ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID-SEMESTER EXAMINATION **SUMMER SEMESTER, 2020-2021**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 100**

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **3 (three)** questions. Answer all **3 (three)** of them. Marks for each question and their corresponding CO and PO are written in the right margin.

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1. a) Solve the following definite integral: $\int_0^{\pi/4} \tan x \ \sec^2 x \, dx$ — 7 (CO1) (PO1)

   b) If the following structure does a 360° rotation around the y axis, what would be the volume created by the shaded region? — 13 (CO2) (PO2)

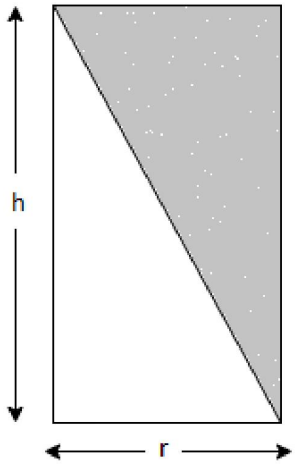

Figure 1. Figure for Question 1.(b)

   c) Show that the average value of $1/x$ over the interval $[a, 2a]$ is of the form C/a, where C is a constant independent of a (Assume $a > 0$). — 8 (CO1) (PO1)

   d) Find the area under the curve **y=1** in the limit of **[0, b]** with the Riemann sum**.** — 6 (CO1) (PO1)

2. a) Solve: $\int \frac{\sqrt{x^2-a^2}}{x} dx$ — 7 (CO1) (PO1)

   b) Assume that $0 < a < b$. Revolve the disk $(x-b)^2 + y^2 = a^2$ around the y-axis. This doughnut shape is known as a ***torus***. — 14 (CO2) (PO2)

      i) Set up the integral for volume using integration ***dx.***

      ii) Set up the integral for volume using integration ***dy.***

iii) A hole of diameter '**a**' is bored through the sphere along the y-axis (from north to south pole, like a cored apple). Find the volume of the resulting "cored" sphere. Draw relevant graphs.

c) If $\frac{d}{dx}\left(F(x)\right) = \frac{1}{1+1/x}$ and ***F(0)=1,*** the mean value theorem implies that $A < F(4) < B$. Find out the boundary values of ***A*** and ***B***. 9 (CO1) (PO1)

d) In case of two curves overlapping each other many times and revolving around the y-axis, which method (disk or shell) should one follow to evaluate the volume? Justify your answer. 3 (CO3) PO2

3. a) Pick a point (x, y) at random at the interval $0 < x < 1 - y^2$. What is the probability that $x > y$? 10 (CO3) (PO2)

b) Determine the area of the region bounded by $y = 2x^2 + 10, y = 4x + 16, x = -2$ and $x = 5$. 15 (CO2) (PO2)

c) What is a transcendental function and an asymptote line? Explain with an example. 5 (CO1) (PO1)

d) To best explain a physical situation, which one is more convenient- arithmetic average or weighted average? Explain briefly. 3 (CO3) (PO2)