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Phys362 Assignment #8

Reflection and transmission coefficients:
(For the following, $n = \frac{n_t}{n_i}$; t for transmission and i for incidence)

- Transverse electric (TE) polarization:

Reflection coefficient: $r_{\text{TE}} = \frac{\cos(\theta_i) - \sqrt{n^2 - \sin^2(\theta_i)}}{\cos(\theta_i) + \sqrt{n^2 - \sin^2(\theta_i)}}$

Transmission coefficient: $t_{\text{TE}} = \frac{2\cos(\theta_i)}{\cos(\theta_i) + \sqrt{n^2 - \sin^2(\theta_i)}}$

- Transverse magnetic (TM) polarization:

Reflection coefficient: $r_{\text{TM}} = \frac{-n^2\cos(\theta_i) + \sqrt{n^2 - \sin^2(\theta_i)}}{n^2\cos(\theta_i) + \sqrt{n^2 - \sin^2(\theta_i)}}$

Transmission coefficient: $t_{\text{TM}} = \frac{2 \cdot n \cdot \cos(\theta_i)}{n^2\cos(\theta_i) + \sqrt{n^2 - \sin^2(\theta_i)}}$

Reflectance and transmittance:

Reflectance: $R = r^2$ (r_{TE} or r_{TM})

Transmittance: $T = n\left(\frac{\cos(\theta_t)}{\cos(\theta_i)}\right) t^2$ (t_{TE} or t_{TM})

but $\cos(\theta_t) = \sqrt{1 - \sin^2(\theta_t)}$ and, by Snell's law, $n_i \cdot \sin(\theta_i) = n_t \cdot \sin(\theta_t)$

$$\Longrightarrow T = n\left(\frac{\sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2(\theta_i)}}{\cos(\theta_i)}\right) t^2 = n\left(\frac{\sqrt{1 - \left(\frac{1}{n}\right)^2 \sin^2(\theta_i)}}{\cos(\theta_i)}\right) t^2 = \frac{n}{n}\left(\frac{\sqrt{n^2 - \sin^2(\theta_i)}}{\cos(\theta_i)}\right) t^2$$

$$\Longrightarrow T = \left(\frac{\sqrt{n^2 - \sin^2(\theta_i)}}{\cos(\theta_i)}\right) t^2$$

(a) Light incident from air ($n_i = 1.0$) onto a pane of glass of index $n_t = 1.48$. We deal with external reflection: $n = n_t / n_i > 1$

(i) Fresnel coefficients of reflection and transmission

In[326]:=

```
(* Define the equations *)

(* Indices of refraction *)
nia = 1.0; nta = 1.48;
na = nta / nia;

(* Reflection and transmission coefficients *)

(* TE polarization *)
rTEa = (Cos[theta] - Sqrt[na^2 - (Sin[theta])^2]) /
    (Cos[theta] + Sqrt[na^2 - (Sin[theta])^2]);
tTEa = (2 * Cos[theta]) / (Cos[theta] + Sqrt[na^2 - (Sin[theta])^2]);
(* TM polarization *)
rTMa = (-na^2 * Cos[theta] + Sqrt[na^2 - (Sin[theta])^2]) /
    (na^2 * Cos[theta] + Sqrt[na^2 - (Sin[theta])^2]);
tTMa = (2 * na * Cos[theta]) / (na^2 * Cos[theta] + Sqrt[na^2 - (Sin[theta])^2]);

(* Graph *)
p1a1 = Plot[rTEa, {theta, 0, Pi / 2}, PlotStyle → Red,
    PlotLegends → {"rTE"}, PlotRange → All, PlotTheme → {"Scientific"},
    FrameLabel → {"Angle of incidence θi [rad]", "Fresnel coefficient"}, PlotLabel →
     "Fresnel coefficients for TE and TM polarization: light in air incident on glass"];
p2a1 = Plot[rTMa, {theta, 0, Pi / 2}, PlotStyle → Orange,
    PlotLegends → {"rTM"}, PlotRange → All];
p3a1 = Plot[tTEa, {theta, 0, Pi / 2}, PlotStyle → Blue,
    PlotLegends → {"tTE"}, PlotRange → All];
p4a1 = Plot[tTMa, {theta, 0, Pi / 2}, PlotStyle → Green,
    PlotLegends → {"tTM"}, PlotRange → All];
Show[p1a1, p2a1, p3a1, p4a1, PlotRange → All]
```

Out[336]=

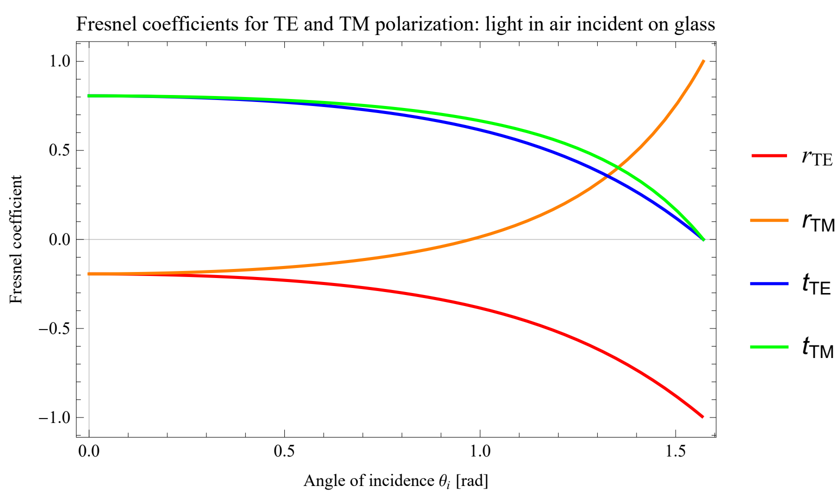


Figure a-1: Fresnel coefficients of reflection and transmission for transverse electric (TE) and transverse magnetic (TM) polarization for light travelling in air (n_i = 1.0) incident on glass (n_t = 1.48).

In[ ]:=
```
(* Graph description *)

(* Brewster angle *)
thetaB = ArcTan[na]
```
Out[ ]=
```
0.976583
```
In[ ]:=
```
thetaB * (180 / Pi)
```
Out[ ]=
```
55.9541
```

Description 1:

We have external reflection: $n_i < n_t$. As expected, as $\theta_i \to 0$, $r_{TE} = r_{TM}$ & $t_{TE} = t_{TM}$. For r_{TE}, $r_{TM} < 0$, the reflected ray has a phase change of π. The brewster angle, at which all the reflected light is TE polarized, and there is no reflected TM polarized light, is $\theta_B = 0.9765$ rad $\approx 56^\circ$. For r_{TE}, $r_{TM} > 0$, the reflected ray has a phase change of 0. At $\theta_i = 90^\circ$, no light is transmitted into the glass, as expected, and both TE and TM polarized light have a π phase shift. There is no phase shift for transmitted light.

(ii) Reflectance and transmittance

```
(* Define the functions *)
```
In[17]:=
```
(* TE polarization *)
RTEa = rTEa^2;
TTEa = ((Sqrt[na^2 - (Sin[theta])^2]) / Cos[theta]) * tTEa^2;
(* TM polarization *)
RTMa = rTMa^2;
TTMa = ((Sqrt[na^2 - (Sin[theta])^2]) / Cos[theta]) * tTMa^2;
```

In[337]:=

```
(* Graph *)
p1a2 = Plot[RTEa, {theta, 0, Pi / 2}, PlotStyle → Red,
   PlotLegends → {"RTE"}, PlotRange → All, PlotTheme → {"Scientific"},
   FrameLabel → {"Angle of incidence θi [rad]", "Reflectance & transmittance"},
   PlotLabel → "Reflectance and transmittance for TE
      and TM polarization: light in air incident on glass"];
p2a2 = Plot[RTMa, {theta, 0, Pi / 2}, PlotStyle → Orange, PlotLegends → {"RTM"},
   PlotRange → All];
p3a2 = Plot[TTEa, {theta, 0, Pi / 2}, PlotStyle → Blue,
   PlotLegends → {"TTE"}, PlotRange → All];
p4a2 = Plot[TTMa, {theta, 0, Pi / 2}, PlotStyle → Green,
   PlotLegends → {"TTM"}, PlotRange → All];
Show[p1a2, p2a2, p3a2, p4a2, PlotRange → All]
```

Out[341]=

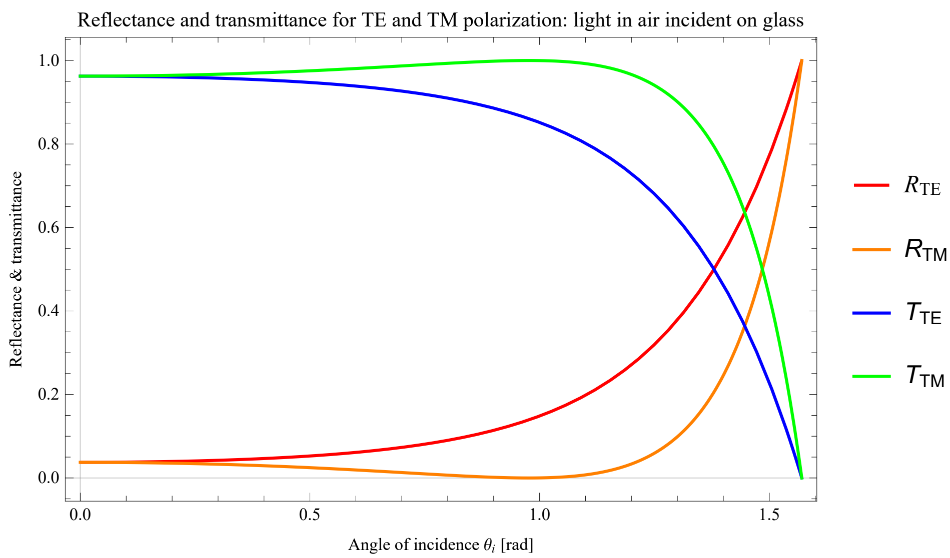


Figure a-2: Reflectance and transmittance for transverse electric (TE) and transverse magnetic (TM) polarization for light travelling in air ($n_i = 1.0$) incident on glass ($n_t = 1.48$).

For a comment on the following chunk of code, see "Note on TM transmittance" on page 12.

In[297]:=

```
(* Graph description *)

(* Solve TM transmittance equation for TTM = 1 *)
red1a = Reduce[TTMa == 1, theta]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[297]= C[1] ∈ ℤ && (theta == -0.976583 + 6.28319 C[1] || theta == 0.976583 + 6.28319 C[1])

In[303]:=
```
(* Solve TM transmittance equation for T_TM > 1 *)
red2a = Reduce[TTMa == 1.001, theta]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[303]=
```
C[1] ∈ ℤ && (theta == (-0.980584 + 0.0536112 ⅈ) + 6.28319 C[1] ||
    theta == (0.980584 - 0.0536112 ⅈ) + 6.28319 C[1] ||
    theta == (-0.980584 - 0.0536112 ⅈ) + 6.28319 C[1] ||
    theta == (0.980584 + 0.0536112 ⅈ) + 6.28319 C[1])
```

In[300]:=
```
(* Find roots of R_TM *)
root1a = Reduce[RTMa == 0, theta]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[300]=
```
C[1] ∈ ℤ && (theta == -0.976583 + 6.28319 C[1] || theta == 0.976583 + 6.28319 C[1])
```

Description 2:

(We have external reflection). As $\theta_i \to 0$, $R_{\mathrm{TE}} = R_{\mathrm{TM}}$, since the corresponding coefficients of reflection are equal. At the brewster angle, $\theta_B = 0.9765$ rad $\approx 56^\circ$, the reflectance for the TM polarized light is zero: the fraction of reflected power for TM goes to zero and the reflected light is TE polarized. At $\theta_i = 90^\circ$, no light is transmitted into the glass: the fraction of transmitted power goes to zero for both TE and TM polarization.

(b) Light incident from the core of an SMF-28 fiber to the cladding (core index = 1.45205; cladding index = 1.44681). We deal with internal reflection: $n = n_t / n_i < 1$

(i) Fresnel coefficients of reflection and transmission

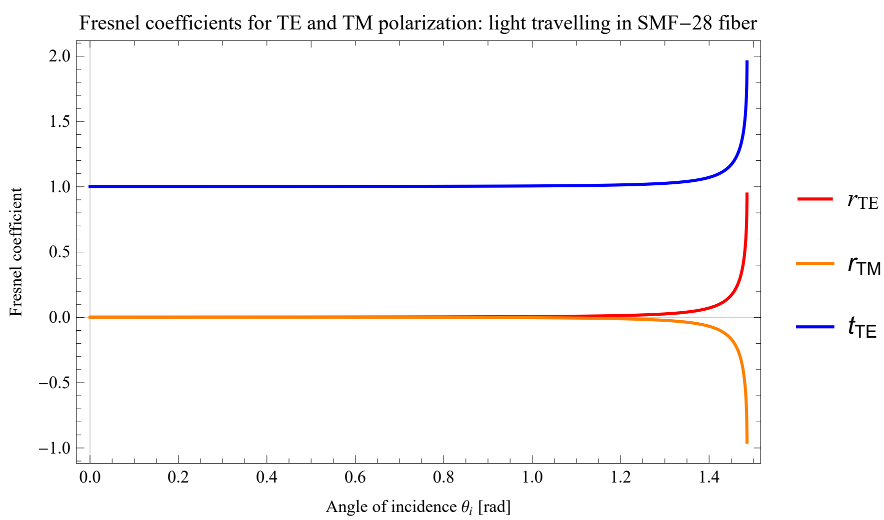
In[342]:=
```
(* Define the indices of refraction *)
nib = 1.45205; ntb = 1.44681;
nb = ntb / nib;

(* Reflection and transmission coefficients *)

(* TE polarization *)
rTEb = (Cos[theta] - Sqrt[nb^2 - (Sin[theta])^2]) /
   (Cos[theta] + Sqrt[nb^2 - (Sin[theta])^2]);
tTEb = (2 * Cos[theta]) / (Cos[theta] + Sqrt[nb^2 - (Sin[theta])^2]);
(* TM polarization *)
rTMb = (-nb^2 * Cos[theta] + Sqrt[nb^2 - (Sin[theta])^2]) /
   (nb^2 * Cos[theta] + Sqrt[nb^2 - (Sin[theta])^2]);
tTMb = (2 * nb * Cos[theta]) / (nb^2 * Cos[theta] + Sqrt[nb^2 - (Sin[theta])^2]);

(* Graph *)
p1b1 = Plot[rTEb, {theta, 0, Pi / 2}, PlotStyle → Red,
   PlotLegends → {"rTE"}, PlotRange → All, PlotTheme → {"Scientific", "BoldColod"},
   FrameLabel → {"Angle of incidence θi [rad]", "Fresnel coefficient"},
   PlotLabel → "Fresnel coefficients for TE and TM
      polarization: light travelling in SMF-28 fiber"];
p2b1 = Plot[rTMb, {theta, 0, Pi / 2}, PlotStyle → Orange, PlotLegends → {"rTM"},
   PlotRange → All];
p3b1 = Plot[tTEb, {theta, 0, Pi / 2}, PlotStyle → Blue,
   PlotLegends → {"tTE"}, PlotRange → All];
p4b1 = Plot[tTMb, {theta, 0, Pi / 2}, PlotStyle → Green,
   PlotLegends → {"tTM"}, PlotRange → All];
(* Show[p1,p2,p3,p4,PlotRange→All] *)
Show[p1b1, p2b1, p3b1, PlotRange → All]
Show[p1b1, p2b1, p4b1, PlotRange → All]
```

Out[352]=



Out[353]=

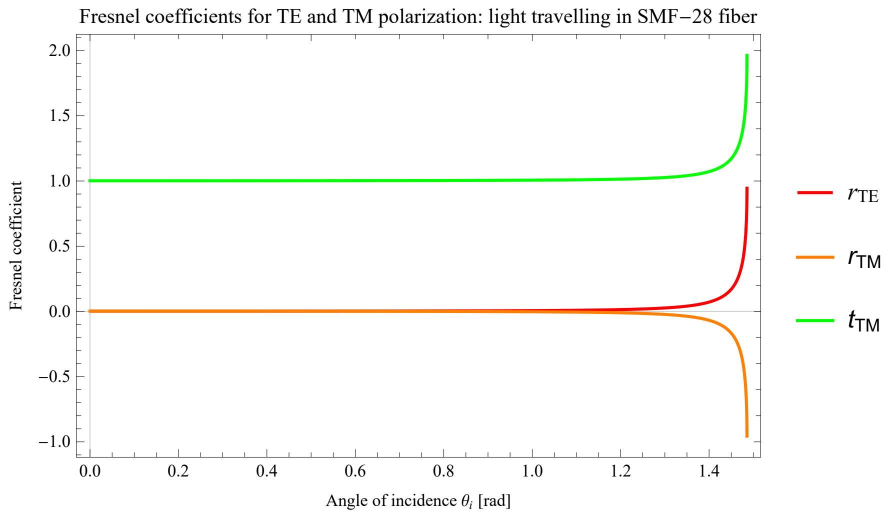


Figure b-1: Fresnel coefficients of reflection and transmission for transverse electric (TE) and transverse magnetic (TM) polarization for light incident from the core of an SMF-28 fiber to the cladding (core index, $n_i = 1.45205$; cladding index, $n_t = 1.44681$). I decided to plot two graphs because t_{TE} has values that are very close to those of t_{TM}, so, at this scale, they look as if they overlap. A zoom in of r_{TE} and r_{TM} at the Brewster angle is provided in Figure b-2.

In[54]:=
```
(* Graph description *)


(* Brewster angle *)
thetaB = ArcTan[nb]
```
Out[54]= 0.783591

In[55]:=
```
thetaB * (180 / Pi)
```
Out[55]= 44.8964

In[56]:=
```
(* Critical angle *)
thetaC = ArcSin[nb]
```
Out[56]= 1.48582

In[57]:=
```
thetaC * (180 / Pi)
```
Out[57]= 85.131

Description 3:

We have internal reflection: $n_i > n_t$. At an angle of incidence of approximately $\theta_B = 45^{\circ}$, the Brewster angle, the reflected light is TE polarized. The critical angle of incidence, above which total internal reflection occurs, is approximately $\theta_C = 85^{\circ}$. As the incident angle goes to zero, the reflection coefficients (TE and TM) tend to be equal, and are equal at zero; the same applies to the transmission coefficients (TE and TM).

Going back to the Brewster angle. From Figure b-2 below (a zoom in of r_{TE} and r_{TM} at the Brewster angle), it can be noticed that for incident $\theta_i < \theta_B$, the reflected light undergoes no phase shift, and that at θ_B, the only light reflected is TE polarized. For $\theta_i > \theta_B$, the TM polarized light suffers a phase shift of π.

Above the critical angle: $\theta_i > \theta_C$. Figure b-1 does not go all the way to normal incidence, this is because at an incident angle larger than the critical angle, the Fresnel coefficients become complex. The same can be noticed in Figure b-3 below.

In[354]:=
```
p1b2 = Plot[rTEb, {theta, Pi / 20, Pi / 3}, PlotStyle → Red,
   PlotLegends → {"rTE"}, PlotRange → All, PlotTheme → {"Scientific", "BoldColod"},
   FrameLabel → {"Angle of incidence θi [rad]", "Fresnel reflection coefficient"},
   PlotLabel → "Reflection coefficients for TE and
     TM polarization: light travelling in SMF-28 fiber"];
p2b2 = Plot[rTMb, {theta, Pi / 20, Pi / 3}, PlotStyle → Orange,
   PlotLegends → {"rTM"}, PlotRange → All];

Show[p1, p2, PlotRange → All]
```

Out[356]=

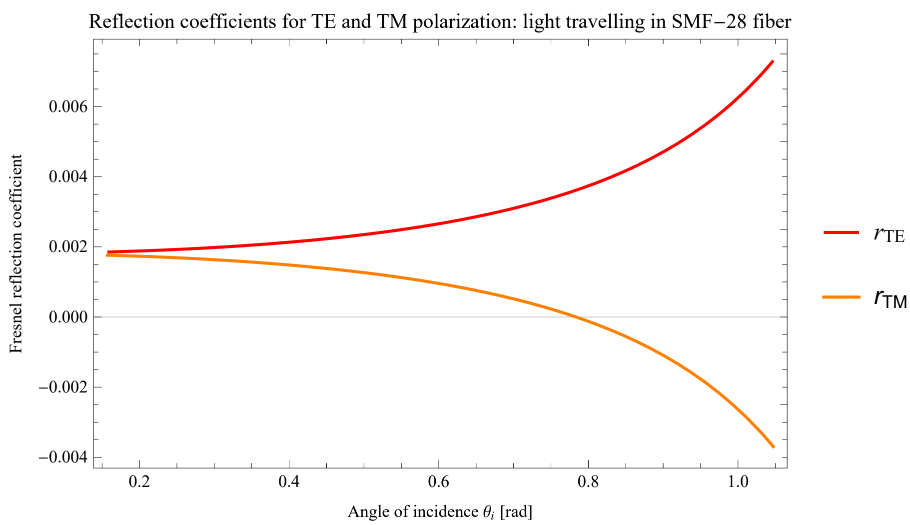


Figure b-2: Fresnel coefficients of reflection and transmission for transverse electric (TE) and transverse magnetic (TM) polarization for light incident from the core of an SMF-28 fiber to the cladding (core index, n_i = 1.45205; cladding index, n_t = 1.44681). Zoom in of Figure b-1.

(ii) Reflectance and transmittance

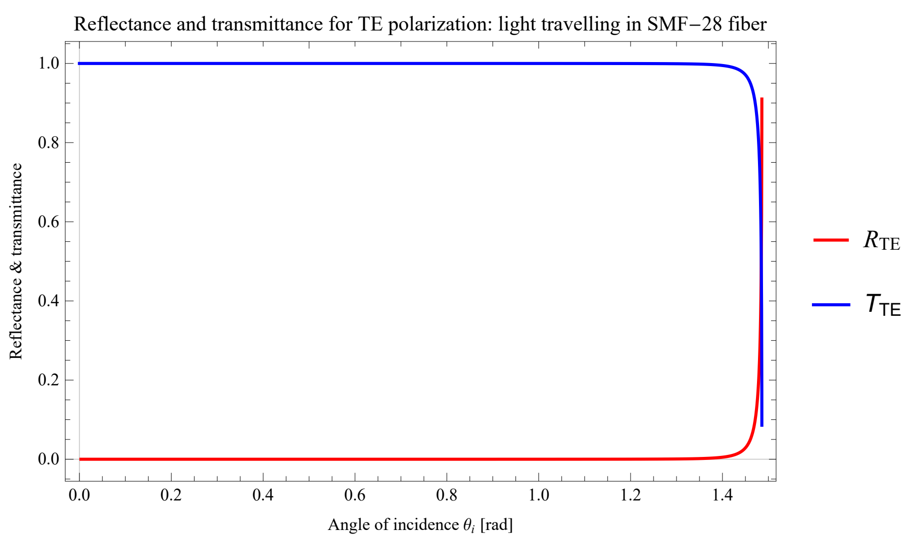
In[357]:=

```
(* Define the functions *)

(* TE polarization *)
RTEb = rTEb^2;
TTEb = ((Sqrt[nb^2 - (Sin[theta])^2]) / Cos[theta]) * tTEb^2;
(* TM polarization *)
RTMb = rTMb^2;
TTMb = ((Sqrt[nb^2 - (Sin[theta])^2]) / Cos[theta]) * tTMb^2;

(* Graph *)
p1b3 = Plot[RTEb, {theta, 0, Pi/2}, PlotStyle → Red,
   PlotLegends → {"R_TE"}, PlotRange → All, PlotTheme → {"Scientific"},
   FrameLabel → {"Angle of incidence θ_i [rad]", "Reflectance & transmittance"},
   PlotLabel → "Reflectance and transmittance for
      TE polarization: light travelling in SMF-28 fiber"];
p2b3 = Plot[RTMb, {theta, 0, Pi/2}, PlotStyle → Orange, PlotLegends → {"R_TM"},
   PlotRange → All, PlotTheme → {"Scientific"},
   FrameLabel → {"Angle of incidence θ_i [rad]", "Reflectance & transmittance"},
   PlotLabel → "Reflectance and transmittance for
      TM polarization: light travelling in SMF-28 fiber"];
p3b3 = Plot[TTEb, {theta, 0, Pi/2}, PlotStyle → Blue, PlotLegends → {"T_TE"},
   PlotRange → All];
p4b3 = Plot[TTMb, {theta, 0, Pi/2}, PlotStyle → Green,
   PlotLegends → {"T_TM"}, PlotRange → All];
(* Show[p1,p2,p3,p4,PlotRange→All] *)
Show[p1b3, p3b3, PlotRange → All]
Show[p2b3, p4b3, PlotRange → All]
```

Out[365]=



Out[366]=

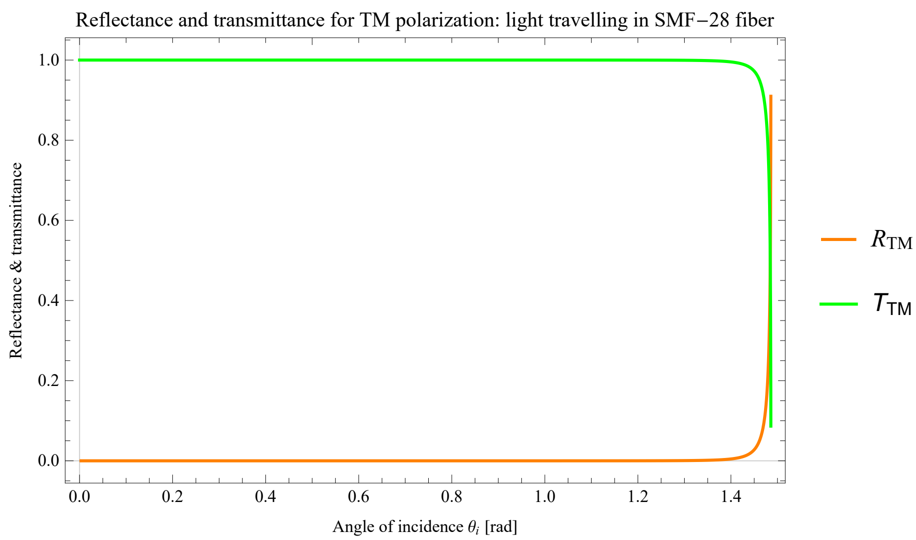


Figure b-3: Reflectance and transmittance for transverse electric (TE) and transverse magnetic (TM) polarization for light incident from the core of an SMF-28 fiber to the cladding (core index, n_i = 1.45205; cladding index, n_t = 1.44681).

A zoom in of Figure b-3 onto r_{TE} & r_{TM} and t_{TM} & t_{TM}:

In[367]:=

```
p1b3 = Plot[RTEb, {theta, Pi / 20, Pi / 3}, PlotStyle → Red,
   PlotLegends → {"R_TE"}, PlotRange → All, PlotTheme → {"Scientific"},
   FrameLabel → {"Angle of incidence θi [rad]", "Reflectance"}, PlotLabel →
    "Reflectance for TE and TM polarization: light travelling in SMF-28 fiber"];
p2b3 = Plot[RTMb, {theta, Pi / 20, Pi / 3}, PlotStyle → Orange,
   PlotLegends → {"R_TM"}, PlotRange → All, PlotTheme → {"Scientific"}];

Show[p1b3, p2b3, PlotRange → All]
```

Out[369]=

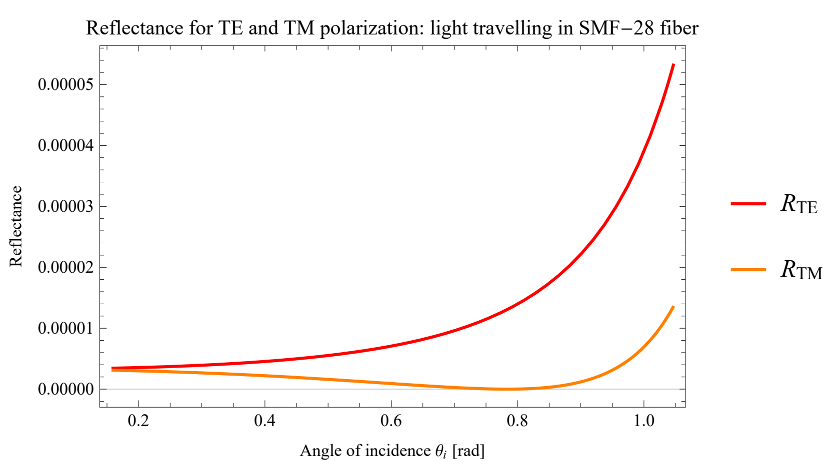


Figure b-4: Zoom into Figure b-3 onto reflectance for TE and TM polarization.

In[370]:=
```
p3b3 = Plot[TTEb, {theta, Pi / 20, Pi / 3}, PlotStyle → Blue,
   PlotLegends → {"T_TE"}, PlotRange → All, PlotTheme → {"Scientific"},
   FrameLabel → {"Angle of incidence θ_i [rad]", "Transmittance"}, PlotLabel →
    "Transmittance for TE and TM polarization: light travelling in SMF-28 fiber"];
p4b3 = Plot[TTMb, {theta, Pi / 20, Pi / 3}, PlotStyle → Green,
   PlotLegends → {"T_TM"}, PlotRange → All];

Show[p3b3, p4b3, PlotRange → All]
```

Out[372]=

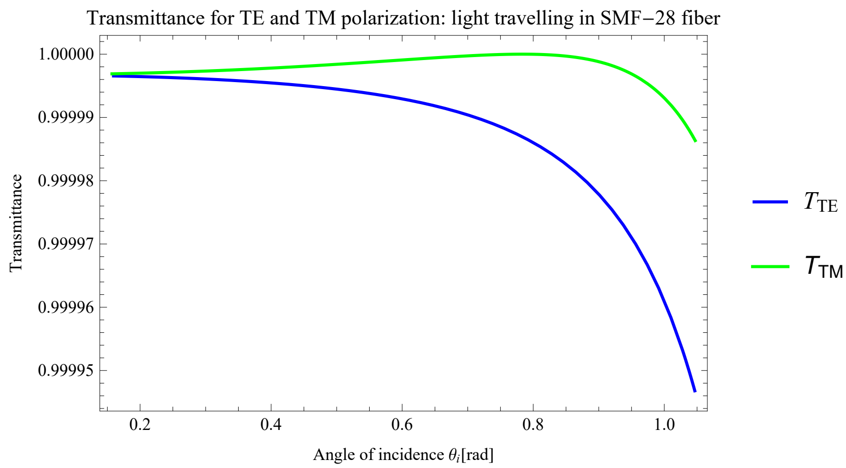


Figure b-5: Zoom into Figure b-3 onto transmittance for TE and TM polarization.

```
(* Graph description *)

(* Solve TM transmittance equation for T_TM = 1 *)
red1 = Reduce[TTMb == 1, theta]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[291]= $C[1] \in \mathbb{Z}$ && (theta == $(-0.783591 + 1.0843 \times 10^{-6}\, i) + 6.28319\, C[1]$ ||
theta == $(0.783591 - 1.0843 \times 10^{-6}\, i) + 6.28319\, C[1]$ ||
theta == $(-0.783591 - 1.0843 \times 10^{-6}\, i) + 6.28319\, C[1]$ ||
theta == $(0.783591 + 1.0843 \times 10^{-6}\, i) + 6.28319\, C[1]$)

```
(* Solve TM transmittance equation for T_TM > 1 *)
red2 = Reduce[TTMb == 1.000001, theta]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[293]= $C[1] \in \mathbb{Z}$ && (theta == $(-0.817968 + 0.126913\, i) + 6.28319\, C[1]$ ||
theta == $(0.817968 - 0.126913\, i) + 6.28319\, C[1]$ ||
theta == $(-0.817968 - 0.126913\, i) + 6.28319\, C[1]$ ||
theta == $(0.817968 + 0.126913\, i) + 6.28319\, C[1]$)

In[273]:=
```
0.817968 * (180 / Pi)
```

Out[273]=
```
46.8661
```

In[294]:=
```
(* Find the roots of R_TM *)
root1 = Reduce[RTMb == 0, theta]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[294]=
```
C[1] ∈ ℤ && (theta == -0.78359 + 6.28319 C[1] || theta == 0.78359 + 6.28319 C[1] ||
    theta == -0.783591 + 6.28319 C[1] || theta == 0.783591 + 6.28319 C[1])
```

Description 4:

Looking at the transmittance, from Figures b-3 to b-5, one can notice the following: below the Brewster angle, the fraction of transmitted power is almost 1 for both TE and TM polarizations, and, for the TM polarized light, it is 1 at the Brewster angle: that is no power is lost in the transmitted TM polarized wave. Looking at the reflectance: the reflected power is almost zero below the Brewster angle for both TE and TM polarized light, it is zero at θ_B for TM polarized light, and at an incident angle $\theta_i > \theta_B$, both the reflected TE and TM polarized waves drastically increase their reflectance; this makes sense, as the incident angle approaches the critical angle, we expect the reflected light to conserve the power of the original ray more and more as it approaches the point of total internal reflection. At total internal reflection, R_{TE} and R_{TM} increase their values dramatically, as can be seen in Figure b-3.

Note on TM transmittance: I am left with one question: it seems that the TM transmittance goes slightly above 1. Why is this so? Could it be due to the numerical approximations of the computer? I don’t find somewhere from where this comes from: the TM reflectance, for instance, has just one root. It seems that this happens for both TE transmittancein the light-glass and SMF-28 fiber cases.