Phys234, 2018, Problem set #8: Due Friday April 13, 8pm

Question 1 & 2 : Will be given at the start of the lab

Question 3:

A simple description for the horizontal ($\hat{\mathbf{x}}$-direction) and vertical ($\hat{\mathbf{z}}$-direction) components of the velocity of a projectile (e.g. cannon ball) is given by the set of coupled equations

$$\frac{dv_x}{dt} = -c\|\mathbf{v}\|v_x$$
$$\frac{dv_z}{dt} = -g - c\|\mathbf{v}\|v_z$$

where $g = 9.8$ m/s^2 is the gravitational acceleration, $c = 0.03$ m^{-1} is a drag coefficient for air resistance, and $\|\mathbf{v}\|$ is the magnitude of the velocity vector given by

$$\|\mathbf{v}\| = \sqrt{v_x^2 + v_z^2}$$

Write a function `ps8q3` that computes a numerical solution of both $v_x(t)$ and $v_z(t)$ between $t = 0$ s and $t = 10$ s, for the following initial condition: $v_x(0) = 20$ m/s and $v_z(0) = 20$ m/s. You should use the built-in Matlab ODE solver `ode45`, with the default tolerance. Give your solution in terms of 2 plots: 1) One plot showing v_x vs t; 2) One plot showing v_z vs t. Don't forget to also submit your m-files containing your right hand side equations (give it the name `rhsq3`).

Question 4:

The Rayleigh oscillator is described by the following non-linear 2nd order ODE:

$$\frac{1}{5}\frac{d^2y}{dt^2} = \frac{dy}{dt} - \left(\frac{dy}{dt}\right)^3 - y$$

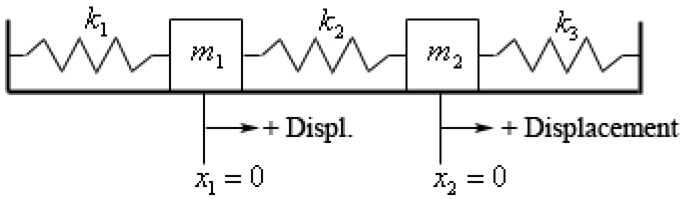
with initial conditions (at $t = 0$):

$$y = 0 \qquad \frac{dy}{dt} = 1$$

Write a function `ps8q4` that computes a numerical solution $y(t)$ of this equation between $t = 0$ and $t = 20$. You should use the built-in Matlab ODE solver `ode45`, with the default tolerance. Give your solution in terms of 2 plots: 1) One plot showing y vs t; 2) One plot showing dy/dt vs t. Don't forget to also submit your m-files containing your right hand side equations (give it the name `rhsq4`).
(Your solution should look like a shark-fin pattern).

Question 5:

Two masses m_1 and m_2 are linked together by 3 springs of elastic constants k_1, k_2 and k_3, as in the image below.



The displacements x_1 and x_2 of the two masses is given by the following two coupled 2nd order ODE's:

$$m_1 \frac{d^2 x_1}{dt^2} = -k_1 x_1 + k_2(x_2 - x_1)$$
$$m_2 \frac{d^2 x_2}{dt^2} = -k_2(x_2 - x_1) - k_3 x_2$$

Use an appropriate change of variables and re-write this system of equations as a set of 4 coupled 1st order ODEs that can be integrated numerically. Then write a function `ps8q5` that computes a numerical solution of these coupled ODE's between $t = 0$ and $t = 100$ for the case $m_1 = 2$, $m_2 = 3$, $k_1 = 1$, $k_2 = 5$, $k_3 = 10$ with initial conditions (at $t = 0$):

$$x_1 = 1 \qquad \frac{dx_1}{dt} = 0 \qquad x_2 = 1 \qquad \frac{dx_2}{dt} = 0$$

You should use the built-in Matlab ODE solver `ode45`, with the default tolerance. Give your solution in terms of 2 plots:

One plot showing x_1 vs t

One plot showing x_2 vs t.

Make sure you label you axes and don't forget to also submit your m-files containing your right hand side equations (give it the name `rhsq5`).

Question 6:

A quantity q varies with position x according to the following second order differential equation:

$$\frac{d^2q}{dx^2} = q^2x^2 - 1$$

Write a function `ps8q6` that computes a numerical solution of q between $x = [0, 1]$ by using the shooting method for the following boundary condition:

$$q(x = 0) = 1 \qquad\qquad q(x = 1) = 1$$

Give your solution in terms of 2 plots:

1) One plot showing q vs x; 2) One plot showing $\frac{\partial q}{\partial x}$ vs x.

As part of your solution, you should create two functions: `rhsq6`, which contains the right-hand side of the system of coupled ODEs, and `shoot6` that performs the "shooting" (i.e. one integration for $x = [0, 1]$ for given set of conditions at $x = 0$ and returning how close the match is to the conditions at $x = 1$).

Question 7:

Question 3 above showed the coupled equations for the horizontal ($\hat{\mathbf{x}}$-direction) and vertical ($\hat{\mathbf{z}}$-direction) components of the velocity of a projectile. The coupled equations describing the positions x and z of the projectile are then

$$\frac{d^2x}{dt^2} = \frac{dv_x}{dt} = -c\|\mathbf{v}\|v_x$$
$$\frac{d^2z}{dt^2} = \frac{dv_z}{dt} = -g - c\|\mathbf{v}\|v_z$$

where $g = 9.8$ m/s^2 is the gravitational acceleration, $c = 0.03$ m^{-1} is a drag coefficient for air resistance, and $\|\mathbf{v}\|$ is the magnitude of the velocity vector given by

$$\|\mathbf{v}\| = \sqrt{v_x^2 + v_z^2}$$

If a projectile is shot from a position $x = 0$ and $z = 0$ at time $t = 0$, what are the velocities v_x and v_z at this initial point such that the projectile would hit a target located at $x = 10$ m and $z = 2$ m at time $t = 5$ s?

Write your solution in a function a `ps8q7` that computes a numerical solution to this problem using a shooting method. Execution of your function `ps8q7` should print on the screen your solution for the initial v_x and v_z, and also show the trajectory of the projectile in a figure of z vs x.

As part of your solution, you should create two functions: `rhsq7`, which contains the right-hand side of the system of coupled ODEs, and `shoot7` that performs the "shooting".