

$$(p \cdot \sin(\theta' - \theta_B) = l_B \cdot \sin \alpha_B)$$

$$p \cdot \sin \beta_B = p' \cdot \sin \alpha_B \quad \dots (1)$$

$$\alpha_B + \boxed{\theta' - \theta_B} + \beta_B = \pi \quad \dots (2)$$

$$(p \cdot \sin(\theta_A - \theta')) = l_A \cdot \sin \alpha_A)$$

$$p \cdot \sin \beta_A = p' \cdot \sin \alpha_A \quad \dots (3)$$

$$\alpha_A + \boxed{\theta_A - \theta'} + \beta_A = \pi \quad \dots (4)$$

$$\left. \begin{array}{l} \theta_B \in [0, \pi] \\ \theta' \in [-\pi, \pi] \\ \theta_B \in [-\pi, \pi] \\ \theta_A \in [-\pi, \pi] \end{array} \right\} \times \left. \begin{array}{l} p \in [0, \infty) \\ p' \in [0, \infty) \end{array} \right\}$$

$$\theta' \in [0, \pi]$$

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$$\theta_B = \frac{l_B - 1}{6} \cdot \pi \quad I_B = l_B - 1 \quad \theta_B = \frac{l_B}{6} \cdot \pi \quad \dots (2)$$

$$\theta_A = \frac{l_A - 1}{6} \cdot \pi \quad I_A = l_A - 1 \quad \theta_A = \frac{l_A}{6} \cdot \pi \quad \dots (3)$$

$$\theta_A / \theta_B - \theta' < \pi \Rightarrow V$$

$$NO: \Rightarrow = 2\pi - (\theta_A / \theta_B - \theta')$$

$$\theta \in [0, \pi] \quad \theta' \in [0, \pi] \quad (p \neq 0/\pi \text{ \& } \alpha \neq 0/\pi)$$

$$\frac{\sin \beta_B}{\sin \beta_A} = \frac{\sin \alpha_B}{\sin \alpha_A} \Rightarrow \sin \beta_B = \frac{\sin \alpha_B}{\sin \alpha_A} \cdot \sin \beta_A \quad \dots (5)$$

$$\theta \in [0, \pi] \quad \theta' \in [0, \pi]$$

$$\beta_B = \arcsin \left( \frac{\sin \alpha_B}{\sin \alpha_A} \cdot \sin \beta_A \right) \quad \dots (5)$$

$$\theta \in [0, \pi] \quad \theta' \in [0, \pi]$$

$$\alpha_A + \alpha_B + \beta_A + \beta_B + \theta_A - \theta_B = 2\pi$$

$$\beta_B = 2\pi - (\beta_A + \alpha_A + \alpha_B + \theta_A - \theta_B) \quad \dots (6)$$

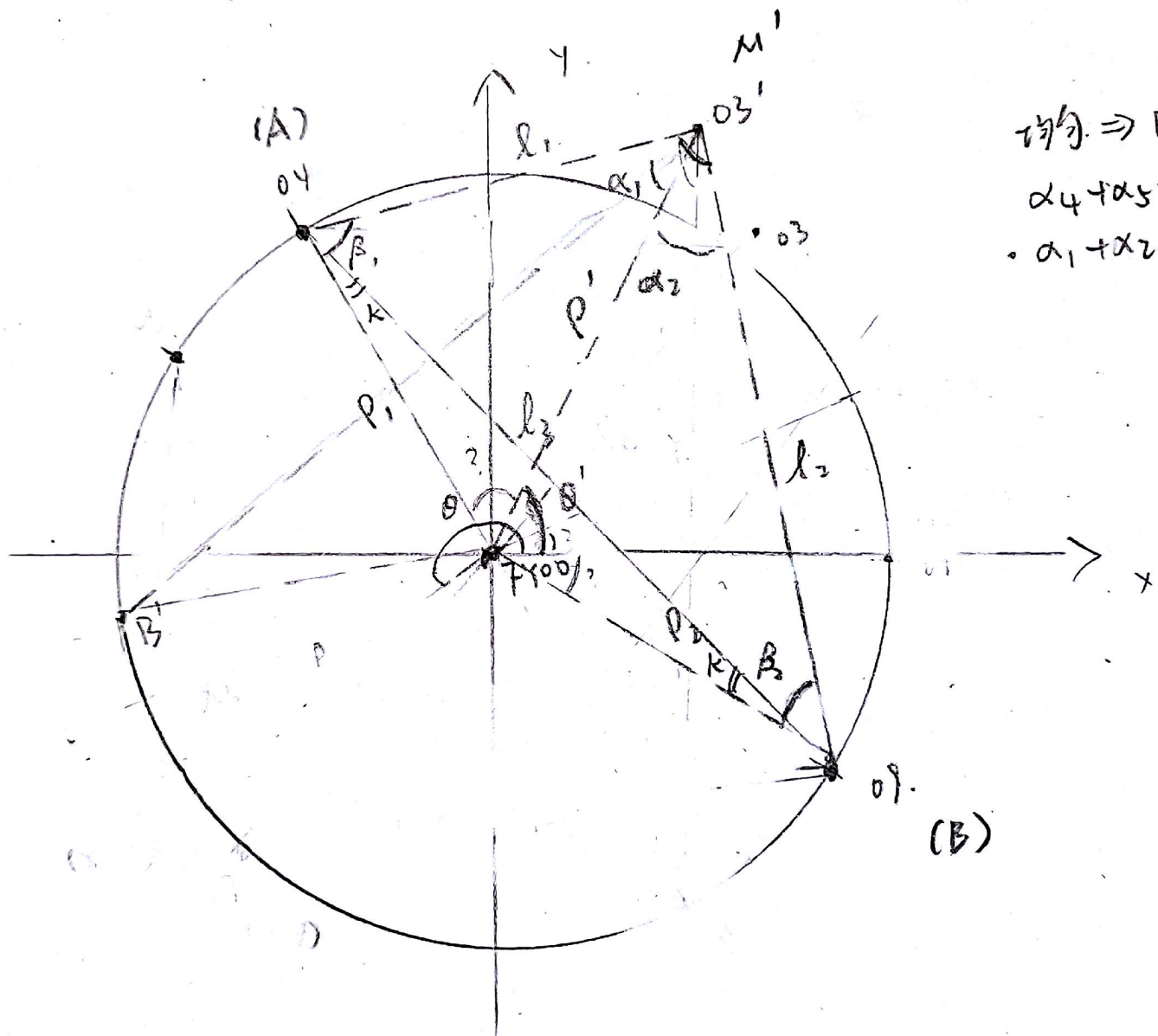
$$\theta \in [0, \pi] \Rightarrow \beta_B$$

$$\alpha_B - \alpha_A + 2\theta' - \theta_B + \beta_B - \theta_A - \beta_A = 0$$

X

$$2\theta' = \alpha_A - \alpha_B + (\theta_A + \theta_B) + \beta_A - \beta_B \Rightarrow \theta' = \frac{1}{2} [\alpha_A - \alpha_B + (\theta_A + \theta_B) + \beta_A - \beta_B] \quad \dots (7)$$

$$p' = p \cdot \frac{\sin \beta_B}{\sin \alpha_B} = p \cdot \frac{\sin \beta_A}{\sin \alpha_A}$$



$\alpha_4 + \alpha_5 = \pi$   
 $\alpha_1 + \alpha_2 = \pi$

①

未定

②

# Question I.

由图1所示, 如以未知点  $M'$  处在第一象限, 而架已知编号的天文位于以  $OM'$  为轴的两侧为例. 其自情况与该情况类似因此不再赘述. 以  $FY00$  为原点,  $FY00$  至  $FY01$  为  $\pi$  已定, 延此极坐标.

对  $\triangle BO'M'$  由正弦定理得

$$\frac{p_B}{\sin \alpha_B} = \frac{p'}{\sin \beta_B} \quad p_A \cdot \sin \beta_B = p' \cdot \sin \alpha_B \quad \dots (1)$$

由内角和关系

$$\alpha_B + \theta_B + \beta_B = \pi \quad \dots (2)$$

$$\min\{[2\pi - (\theta_B - \theta')], \theta_B - \theta'\} =$$

对  $\triangle AO'M'$  由正弦定理得

$$\frac{p_A}{\sin \alpha_A} = \frac{p'}{\sin \beta_A} \quad p_B \cdot \sin \beta_A = p' \cdot \sin \alpha_A \quad \dots (3)$$

由内角和关系

$$\alpha_A + \beta_A + \min\{\theta_A - \theta', [2\pi - (\theta_A - \theta')]\} = \pi \quad \dots (4)$$

$$p_A = p_B = p \quad \dots (5)$$

设  $M'(p', \theta')$ ,  $A(p_A, \theta_A)$ ,  $B(p_B, \theta_B)$

以下分情况讨论

I 当  $\theta_B - \theta' \leq \pi$  时.

$$\text{②式} \text{ 为 } \alpha_B + \theta_B - \theta' + \beta_B = \pi$$

II 当  $\theta_B - \theta' > \pi$  时

$$\text{②式} \text{ 为 } \alpha_B + 2\pi - (\theta_B - \theta') + \beta_B = \pi$$

III 当  $\theta_A - \theta' \leq \pi$  时

$$\text{④式} \text{ 为 } \alpha_A + \beta_A + \theta_A - \theta' = \pi$$

(IV) 当  $\theta_A - \theta' > \pi$  时.

$$\text{④式} \text{ 为 } \alpha_A + \beta_A + 2\pi - (\theta_A - \theta') = \pi$$

共有四种情况.

I, II, III, IV.

a. 当 I, II 时.

由①②得

$$\beta_B = \begin{cases} \dots \end{cases} \quad \text{⑤}$$

$$\beta_A = \alpha_B + \theta_B + \beta_B - \alpha_A - \theta_A \quad \dots \text{⑥}$$

b. 当 I, IV 时.

由①④得.

$$\beta_A = \theta_A - (\alpha_B + \theta_B + \alpha_A + \beta_B) \quad \dots \text{⑦}$$

c. 当 II, III 时.

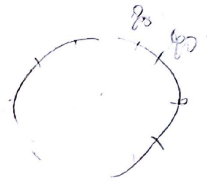
由①③得.

$$\beta_A = \theta_B - (\alpha_B + \beta_B + \alpha_A + \theta_A) \quad \dots \text{⑧}$$

d. 当 II, IV 时.

由①④得.

$$\beta_A = \theta_A - \theta_B + \beta_B + \alpha_B - \alpha_A \quad \dots \text{⑨}$$



由①③⑤

$$\sin \beta_A = \frac{\sin \alpha_A}{\sin \alpha_B} \cdot \sin \beta_B \quad \text{--- ⑥}$$

$$\Rightarrow \beta_A = \arcsin \left( \frac{\sin \alpha_A}{\sin \alpha_B} \cdot \sin \beta_B \right) \quad \text{--- ⑦}$$

易知:

易知  $\beta_A \in (0, \pi)$ ,  $\beta_B \in (0, \pi)$

由⑦和⑥可知  $\beta_B$  的值

但是并没有讨论过. 通过曲线相交求导情况.

求出  $\beta_B$  后

由④和③求得

$$\rho' = \frac{\rho \cdot \sin \beta_B}{\sin \alpha_B}$$

$$\theta' = \begin{cases} \pi - \alpha_B - \beta_B + \theta_B, & 0 < \theta' - \theta_B \leq \pi \\ \theta_B - \alpha_B - \beta_B - \pi, & \pi \leq \theta' - \theta_B \leq 2\pi \end{cases}$$