ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Mid-Semester Examination   Winter Semester, A.Y. 2019-2020
Course No.: Math 4121   Time: 90 Minutes
Course Title: Mathematics I   Full Marks: 75

There are 4 (four) questions. Answer any 3 (three) questions. Marks in the margin indicate full marks. Programmable calculators are not allowed. Do not write on this question paper.

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1. a) Find the value of k for which $6x^2 - 5xy - 6y^2 + 14x + 5y + k = 0$ represents a pair of straight lines. Find the separate equation of straight lines. **13**

    b) Remove xy term from the equation $17x^2 + 18xy - 7y^2 = 1$ by using the invariants of transformation. **12**

2. a) If $y = e^{ax} \sin bx$ find y_n. **13**

    b) If $\ln y = \tan^{-1} x$ then show that
    $$(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$$ **12**

3. a) (i) State and prove L' Hospital's theorem. **13**

        (ii) Evaluate $\underset{x \to \frac{1}{2}\pi}{Lt}\ \sec x\left(x\sin x - \frac{1}{2}\pi\right)$.

    b) If $u = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$ and $\dfrac{\delta^2 u}{\delta x^2} + \dfrac{\delta^2 u}{\delta y^2} + \dfrac{\delta^2 u}{\delta z^2} = 2$ then find the value of $a + b + c$. **12**

4. a) If $lx + my = 1$ touches the curve $(ax)^n + (by)^n = 1$ then show that $(l/a)^{\frac{n}{n-1}} + (m/b)^{\frac{n}{n-1}} = 1$. **13**

    b) If (α, β) are the coordinates of the centre of curvature of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at (x,y) then show that $\alpha + \beta = 3(x + y)$. **12**