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PHYS 362 Assignment #2

Q1

The problem gives the following values for the variables:

Wavelength: $\lambda = 632$ nm
Beam area: $A = 4\ \text{mm}^2 = 4 \cdot 10^{-6}\ \text{m}^2$
Average power: $P = 30\ \mu\text{W} = 30 \cdot 10^{-6}$ W
Electric field: $E = E_0\, sin(kx - \omega t)$

(a) The power density (W/m^2) can be found from the magnitude of the Poynting vector, $\vec{S} = \epsilon_0\, c^2\, \vec{E} \times \vec{B}$:

$$\vec{S} = \epsilon_0\, c^2\, (\vec{E} \cdot sin(kx - \omega t))\, (\vec{B} \cdot sin(kx - \omega t))$$
$$|\vec{S}| = \epsilon_0\, c^2\, E_0\, B_0\, sin^2(kx - \omega t)$$

where $\omega = \frac{2\pi c}{\lambda}$

Choosing the location of the specimen to be at $x = 0$:

$$|\vec{S}| = \epsilon_0\, c^2\, E_0\, B_0\, sin^2(kx - \omega t)$$
$$= \epsilon_0\, c^2\, E_0\, B_0\, (-1)^2\, sin^2(\omega t)$$
$$\Rightarrow |\vec{S}| = \epsilon_0\, c^2\, E_0\, B_0\, sin^2(\omega t)$$

The average irradiance is $I = \frac{P}{A}$

but $I = \langle|\vec{S}|\rangle = \epsilon_0\, c\, \langle E_0\, B_0\, sin^2(\omega t)\rangle$

$= \frac{1}{2}\, \epsilon_0\, c\, E_0{}^2$

$\Rightarrow \frac{P}{A} = \frac{1}{2}\, \epsilon_0\, c\, E_0{}^2$

$\therefore E_0 = \sqrt{\frac{2P}{A\, \epsilon_0\, c}}$

$\rightarrow |\vec{S}| = \epsilon_0\, c \cdot \frac{2P}{A\, \epsilon_0\, c}\, sin^2(\omega t)$

$\therefore\ |\vec{S}| = \frac{2P}{A}\, sin^2(\omega t)$

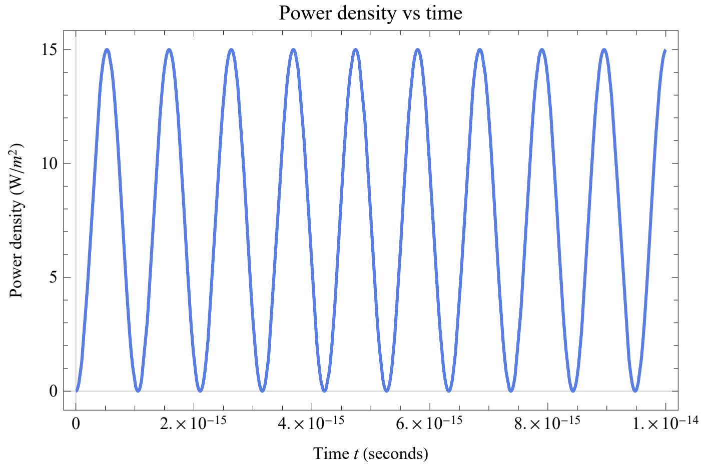
The following is the graph of the power density received by the specimen as a function of time from the moment the beam is turned on out to a time of 10 fs ($10 \cdot 10^{-15}$ seconds):

In[33]:=
```
(* assign values to the variables *)
P = 30 * 10^-6; A = 4 * 10^-6; omega = ((3 * 10^8) * (2 * Pi))/((632 * 10^-9));
epsilon0 = 8.85 * 10^-12; c = 3 * 10^8; E0 = ((2 * P) / (A * epsilon0 * c)) ^0.5;

(* define the function for the power density (magnitude of the Poynting vector) *)
S = (2 * P / A) * (Sin[omega * t]) ^2; (* where t = time*)

(* plot the graph *)
Plot[S, {t, 0, 10 * 10^-15}, FrameLabel → {"Time t (seconds)", "Power density (W/m^2)"},
 PlotTheme → {"Scientific", "BoldColor"}, PlotLabel → "Power density vs time"]
```

Out[35]=



(b) The time averaged Poynting vector of the sample is the irradiance, and it is the following:

$$I = \langle|\vec{S}|\rangle = \epsilon_0\, c\, E_0{}^2 \langle sin^2(\omega t)\rangle$$

The average of a function is given by: $\frac{1}{b-a}\int_a^b f(x)\, dx$, so

$$I = \epsilon_0\, c\, E_0{}^2 \cdot \frac{1}{t}\int_0^t sin^2\ (\omega s)\ ds$$

(i) For $\Delta t = 0$:

$$I = \epsilon_0\, c\, E_0{}^2 \cdot lim_{t\to 0}\left(\frac{1}{t}\int_0^t sin^2\ (\omega s)\ ds\right) = 0 \text{ // Answer}$$

The code is as follows:

In[27]:=
```
(* assign values to the variables *)
P = 30 * 10^-6; A = 4 * 10^-6; omega = ((3 * 10^8) * (2 * Pi)) / (632 * 10^-9);
epsilon0 = 8.85 * 10^-12; c = 3 * 10^8; E0 = ((2 * P) / (A * epsilon0 * c))^0.5;

(* define the integral of function to be averaged *)
int = (epsilon0 * c * E0^2) * Integrate[(Sin[omega * s])^2, {s, 0, t}];

(* find the limit as delta_t approaches 0 of
 the average of the function inside the integral (int) *)
Limit[int / t, t → 0]
```

Out[29]= 0.

(ii) For $\Delta t = \infty$:

$$I = \epsilon_0\, c\, E_0{}^2 \cdot lim_{t\to\infty}\left(\frac{1}{t}\int_0^t sin^2\ (\omega s)\ ds\right) = 7.5\ \text{W/m}^2 \text{ // Answer}$$

The code is as follows:

In[24]:=
```
(* assign values to the variables *)
P = 30 * 10^-6; A = 4 * 10^-6; omega = ((3 * 10^8) * (2 * Pi)) / (632 * 10^-9);
epsilon0 = 8.85 * 10^-12; c = 3 * 10^8; E0 = ((2 * P) / (A * epsilon0 * c))^0.5;

(* define the integral of function to be averaged *)
int = (epsilon0 * c * E0^2) * Integrate[(Sin[omega * s])^2, {s, 0, t}];

(* find the limit as delta_t approaches Infinity
 of the average of the function inside the integral (int) *)
Limit[int / t, t → Infinity]
```

Out[26]= 7.5

(c) To calculate the average irradiance, we have to determine the same irradiance I as a function of the change in time:

$I = \langle|\vec{S}|\rangle = \epsilon_0\, c\, E_0{}^2 \langle sin^2(\omega t)\rangle$. Using the equation for the average of a function $\frac{1}{b-a}\int_a^b f(x)\, dx$, we get:

$$I = \epsilon_0\, c\, E_0{}^2 \cdot \frac{1}{\Delta t}\int_0^{\Delta t} sin^2(\omega \cdot s)\, ds$$

Do the substitution $\omega s = \text{u} \Longrightarrow ds = du/\omega$:

$$I = \epsilon_0\, c\, E_0{}^2 \cdot \frac{1}{\omega \cdot \Delta t}\int_{u=0}^{u=\omega\cdot\Delta t} sin^2(u)\, du$$

Define $T \equiv \omega \cdot \Delta t$:

$$I = \epsilon_0\, c\, E_0{}^2 \cdot \frac{1}{T}\int_0^T sin^2(u)\, du$$

Using $E_0{}^2 = \frac{2 \cdot P}{A\, \epsilon_0\, c}$:

$$I = \frac{2 \cdot P}{A} \cdot \frac{1}{T} \int_0^T sin^2(u)\, du$$

The following code does the integration and graphing:

In[36]:=
```
(* assign values to the variables *)
P = 30 * 10^-6; A = 4 * 10^-6; omega = ((3 * 10^8) * (2 * Pi)) / (632 * 10^-9);
epsilon0 = 8.85 * 10^-12; c = 3 * 10^8; E0 = ((2 * P) / (A * epsilon0 * c))^0.5;

(* compute the integration of the average irradiance *)

(* define the the integral of the function to be averaged *)
int = (2 * P / A) * Integrate[(Sin[u])^2, {u, 0, T}];

(* compute the average of the function inside the integral (int) and plot *)
LogLinearPlot[int / T, {T, 0, 100},
 FrameLabel → {"T = ω·Δt (radians)", "Average Irradiance (W/m^2)"},
 PlotTheme → {"Scientific", "BoldColor"},
 PlotLabel → "Average irradiance vs change in time"]
```

Out[38]=