

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

WINTER SEMESTER, 2020-2021

DURATION: 1 Hour 30 Minutes

FULL MARKS: 100

Math 4141: Differential Calculus and Geometry

Answer all **three (3)** questions. Each question is worth $33\frac{1}{3}$ marks

Figures in the right margin indicate marks along with COs and POs.

N.B. The name of the pdf must be in the following format <Student ID Course Code MID> .

Calculators (non-programmable) are allowed in the exam.

If it becomes evident that you have copied any answer from any other source without prior instruction, evaluators can reject that answer altogether at the time of evaluation.

1.

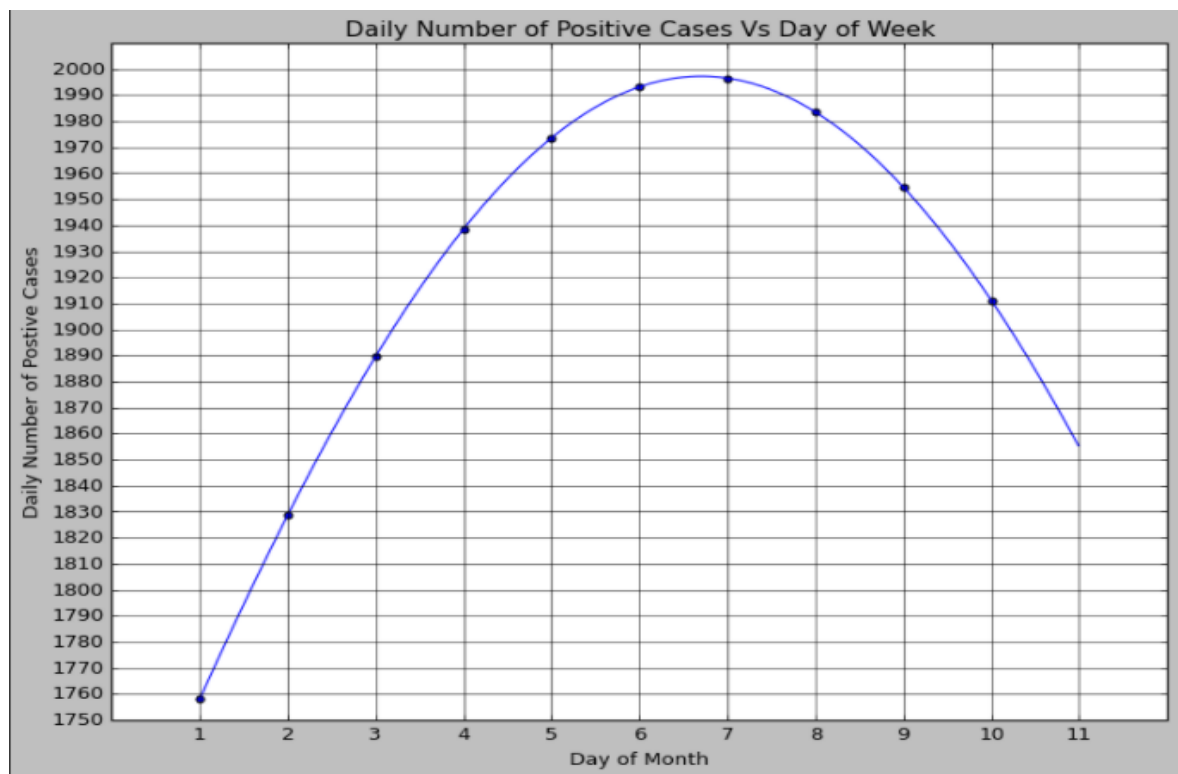


Figure 1

- a) In Figure 1, the daily number of positive cases of coronavirus (y) for the first 11 days (x) of the month of August in Bangladesh are given. Given this plot, calculate the following: (8 $\frac{1}{3}$)
(CO1, CO4)

- i) Average rate of positive cases per day between days **2** and **3**
- ii) Average rate of positive cases per day between days **3** and **4**

- b) After some mathematical modelling, you find out that the graph in Figure 1 can be modelled using the following equation ($x = \text{Day}$, $y = \text{Daily Infection Rate}$): (10)
(CO2)

$$y = 500 \times (3^{\sin(x/10)} + \ln(\frac{1}{x}) + 1)$$

Using this equation, calculate the instantaneous rate of positive cases on the **8th** day of the month. What would be the equation of the tangent to the given slope at $x=8$?

- c) Using the definition of derivatives, calculate the derivative with respect to x for the following function: (8)
(CO2)

$$y = (x - 3)^2 + 2$$

- d) Given $f(x) = \frac{x\sqrt{x^2-1}}{(x+1)^{2/3}}$, find out the value of $\frac{d}{dx}f^{-1}(x)|_{x=3}$ (the derivative of the inverse function at $x=3$) (7)
(CO1)

2. a) Evaluate the following limits:

i) $\lim_{x \rightarrow \infty} \frac{3x^2+3}{5x^2+7}$ (5)
(CO2)

ii) $\lim_{x \rightarrow \sqrt{2}} \frac{x^2-2}{x-\sqrt{2}}$ (5)
(CO2)

- b) Let,

$$f(x) = \begin{cases} \frac{x^3-8x^2+15x}{x^2-3x} & , \text{ if } x \neq 0 \text{ and } x \neq 3 \\ x & , \text{ if } x = 0 \text{ or } x=3 \end{cases}$$
 (8)
(CO1)

Is the function continuous at points $x = 0$ and $x = 3$? Draw a sketch of the function.

- c) Find $\frac{dy}{dx}$ for $y = x \sin^{-1}(\ln x) + \sqrt{1-x^2}$ (7)
(CO2)

- d) Is the following piecewise defined function differentiable at the origin $x=0$? Show necessary calculation. Based on this information about differentiability, could you say anything about whether the function is continuous at $x=0$? (8 $\frac{1}{3}$)
(CO1)

$$f(x) = \begin{cases} x - 1 & , \text{ if } x \leq 0 \\ \sin(x) - 1 & , \text{ if } x > 0 \end{cases}$$

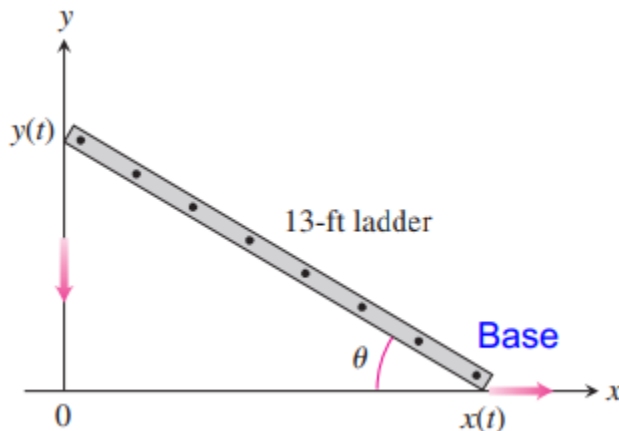
3. a) In the curve defined by the equation $x^3 + y^3 - 9xy = 0$, find out the equation for all the **horizontal** tangents. (Horizontal tangent implies the slope of the tangent is 0) (12)
(CO3)

- b) Find a linear approximation centered around $x=4$ for the following function and approximate the value of the function at $x=3.9$. (8)
(CO1)

$$f(x) = 1 + \sqrt[5]{x-2} + \ln(x)$$

- c) Find differential dy given $y = \ln(1 + x^2)$ (5)
(CO1)

- d) (8 $\frac{1}{3}$)
(CO5)



A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 feet from the house, the base is moving at a rate of 5 ft/sec.

- How fast is the angle θ between the ladder and the ground changing?
- How fast is the ladder sliding down the wall?