CIS 623 Exercises 1: Warm up

Complete this by class time Monday January 22

Background

Below we develop a series of little programs about lines in the plane. We start with reviewing some elementary math. See http://mathforum.org/dr.math/faq/formulas/faq.ag2.html#twolines for a bit more background.

Lines and their equations. Each line in the x-y-plane is given by an equation of the form:

$$a \cdot x + b \cdot y + c \quad = \quad 0$$

where a, b, and c are real numbers and at least one of a and b are nonzero. Some examples:

(1) $x - 1 = 0$ is the vertical line passing through the point $(1, 0)$.[1]

(2) $y + 6 = 0$ is the horizonal line passing through the point $(0, -6)$.[2]

(3) $5 \cdot x - 2 \cdot y - 5 = 0$ is the line with slope 2.5 that passes through the point $(1, 0)$.[3]

Note that any line is given by *many* equations.[4]

Degenerate lines. An equation of the form $0 \cdot x + 0 \cdot y + c = 0$ is called *degenerate*.[5] By convention, we shall consider a degenerate equation as describing a *degenerate line* (that is, a nonsense line). That way, every equation of the form $a \cdot x + b \cdot y + c = 0$ names some sort of line, even if it turns out to be nonsensical.

[1] So: $a = 1$, $b = 0$, $c = -1$.

[2] So: $a = 0$, $b = 1$, $c = 6$.

[3] So: $a = 5$, $b = -2$, $c = -5$.

[4] E.g.: the line of example 3 is also given by each of:

$$\begin{aligned} 10 \cdot x - 4 \cdot y - 10 &= 0 \\ -2.5 \cdot x + y + 2.5 &= 0 \\ x - 0.4 \cdot y - 1 &= 0 \end{aligned}$$

[5] E.g., $a = b = 0$. When $c = 0$, then every point satisfies the equation. When $c \neq 0$, no point satisfies the equation.

Horizonal and vertical lines and intercepts. Suppose $a \cdot x + b \cdot y + c = 0$ names a nondegenerate line L.

- L is *horizonal* line exactly when $a = 0$.
- L is *vertical* line exactly when $b = 0$.
- If L is not horizonal, it crosses the x-axis at a single point: $(-c/a, 0)$. This is called the *x-intercept*.
- If L is not vertical, it crosses the y-axis at a single point: $(0, -c/b)$. This is called the *y-intercept*.

Pairs of lines. Suppose two nondegenerate lines L_1 and L_2 named by equations $a_1 \cdot x + b_1 \cdot y + c_1 = 0$ and $a_2 \cdot x + b_2 \cdot y + c_2 = 0$, respectively.

- L_1 and L_2 are *parallel* exactly when $a_1 \cdot b_2 = a_2 \cdot b_1$.[6]

[6] The two equations might name the same line. For convenience we'll say that a line is parallel to itself. Moreover, we say that two degenerate lines are parallel to each other.

- L_1 and L_2 intersect in a single point exactly when the lines are *not* parallel, in which case the point of intersection is (x, y) where:

$$x = \frac{b_1 \cdot c_2 - b_2 \cdot c_1}{a_1 \cdot b_2 - a_2 \cdot b_1}. \qquad y = \frac{a_2 \cdot c_1 - a_1 \cdot c_2}{a_1 \cdot b_2 - a_2 \cdot b_1}.$$

- Here is one way to test whether two equations name the same line.[7] First we check whether the two lines are parallel. If they are not parallel, then they must be different lines. If they are parallel and vertical, then they are equal exactly when they have the same x-intercept. If they are parallel and not vertical, then they are equal exactly when they have the same y-intercept.

[7] There are other ways to do this test.

Problems

The following depends on the first half of Chapter 1 of *Learn You a Haskell for Great Good*. The starter file for this assignment is:

http://www.cis.syr.edu/courses/cis623/code/lines.hs

which contains the beginnings of each of the following functions.[8]

[8] Each function in the file has a silly definition that you will have to fix.

Advice. In doing the problems for this assignment, change one or two definitions in the `lines.hs` file at a time, debug the definitions, and then go on to the next batch. Changing them all and dealing with the boat-load of errors that probably results is too much of a nuisance.

N.B. Haskell can handle pairs of numbers as input and output values. For instance, the function definition

```
north1 (x,y) = (x,y+1)
```

is perfectly fine Haskell.

◆ ***Problem 1*** ◆

Define a function `degenerate` such that

```
degenerate a b c
```

tests whether the equation $a \cdot x + b \cdot y + c = 0$ is degenerate.

◆ ***Problem 2*** ◆

Define a function `onLine` such that

```
onLine (x1,y1) a b c
```

tests if the point $(x1, y1)$ satisfies the equation $a \cdot x + b \cdot y + c = 0$.

◆ ***Problem 3*** ◆

Define a function `horizonal` such that

```
horizonal a b c
```

tests whether $a \cdot x + b \cdot y + c = 0$ names a horizonal line.[9]

[9] Return `False` on degenerate lines.

◆ ***Problem 4*** ◆

Define a function `vertical` such that

```
vertical a b c
```

tests whether $a \cdot x + b \cdot y + c = 0$ names a vertical line.[10]

[10] Return `False` on degenerate lines.

◆ ***Problem 5*** ◆

Define a function `xIntercept` such that

```
xIntercept a b c
```

returns the x-coordinate of the x-intercept of the line named by $a \cdot x + b \cdot y + c = 0$, when the line is not degenerate and not horizonal. Return 0.0 when the line is degenerate or horizonal.

◆ ***Problem 6*** ◆
Define a function `yIntercept` such that

```
yIntercept a b c
```

returns the y-coordinate of the y-intercept of the line named by $a \cdot x + b \cdot y + c = 0$, when the line is not degenerate and not vertical; otherwise, return 0.0.

◆ ***Problem 7*** ◆
Define a function `parallel` such that

```
parallel a1 b1 c1 a2 b2 c2
```

tests if the equations $a1 \cdot x + b1 \cdot y + c1 = 0$ and $a2 \cdot x + b2 \cdot y + c2 = 0$ name parallel lines.

◆ ***Problem 8*** ◆
Define a function `intersect` such that

```
intersect a1 b1 c1 a2 b2 c2
```

tests whether the two lines named by the equations $a1 \cdot x + b1 \cdot y + c1 = 0$ and $a2 \cdot x + b2 \cdot y + c2 = 0$ intersect in single point.[11]

[11] Return `False` if either line is degenerate.

◆ ***Problem 9*** ◆
Define a function `intersectionPt` such that

```
intersectionPt a1 b1 c1 a2 b2 c2
```

returns the x-y-coordinates of the intersection point of the two lines named by the equations $a1 \cdot x + b1 \cdot y + c1 = 0$ and $a2 \cdot x + b2 \cdot y + c2 = 0$, provided these lines do intersect in a single point; otherwise return $(0.0, 0.0)$.

◆ ***Problem 10*** ◆
Define a function `lineEqual` such that

```
lnEqual a1 b1 c1 a2 b2 c2
```

test if equations $a1 \cdot x + b1 \cdot y + c1 = 0$ and $a2 \cdot x + b2 \cdot y + c2 = 0$ name the same line.