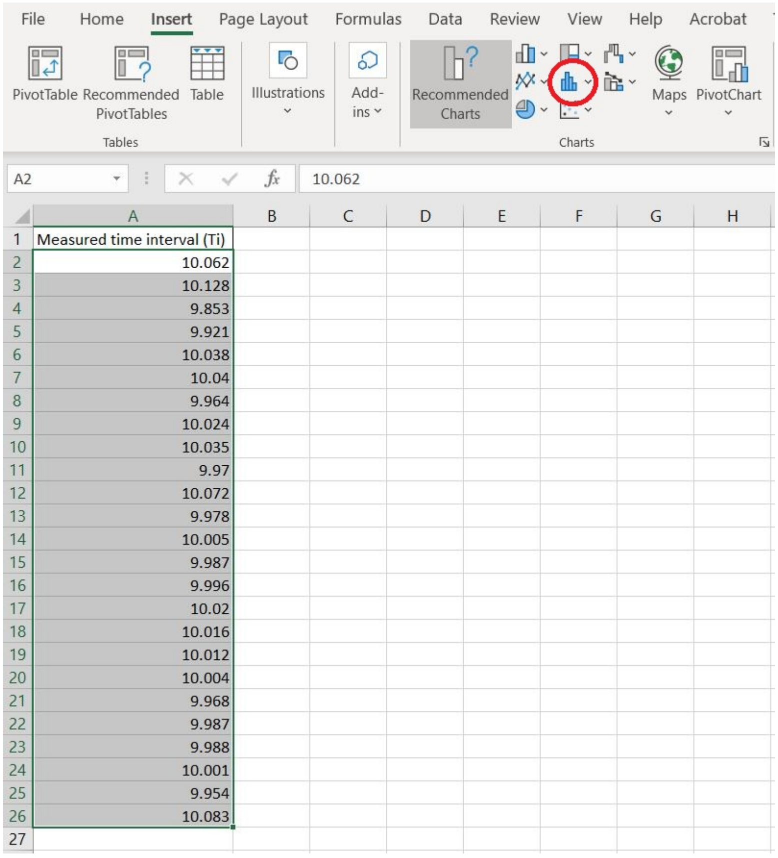


Figure 20

Adjusting the bin width:

1) *Left click on the horizontal axis of the graph and select 'Format Axis'*
2) *Adjust the bin width to 0.04*

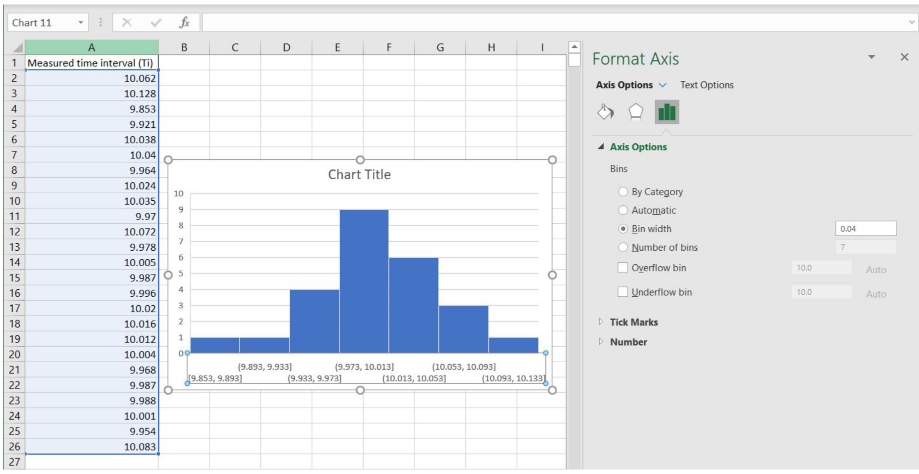


Figure 21

- Clearly mark the points of T and $T \pm \sigma$ for your measurements. These quantities can be shown to be the best estimate of your measurement. The region included in the range $\pm\sigma$ should contain about 68% of your data points if your errors are random and consequently the distribution of your measurements is normal/Gaussian (Taylor [1], Chapter 5). What fraction of your data lies within this range?

  – *Done by hand: 76% of the data is within a standard deviation from the mean.*

- Repeat these studies where you have used your non-favoured hand forefinger to push the button. What effect on the measurements did you see?

  – *The expectation is that the standard deviation will become bigger for each N-value.*

4.4 Drawing conclusions from the timing measurements

Check to see if there is a (statistically) significant discrepancy in the time measured by the large clock. We want to check the hypothesis that the large digital clock is running correctly. Statistical significance is tested not by just looking at the size of the difference and saying “that seems small” or “looks big to me”. Rather we compare the relative size of the difference with the uncertainty of our measurement. If the difference isn't substantially larger than our uncertainty, then we say that our statistical analysis left us with information insufficient to reject our original hypothesis, and we say that the differences are not statistically significant. We make the test quantitative by calculating the size of the discrepancy from expectations in units of the uncertainty of that difference.

The discrepancy we want is that between the average time counted, $\overline{T}$, with its expected value:

$T_{exp} = (actual\ count/s) \times (the\ number\ of\ counts\ in\ your\ interval)$.

Your instructor will give you the actual counts/sec which was set before the start of the experiment. Following the uncertainties summary in the Reference Guide, we use $D = \overline{T} - T_{exp}$. We use σ_m for δD, since that is our uncertainty in how well we know T, and there is no uncertainty in our prediction, T_{exp}. Then:

$$t = \frac{|\overline{T} - T_{exp}|}{\sigma_m} \tag{4}$$

Here, in accord with standard statistical notation, t has the meaning of the number of the standard deviations of the mean needed to cover the difference between the mean and expected times. In this equation, t is not a time: it has no units since both the numerator and denominator are in seconds. The statistic t is handy because we can make such a comparison for any measurement: the numerator D and denominator δD always have the same units, so t always means "number of standard deviations", a pure number on the same (dimensionless) scale, no matter what quantity was originally being measured. Following the handout, and common statistical practice, we will say the deviation is statistically significant if $|t| > 2$, that is the discrepancy is more than twice as big as our uncertainty.

$t = \frac{10.0042 - 1\times 10}{0.054} = 0.08$

(using the values from Excel spreadsheet)

As you will see later, this should happen by chance only about 5% of the time. Notice also if we make a sloppy measurement with a huge uncertainty δD, it will be hard to ever learn anything: it would be rare to have a large enough discrepancy to find a statistically significant difference from our starting assumption.

Based on this calculation, is $\overline{T}$ compatible with T_{exp}? That is, is the discrepancy (statistically) significant?

Assuming that T_{exp} = 10, the discrepancy is of 0.07; not significant.

- Calculate the time interval per clock tick (how is this related to $\overline{T}$). Does it appear that the clock tick is 1.0 seconds, as we assumed at the beginning? What is the uncertainty in the time interval per clock tick? Show your work.

  $time\ interval\ per\ clock\ tick = \frac{10.004}{10} = 1.0004\ seconds$

  $uncertainty = \frac{10.004}{10} \times \sqrt{\left(\frac{0.054}{10.004}\right)^2} = 0.0054 seconds$

  (using the values from the Excel spreadsheet)

- Suppose you had recorded only your first 3 measurements. What would you have concluded about the existence of a significant discrepancy? Were the remaining measurements necessary in your opinion? Give a quantitative justification.

  The following calculations use the data for 3 measurements:

  $t = \frac{10.0014-1\times10}{0.144} = 0.01$

  (using the values from Excel spreadsheet)

  The uncertainty in the measurements of the time interval for 10 ticks is too big to extract significant statistical information

  The remaining measurements were necessary, and would expect the value of t will get larger if more measurements are taken. If I had to repeat the experiment, I would probably take 50 to 100 measurements. The following table shows how t varies according to $N = 3, 5, 10, 25$ measurements:

N	t
3	0.0998
5	0.00359
10	0.0448
25	0.0776

References

[1] Taylor, John R. (1997). An introduction to error analysis: the study of uncertainties in physical measurements. 2nd. ed. University Science Books: USA.