Millikan experiment for detection of fractional charges via deflection due to electric field in vertical fall - report 2

A. A. Salazar[1,*]

[1]*Department of Physics, University of Alberta, Edmonton, AB, Canada T6G 2G7*

I. DESCRIPTION

This study explores the possibility of detecting fractional charges through a modified Millikan experiment. It can be divided into two parts: (1) detection of fractional charges from orbital motion of a 25-micron diameter glass sphere around a vertical charged pillar, and (2) from its horizontal deflection in vertical fall. The setup for the two is the exactly the same one, with the exception that in the first case the sphere is given an initial velocity.

The setup consists of a central pillar and a concentric tube. The central pillar is given a certain surface charge; given its size in comparison to the sphere's, one can consider the central pillar to be infinitely long, and so, approximate the electric field lines to point radially from the axis of symmetry of the concentric cylinders. For the first part, the sphere is given an initial velocity with a tangential component to the edge of an (imaginary) horizontal concentric disk. For the second part, the sphere is dropped with an initial zero velocity and is let free fall. (See Figure 1.)

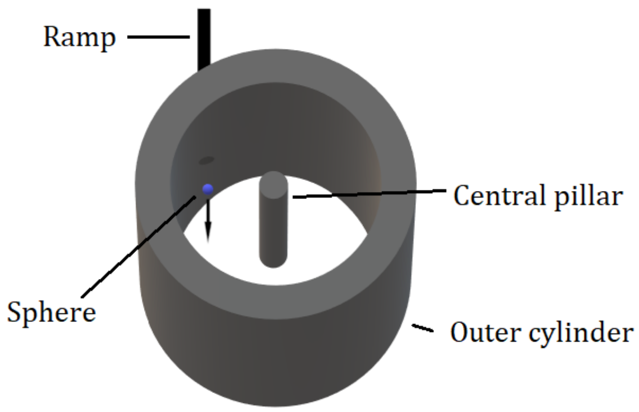

FIG. 1:

II. METHOD

The study was performed with computer simulations. Each of the two parts of the study mentioned above can be further divided into two: the sphere falling in air or in vacuum. Following are the equations used for the simulations; consult Appendix A to see the equations and values used for each parameter. The full derivations can be found in Appendix B.

- In air

Equations of motion:

The Lagrangian and equation for angular momentum are the following, respectively:

$$\mathcal{L} = \frac{1}{2}m_{eff}(\dot{r}^2 + r^2\dot{\theta}^2) - V(r), \tag{1}$$

$$\frac{dL}{dt} = \frac{d}{dt}(m_{eff}r^2\dot{\theta}) = \tau, \tag{2}$$

where m_{eff} is the effective mass of the glass sphere, r is the radial distance from the center of the central pillar, $\dot{\theta} = \frac{v_\theta}{r}$ is the angular velocity, where v_θ is the tangential velocity for circular motion. $V(r)$ is the potential energy of the system, and $\tau = rF_\theta$ is the torque applied to the orbiting sphere. $F_\theta = -Km_{eff}v_\theta$ is given by Stokes' law.

One solves the Lagrangian to find:

$$m_{eff}\ddot{r} = m_{eff}r\dot{\theta}^2 - V'(r), \tag{3}$$

where $-V'(r) = F_r$ the radial force. There are non-conservative forces, and these must be taken into account: $F_r = -qE - Km_{eff}\dot{r} + \rho V a_f$, where the first, second and third are the electric field, Stokes' and buoyancy terms, respectively. The electric field can be found using Gauss's law. This is $E = \frac{r_1\sigma_1}{r\epsilon_0}$, where r_1 is the radius of the central pillar, and σ_1 is its surface charge density; ϵ_0 is the vacuum electric permittivity. We obtain the following system of ODEs, which can be solved numerically in the computer:

$$\begin{cases} \ddot{r} = \frac{1}{r}[\dot{v_\theta}^2 + \frac{qr_1\sigma_1}{\epsilon_0 m_{eff}}(\frac{\rho V}{m_{eff}} - 1)] - K\dot{r} \\ \dot{v_\theta} = -(K + \frac{\dot{r}}{r})v_\theta. \end{cases} \tag{4}$$

The vertical displacement is given by:

$$y(t) = \frac{g}{K}[t + \frac{1}{K}(e^{-Kt} - 1)](1 - \frac{\rho}{\sigma - \rho}) - \frac{v_{y_0}}{K}(e^{-Kt} - 1), \tag{5}$$

where g is the acceleration due to gravity, ρ is the density of air, and σ is the density of glass.

*Electronic address: aasalaza@ualberta.ca

- In vacuum

In vacuum the angular momentum is conserved and the only radial force is the central force from the electric field. So, Equation 2 and the Lagrangian read:

$$\begin{cases} \ddot{r} = \frac{1}{r}(v_\theta^2 - \frac{qr_1\sigma_1}{\epsilon_0 m_{eff}}) \\ \dot{v_\theta} = -v_\theta \frac{\dot{r}}{r}, \end{cases} \tag{6}$$

and the vertical displacement is given by:

$$y(t) = v_{y_0} t + \frac{1}{2} g t^2 \tag{7}$$

For free fall cases, one just needs to give the variables v_{θ_0} and v_{y_0}, the initial tangential and vertical velocities, respectively, the values of zero.

For this experiment, it would be good to completely terminate the electric fields at the outer tube. For this, the surface charge density on the outer tube must be $\sigma_2 = -\frac{r_1}{r_2}\sigma_1$, where r_2 is its radius. This gives an electric potential between the pillar and tube of:

$$V = -\frac{r_1\sigma_1}{\epsilon_0} ln(\frac{r_1}{r_2}). \tag{8}$$

III. EQUIPMENT

- Large amount of 25-micron diameter glass spheres
- central pillar and outer tube of radii 0.5 cm and 10 cm, respectively
- For circular motion: a ramp to give the sphere an initial tangential velocity
- In vacuum: cylindrical sealed container to keep things inside in vacuum
- Rings of 3 centi-meters in radius and probably 30-micron thick placed horizontally at certain vertical positions determined by the computer simulations. The spheres will start at a radial distance from the central pillar of 5 cm, and they would be caught at a radial distance of 3 cm. (Note: these distance were used for the generation of all the results that follow.)

IV. RESULTS

- Circular motion

In air:

Ideally one would like the central pillar to have the same surface charge density at all times. A reasonable surface charge density is ≈ 0.000025 coulombs/m^2, which gives an electric potential between the central pillar and outer tube of about 50 kilo-volts. The main problem with air is that circular motion is hardly obtained. As shown in Figures 2 & 3, the sphere's tangential velocity of circular motion reaches the value of zero at approximately 0.1 seconds after the beginning of its circular motion around the central pole. In addition, the air resistance seems to overcome the force of attraction from the central pillar: even when there is no circular motion present, the change sphere's distance from the central pillar seems to be approximately constant. It would be good to get rid of the circular motion and just look at the free fall of the sphere (see below).

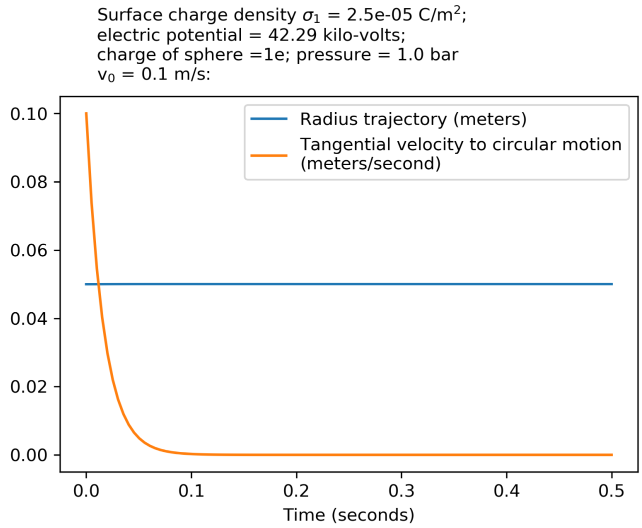

FIG. 2:

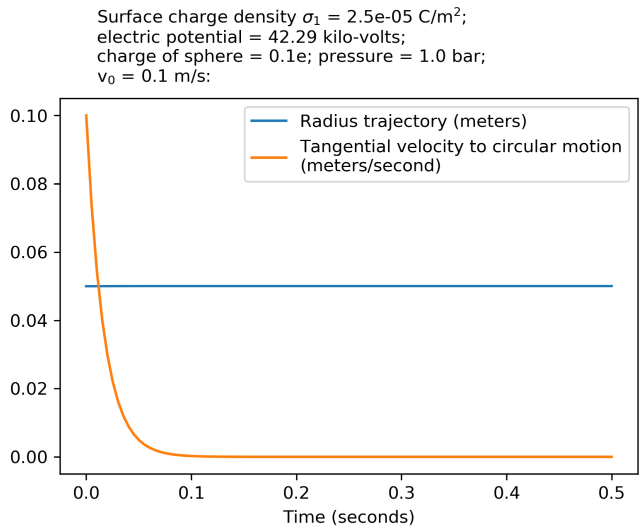

FIG. 3:

In vacuum:

One is able to see orbital motion in vacuum, as shown in Figures 4 & 5. First, look at Figure 4; one could restrict the fall time to approximately 0.5 seconds and be able to see a significant change in the radius of the orbit (this for different fractional charges) in a fall of approximately 1.2 m. This is not bad, since one should be able to build a vacuum chamber of this height (with a base of approximately 15 × 15 cm). The main problem arises when one looks at the voltage required: ≈ 340 mega-volts. Unless one can produced such a high voltage between the central pillar and outer tube, one could not possibly see any significant orbital motion without further decreasing the initial tangential velocity to the orbital motion of the sphere, as shown in Figure 5. The problem with the results shown in this Figure 5 is that in order to be able to see a change in radius of 2 cm, one has to wait for approximately 25 seconds, which means a fall of 3 km! (according to Eq. 7). I thought of trapping the orbiting sphere between two horizontally-oriented parallel plate capacitors that slow down the fall of the sphere. For a sphere of charge 1 e, in order to completely cancel the acceleration due to gravity, one has to generate an electric field of about 4 Mvolts/mm!

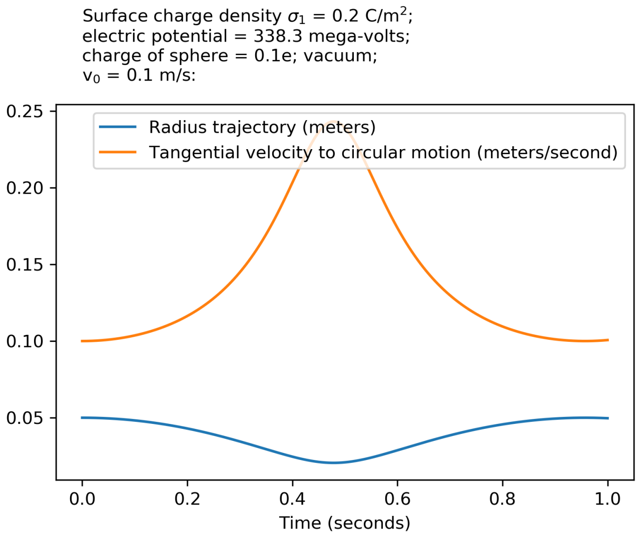

FIG. 4:

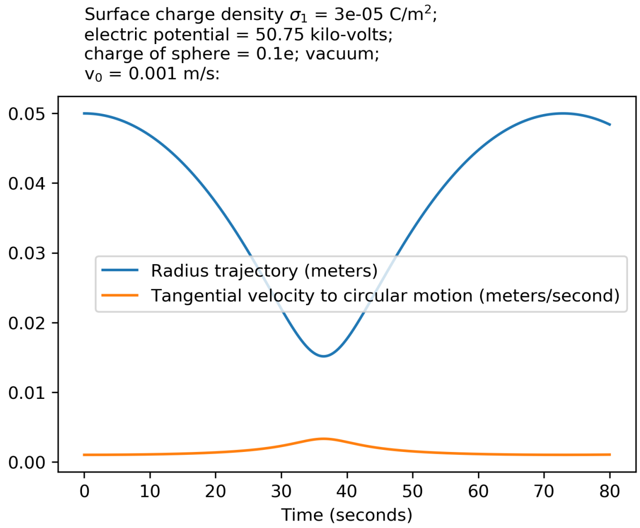

FIG. 5:

- Free fall

One can completely forget about circular motion and look into letting the spheres free fall. The results for free fall are shown in Table I; here the sphere starts at a distance of 5 cm from the central pillar. Because the electric force on the sphere due to the central pillar is a central force, one could increase the radial displacement by letting the sphere fall from a distance less than 5 cm from the central pillar. Note that the change in radius and vertical displacements are very large compared to any deflection that could arise from Brownian motion when the spheres fall in air (see first report on parallel plates to get an idea of the magnitude of the deflection due to Brownian motion [2]). From the results in Table I, one can only hope to detect charges of 1.0e and 0.1e, the radial deflections for the other charges are too small (smaller than the diameter of the sphere).

Surface charge density on central pillar: 3e-05 coulombs/m^2 Electric potential: 50.75 kilo-volts Fall time: 60 seconds			
Charge	Pressure [bar]	Fall height [meters]	Radial displacement [mm] | [sphere diameters]
1.0e	1.0	9.800	1.415 | 56.61
	2.0	9.435	1.361 | 54.45
	5.0	9.209	1.329 | 53.14
	10.0	9.123	1.317 | 52.68
	20.0	9.509	1.310 | 52.38
	50.0	8.957	1.300 | 52.01
0.1e	1.0	9.800	0.1385 | 5.540
	2.0	9.435	0.1333 | 5.334
	5.0	9.209	0.1302 | 5.208
	10.0	9.123	0.1291 | 5.163
	20.0	9.509	0.1284 | 5.135
	50.0	8.957	0.1275 | 5.100
0.01e	1.0	9.800	0.01382 | 0.5528
	2.0	9.435	0.01331 | 0.5323
	5.0	9.209	0.01299 | 0.5198
	10.0	9.123	0.01288 | 0.5153
	20.0	9.509	0.01281 | 0.5125
	50.0	8.957	0.01272 | 0.5090
0.001e	1.0	9.800	0.001382 | 0.05527
	2.0	9.435	0.001330 | 0.05322
	5.0	9.209	0.001299 | 0.05197
	10.0	9.123	0.001288 | 0.05152
	20.0	9.509	0.001281 | 0.05124
	50.0	8.957	0.001272 | 0.05089

TABLE I: Radial displacement of glass sphere in a free fall of 60 seconds for different charges and pressures. Initial distance from central pillar: 5 cm.

Compare the results for spheres falling in air to the ones obtained for the spheres free falling in vacuum; see Table II and Figure 6. The main problem that arises from letting the spheres fall in vacuum is that the vertical distances the spheres would travel before they reach a distance from the central pillar of 2 cm from an initial radial distance of 5 cm meters are very large.

Vacuum Surface charge density on central pillar: 3e-05 coulombs/m^2 Electric potential: 50.75 kilo-volts Fall time: 10 seconds Fall height: 490 meters	
Charge	Radial displacement [mm] | [sphere diameters]
1.0e	50.00 | 2000
0.1 e	4.207 | 168.3
0.01e	0.4153 | 16.61
0.001e	0.04148 | 1.659

TABLE II: Radial displacement in a free fall of 10 seconds in vacuum for different charges. Initial distance from central pillar: 5 cm.

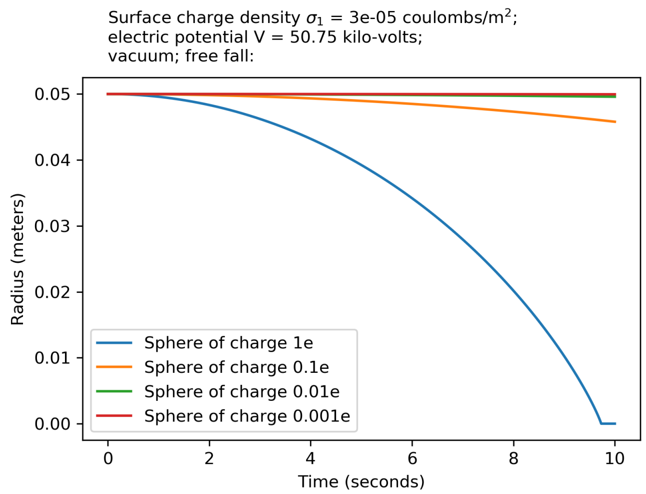

FIG. 6:

The results in Table II are not bad if one could keep the fall height to a minimum by placing two horizontally-oriented parallel plates that generate an electric field that slows down the fall of the spheres. If one was able to generate a vertical electric fields of 4 mega-volts/mm or larger, one could focus on cancelling the gravitational force on spheres of particular charge values. One would look only at those spheres that, in addition of not falling, would reduce their radius by a certain amount. The zero fall and the reduction in radius would be two indicators of the charge of the sphere.

V. CONCLUSION

If one is able to make the spheres levitate, the best option would be to perform the experiment in a vacuum chamber. Balancing the electric field against the gravitational pull for each single sphere would not be necessary. For example, one could focus on finding the spheres that have a charge of $0.1e$ by setting the vertical electric field to a value that would make the spheres of this charge levitate. In addition, the sphere would change its radius by 2 cm in a definite amount of time (predicted by Eq. 6).

VI. APPENDIX

A.

Parameter list

Constants:
Electron charge, $e = 1.602176634\text{e-}19$ C
Acceleration due to gravity, $g = 9.806$ m/s

Air:
Pressure (varies for each situation)
Individual gas constant, $R_{air} = 287.15$ J/(kg mol)
Temperature, $T = 20^o$C
Density, $\rho = P/(R_{air}T)$ [kg/m^3
(Dynamic) viscosity, $\eta = 1.8\text{e-}05$ kg / (m s)

Glass sphere:
Radius, $a = 12.5\text{e-}06$
Density, $\sigma = 8000$ kg/m^3
Volume, $V = \frac{4/3}{\pi}a^3$ [m^3]
Mass, $m = V\sigma = 6.544\text{e-}11$ kg
Effective mass (in air), $m_{eff} = V(\sigma - \rho)$ [kg]
Charge, $q = [1.0e, 0.1e, 0.01e, 0.001e]$

Apparatus:
Radius of central pillar, $r_1 = 0.05$ m Radius of outer cylinder, $r_2 = 0.35$ m Surface charge density on central pillar (varies for each situation), σ_1 Surface charge density on outer tube (terminates electric field lines from central pillar), $\sigma_2 = -\frac{r_1}{r_2}\sigma_1$
Electrostatics:
Vacuum electric permittivity, $\epsilon_0 = 8.8541878128\text{e-}12$ F/m Electric field between central pillar and outer tube, $E = \frac{r_1 sigma_1}{r\epsilon_0}$ Electric potential between central pillar and outer tube, $V = -\frac{r_1\sigma_1}{\epsilon_0}ln(\frac{r_1}{r_2})$
Other constants and factors [1]:
$B = 0.100$
$k = [1 + B/(aP)]^{-1}$
$K = 9\eta/[2a^2(\sigma - \rho)]k$

B.

Full derivation of equations

- Circular motion

Lagrangian:

$$\mathcal{L} = T - V$$

$$\mathcal{L} = \frac{1}{2}m_{eff}\dot{r}^2 + \frac{1}{2}I\omega^2 - V(r)$$

$$\mathcal{L} = \frac{1}{2}m_{eff}(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$$

Solution:

$$\frac{d}{dt}(\frac{\partial\mathcal{L}}{\partial\dot{r}}) = \frac{\partial\mathcal{L}}{\partial r}$$

$$m_{eff}\ddot{r} = m_{eff}r\dot{\theta}^2 - V'(r)$$

$$m_{eff}\ddot{r} = m_{eff}r\dot{\theta}^2 + F_r$$

Angular momentum:

$$\frac{dL}{dt} = \tau$$

$$\frac{d}{dt}(m_{eff}rv_\theta) = rF_\theta$$

$$\frac{d}{dt}(m_{eff}r^2\dot{\theta}) = rF_\theta$$

The force vector is:

$$\overline{\mathbf{F}} = F_r\hat{\mathbf{r}} + F_\theta\hat{\theta}$$

The used convention is negative for radially inwards and positive for counterclockwise.

<u>In air</u>:

The force vector components are:

$$F_r = -F_{electric} + F_{airdrag} + F_{buoyancy}$$

$$F_r = -qE - Km_{eff}\dot{r} + \rho V(\frac{qE}{m_{eff}})$$

$$F_\theta = F_{airdrag}$$

$$F_\theta = -Km_{eff}v_\theta$$

$E = \frac{r_1\sigma_1}{r\epsilon_0}$, obtained from Gauss's law:

$$\oint_S \overline{\mathbf{E}} \cdot d\overline{\mathbf{A}} = \oint_S E \cdot dA = E(2\pi r\ell) = \frac{Q_{enc}}{\epsilon_0} = \frac{2\pi r_1\ell\sigma_1}{\epsilon_0}$$

We get a system of ODEs:

$$\begin{cases} \ddot{r} = \frac{1}{r}[v_\theta^2 + \frac{qr_1\sigma_1}{\epsilon_0 m_{eff}}(\frac{\rho V}{m_{eff}} - 1)] - K\dot{r} \\ \dot{v_\theta} = -(K + \frac{\dot{r}}{r})v_\theta \end{cases}$$

<u>In vacuum</u>:

The force vector components are:

$$F_r = -F_{electric} = \frac{qr_1\sigma_1}{r\epsilon_0}$$

$$F_\theta = 0$$

We obtain the following system of ODEs:

$$\begin{cases} \ddot{r} = \frac{1}{r}(v_{theta}^2 - \frac{qr_1\sigma_1}{\epsilon_0 m_{eff}}) \\ \dot{v_{theta}} = -v_\theta \frac{\dot{r}}{r} \end{cases}$$

- Free Fall

In air:

Radial equation of motion:

$$m_{eff}\ddot{r} = -F_{electric} + F_{airdrag} + F_{buoyancy}$$

$$m_{eff}\ddot{r} = \frac{qr_1\sigma_1}{r\epsilon_0 m_{eff}}(\frac{\rho}{\sigma-\rho} - 1) - K\dot{r}$$

In vacuum:

Radial equation of motion:

$$m_{eff}\ddot{r} = -\frac{qr_1\sigma_1}{r\epsilon_0 m_{eff}}$$

- Vertical motion [2]

In air:

$$y(t) = \frac{g}{K}[t + \frac{1}{K}(e^{-Kt} - 1)](1 - \frac{\rho}{\sigma-\rho}) - \frac{\dot{y_0}}{K}(e^{-Kt} - 1)$$

In vacuum:

$$y(t) = \dot{y_0}t + \frac{1}{2}gt^2$$

- Electrostatics

Electric field:

Using Gauss's law, one can find the electric field between the central pillar and the outer cylinder:

$$\oint_S \overline{\mathbf{E}}_{\mathbf{in}} \cdot d\overline{\mathbf{A}} = \oint_S E_{in} \cdot dA = E_{in}(2\pi r\ell) = \frac{Q_{enc}}{\epsilon_0} = \frac{2\pi r_1 \ell \sigma_1}{\epsilon_0}$$

One wants to terminate the electric field lines from the central pillar at the outer tube; use Gauss's law:

$$\oint_S \overline{\mathbf{E}}_{\mathbf{out}} \cdot d\overline{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} = \frac{2\pi\ell(r_1\ell\sigma_1 - r_2\sigma_2)}{\epsilon_0} = 0$$

$$\Rightarrow \sigma_2 = -\frac{r_1}{r_2}\sigma_2$$

We get:

$$\overline{\mathbf{E}} = \begin{cases} -\frac{r_1\sigma_1}{r\epsilon_0}\hat{\mathbf{r}}, \text{if } r_1 < r < r_2 \\ \overline{\mathbf{0}}, \text{if } r > r_2 \end{cases}$$

Electric potential:

$$V = -\int \overline{\mathbf{E}} \cdot d\overline{\mathbf{r}} = -\int_{\infty}^{r_2} \overline{\mathbf{E}}_{\mathbf{out}} \cdot d\overline{\mathbf{r}} - \int_{r_2}^{r_1} \overline{\mathbf{E}}_{\mathbf{in}} \cdot d\overline{\mathbf{r}}$$

$$V = -\int_{r_2}^{r_1} \frac{r_1\sigma_1}{r\epsilon_0}\, dr$$

$$V = -\frac{r_1\sigma_1}{\epsilon_0} ln(\frac{r_1}{r_2})$$

[1] L. W. McKeehan. (1911). The terminal velocity of fall of small spheres in air ate reduced pressures. University of Minesota.

[2] A. A. Salazar. Millikan experiment for detection of fractional charges via horizontal deflection due to electric field in vertical fall - report 1. University of Alberta.