Homework 1: Conceptual Foundations

November 6, 2021

Due Date: Sunday, 14 Nov 2021 Note: You must email your homework before the deadline to be graded. Part 1 can be submitted as per your convenience (scanned handwork or a word/ pdf document), whereas Part 2 has to be submitted as the notebook file. Both parts should be in a zipped folder (format: **H01_yourfullname**), emailed to **asim.dilawar@mcs.edu.pk**.

1 Part I: Mathematical Foundations (20)

1. Compute the following sums and products:
   (a) $\sum_0^N 1$
   (b) $\sum_0^N C$
   (c) $\sum_{k=1}^K \sum_{t=1}^T 1$
   (d) $\sum_{k=1}^K \sum_{t=1}^T 0.5^k$
   (e) $\sum_{k=1}^\infty \sum_{t=1}^T 0.5^k$
   (f) $\prod_{i=1}^M \frac{1}{\theta}$
   (g) $\prod_{k=1}^K \frac{k}{k+1}$
   (h) $\ln\left(\prod_{k=1}^K e^k\right)$
2. For $f(x) = \max(0, x)$, answer following:
   (a) $\lim_{x\to-\infty} f(x) = ?$
   (b) $\lim_{x\to+\infty} f(x) = ?$
   (c) What are the intervals of x where $f(x)$ is differentiable, i.e. $\frac{d}{dx} f(x)$ exists.
   (d) What are the intervals of x where $f'(x) = \frac{d}{dx} f(x) > 0$.
3. What is the length (euclidean norm) $\|\mathbf{x}\|$ of the vector $\mathbf{x} = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$?
4. What is the angle (in radians) between $\mathbf{x}_1 = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} -0.15 \\ 0.2 \end{bmatrix}$?

5. Are $\mathbf{x}_1$ and $\mathbf{x}_2$ in above question orthogonal to each other?
6. If probability density function (pdf) of a Gaussian random variable X is given by $f_X(x) = \frac{n}{3\sqrt{2\pi}} \exp\left(-\frac{n^2(x-2)^2}{18}\right)$, then what is the mean μ and variance σ^2 of X?
7. Given X from Question 6, can you write the pdf of the random variable Y when $Y = 2X$?
8. What is the rank of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 0.5 & 1 & 0.5 \\ 2 & 1 & 2 \end{bmatrix}$?
9. Compute $\mathbf{A}^{-1}$?
10. Let $\mathbf{A} = \begin{bmatrix} 3 & 0 \\ 0.5 & 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Suppose furthermore that $\mathbf{Av} = \lambda_1 \mathbf{v}$ and $\mathbf{Aw} = \lambda_2 \mathbf{w}$. What will be the value of $\lambda_1$and λ_2?

2 Part 2: Coding Exercise (5)

Refer to the notebook **H01-kNN-MNIST-Classification.ipynb** in the homeworks folder.

