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Phys 362 Assignment #3

1. Situation:

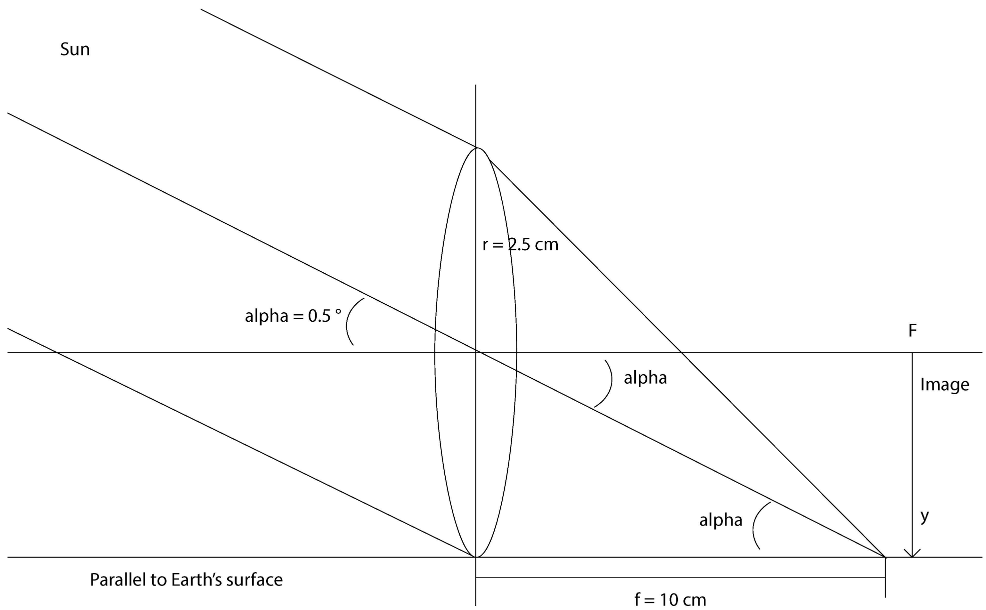

Fig. 1 (not to scale; angle is exaggerated, and y is not necessarily equal to r)

In Fig. 1 the rays of the sun can be considered as coming from infinity and thus parallel to each other: they will coincide/intersect at a focal distance of the lens. We are assuming a thin lens.

Irradiance of the Sun on Earth ~ 1.0 kW/m^2
Radius of lens: r = 0.025 m

⇒ Power: $P = 1.0 \times 10^3\ W/m^2 \cdot \pi (0.025\ m)^2 = \frac{5}{8} \pi\ W$

Size of the image formed (from Fig. 1):
$y = f \cdot tan(\alpha) = (0.10\ m) \cdot \tan(0.5^\circ)$
where *f* is the focal length

Irradiance of image:
$I_i = \frac{P}{\pi \cdot (y/2)^2} = 3.283 \cdot 10^6\ W/m^2 \simeq 3300\ kW/m^2$ // Answer

The calculation is shown in the code below:

```
(* Assign values to the variables *)
f = 0.10; alpha = 0.5 * (Pi / 180);

P = 5 * Pi / 8; y = f * Tan[alpha];

(* Calculate the irradiance of the image *)
Ii = P / (Pi * (y / 2) ^2)
```

Out[ ]= 3.28264×10^{6}

2. Situation:

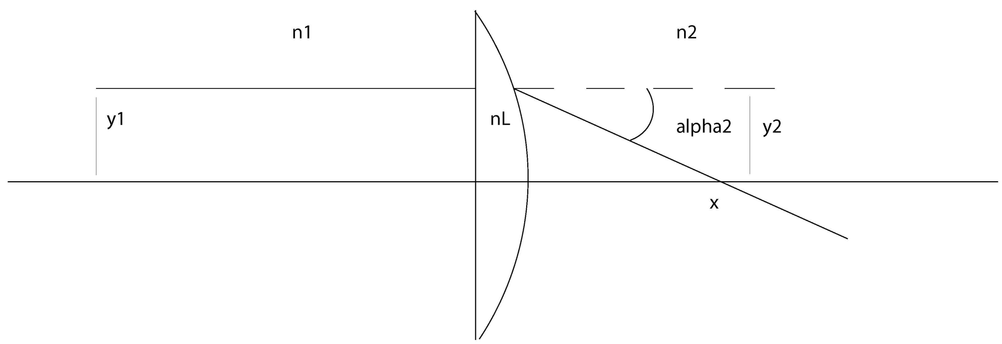

Fig. 2

(a) The matrix calculation is the following:

$$M = T_1 \cdot R_1 = \begin{pmatrix} 1 & 0 \\ \frac{n_L - n}{n \cdot R} & \frac{n_L}{n} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The result is as follows:

$$\begin{pmatrix} y_2 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \alpha_1 \end{pmatrix}$$

i.e. $y_2 = a \cdot y_1 + b \cdot \alpha_1$

$\alpha_2 = c \cdot y_1 + d \cdot \alpha_1$

with y_2 and α_2 being the exit elevation and angle of the ray that entered parallel to the O. A., respectively.

The following code does the calculation:

In[•]:=
```
(* Assign values to variables (provided in the problem) *)
alpha1 = 0; y1 = 1; n = 1; n1 = n; n2 = n; R = -3; nL = 1.5;

(* Define matrices *)
R1 = {{1, 0}, {(nL - n) / (n * R), nL / n}};
T1 = {{1, 0}, {0, 1}};

(* Perform the multiplication *)
M = R1.T1
```

Out[•]=
```
{{1., 0.}, {-0.166667, 1.5}}
```

In[•]:=
```
(* Calculate the exit elevation and angle of the ray *)
a = Part[M, 1, 1];
b = Part[M, 1, 2];
c = Part[M, 2, 1];
d = Part[M, 2, 2];
y2 = a * y1 + b * alpha1
alpha2 = c * y1 + d * alpha1
```

Out[•]=
```
1.
```

Out[•]=
```
-0.166667
```

∴ The exit elevation is 1 cm above the optic axis, and the exit angle with respect to the optic axis is -1/6 rad ≈ -0.1667 rad ≈ -9.55° (so the ray is refracted towards the O. A.). // Answer

(b) The focal length is given by $f_2 = -1/c$ (c from the matrix M above).

The calculation:

In[•]:=
```
(* Determine the focal length *)
f2 = -1/c
```

Out[•]=
```
6.
```

∴ The ray described in part (a) intersects the optic axis at a distance x = 6 cm form the central plane (i.e. from the planar surface). // Answer

3. Situation:

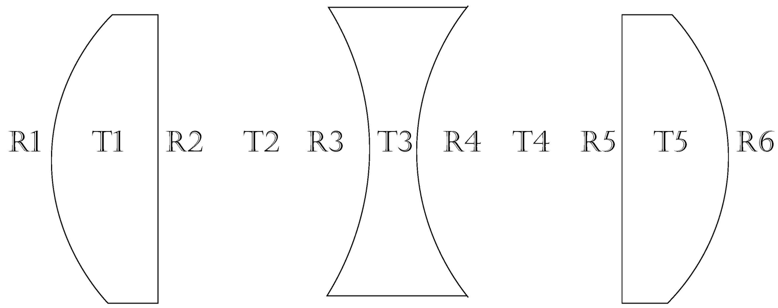

Fig. 3

The resultant matrix is given by:

$$\mathrm{M} = \mathrm{R}_6 \cdot \mathrm{T}_5 \cdot \mathrm{R}_5 \cdot \mathrm{T}_4 \cdot \mathrm{R}_4 \cdot \mathrm{T}_3 \cdot \mathrm{R}_3 \cdot \mathrm{T}_2 \cdot \mathrm{R}_2 \cdot \mathrm{T}_1 \cdot \mathrm{R}_1$$

$$\mathrm{M} = \begin{pmatrix} 1 & 0 \\ \frac{n_3 - n}{n \cdot r_6} & \frac{n_3}{n} \end{pmatrix} \cdot \begin{pmatrix} 1 & t_3 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{n - n_3}{n_3 \cdot r_5} & \frac{n}{n_3} \end{pmatrix} \cdot \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{n_2 - n}{n \cdot r_4} & \frac{n_2}{n} \end{pmatrix} \cdot \begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{n - n_2}{n_2 \cdot r_3} & \frac{n}{n_2} \end{pmatrix} \cdot \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n}{n * r_2} & \frac{n_1}{n} \end{pmatrix} \begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{n - n_1}{n_1 \cdot r_1} & \frac{n}{n_1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

We are looking for the back focal length for the right most surface. This one is given by: $q = -\frac{A}{C}$.

The following code does the calculation:

In[◦]:=
```
(* Assign values to the variables (provided in the problem) *)
r1 = 19.4; r2 = -128.3; r3 = -57.8; r4 = 18.9; r5 = 211.3; r6 = -66.4;
t1 = 4.29; t2 = 0.93; t3 = 3.93;
n1 = 1.6110; n2 = 1.5744; n3 = 1.6110;
(* Assume the the lenses are surrounded by air *)
n = 1;
d1 = 1.63; d2 = 12.90;

(* Define matrices *)
R6 = {{1, 0}, {(n3 - n) / (n * r6), n3 / n}};
T5 = {{1, t3}, {0, 1}};
R5 = {{1, 0}, {(n - n3) / (n3 * r5), n / n3}};
T4 = {{1, d2}, {0, 1}};
R4 = {{1, 0}, {(n2 - n) / (n * r4), n2 / n}};
T3 = {{1, t2}, {0, 1}};
R3 = {{1, 0}, {(n - n2) / (n2 * r3), n / n2}};
T2 = {{1, d1}, {0, 1}};
R2 = {{1, 0}, {(n1 - n) / (n * r2), n1 / n}};
T1 = {{1, t1}, {0, 1}};
R1 = {{1, 0}, {(n - n1) / (n1 * r1), n / n1}};

(* Do the multiplication *)
M = R6.T5.R5.T4.R4.T3.R3.T2.R2.T1.R1
```

Out[◦]=
```
{{0.808783, 22.8104}, {-0.0115688, 0.910147}}
```

We obtained the matrix:

$$M = \begin{pmatrix} 0.8088 & 22.81 \\ -0.01157 & 0.9101 \end{pmatrix}$$

Calculate q:

In[◦]:=
```
(* Calculation of back focal length for right most surface *)
a = Part[M, 1, 1];
c = Part[M, 2, 1];
q = -a / c
```

Out[◦]=
```
69.9108
```

∴ The sensor has to be 69.9 mm ≃ 70 mm behind the last lens surface's vertex (last output plane). // Answer