Phys234, 2018, Problem set #7: Due Monday April 2, 8pm

Question 1 & 2 : Will be given at the start of the lab

Question 3:

Use the built-in functions `quad` to evaluate the integral

$$I = \int_0^\infty e^{-x}\,\cos(e^{-x})\,dx$$

This integral has an analytical solution:

$$I_{exact} = \int_0^\infty e^{-x}\,\cos(e^{-x})\,dx = \left[-\sin(e^{-x})\right]_0^\infty = \sin(e^{-0}) - \sin(e^{-\infty}) = \sin(1)$$

You will need to pick a *numerical* value to use as a replacement for "infinity" for the upper limit of the integral (you may need to experiment with different values). Use an error tolerance of 10^{-16} in `quad`. Write your solution in a file `ps7q3`. Execution of your function `ps7q3` should give the result of your integration and also report the choice of your upper integral limit such that your numerical integration equals $\sin(1)$. Also it should print the number of function evaluations that were done (see `help quad`).

Question 4:

Use the function `gaussQuad.m` to evaluate the integral

$$I = \int_a^b f(x)\,dx$$

where $a = -3$, $b = 3$ and $f(x)$ is the cubic-spline interpolation function (with "not-a-knot" end condition) built from the following set of data points:

i	1	2	3	4	5	6	7	8	9	10	11
x	-3.0	-2.4	-1.8	-1.2	-0.6	0	0.6	1.2	1.8	2.4	3.0
y	0.25	0.42	0.78	1.45	1.59	1.37	2.56	4.96	1.57	0.62	0.33

Calculate your answer in a function called `ps7q4.m`. Choose a combination of number of panels N and the number of Gauss-Legendre points n that would guarantee that the evaluation of I is **exact**. As added commented text at the end of your function `ps7q4.m`, explain the rational behind your choice of N and n. Execution of your function `ps7q4.m` should print the value of the integral on the screen.

Question 5:

As we saw in class, Simpson's method involves a *basic* rule which requires 3 function evaluations at 3 equally spaced points in one panel. A slightly different *basic* rule for Simpson's method is the so-called "3/8 rule", which requires 4 function evaluations at 4 equally spaced points in one panel. The basic rule for this case is:

$$\int_{x_1}^{x_4} f(x)\,dx \approx \frac{3h}{8}\Big[f(x_1) + 3f(x_2) + 3f(x_3) + f(x_4)\Big], \qquad \text{where } h = \frac{x_4 - x_1}{3}$$

Your task is to build a *composite* rule from this *basic* rule in order to evaluate the integral of $f(x)$ between $[a, b]$, divided into N number of panels. It may be helpful to first figure out the total number of points on which we need to evaluate the function $f(x)$. You may also find helpful to write down on paper the composite rule for this 3/8 Simpson's method before you start coding it.

Generate a function `simpson38.m` that implements this composite rule. (It is a good idea to start with `simpson.m` and to modify it accordingly.) The first line of your `simpson38.m` m-file should follow the same format as `simpson.m`:

```
function I = simpson38(fun,a,b,npanel)
```

To test your function, write a function `ps7q5` that computes the integral (which has the following analytical solution)

$$I = \int_0^{\pi/2} e^{5x}\cos(2x)\,dx = -\frac{5}{29}\left(e^{5\pi/2} + 1\right)$$

and uses your function `simpson38.m` for $N = 100$ panels and $N = 200$ panels. For each case, also compute the error (i.e. the difference between the *exact* value of the integral and your numerical approximation). Execution of your function `ps7q5` should print out on the screen both the numerical evaluation of the integral and the error for each choice of N. Make sure that the print-out of your answer for the error is in scientific notation and contains at least 4 significant digits (i.e. the error does not print as 0.0000).