

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

SUMMER SEMESTER, 2017-2018

DURATION: 1 Hour 30 Minutes

FULL MARKS: 75

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 4 (four) questions. Answer any 3 (three) of them.

Figures in the right margin indicate marks.

1. a) Describe the worst-case time complexity of the odd-even sort algorithm in terms of the number of comparisons used. Express the complexity in Big-O notation. 10

The algorithm for odd-even sort algorithm is given in Figure 1.

```

procedure oddEvenSort( $a_1, \dots, a_n$  : real numbers with  $n \geq 2$ )
  isSorted := False
  while isSorted = False do
    isSorted := True
     $i := 1$ 
    while  $i < n$  do
      if  $a_i > a_{i+1}$  then
        interchange  $a_i$  and  $a_{i+1}$ 
        isSorted := False

       $i := i + 2$ 

     $i := 2$ 
    while  $i < n$  do
      if  $a_i > a_{i+1}$  then
        interchange  $a_i$  and  $a_{i+1}$ 
        isSorted := False

       $i := i + 2$ 

  return
  
```

Figure 1: Code listing for Question 1(a).

- b) Express the negation of the following propositions using quantifiers, and then express them in English. 15
- i. Some drivers do not obey the speed limit.
 - ii. All Swedish movies are serious.
 - iii. No one can keep a secret.
 - iv. There is someone in this class who does not have a good attitude.
 - v. Every bird can fly.
2. a) Give big-O estimates (with the values of C and n_0) for the following functions: 12
- i. $(n^2 + 5)(n - 1)$
 - ii. $(n \lg n + 1)^2 + (\lg n + 1)(n^2 + 1)$
 - iii. $3n \lg(n!) + (n^2 + 4) \lg(n)$

- b) Draw Venn Diagrams showing the followings: (Identify all parts of the diagram with proper notations) 8
- $A \cup B \subset A \cup C$, but $B \cap C = \{\}$
 - $A \cap B \subset A \cap C$, but $B \cap C = \{\}$
- c) Write the following complexities in ascending order : 5
- $\theta(n^2)$, $\theta(b^n)$, $\theta(n \log n)$, $\theta(n!)$ and $\theta(1)$
- 3 a) Use logical equivalence to show that following propositions are contradiction: 6
- $\neg(p \vee \neg(p \wedge q))$
 - $\neg(((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r))$
- b) Use truth table to show that: 6
- $(p \wedge (p \rightarrow q) \rightarrow \neg q)$ is a contingency.
 - $((p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r))$ is a tautology.
- c) There are two restaurants next to each other. One has a sign says "Good food is not cheap" and other has a sign that says "Cheap food is not good". Are the signs saying the same thing? Justify your answer using predicates, quantification etc. 7
- d) Given that $h(x) = 3x$ and $g(t) = -2t - 2 - h(t)$ and $f(n) = -5n^2 + h(n)$, calculate $h(g(8) + f(2))$. 6
- 4 a) Show that if n is an integer and $n^2 + 5$ is odd, then n is even using 12
- A proof by contraposition.
 - A proof by contradiction
- b) Given premises : 8
- "Students who pass the course either do the homework or attend lecture;"
- "Mahid did not attend every lecture;"
- "Mahid passed the course."
- Using rules of inference prove the conclusion that "Mahid have done the homework".
- c) Theorem: If n^2 is positive, then n is positive. 5
- Proof: Suppose that n^2 is positive. Because the conditional statement "If n is positive, then n^2 is positive" is true, we can conclude that n is positive.
- Is there any problem with the proof of this theorem? Give proper arguments to support your answer.

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- a) Show that the premises 'There is a student such that if he knows programming, then he knows Java.' and 'All students know programming.' imply the conclusion 'There is a student who knows either Java or C++.' Write down the name of each of the rules of inference that you use. 9
 - b) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with n cars made in the n th month. 3×3
 - i. Set up a recurrence relation for the number of cars produced in the first n months by this factory.
 - ii. How many cars are produced in the first year?
 - iii. Find an explicit formula for the number of cars produced in the first n months by this factory.
 - c) Prove the following statement using contradiction: 7
 'For all rational number x and irrational number y , the sum of x and y is irrational.'
2. a) Describe the worst-case time complexity of the bubble sort algorithm in terms of the number of comparisons used. The algorithm for bubble sort is given in Figure 1. 10
- ```

procedure bubblesort(a_1, \dots, a_n : real numbers with $n \geq 2$)
for $i := 1$ to $n - 1$
 for $j := 1$ to $n - i$
 if $a_j > a_{j+1}$ then interchange a_j and a_{j+1}
 { a_1, \dots, a_n is in increasing order}

```
- Figure 1: Code listing for question 2a.
- b) Prove the following statement using contraposition: 7  
 'If  $x^2 - 6x + 5$  is even then  $x$  is odd'.
  - c) Prove that there are infinitely many primes. 4
  - d) Show that if  $n \mid m$ , where  $n$  and  $m$  are integers greater than 1, and if  $a \equiv b \pmod{m}$ , where  $a$  and  $b$  are integers, then  $a \equiv b \pmod{n}$ . 4

3. a) Give big-O estimates for the following functions: 10

- i.  $(n^2 + 8)(n + 1)$
- ii.  $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$

- b) Draw Venn Diagrams showing: 8

- i.  $A \cup B \subset A \cup C$ , but  $B \not\subset C$
- ii.  $A \cap B \subset A \cap C$ , but  $B \not\subset C$



- c) Suppose there are signs on the doors to two rooms. The sign on the first door reads "In this room there is a lady, and in the other one there is a tiger"; and the sign on the second door reads "In one of these rooms, there is a lady, and in one of them there is a tiger." Suppose that you know that one of these signs is true and the other is false. Behind which door is the lady?
4. a) Express each of these statements using predicates and quantifiers.
- At least two students like sports, though not everybody likes it.
  - The sum of an even integer and an odd integer is odd.
  - Not all cars made by Toyota are durable.
  - No one in your school owns both a bicycle and a motorcycle.
- b) There are two restaurants next to each other. One has a sign says "Good food is not cheap" and other has a sign that says "Cheap food is not good". Are the signs saying the same thing? Justify your answer using predicates, quantification etc.
- c) Given that  $h(x) = 3x$  and  $g(t) = -2t - 2 - h(t)$  and  $f(n) = -5n^2 + h(n)$ , calculate  $h(g(8))$ .

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DURATION: 1 Hour 30 Minutes

SUMMER SEMESTER, 2015-2016

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There are 4 (four) questions. Answer any 3 (three) of them.  
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- a) Once upon a time there was a king named Ozymandias. He was a brave king and ruled a big kingdom. He was happy and relaxed until his only and beloved daughter fall ill. So ill that she took the death bed. The king became restless and ordered his men to bring all the famous physicians of that time. Many physicians came, but in vein. None of them could even diagnose the disease of the princess. Everyone lost their hope. Meanwhile an undistinguished and absurdly dressed physician came to the palace. He spent three days to diagnose the disease. After being sure he said, "I can treat the Princess to cure." Most of the people was doubtful about his capability. But he was their only chance. The weird physician treated her for 40 days. Proving peoples doubt wrong, the prince was on her feet again. The King was the happiest person among all. He said to the physician, "Ask for your reward. Whatever you wish will be granted." The physician was humble and straight. He asked for 3 gold coin for each days of diagnosis. But for the treatment period he gave a strange rule. The first day would cost 1 gold coin. For each of the other treatment days, the payment would be double of the previous day. People were laughing at him thinking he demanded very less. Being good at math, the physician knows how much he will get. Calculate the total amount of the payment. 10
- b) Give a proof by contradiction of the theorem "If  $3n+2$  is odd, then  $n$  is odd." 8
- c) For the following set of premises, what relevant conclusion or conclusions can be drawn? 7  
Mention the rules of inference used to obtain each conclusion from the premises.
- "All foods that are healthy to eat do not taste good."
  - "Tofu is healthy to eat."
  - "You only eat what tastes good."
  - "You do not eat tofu."
  - "Cheeseburgers are not healthy to eat."
- a) Express each of these statements using predicates and quantifiers. 12
- i. A passenger on an airline, qualifies as an elite flyer, if the passenger flies more than 25,000 miles in a year or takes more than 25 flights during that year.
  - ii. A student must take at least 60 course hours, or at least 45 course hours and write a master's thesis, and receive a grade no lower than a B in all required courses, to receive a master's degree.
  - iii. Whenever there is an active alert, all queued messages are transmitted.
  - iv. Each participant on the conference call whom the host of the call did not put on a special list was billed.
- b) What is a Power Set? What is the power set of the set  $\{\emptyset\}$ ? 5



- c) There are three people (Alex, Brook, and Cody), one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth.

Alex says: "Cody is a knave."

Brook says: "Alex is a knight."

Cody says: "I am the spy."

Who is the knight, who the knave, and who the spy? Explain your answer.

3. a) What is the worst-case complexity of the insertion sort in terms of the number of comparisons made? The algorithm for insertion sort is given in figure 1,

```

Procedure insertion sort(a_1, a_2, \dots, a_n : real numbers with $n \geq 2$)
For $j := 2$ to n
 $i := 1$
 while $a_j > a_i$
 $i := i + 1$
 $m := a_j$
 for $k := 0$ to $j - i - 1$
 $a_{j-k} := a_{j-k-1}$
 $a_i := m$
 { a_1, \dots, a_n is in increasing order}

```

Figure 1: Code listing for question 3.a)

- b) Give big-O estimates for the following functions:

i.  $f(n) = \log(n!)$

ii.  $f(x) = (x+1)\log(x^2+1) + 3x^2$

4. a) Write short notes on:

i. One-to-one function

ii. Big theta

iii. The Division Algorithm

- b) Use the extended Euclidean algorithm to express  $\gcd(144, 89)$  as a linear combination of 144 and 89.

- c) Show that if  $a, b, k$ , and  $m$  are integers such that  $k \geq 1$ ,  $m \geq 2$ , and  $a \equiv b \pmod{m}$ , then  $a^k \equiv b^k \pmod{m}$ .