

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Software Engineering (SE)

MID SEMESTER EXAMINATION**SUMMER SEMESTER, 2017-2018****DURATION: 1 Hour 30 Minutes****FULL MARKS: 100****Math 4241: Integral Calculus and Differential Equations**

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

1. a) What are anti-derivative and integral curves? Find the anti-derivative of $\sin x$ and draw the integral curves. 12
 b) Find the area function $A(x)$ between the graph $f(x) = 3x - 3$ and the interval $[2, x]$ and show that $A'(x) = f(x)$. 6
 c) Evaluate the following integrals : 15.33
 - i. $\int x^3 \sqrt{x^2 + 1} dx$
 - ii. $\int \frac{1}{x^2} \sqrt{2 - \frac{1}{x}} dx$
 - iii. $\int \frac{e^x}{4 + e^{2x}} dx$

2. a) A particle moves on a coordinate line with acceleration $a = \frac{d^2s}{dt^2} = 15\sqrt{t} - \frac{3}{\sqrt{t}}$ subject to the condition $s'(1) = 4$ and $s(1) = 0$, then answer the following questions: 8.33
 - i. The velocity v in terms of t .
 - ii. The position s in terms of t .
- b) Find the reduction formula $\int \cos^n x dx$ and using reduction formula evaluate $\int \cos^5 x dx$ 10
 c) Evaluate the following integrals 15
 - i. $\int \tan^{-1} x dx$
 - ii. $\int \cos^3 x \sin^2 x dx$
 - iii. $\int \sec^3 x \tan x dx$

3. a) Solve the followings: 12
 - i. $(x^2 + 4) \frac{dy}{dx} = 3, y(2) = 0$
 - ii. $(t^2 + 2t) \frac{dx}{dt} = 2x + 2 \quad (t, x > 0), x(1) = 1$
- b) Use the Heaviside method to evaluate $\int \frac{x+3}{2x^3 - 8x} dx$ 10
 c) Evaluate the following integrals: 11.33
 - i. $\int \frac{e^t}{e^{2t} + 3e^t + 2} dt$
 - ii. $\int \frac{1}{x^{\frac{3}{2}} - \sqrt{x}} dx$

4. a) Find the area of $f(x) = x^3$ on the interval $[2, 6]$ using right endpoints. 10
 b) Sketch the region whose are represented by the definite integral and evaluate the integral using appropriate formula. 10
 - i. $\int_{-1}^3 |2x - 1| dx$
 - ii. $\int_0^1 \sqrt{1 - x^2} dx$
- c) If $f(x) = \begin{cases} 2x & x \leq 1 \\ 2 & x > 1 \end{cases}$ then find 13.33
 - i. $\int_0^1 f(x) dx$
 - ii. $\int_{-1}^1 f(x) dx$
 - iii. $\int_1^{10} f(x) dx$
 - iv. $\int_{\frac{1}{2}}^5 f(x) dx$

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SUMMER SEMESTER, 2016-2017

FULL MARKS: 100

Math 4241: Integral Calculus and Differential Equations

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There are 4 (four) questions. Answer any 3 (three) of them.

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- a) Define integration, integrand and integral. Explain with examples, why it is necessary to add the integral constant to the result in indefinite integral. 8
- b) An environmentalist finds that a certain type of tree grows in such a way that its height $h(t)$ after t years is changing at the rate of $0.2 t^{2/3} + \sqrt{t}$ feet per year. If the tree was 2 feet tall when it was planted, how tall will it be in 27 years? 13.33
- c) Evaluate the followings : 12
 - i. $\int 3x^5 \sqrt{x^3 + 1} dx$, ii. $\int x \sin^{-1} x dx$, iii. $\int e^{-3x} \cos 4x dx$,
iv. $\int \cos^3 x \sin^3 x dx$
2. a) Obtain the reduction formula for $\int \sin^n x dx$ and then evaluate $\int \sin^4 x dx$. 13.33
- b) Express the integrand as a sum of partial fractions, using Heaviside cover up and any other method, and then evaluate the integral $\int \frac{2x+1}{x^2-7x+12} dx$. 20
3. a) What is the physical meaning of $\int_a^b f(x) dx$. Show that the definite integral can be written as a limit of a finite sum. Also write the properties of definite integral. 13.33
- b) Find the area under the graph of the function $f(x) = \frac{1}{x}$ between $x=1$ to $x=5$ using lower sum, upper sum and mid-point approximations considering four rectangles of equal width. Finally, find the percentage of error in each case and comments on your results. 20
4. a) Suppose an industrial plant in Gazipur city spills pollutant into a river. The Pollutant spreads out as it carried downstream by the current of the river and three hours after the spill, it forms the following pattern as shown in Figure 1. Moreover, the same data are given in the table (all dimensions are in meter): 13.33

Distance:	0	5	10	15	20	25	30	35	40
Height :	0	5	6	8	11	10	11	10	5

Using appropriate approximation, find the polluted area of the river.

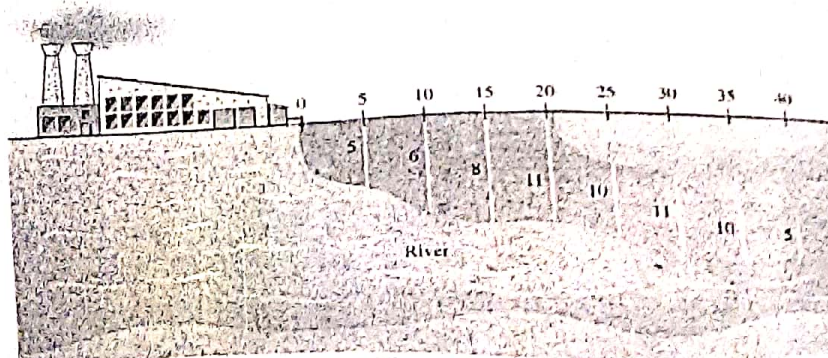


Figure 1.

- b) Evaluate $\int_{-1}^1 (1 + \sqrt{1-x^2}) dx$. Graph the integrand and then compare your result using appropriate formula from geometry. 20

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DURATION: 1 Hour 30 Minutes

SUMMER SEMESTER, 2015-2016

FULL MARKS:100

Math 4241: Integral Calculus and Differential Equation

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Give the figure or figures where necessary.

There are 4 (four) questions. Answer any 3 (three) of them.

Figures in the right margin indicate marks.

1. a) Define integration, integrand, and integral. Explain with examples, why it is necessary to add the integral constant to the result in indefinite integral. 10
- b) Suppose that x months from now, the population $P(x)$ of Gazipur City will be changing at the rate of $(500+0.15x)^{3/2}$ people per month. If the current population is 995000 then what will be the populations after 5 years from now? 8.33
- c) Evaluate the following: 15
 - i. $\int x \sin^{-1} x \, dx$
 - ii. $\int e^{-3x} \cos 4x \, dx$
 - iii. $\int \sin^3 x \cos^3 x \, dx$
2. a) Evaluate the following using the method of trigonometric substitutions: 12
 - i. $\int \frac{1}{a \sin x + b \cos x} \, dx$
 - ii. $\int \frac{\sin x}{\sqrt{1+\sin x}} \, dx$
- b) Obtain the reduction formula for $\int \cos^n x \, dx$, and then evaluate $\int \cos^5 x \, dx$. 12
- c) Evaluate $\int \frac{2x^2-1}{(4x-1)(x^2+1)} \, dx$ by using partial fractions. 9.33
3. a) Find the area that is above the x -axis but below the curve $f(x) = 4 - x^2$ and the left side bounded by the lines $x=0$ and the right side by the line $x=1$ with $n=5$ equal subintervals using left hand, right hand and mid-point approximations of integration. 20
 - i. Make a sketch of the region
 - ii. Find the approximate area in each case
 - iii. Find the exact area
 - iv. Finally, find the percentage of error in each case and then write your comments on your results.
- b) Write the statement of fundamental theorem of definite integral. Evaluate $\int_0^4 f(x) \, dx$, 13.33

where $f(x) = \begin{cases} \sqrt{x}, & 0 \leq x < 1 \\ \frac{1}{x^2}, & x \geq 1 \end{cases}$
4. a) Evaluate the following: 15
 - i. $\int_1^{\sqrt{2}} \frac{dx}{x^2 \sqrt{4-x^2}}$
 - ii. $\int_0^1 \sin^{-1} x \, dx$
 - iii. $\int_1^3 \frac{1}{\sqrt{x}(x+1)} \, dx$
- b) State the Mean-Value theorem and verify it for the function $f(x) = \sin x$ in the interval $-\pi \leq x \leq \pi$. 10
- c) Evaluate and sketch the region whose area is represented by the integral $\int_{-1}^5 |x-3| \, dx$ and then verify it using appropriate formula from geometry. 8.33