MOLPOP Line Emission Output

Moshe; May 12, 2017

The entire line flux at position τ is

$$F(\tau) = \int_{-\infty}^{\infty} F_\nu(\tau) d\nu = \Delta\nu \int_{-\infty}^{\infty} F_\nu(\tau) dx \tag{1}$$

We introduced the line cooling rate per unit area (erg cm^{-2} s^{-1}) and the cooling factor $\jmath$ via

$$\Lambda = F(\tau_t) - F(0) = 4\pi\Delta\nu\,\jmath \tag{2}$$

taking account of emission from the two faces of the slab (eq. 4, 2006 CEP paper). As we show in eq. 93 of our CEP notes (where we used $\mathcal{E}$ for Λ), in the case of 1–zone we have

$$\jmath = S\tau_t\beta(\tau_t) = \epsilon\ell\beta(\tau_t) \tag{3}$$

where ϵ is the emission coefficient and ℓ is the slab thickness. Therefore, the $u \to l$ transition has

$$\Lambda_{u,l} = 4\pi\Delta\nu\epsilon_{u,l}\beta_{u,l}\ell = h\nu_{u,l}A_{u,l}\beta_{u,l}n_u\ell = h\nu_{u,l}A_{u,l}\beta_{u,l}\frac{n_u}{n}N \tag{4}$$

where n_u is the population of the upper level, n is the overall molecular density and N is its column density. Since N is the overall number of molecules per unit area, the cooling rate per molecule (erg s^{-1} per molecule) is

$$\texttt{cool}_{u,l} = \frac{\Lambda_{u,l}}{N} = h\nu_{u,l}A_{u,l}\beta_{u,l}\frac{n_u}{n} \tag{5}$$

and the flux is

$$F(\tau_t) = -F(0) = \tfrac{1}{2}\Lambda_{u,l} = \tfrac{1}{2}\texttt{cool}_{u,l}N \tag{6}$$

MOLPOP Implementation

We use the variables `x(i)` so that

$$\texttt{n(i)} = \texttt{nmol} * \texttt{x(i)} * \texttt{we(i)} \tag{7}$$

is the population of level i, with `we(i)` its weight factor. Therefore

$$\texttt{n(i)A(i,j)} = \texttt{nmol} * \texttt{x(i)} * [\texttt{we(i)A(i,j)}] \tag{8}$$

and in MOLPOP we absorb the weight factor into the A-coefficient. So after that is done,

$$\texttt{cool(i,j)} = \texttt{ems(i,j)} * \texttt{esc(i,j)} * \texttt{x(i)} \tag{9}$$

(in subroutine `lines`) where

$$\texttt{ems(i,j)} = \texttt{h}\nu\texttt{(i,j)} * \texttt{A(i,j)} \tag{10}$$

is defined right after A is scaled with `we(i)` at the end of subroutine `INPUT`. Therefore, the flux should be defined as

$$\mathtt{F(i,j)} = \frac{1}{2}\mathtt{cool(i,j)} * \mathtt{nmol} * \mathtt{R} \tag{11}$$

This will give the result in erg cm^{-2} s^{-1}. Switch to MKS: W = 10^7 erg s^{-1} and m^2 = 10^4 cm^2 so W m^{-2} = 10^{7-4} erg cm^{-2} s^{-1} and erg cm^{-2} s^{-1} = 10^{-3} W m^{-2}. Therefore

$$\mathtt{F(i,j)} = \mathtt{aux} * \mathtt{cool(i,j)} \qquad \mathtt{W\,m}^{-2} \tag{12}$$

where

$$\mathtt{aux} = .5 * 10^{-3}\,\mathtt{nmol} * \mathtt{R} \tag{13}$$