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PHYS 362 Assignment 1
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Q1

$E = -2.00(V/m) \cdot e^{i(1.00 \cdot 10^7 z + 2.00 \cdot 10^{15} t)}$

(a) Solution:

From the equation, $\omega = 2.00 \cdot 10^7$ rad/sec, $k = 1.00 \cdot 10^{15}$ rad/m.

$v = \frac{\omega}{k} = \frac{c}{n}$

$\Rightarrow n = \frac{ck}{\omega} = (3 \cdot 10^8 \text{ m/s})(\frac{1.00 \cdot 10^7}{2.00 \cdot 10^{15}} \text{ s/m}) = 3/2 = 1.5$

$\therefore$ The index of refraction of the medium in which the wave is travelling is $n = 1.5$. //Answer

(b) Solution:

The equation of irradiance is $I = \frac{1}{2} \cdot v \cdot \epsilon \cdot {E_0}^2$.

$v = c/n$ is the phase velocity
$\epsilon = n^2 \cdot \epsilon_0$ is the electric permittivity

$\Rightarrow I = \frac{1}{2} c n \epsilon_0 {E_0}^2$
$= \frac{1}{2}(3 \cdot 10^8 \text{ m/s})\frac{3}{2}(8.85 \cdot 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)(-2.00 \text{ V/m})^2$
$= 7.96 \cdot 10^{-3} \text{ W/m}^2$

$\therefore$ The irradiance of the wave is $I = 7.96 \cdot 10^{-3}$ W/m^2. //Answer

(c) Answer: The irradiance of a 0.5 mW laser pointer is 4 times the order of magnitude of the irradiance of a 220 Watt light bulb seen 10 m away, which is $\approx 8.75 \cdot 10^{-3}$ W/m^2. The irradiance of the electromagnetic wave of this problem ($7.96 \cdot 10^{-3}$ W/m^2) is close to this value, so, compared to a 0.5 mW laser pointer, this electromagnetic wave will appear deam to the human eye.

Q2

$\Psi_1 = Asin(kx + \omega t - \pi/5)$
$\Psi_2 = Asin(kx - \omega t - \pi/6)$

(a) Solution:

The waves are travelling in opposite directions.

(i) $\Psi_R = \Psi_1 + \Psi_2 =$Im($\tilde{\Psi_1}$)+Im($\tilde{\Psi_2}$) $\leftarrow$ Because we are given sines.

Use the following identity: $sin(\alpha) + sin(\beta) = 2sin(\frac{\alpha+\beta}{2})cos(\frac{\alpha-\beta}{2})$

$\Rightarrow Asin(kx + \omega t - \pi/5) + Asin(kx - \omega t - \pi/6 =$
$= A[2sin(\frac{kx+\omega t-\pi/5+kx-\omega t-\pi/6}{2})cos(\frac{kx+\omega t-\pi/5-kx+\omega t+\pi/6)}{2}]$
$= 2Asin(\frac{2kx-\frac{11\pi}{30}}{2})cos(\frac{2\omega t-\frac{\pi}{30}}{2})$
$= 2Asin(kx - \frac{11\pi}{60})cos(\omega t - \frac{\pi}{60})$

$\therefore$ The resulting wave is $\Psi_R = 2Asin(kx - \frac{11\pi}{60})cos(\omega t - \frac{\pi}{60})$. //Answer

(ii) The plot of the wave can be done by defining the following two variables:

$X \equiv kx; T \equiv \omega t,$

so that we get a kx-axis, and the different values of time will be $t = T/\omega$, for a chosen T.

The resultant wave looks like: $\Psi_R = 2Asin(X - \frac{11\pi}{60})cos(T - \frac{\pi}{60})$.

The graph of the resultant wave at 8 different times is the following:

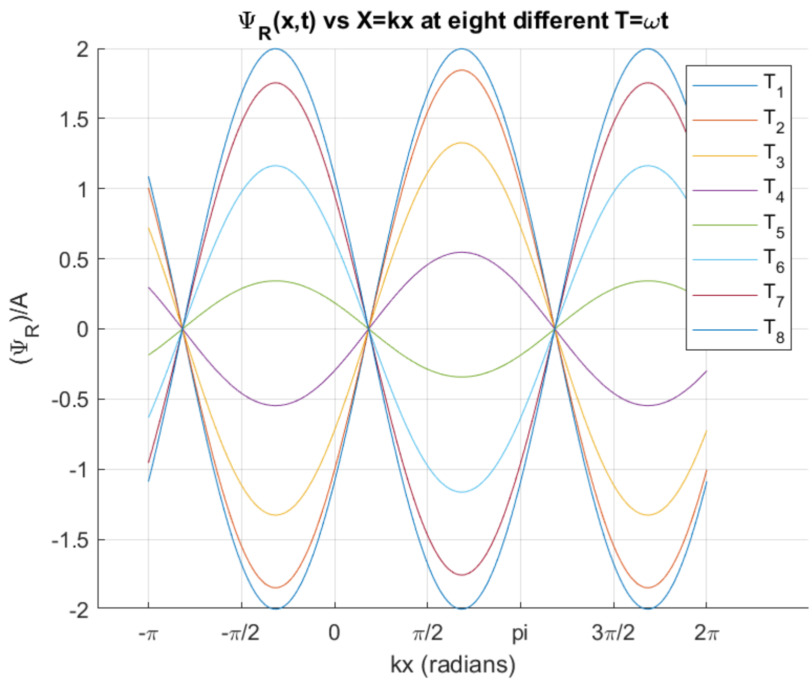

Figure 1: $\Psi_R = 2Asin(X - \frac{11\pi}{60})cos(T - \frac{\pi}{60})$ graphed at 8 different times. $X = kx$; $T = \omega t$; $T_1 = 0$ rad, $T_2 = 0.4488$, $T_3 = 0.8976$, $T_4 = 1.3464$, $T_5 = 1.7952$, $T_6 = 2.2440$, $T_7 = 2.6928$, $T_8 = \pi$ rad. The values of T_i are equally separated.

(iii) There is a node at $\Psi_R = 0$, i.e. $kx - \frac{11\pi}{60} = m\pi$, $m = 0, \pm 1, \pm 2, ...$

We want the node nearest to the origin: $kx = \frac{11\pi}{60}$

$\Rightarrow x = \frac{11\pi}{60k}$

$\therefore$ The value of the node nearest to the origin is $x = \frac{11\pi}{60k}$. //Answer

Q3

$B_y = 18 \cdot 10^{-18} sin(4\pi 10^6(z - 3 \cdot 10^8 t))$

(i) $B_{o,y} = 18 \cdot 10^{-8} = \frac{E_{o,x}}{c}$

$\Rightarrow E_{o,x} = (18 \cdot 10^{-8}\text{ T})(3 \cdot 10^8\text{ m/s }) = 54\text{ V/m}$

$\rightarrow E_y = E_{o,x} sin(...)$

Let z be the direction of travel and let E-field be x-polarized:

$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$

$k = 4\pi 10^6$ rad/m, $v = c$; $kv = \omega$

$B_y = B_{o,y} sin(k(z - vt))$

$-\frac{\partial B_y}{\partial t}$ T$= -B_{o,y}(-kv)cos(k(z - vt))$
$= \omega B_{o,y} cos(k(z - vt))$

$\Rightarrow \frac{\partial E_x}{\partial z} = \omega B_{o,y} cos(k(z - vt))$
$E_x = \int \omega B_{o,y} cos(k(z - vt))dz$
$= \omega B_{o,y} \frac{sin(k(z-vt))}{k} + g(x, y)$
$= cB_{o,y} sin(kz - \omega t)$

$\therefore$ An expression for the E-field is: $E = 54$ V/m $sin(4\pi 10^6(z - 3 \cdot 10^8 t))$, $E_y = E_z = 0$. //Answer

(ii) $\epsilon = n^2 \epsilon_o$, $n^2 = \kappa =$ dielectric constant

$\frac{\omega}{k} = \frac{c}{n} = \frac{c}{\sqrt{\kappa}}$, $k = \frac{2\pi}{\lambda}$, $\omega = (4\pi 10^6)(3 \cdot 10^8)$rad/s

$\rightarrow \frac{\lambda\omega}{2\pi} = \frac{c}{\sqrt{\kappa}}$
$\Rightarrow \lambda = \frac{2\pi c}{\omega\sqrt{\kappa}}$

Free space: $\kappa = 1$:

$\lambda_{freespace} = \frac{2\pi(3 \cdot 10^8)}{12\pi 10^{14}}$ m/rad $= 0.5 \cdot 10^{-6}$ m $= 500$ ηm

For $\kappa = 2.1$:

$\lambda_{\kappa=2.1} = \frac{\lambda_{freespace}}{\sqrt{2.1}} = 345$ ηm

$\therefore$ The wavelength of the wave in free space is $\lambda_{freespace} = 500$ ηm; the wavelength of the wave in a medium with a dielectric constant of 2.1 is $\lambda_{\kappa=2.1} = 345$ ηm. //Answer