Laser-Wakefield Electron Accelerators

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Abstract

In the late 1970's, Tajima and Dawson proposed a method of accelerating electrons using the large electric field gradients that plasmas are capable of sustaining. They showed that when a large amplitude laser pulse is sent through a plasma it can create a co-propogating longitudinal plasma wave capable of having electric field gradients of multiple $\mathrm{GeV\,cm^{-1}}$– orders of magnitude larger than that of conventional particle accelerators.

In this work, we review the state-of-the-art experimental progress being made in creating a fully functioning electron accelerator based on laser plasma acceleration (LPA) technology. We discuss how the energy in an electric field of a high intensity pulsed laser can be converted into an electric field gradient that can be used to accelerate electrons. We will review the physics of the laser-plasma interaction, and discuss the limitations involved. We will finish with a review of the two most advanced experimental efforts to advance LPA technology.

Contents

1 Introduction

The field of laser plasma acceleration is motivated by the goal of having table-top electron accelerators capable of producing electron beams with hundreds of GeV. This is achievable because the acceleration gradients that plasmas are capable of sustaining are many orders of magnitude larger than conventional particle accelerator. Much like the introduction of the personal computer in the era of massive supercomputers, LPWA will put the technology of electron acceleration to large energies in the hands of many.

The ability to accelerate particles to high energies is a cornerstone in many areas of science. Although the recent discovery of the Higgs boson[3] at the LHC demonstrated the value of ion-accelerators capable of TeV energies, a more direct comparison to the regimes acheivable to LPWA technology would be to linacs– such as Stanford Linear Accelerator Center (SLAC), which can excite electrons up to 50 GeV.

Although historically important to the development of particle physics the most broad application of linacs such as SLAC is the use of accelerated electrons for an free-electron x-ray laser; the development of which has allowed investigation of structures from biology to solid state physics, as well as pioneering techniques in medical imaging.[14]

But, much like the supercomputers of the 80's, the future of the field is limited by the size and expense of these facilities[1]. Laser plasma wakefield accelerators offer an option that is compact, and inexpensive by comparison. ALthough LPWA will never supplant linac's[2] they offer a complimentary approach at a fraction of the cost and real-estate. The prospect of a tabletop free-electron laser has sparked intense interest in the field of laser plasma acceleration, and there is a dynamic field that is constantly improving the quality of the accelerated electron beam.

The quest to produce high-energy electrons has four main goals: getting high-

[1] The famous cancellation of the Superconducting Super Collider in Texas due to budget problems in one dramatic example

[2] This is because the beam produced by LPWA is not continuous; the nature of LPWA schemes means that the electrons beam produced will be an intense bunch, rather than a continuous beam like SLAC.

energy electrons; having a narrow energy distribution; producing collumated beams; and having a large number of electrons produced.

In the context of free-electron lasers, these are important for the following simple reasons: the frequency of the x-rays scales with the square of the energy of the electrons, so the higher the energy the smaller length scales we can probe; the narrower the electron energy distribution, the narrower the x-ray linewidth; and the beam quality (number of electrons and angular spread) will also directly effect the x-ray laser coherence.

1.1 History

In the 1970's, Tajima and Dawson proposed shooting a high-intensity laser at a plasma[16]. They predicted that the laser would generate a wakefield very similiar to a boat moving through water and electrons could be accelerated by the wakefield. Unfortunately, the lasers of the day were unable to get to the high-intensities required. It was only the invention of the chirped-pulse laser (CPA) fifteen years later that allowed serious progress in the LPA field to occur[1]. In order to excite the plasma wave, you had to use a laser at the resonant frequency, which scales with the square-root of the plasma density. For typical laboratory plasmas, this resonant frequency was out of reach for the laser's of the time. Several LPA schemes were devised to overcome this, the two major ones being the plasma beatwave accelerator (PBWA), and the self-modulated laser. Although the mechanics of these schemes differed the

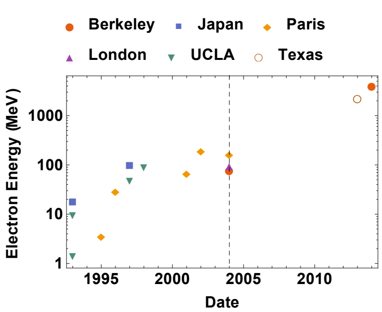


Figure 1: The progress of laser plasma wakefield acceleration by the total energy of the electrons. The dashed line shows the advent of quasi-monoenergetic electrons, until that point the electron bunches had large thermal tails. *This data was gathered from the web of science abstract list*

underlying principle was to use the beatwave generated by spatio-temporally overlapping two lasers to excite the plasma. This effort was moderately succesful: in 1995 electrons were succesfully accelerated to energies in excess of 40 MeV. Unfortunately, these electron beams had large Maxwellian energy distributions, making them untenable for practical applications.

In 2002, Pushkin predicted the bubble regime, which would solve many of the problems historyically faced by LPWA schemes. By using a laser so powerful it completely expells electrons from a region, the bubble regime was predicted to be able to produce mono-energetic electron energies. However, it would require the discovery and development of CPA technology that would allow lasers to reach the intensities required to bubble.

In 2004, this approach bore fruit, as three papers published simultaneously in Nature[13, 6, 7], demonstrated quasi-monoenergetic electron bunches in the bubble regime. Their results were soon extended to achieve energies of 1 GeV in 2006 [12].

In 2013, a group at UT Austin produced a collumated, quasi-monoenergetic electron beam at 2.3 GeV[18], and in 2014 the Esarey group at UC Berkeley produced a 4 GeV[11] beam. These recent developments bring the field within striking distance of the LCLS at SLAC, which uses electrons accelerated to 17.4 GeV.

In the following section, we will discuss the physics of Laser-Plasma-Acceleration, and then discuss the two most recent experiments from UT Texas and UC

Berkeley.

2 The Physics of Laser-Plasma-Acceleration

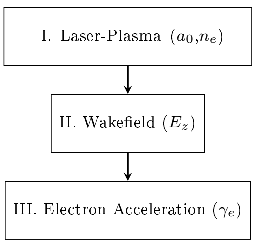


Figure 2: The LWFA process.

The LPWA process can be divided into three main topics, as shown in Figure 2. First the intense laser (charaterized by the normalized field strength ($a_0 = e\mathbf{A}/m_e c^2$) interacts with the plasma (characterized by the plasma density (n_e) producing a longitudinal density modulation; this gives rise to a longitudinal electric field (E_z); which will in turn accelerate the electrons to a relativistic energy γ_e.This section will review each in turn.

Due to the complicated nature of the theory of LPWA in 3D relativistic fields, we will mainly discuss these topics in the linear regime, and show the numerical results of their extension into the 3D relativistic regime.

2.1 Laser-Plasma Interaction

The interaction of the exciting laser pulse with the plasma is governed by Lorentz force[**?**]. As the mass of the ions in the plasma is many orders of magnitude larger than the electrons, it is valid to approximate the behaviour of the plasma as a fluid of mobile electrons against a background of ions.

With this simple picture, the motion of the electrons will be governed by the Lorentz force law, the continunity equation and Poisson's equation. If the intensity of the laser is small enough, then these equations can be linearalized, and are refered to the cold-fluid equations[9]. In this electron-fluid regime, the Lorentz force looks like:

$$\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{p} = e\left(\boldsymbol{\nabla}\Phi + \frac{\partial \mathbf{A}}{\partial t} - \mathbf{v} \times \boldsymbol{\nabla} \times \mathbf{A}\right) \tag{1}$$

Where $\mathbf{p}$, $\mathbf{v}$ are the electron's momentum and velocity, and Φ and $\mathbf{A}$ are the scalar and vector potentials of the total field.

To first order, the movement of this fluid will be dominated by the force of the electric field of the laser on the electrons, accelerating them in the polarization plane. This is called the 'quiver' momentum[1]. This quiver momentum will not produce a net acceleration on the electrons.

If the Lorentz force law is expanded out to second order, a term will appear that is proportional to the intensity gradient, known as the ponderomotive force. This can be thought of as the radiation pressure of the laser pulse, and will act to push electrons away from the local space of the laser packet. It is this ponderomotive force that will be directly used to drive the density waves that all LPA schemes use. This is analogous to the physical situation of shooting a cannonball underwater– it will excite a density wave that copropogates with it.

By combining the Lorentz force with Possion's equation and the fluid-continunity

[1] So named because the electron will undergo rapid oscillations while its time averaged acceleration will be zero, so it will appear to be quivering

equation, a closed set of equations in one dimension can be found and solved to give the behaviour of the plasma electrons:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{\mathrm{p}}^2\right) = c^2\nabla^2\frac{a^2}{2} \tag{2}$$

where we have introduced the normalized vector potential $a = eA/m_e c$, and the resonant frequency of the electron fluid:

$$\omega_{\mathrm{p}} = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}, \tag{3}$$

Where m_e, n_e, and e are the mass, density and charge of the electron, respectively.

The resonant frequency of the plasma will give an experimental constraint. As with all driven-oscillator type systems, to drive the oscillator at resonance, you need to force it at the natural frequency of the system. We want high-intensity waves, as their amplitude will ultimately determine the accelerating field E_z. In order to excite resonance then $\tau_{\mathrm{laser}}\omega_p \approx \pi$. The resonance condition is further illustrated in Figure **??**. In this regime, the density of the electrons behaves like a wave that co-propogates with the laser pulse. We can connect the variations in density to the electric field produced using Possion's equation, which reduces to

$$\mathbf{k} \cdot \mathbf{E} = \frac{\delta n_e}{\epsilon_0} \tag{4}$$

This shows there will be a co-propogating electric field, which points along the propogation direction, and oscillates π out of phase from the density w This field E_z is the one that will be used to accelerate electrons. This process can be generalized into a non-linear regime by simply eliminating the assumption that $a \ll 1$, and an analytic solution can be found. However, in three dimensions the equations become intractable and numerical simulation is required to solve them. The solutions in one-dimension are plotted in Figure 3a for a range of parameters.

It is worth noting that modern LWFA schemes operate in the bubble regime, where the laser pulse is so intense that it completely expels the electrons from its local space– putting us firmly in the non-linear 3D regime. This means that the majority of theoretical work currently being done is by using numerical methods. An example is given in Figure 4

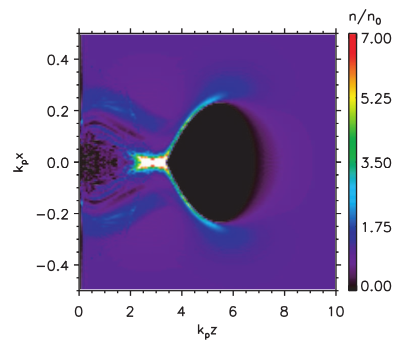
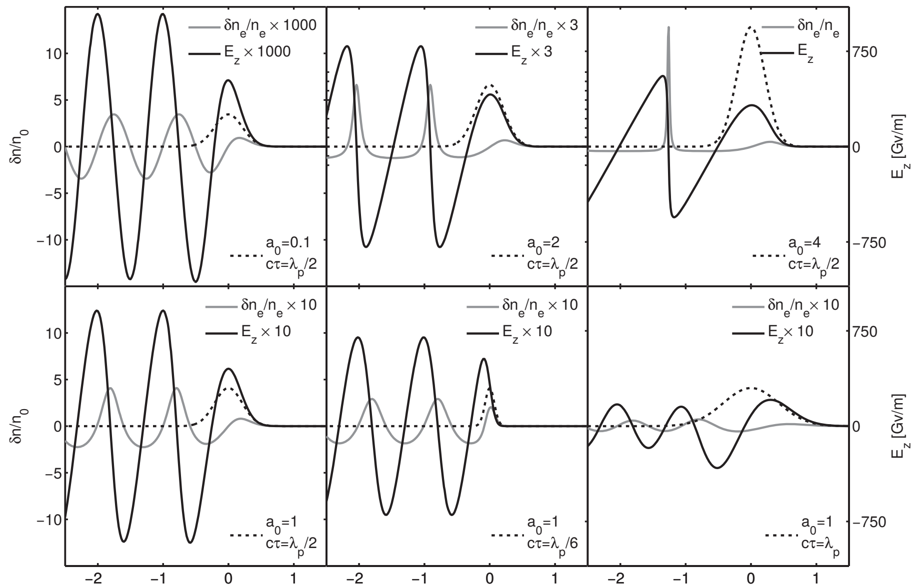


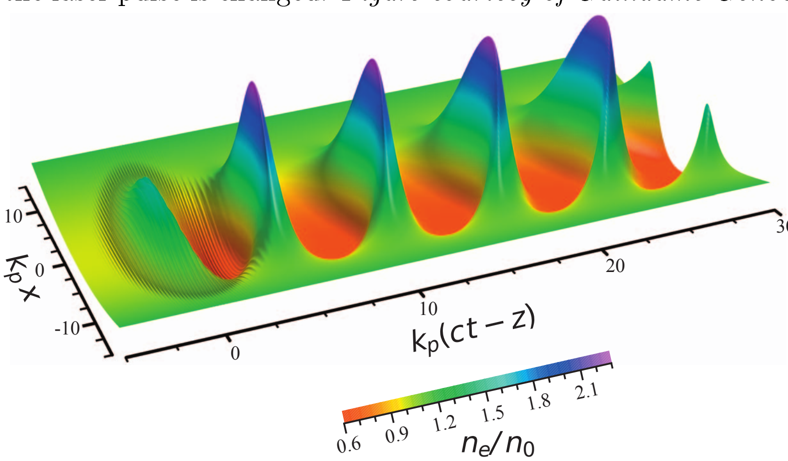
Figure 4: An example of the bubble regime created by a laser pulse with $a = .3$). The laser is moving toward the right.[4]

2.2 Electron Trapping, Injection, and Acceleration

In order to be accelerated by the co-propogating electric field created by the plasmon, the electrons need to be travelling at some minimum speed. Transforming to the rest-frame of the plasmon, only those electrons that are moving slowly enough will be trapped by the electrostatic field. The phase-space trajectories of various test electrons are shown in Figure 5, illustrating that only for a specific range of the electron momentum will they actually be able to be trapped. The background electrons will not meet



(a) Showing plasmons generated with varying strengths of the peak amplitude of the laser pulse.[8] The laser pulse is the dashed line, the density perturbation is the grey line, and the longitudinal electric field E_z is the black curve. The x-axis is showing the normalized co-ordinate $\zeta = kz - wt$, which shows the evolution of the phase-front. Different scenarios are shown: from left to right, the normalized field strength variable ($a = eA/m_e c^2$) is varied, and from top to bottom, the duration of the laser pulse is changed. *Figure courtesy of Guillaume Genoud, Lund Univeristy*



(b) Numerical simulation for a 2D, non-linear model. Many of the features in the 1D model remain– the signature 'leaning' of the density pulse as it becomes non-linear being the most striking.

these requirements in the linear regime. This can be overcome by having an external injection source of electrons, but this has the obvious disadvantage of already requiring electrons with large energies. As we discuss below, there is a way to use accelerate the background electrons.

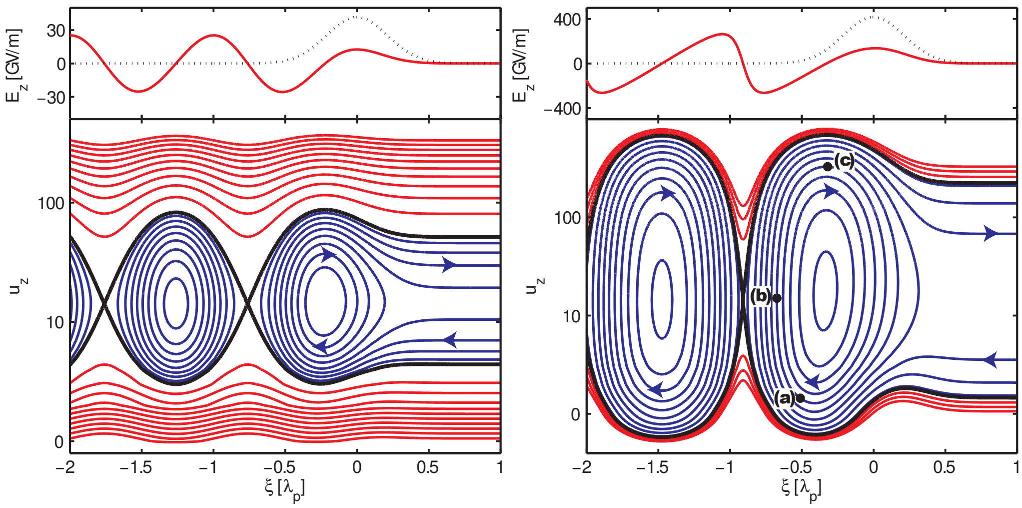


Figure 5: The various trajectories of electrons at different initial momentum ($u_z = p/m_e c^2$) in the reference frame of the laser pulse, which is moving at an relativisitc energy $\gamma = 13$ in the lab frame. For specific initial electron energies the electrons will be trapped and accelerated by the wakefield.

If the amplitude of the laser field is strong enough to excite the plasmon into a non-linear regime, then there are ways for the background electrons to be accelerated. This process is refered to as self-injection. [3]

[3] Although self-injection happens in non-linear fields in both 1D and 3D, it occurs by different physical processes[5]. We will be focusing on the 3D case in this review

Looking at Figure **??**, the non-linear regime plasmons are steeper on one edge. This is due to the fact that the background electrons are having enough energy imparted to them to affect the shape and properties of the plasmon. This process, refered to as wavebreaking, is useful for two reasons:

it enhances the overall field and allows the background electrons to meet the velocity requirements to be trapped by the plasmon. The bubble regime, in which the laser is powerful enough to completely expel the electrons from a central region of the pulse, is an extreme example of wavebreaking in plasmons. Figure 6 is a schematic of the bubble regime.

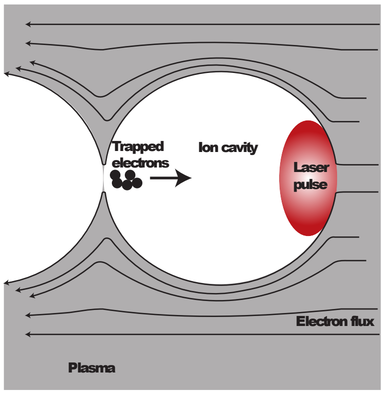


Figure 6: A schematic of the bubble regime.[8]

It can be shown using a combination of numerical and analytical analysis that in the bubble regime there is a specific requirement for the radius of the bubble for self-injection to occur:$R/\sqrt{2} > \omega_0/\omega_p$, where R is the radius of the bubble, and ω_0 is the laser frequency.[10].

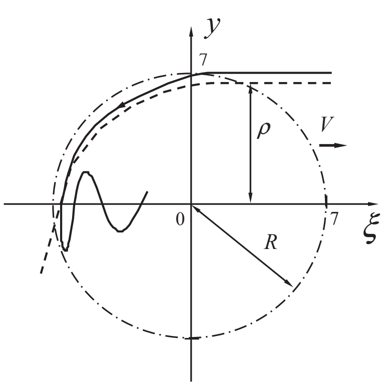


Figure 7: Showing a trapped, and untrapped trajectory of an electron. The bubble parameters are $R = 7$, $\gamma_0 = 4$

However, it is easy to gain a phenomolgical explanation of the dynamics of self-injection in the bubble regime. The bubble can be thought of as an ion-cavity moving relativistically through an electron fluid. A small sheath of electrons will form as a boundary layer between the ion-cavity and the surrounding electron fluid. This sheath will be in contact with the relativistic fields of the cavity for the longest time, and is the best candidate to be accelerated. If the laser defocuses, then the bubble will grow. If it grows on a time scale that is quick enough, electrons that previously wouldn't have the right initial orbit are now have a small enough relative velocity to be trapped. Although the dynamics of the laser evolution in the plasma are complicated, it will undergo cycles of focusing and defocusing due to diffraction and self-focusing phenomena– discussed in more detail in Section **??**. In this way, the bubble will breath, trapping and accelerating electrons.

2.3 Electron Acceleration

The plasma bubble will set up a very high $\mathrm{GeV\,cm^{-1}}$ acceleration field, however what will limit the total energy gain is how far the electron can be accelerated for. There are three main lengths involved with accelerating the electrons.

The first, $L_{\text{Dephasing}}$ is analogous to what happens when a surfer outruns the wave they are on; no longer being accelerated, they slow down as their energy is dissapated to the waves. Similarly, electrons can outrun the plasma bubble.

The second, $L_{\text{PulseDepletion}}$, occurs because the interaction with of the laser-plasma will dissipate the energy of the initial laser pulse. The energy in the initial laser will be transfered to the plasma wake, where it will ultimately be disappaited as heat.

The third length, $L_{\text{Diffraction}}$ is the most important, as it is the limiting length scale. This length scale is due to the inherent diffraction of lasers. In order to achieve the intense energies neccesary for the bubble regime, the lasers need to be focused down to a specific spot size. As soon as the minimum spot size is reached, the laser will begin to diffract, the length scale where the laser is approximately the spot size is the Rayleigh length. In a plasma this becomes more complicated, as the plasma can and will act like a lens. This leads to a feedback effect, where the intense laser pulse will change the plasma density, which in turn changes the intense laser pulse, and so on. The majority of effort in current wakefield accelerator programs is to overcome this issue.

In Figure 8, we can see the length scales multiplied by the accelerating electric field– giving the total energy possible if an electron was accelerated over that distance. Clearly, the limiting length scale is diffraction.

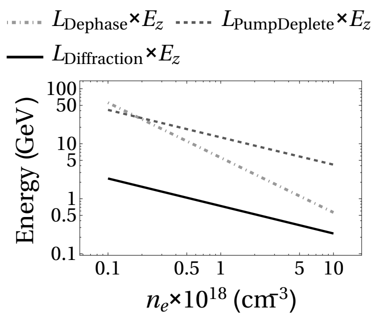


Figure 8: The three length scales involved with accelerating electrons: L_{Dephase} where the electron outruns the wave, self-limiting the total energy gained; $L_{\text{PumpDepletion}}$ where the incident energy in the laser pulse is completely transfered to the wakefield, and the laser can no longer sustain the bubble regime; and $L_{\text{Diffraction}}$ the inherent diffraction of the laser pulse. All lengths are scaled by an accelerating field using parameters from the Texas experiment[18], to show the total possible energy an electron could gain.

There are several strategies for extending the length of the laser-plasma interaction: two that we will mention are relativistic self-focusing, and external plasma-waveguide solutions. These are the two main solutions currently being pursued by the UT Texas group and the Berkeley group, respectively.

3 Experimental Set-Up and State-of-the-Art

In this review, we will focus on the experimental efforts of two groups, UT Austin Texas, and University of California Berkeley. Although there are many more groups doing interesting work in the field of LWFA, these two groups are the main ones actively pursuing the goal of high-energy electron accleration.

The Texas group uses very high-intensity laser pulses and exploits the phenomena of relativistic self-guiding to cancel out the inherent diffraction of the laser. The Berkeley group uses low-intensity laser pulses and plasma waveguide channels to overcome the diffraction issue. A more in depth review of each group and method is presented below.

3.1 Texas: Relativistic Focusing

In 2013, the Texas group reported a collumated beam of 2 GeV, which blew current records out of the water[18].

Their approach used the new petawatt laser facility at UT Austin. The large amplitudes that the laser was capable of generating placed them firmly in the relativisitic self-guiding regime.

3.1.1 Relativistic Self-Guiding

The group velocity of the laser pulse will be set by the index of refraction of the medium, which in turn is $eta = v_g/c = c^{-1}\frac{\mathrm{d}w}{\mathrm{d}k}$. The dispersion relation for a plasma is shown in Figure **??**. Thus,

$$eta = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}. \tag{5}$$

If eta has a density distribution where it is larger on the optical axis, then a guiding plasma lens will be formed that can counteract the effects of diffraction. As we will discuss below, one can use relativistic effects to achieve this.

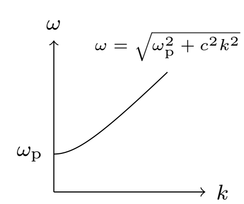


Figure 9: The plasma dispersion relation. We will be dealing with plasmas where $\omega_{\mathrm{p}}/\omega << 1$, so to first order the laser will be dispersionless.

Remembering back to Section **??**, the first order effect of the high-intensity laser pulse will make the electrons quiver in the polarization plane.If the laser is intense enough, the electrons will be quivering relativistically, and will have maximum energy as they pass through the optical axis of the pulse. This means that their mass will be a maximum on the optical axis. This can be roughly seen to change the index of refraction, as:

$$\eta(m_e) \rightarrow \eta(\gamma m_e) \approx 1 - \frac{\omega_p^2}{2\omega^2}\frac{\delta n(r)}{\gamma} \tag{6}$$

where $\delta n(r)$is a density variation, and *gamma* will be a maximum on axis.[**?**] This gives rise to the focusing behaviour discussed above.

This sets a very harsh experimental parameter, as the fields have to be relativistic enough to be able to counter the effects of diffraction. It can be shown that this limit is that the power in the field is P(GW) = 17.4 $(\omega/\omega_p)^2$[**?**].

In reality, the dynamics are much more complicated, and require detailed numerical simulations, one such example is shown in Figure 10[18]. This is partially due to the fact that the front and back of the waves will not satisfy the relativistic criterion, and diffract away– dynamically changing the pulse shape, which can have an effect.

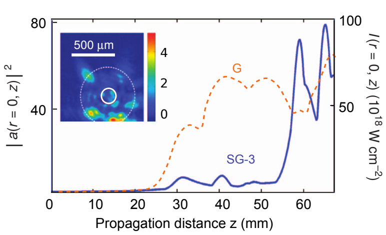


Figure 10: Simulations done by the Texas group using the WAKE code showing clear features of self-focusing.[18] As the normalized laser-intensity gets larger, the pulse is contracting–concentraing more of its energy over a smaller area. Interestingly, the self-focusing exhibits a periodic structure– going through two cycles of diffraction-focusing for the super-gaussian pulse.

3.1.2 Experimental Setup and Results

Shown in Figure 11 is the experimental setup for Texas. At its heart, it is a high-intensity laser that is hitting an ionized gas. The majority of the experiment is the diagnostics, which allow the team to determine the energy and spread of the electrons.

The Texas group is now hard at work trying to find a strategy to increase the energy of the electrons. Although their initial result is very promising, there isn't a clear way forward. This is due to the very complicated dynamics of the highly non-linear laser-plasma interaction that leads to self-focusing. As shown in Figure 10, the behaviour of the self-focusing is very dependent on initial conditions (laser intensity, laser pulse shape, etc.). As no analytic model can fully encompass its behaviour, numerical simulation is required to understand the self-focusing behaviour required to produce accleration lengths required for high energy electrons.

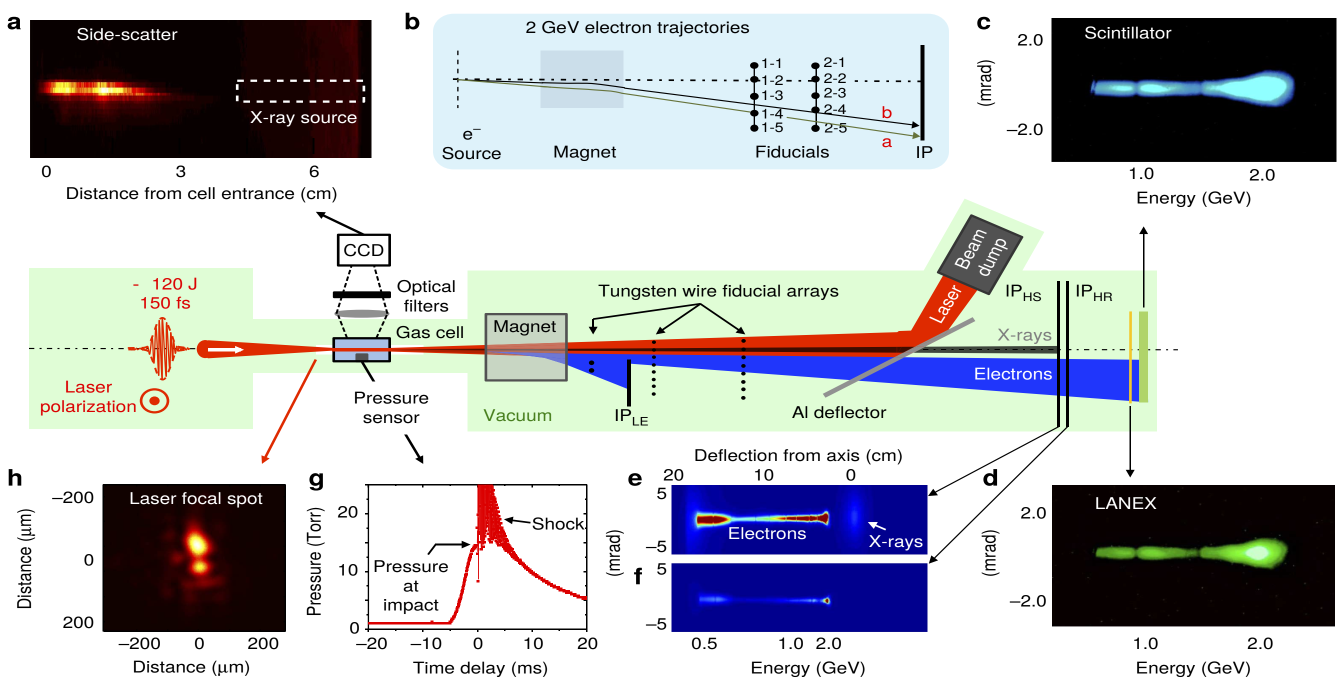


Figure 11: The experimental setup of the Texas group.[18] *This figure is reproduced from Nat. Commun. Vol. 4, 2013*

Their current work is focused on visualizing and understanding the propogation of high-intensity laser light through their plasma, and they have developed several novel techniques.

3.2 Low-Intensity, Quasi-Linear Plasmas with Waveguides

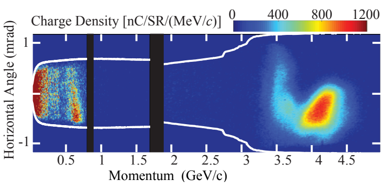


Figure 12: The energy spectrum for the recent Berkeley result.

In 2014, the Berkeley group reported a collumated electron beam with peak energy of 4.2 GeV.

The Berkeley has, for a while now, pursued the strategy of low-intensity laser pulses that are guided through channels. Their most current results use a capillary discharge channel. The physics of focusing remain the same: there is a radial density profile that has the same profile as a focusing lense. Instead of the density profile coming from relativistic effects, like the Texas group, the Berkeley group uses a plasma channel guide, which is a simple tube containing hydrogen gas, with a metal plate at each end. A voltage is applied across the length of the tube, ionizing the gas. The gas will cool down more rapidly along the edges of the tube, and this will give rise to an approximately parabolic density profile of the hydrogen gas[2, 15]. It is this parabolic density profile which will act as the focusing lens of the plasma wave. The evolution of a laser pulse in a plasma channel is shown in Figure 13.

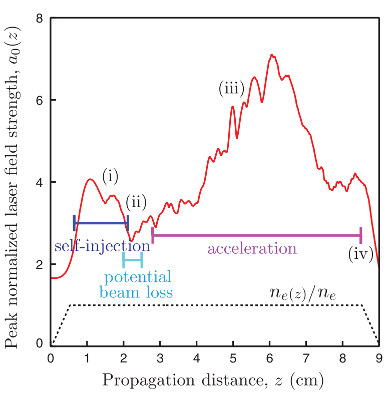


Figure 13: Evolution of the peak normalized intensity of the laser pulse, ($a_0(z)$ done using a particle-in-cell simulation for a top-hat laser pulse with energy 16 J, through a 9 cm plasma waveguide. *From Leemans et. al. 2014 [11]*

The benefits over the Texas strategy is that they gain signifigantly on energy conversion. Because they don't need to enter into the self-focusing regime, they can use much less powerful lasers, and compensate by accelerating the electrons over a larger distance. As well, there are some concerns in the field that the relativistic self-focusing process is too unstable to guide the beam over the necesariy distances[4], but some progress has been made to extend the distances that relativistic self-guiding can be used over[17].

4 Conclusion

In this review, we have highlighted two approaches for electron acceleration: the high-intensity, relativisticly guiding approach used by Texas, and the low-intensity, plasma-channel guided approached used by Berkeley. For further progress to be made in the field, there needs to be a better understanding of the evolution of the laser-plasma interaction and evolution. It is worth noting that the bubble regime– the regime used by all groups trying to achieve GeV acceleration– was first seen in simulations.

New imaging techniques at UT Texas, which allow in vivo imaging of the evolution of the laser and plasmon, as well as novel theorertical and computational approaches are the current frontier. The raw power exists to accelerate electrons to high GeV energies, all we need to do is control it.

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