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B.Sc. Engg. CSE 2nd Semester 01 November 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

FINAL SEMESTER EXAMINATION SUMMER SEMESTER, 2017-2018

DURATION: 3 Hours FULL MARKS: 150

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (Eight)** questions. Answer any **6 (Six)** of them.

Figures in the right margin indicate marks.

1. a) Suppose that a and b are integers where, $a \equiv 11\ (mod\ 19)$ and $b \equiv 3\ (mod\ 19)$, find the integer c with $0 \leq c < 19$ such that, 3×3
   i. $c \equiv a^3 + b^3 \quad (mod\ 19)$
   ii. $c \equiv 2a + 3b \quad (mod\ 19)$
   iii. $c \equiv a^3 - 4b^3 \ (mod\ 19)$

   b) Find the Octal and Hexadecimal expansion of $(345678)_{10}$. 2×4

   c) Compute the summation and multiplication of the following numbers, 8
   i. $(11001)_2$ and $(11011)_2$
   ii. $(10001)_2$ and $(10101)_2$

2. a) Mr. Luke Skywalker is a leader of the resistance against the empire. He is stranded on a distant planet and now needs help from the resistance for rescuing him. Luke wants to send the message "SABOTAGE EMPIRE" to his friend R2D2 far away in planet Nebula so that the Empire does not understand the message he is sending. Luke has with him his robot friend C-3PO who is able to encrypt and send the message. Your task is to help C-3PO encrypt the message using a hybrid encryption involving Caesar's Cipher first and then Transposition Cipher with the following details in mind, 10
   i. $\sigma = \{1, 2, 3, 4\}$
   ii. $\sigma(1) = 3, \sigma(2) = 1, \sigma(3) = 4, \sigma(4) = 2$

   b) Prove that, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$ is valid using mathematical induction. 9

   c) The most commonly used procedure for generating *pseudorandom* numbers is the *linear congruential method*. We choose four integers: the modulus m, multiplier a, increment c, and seed x_0, with $2 \leq a < m, 0 \leq c < m$, and $0 \leq x_0 < m$. We generate a sequence of pseudorandom numbers $\{x_n\}$, with $0 \leq x_n < m$ for all n, by successively using the recursively defined function 6

   $$x_{n+1} = (ax_n + c)\ mod\ m.$$

   Based on the following information and considering $m = 9$, $a = 7$, $c = 4$, $x_0 = 3$, find the sequence of pseudorandom numbers generated by the *linear congruential method*.

3. a) Give a recursive algorithm for finding the reversal of a bit string. Using that algorithm find the reverse of the following bit strings. Show step by step execution of your algorithm. $7 + 4 + 4$
   i. 101101
   ii. 101010

   b) If x is a real number then prove that, $\lfloor 2x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{4} \right\rfloor$ 10

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4. Consider the following adjacency matrix of a town map

$$\begin{array}{c|ccccccc} \therefore & a & b & c & d & e & f & g \\ \hline a & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 0 & 2 & 0 \\ c & 0 & 1 & 0 & 1 & 2 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 1 & 0 & 2 \\ e & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ f & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{array}$$

From the above information answer the following:

a) Draw the graph that is represented by the adjacency matrix. 3

b) What is an Euler path? Does the above graph have an Euler path or circuit? Explain your answer logically. 2+5

c) If this graph has an Euler path then find an Euler path for travelling from town a to town e. 3

d) With the help of Dirac's and Ore's Theorem find whether the graph has a Hamilton circuit or a Hamilton path or both. 6

e) From the above graph prove that *"An undirected graph has an even number of vertices of odd degree."* 6

5. a) Consider the following adjacency matrix for a directed graph

$$\begin{array}{c|ccccc} \therefore & a & b & c & d & e \\ \hline a & 1 & 1 & 1 & 0 & 1 \\ b & 0 & 0 & 0 & 1 & 0 \\ c & 0 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 1 & 0 & 0 & 1 & 1 \end{array}$$

From the above information do the following:

i. Draw the graph represented by the matrix. 2

ii. Mathematically show that $\sum_{v\in V} \deg^-(v) = \sum_{v\in V} \deg^+(v) = |E|$. 3

iii. Does this graph have a Hamilton path and a circuit? Justify your answer. If there is any Hamilton path and/or circuit then write down the Hamilton path and/or circuit. 5+2

b) Determine whether the pair of graphs in Figure 1 are isomorphic. 10

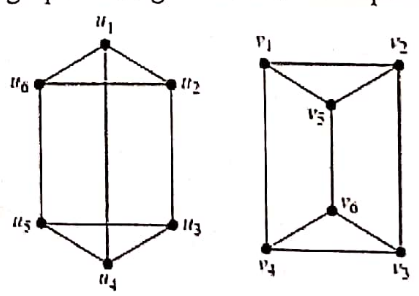

Figure 1: Graph for 5Question .(b)

c) Draw the following graphs Q_3, $K_{3,3}$, W_8, and determine their chromatic number χ. 3

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a) From Figure 2 generate the sequence of nodes in the following methods of tree traversal: 3×3

i. Pre-order traversal
ii. In-order traversal
iii. Post-order traversal

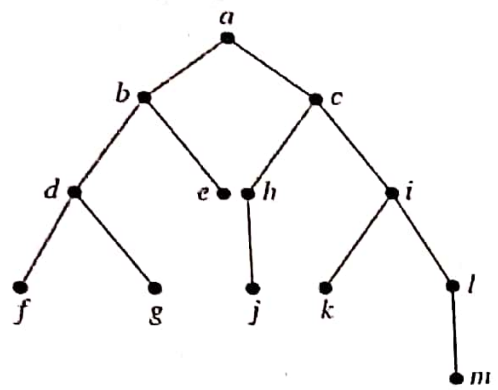

Figure 2: Graph for Question 6.(c)

b) From the expression $((x + y) \uparrow 2) + ((x - 4)/3)$ do the following: $3 + 9$

i. Draw the tree that represents this expression.
ii. Generate the prefix, infix and postfix notation for this expression.

c) Considering Figure 2, find out the following: $2 + 2$

i. Descendants of nodes b and i.
ii. Ancestors of nodes m and j

7. a) Using rules of inference show that the premises "*A student in this class has not read the book,*" and "*Everyone in this class passed the first exam*" imply the conclusion "*Someone who passed the first exam has not read the book.*" 7

b) Draw the combinatorial circuit for the following expressions: 2×3

i. $(p \vee (q \wedge \neg r)) \wedge (\neg q \vee \neg r)$
ii. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

c) Consider that f and g are the functions from the set of integers to the set of integers defined by $f(x) = \frac{2x+3}{x+2}$ and $g(x) = \frac{3x+2}{x-3}$.

Answer the following based on this information:

i. What is the composition of f and g? 2
ii. What is the composition of g and f? 2
iii. Determine whether $f(x)$ and $g(x)$ are one-one functions. 8

8. a) A father tells his two children, a boy and a girl, to play in their backyard without getting dirty. However, while playing, both children get mud on their foreheads. When the children stop playing, the father says "At least one of you has a muddy forehead," and then asks the children to answer "Yes" or "No" to the question: "Do you know whether you have a muddy forehead?" The father asks this question twice. What will the children answer each time this question is asked, assuming that a child can see whether his or her sibling has a muddy forehead, but cannot see his or her own forehead? Assume that both children are honest and that the children answer each question simultaneously. 7

b) Devise an algorithm for computing the quotient and remainder in a division operation. 10

c) Using the algorithm in 8.(b) find the quotient and remainder if divisor is 5 and dividend is -21. You have to show step by step operation of the algorithm. 8

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION SUMMER SEMESTER, 2016-2017

DURATION: 3 Hours FULL MARKS: 150

CSE 4203: Discrete Mathematics

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There are 8 (eight) questions. Answer any 6 (six) of them.

Figures in the right margin indicate marks.

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1. a) An iterative algorithm for computing Fibonacci numbers is given in Figure 1.

```
procedure iterative fibonacci(n: nonnegative integer)
if n = 0 then return 0
else
    x := 0
    y := 1
    for i := 1 to n - 1
            z := x + y
            x := y
            y := z
    return y
{output is the nth Fibonacci number}
```

Figure 1: Pseudocode for question 1(a).

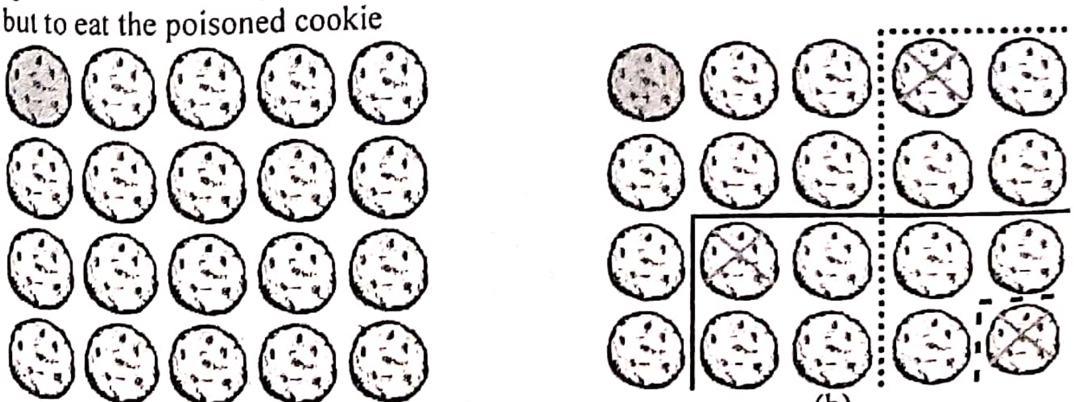
i. Determine the number of comparisons used by the algorithm for any input n. 6

ii. Determine the complexity of the algorithm from the number of comparisons. Mention the witness values you have used for your calculation 4

b) Differentiate among Big-O, Big-Omega and Big-Theta notations. 9

c) Arrange the functions $\sqrt{n}, 1000\log n, n\log n, 2n!, 2^n, 3^n, n^2/1000000$ in a list so that each function is big-O of the next function. 6

2. a) Chomp is a two-player game where cookies are laid out on a rectangular grid. The cookie in the top left position is poisoned, as shown in Figure 2(a). The two players take turns making moves; at each move, a player is required to eat a remaining cookie, together with all cookies to the right and/or below it (see Figure 2(b), for example). The loser is the player who has no choice but to eat the poisoned cookie



(a) (b)

Figure 2: Example for question 2(a).

Using strong induction prove that the first player has a winning strategy if the initial board is two squares wide, that is, a $2 \times n$ board. [Hint: The first move of the first player should be to chomp the cookie in the bottom row at the far right.]

b) Prove that $3^n < n!$ if n is an integer greater than 6.

3. a) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

b) Four friends have been identified as suspects for an unauthorized access into a computer system. They have made the following statements to the investigating authorities: Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana said "Carlos lied when he said I did it."

If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.

c) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using contraposition.

4. a) Use the back-substitution method to find all solutions to the system of congruences $x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, and $x \equiv 4 \pmod{11}$.

b) Show that if $a \equiv b \pmod m$ and $c \equiv d \pmod m$, where a, b, c, d, and m are integers with $m \geq 2$, then $a - c \equiv b - d \pmod m$.

c) Prove that if n is a composite integer, then n has a prime divisor less than or equal to $\sqrt{n}$.

5. a) Determine how many bit strings of length seven either begin with two 0s or end with three 1s with the help of a tree diagram.

b) Derive an equation for determining the number of r-combinations of a set with n elements where n is a nonnegative integer and r is an integer with $0 \leq r \leq n$ from the equation of r-permutation.

c) For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

6. a) Prove that if a and b are positive integers with $a \geq b$, then the number of divisions used by the Euclidean algorithm to find gcd(a, b) is less than or equal to five times the number of decimal digits in b.

b) Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

c) Use a merge sort to sort $b, d, a, f, g, h, z, p, o, k$ into alphabetic order. Show all the steps used by the algorithm.

7. a) Suppose that the ciphertext DVE CFMV KF NFEUVI, REU KYRK ZJ KYV JVVU FK JTZVETV was produced by encrypting a plaintext message using a shift cipher. What is the original plaintext? [Hint: the most common letters in English language are E, T, A, O, I, N, S, H, R]

b) Discuss the advantages and disadvantages of mathematical induction.

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Represent the following graph with an adjacency list and also an adjacency matrix: 7

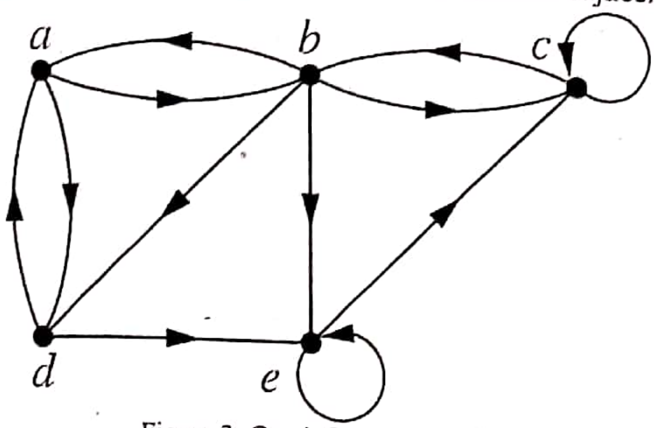

Figure 3: Graph for question 7(c).

Differentiate between the following:

i. Vertex Cut and Cut Vertices 4

ii. Edge Cut and Cut Edges 4

Consider two sets A and B, where the elements of A are the id of 2^{nd} semester students and elements of set B contains age of all students. If function from set A to set B $f(a)=b$ indicates that age of student a is b. 12

For each of the case bellow whether these functions are one-to-one, onto or bijection. Justify your answer.

i. More than one student having the same age.

ii. Each student having unique age.

iii. Set B contains age of the students of whole university, not only for 2^{nd} year.

Will this relation still be called a function if set A contains name of a student instead of id? Explain your answer.

Give a recursive definition of a extended binary tree. 5

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...Engg. / HD CSE 2nd Semester (64)

23 October 2016

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

...ESTER FINAL EXAMINATION

SUMMER SEMESTER, 2015-2016

...RATION: 3 Hours

FULL MARKS: 150

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (eight) questions. Answer any 6 (six) of them.

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1. a) A band of 17 pirates stole a bag of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The wealth was redistributed, but this time an equal division left 10 coins. Again, another argument developed in which another pirate was killed. But now the total fortune was evenly distributed among the survivors. What is the least number of coins that could have been stolen? 10

   b) Show that if m is an integer greater than 1 and $a*c \equiv b*c(mod\ m)$, then $a \equiv b(mod\ m/gcd(c, m))$. 8

   c) Find 5^{2003} mod 13 and 23^{1002} mod 41. 7

2. a) Ten friends have several interests in common, as shown in table 1. 15

Table 1: Data Table for Question 2(a).

Basketball	Classic rock	Film	Painting	Political discussion	Wine tasting
Andrew Leah Megan Tanya	David Steve Whitney	Andrew Eric Megan Sarah	David Joe Sarah Whitney	Joe Leah Steve Whitney	Eric Megan Tanya

They decided to get together regularly to enjoy their interests. There is an activity center they can rent by the hour that has many rooms (and a gym), but they must rent the whole center, so they would like to schedule as many interest groups at the same time as possible. Of course, they cannot schedule two groups at the same time if there is someone who wants to participate in both groups. Suppose that each interest group wants to meet for two hours each week.

   i. Draw the conflict graph for this situation.
   ii. Color the vertices of the conflict graph according to the rules for graph coloring.
   iii. How should the interest groups be scheduled? How many hours a week must the activity center be reserved in total?

   b) Define Dual graph and chromatic number. Find the chromatic number of the following graphs: 4+6

   i. K_n
   ii. W_n
   iii. C_n
   iv. Q_n

3. a) Define Proposition and Propositional Variable.
   b) If a sequence in the form of $s_n = a(2^n) + b*n + c$ is $\{s_1, s_2, \ldots\} = \{6, 11, 18, 29, 48, 83, 150$ then what are the values of a, b, and c?
   c) Give a good big-O estimate for each of the followings:
      i. $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$
      ii. $n^{2^n} + n^{n^2}$

4. a) Prove that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$
   b) Determine whether the following relations are equivalence relation:
      i. Let $a = (x1, y1)$, $b = (x2, y2)$ are ordered pairs of integers; $(a, b) \in R$ if $x1y2 = x2y1$.
      ii. Let x, y be integers; $(x, y) \in R$ if $|x - y| = 2$,
      iii. Let R be the relation on the set of all URLs (or Web addresses) such that xRy if and only if the Web page at x is the same as the Web page at y.
   c) Define n-ary relation and transitive closure. What is the transitive closure for $\emptyset$?

5. a) Prove that the product of any three consecutive integers is divisible by 6.
   b) Consider two sets A and B, where the elements of A are the id of 2nd semester students and elements of set B contains age of all students. If function from set A to set B $f(a)=b$ indicates that age of student a is b.
      For each of the case below whether these functions are either one-to-one, onto or bijection. Justify your answer.
      i. More than one student having the same age.
      ii. Each student having unique age.
      iii. Set B contains age of the students of whole university, not only for 2nd year.
      Will this relation still be called a function if set A contains name of a student instead of id? Explain your answer.
   c) Using truth table find out whether this statement is tautology or not:
      $(p \wedge q) \rightarrow (p \rightarrow q)$.

6. Given a 7×7 matrix X whose i,j th entry is 1 if i+1 divides j+1 or j+1 divides i+1, $i \neq j$; whose i,j th entry is 2 if $i=j$; and whose i,j th entry is 0 otherwise.
   a) If X is an adjacency matrix representing graph G, then draw that graph.
   b) If X represents a relation, is it an equivalence relation?
   c) Many puzzles ask you to draw a picture in a continuous motion without lifting a pencil so that no part of the picture is retraced and to finish the drawing at the vertex where you started. To solve such puzzles we use Euler circuits and paths. In Figure 1 there is a graph. Your task is to determine whether the graph in Figure 1 have Euler Circuit? If yes, then show the steps to find it.

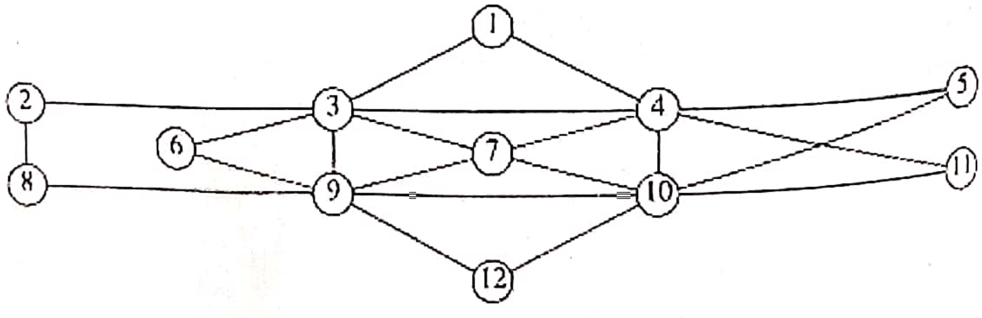

Figure 1: Graph for Question 6(c).

If a football game remains tied even after the extra time we go for tie breaker. This year, BFF has modified the rules a bit. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious. 12

How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

b) How many nonnegative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 < 40$ if we must satisfy $x_1 \geq 5$, $x_2 \geq 4$, $x_3 \geq 3$, $x_4 \geq 2$, $x_5 \geq 1$? 8

c) A key in the Vigenère cryptosystem is a string of English letters, where the case of the letters does not matter. How many different keys for this cryptosystem are there with three, four, five, or six letters? 5

a) Construct a Huffman code for the letters of the English alphabet where the frequencies of letters in typical English text are as shown in Table 2. 12

Table 2: English letter frequency

Letter	Frequency	Letter	Frequency
E	12.02	M	2.61
T	9.10	F	2.30
A	8.12	Y	2.11
O	7.68	W	2.09
I	7.31	G	2.03
N	6.95	P	1.82
S	6.28	B	1.49
R	6.02	V	1.11
H	5.92	K	0.69
D	4.32	X	0.17
L	3.98	Q	0.11
U	2.88	J	0.10
C	2.71	Z	0.07

b) Define ancestors. Show ***inorder traversals*** of the tree in Figure 2. 7

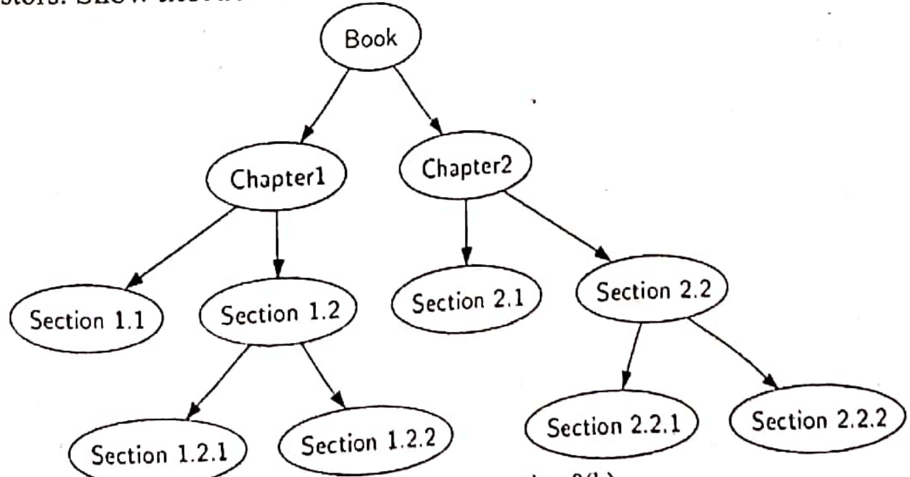

Figure 2: Tree for Question 8(b).

c) What is the value of each of the following expressions: 6

i. $+ - \uparrow 3\ 21 \uparrow 2\ 3 / 6 - 4\ 2$ (Prefix)

ii. $3\ 2 * 2 \uparrow 5\ 3 - 8\ 4 / * -$ (Postfix)