

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

SUMMER SEMESTER, 2020-2021

DURATION: 1 HOUR 30 MINUTES

FULL MARKS: 75

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 3 (three)** questions. Marks of each question and corresponding COs and POs are written in the right margin with brackets.

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|----|----|--|----|------------------------|
| 1. | a) | Show that the Halting Problem is unsolvable, with proper illustration. | 5 | |
| | | | | (CO1,
CO2)
(PO2) |
| | b) | Give as good a big-O estimate as possible for each of the following functions” | 8 | |
| | | i. $(n! + 2^n)(n^3 + \log(n^2 + 1))$ | | |
| | | ii. $(n^n + n \cdot 2^n + 5)(n! + 5^n)$ | | (CO2)
(PO2) |
| | c) | Three friends p, q , and r of the same university, intend to go out on a short tour. However, everything was going fine until p and q fell apart due to differences in opinion regarding the tour-destination. r on the other hand, has no clash with either of them. However, if he is free, he prefers the destination that q proposed over p . Given the clash between p and q , consider the following two propositions. | 12 | (CO2,
CO3)
(PO2) |

Proposition 1:

It appears that, r will accompany whenever q wants to go on a tour. If that is the case, it may be concluded that r will go and p will avoid the tour. In short, the tour will take place.

Proposition 2:

Alternatively speaking, the tour will take place when all the following conditions are satisfied –

- i. q goes on a tour alone or he may take r along with him and not p .
- ii. Both r and p do not go on a tour.

From your understanding of propositional equivalence, verify whether the two propositions mentioned above are equivalent or not.

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|----|----|---|----|---------------|
| 2. | a) | Consider these statements: | 12 | |
| | | <i>“All hummingbirds are richly colored.”</i> | | |
| | | <i>“No large birds live on honey.”</i> | | (CO2,
CO3) |
| | | <i>“Birds that do not live on honey are dull in color.”</i> | | (PO2) |
| | | <i>“Hummingbirds are small.”</i> | | |

Where,

$P(x)$: “ x is a hummingbird”

$Q(x)$: “ x is large.”

$R(x)$: “ x lives on honey.”

$S(x)$: “ x is richly colored.”

After expressing the statements logically, use rules of inference to prove that the last statement is a valid conclusion when the first three are premises.

- b) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = a_{n-1} + 2 \cdot a_{n-2} + 4 \times 2n - 9$ if,
- $a_n = 3 \cdot (-1)^n + 2^n - n + 2$ (CO1)
 - $a_n = 7 \cdot 2^n - n + 2$ (PO1)
- c) Find the solution to the recurrence relation $a_n = 2 \cdot n \cdot a_{n-1}$ and initial condition $a_0 = 1$. 5
(CO1, CO2, PO1)
3. a) A wonderland has a shooting booth where visitors get to shoot a balloon for free only once per day. If they succeed, they get a gift voucher of 30 Tk for the free round. Each extra shooting round can be purchased for 1 Tk if they succeed in the free round. If they succeed in any paid round, 5 Tk will be donated to charity from their winning money for each paid round. A visitor can end the game after playing any number of rounds as they wish. At the end of the game, they will get a gift voucher worth 7 times the square of total amount paid by the visitor. Now, consider that you are a visitor to this shooting booth and you succeed in all of the 21 rounds you played including the free round. Answer the following questions based on this scenario. 15
(CO2, CO4, PO1, PO2, PO3)
- Model this scenario with a quadratic equation $f(x)$ where, $x \in \mathbf{Z}^+$ and $x < 20$ and give a **big-O** estimate of $f(x)$.
 - Can a **big-Θ** estimate of $f(x)$ also be calculated using the same $g(x)$ as **big-O**?
 - Calculate the amount of money you have won at the end of the game.
- b) The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. 5
(CO2, CO4, PO1)
- Can you determine who the murderer was if
- One of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?
 - Innocent men do not lie?
- c) Give a proof by contradiction of the theorem, "If $3n + 2$ is odd, then n is odd." 5
(CO1, CO2, PO2)