Summer NSERC Research Project 2010-2011

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1 Introduction

This report aims to show that under certain circumstances, a neutral bose system can look like a charged, fermi system.

I will be showing circumstantial evidence, as well as definative results that show numerically show that similar behaviour is seen.

1.1 Problems

I will list the problems/confusions/ambiguities here.

1.1.1 Prefactor

The prefactor isn't correct. The kubo calculation that I am using looks like:

$$(\sigma^{xy})_\alpha = -iL^2\hbar \sum_{\beta\neq\alpha} \frac{<\alpha|j_x|\beta><\beta|j_y|\alpha> - <\alpha|j_y|\beta><\beta|j_x|\alpha>}{[E_\beta(t) - E_\alpha(t)]^2} \tag{1}$$

with the $\vec{j}$ equal to:

$$j_x = \sum_{h=0}^{Q-1} -2\sin(k_x^0 + 2\pi\frac{P}{Q}h)a(k_x^0, k_y)a^\dagger(k_x^0, k_y)$$

$$j_y = \sum_{h=0}^{Q-1} -i\exp(k_y)a(k_x - 2\pi\frac{P}{Q}(h-1), k_y)a^\dagger(k_x^0, k_y) + i\exp(ik_y)a(k_x + 2\pi\frac{P}{Q}(h+1), k_y)a^\dagger(k_x, k_y)$$

The actual code I used is here: The part that will calculate j_x:

```
!FORTRAN J_X CALCULATOR!!!!!!!!!!
do m = 1,Q
        if(xx(i,m) .ne. 0)THEN
        !interactions have been added
        Hx(i,k) = Hx(i,k) -1.0*dble(xx(i,m))*2.0*sin(kx+2.0*PI*
           dble(P)*dble(m)/dble(Q))
        ENDIF
ENDDO
```

Where xx(i,m) contains the prefactor due to the bose enhancement factor, P/Q is the magnetic flux, and Hx(i,k) is the matrix that will hold the representation of the j_x operator. This matrix will be diagonal. The matrix is also summed over i,k. So the code snippet above will calculate the j_x matrix element (i,k).

The part that calculates j_y:

```
!FORTRAN J_Y CALCULATOR!!!!!!!!!!!
!pseudo code: IF (CONDITIONS SUCH THAT i,k = i,i+1) THEN
Hy(i,k) = Hy(i,k)+J*prefac*exp(J*ky)
 !pseudo code: IF (CONDITIONS such that i,k = i,i-1) THEN
Hy(i,k) = Hy(i,k)-J*prefac*exp(-J*ky)
```

So, when those conditions are used, when I actually calculate the Kubo formula, Eq. (1) for a single alpha and then sum over all alphas, I get:

$$\sum_{\alpha} \frac{\sigma_{\alpha}^{xy}}{L^2\hbar} = M\frac{L^2Q}{2\pi} \tag{2}$$

where $M \in \mathbb{Z}$, it is the quantized kubo conductance that I am looking for. However, the code gives me this funny prefactor. So when I divide my results by L^2Q and multiply by 2π, I get the predicted integer value. Note: on the left hand side, I have $\frac{\sigma_{\alpha}^{xy}}{L^2\hbar}$ because my code actually only evaluates:

$$\sum_{\beta\neq\alpha} \frac{<\alpha|j_x|\beta><\beta|j_y|\alpha> - <\alpha|j_y|\beta><\beta|j_x|\alpha>}{[E_{\beta}(t) - E_{\alpha}(t)]^2}$$

1.2 To Do

2 Main Evidence

This section will be showing the evidence of a gap in the energy. Show the graphs that show the single particles occupation as a function of the temperature. Show that the system really behaves like a fermi system, and that a gap will always been seen to exist at low temperatures.

2.1 Energy Gaps

Figure 1: Intro figure showing the energy levels for the single particle. Link to appendix that shows how to use the code to generate this figure. Also include the derivation of the hamiltonian in the appendix.

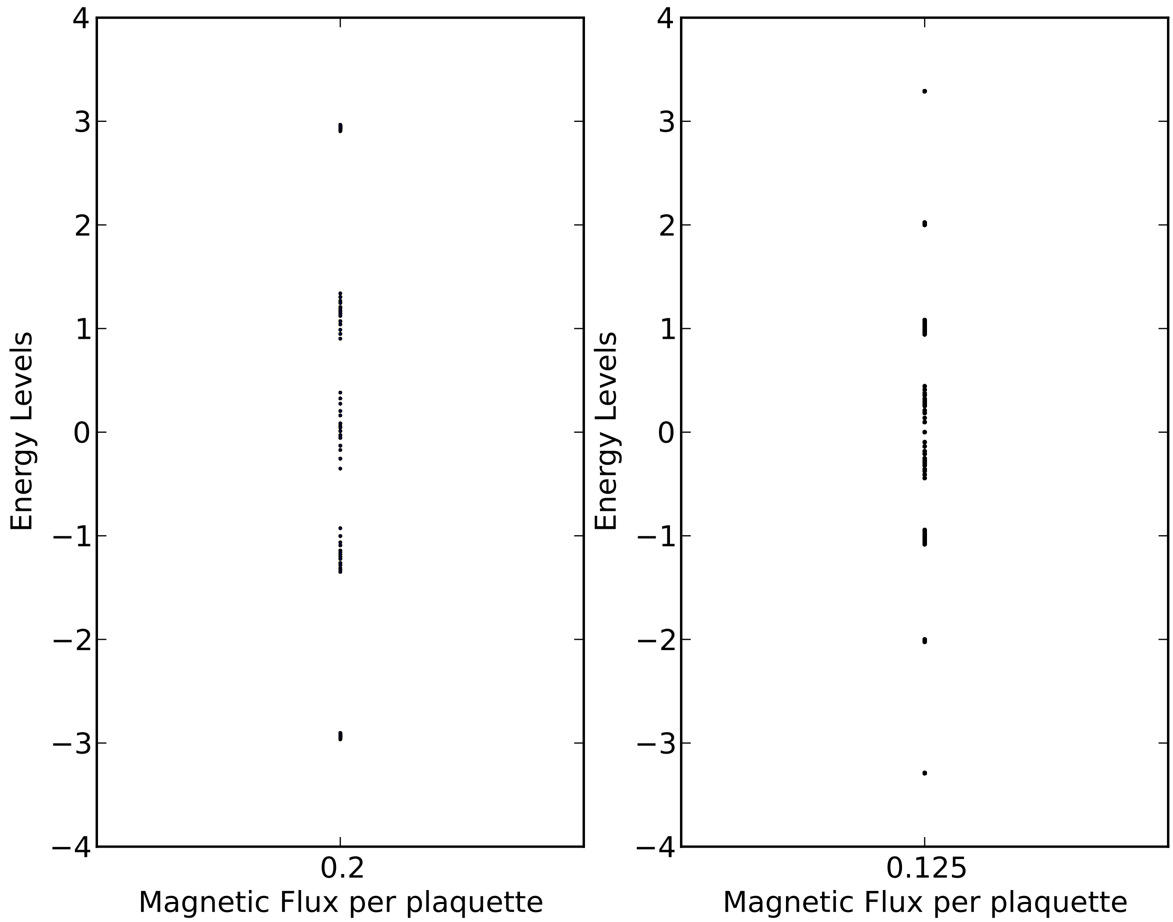

Figure 1: This shows the spectrum of a 2-D system of electrons in a periodic potential with a magentic field perpendicular to the plane of the system. I have isol the interesting cases where the Flux per plaquette is 1/5 and 1/8. These values both have nice structure, as they clearly have a gap between two of the lowest energy subbands.

Figure2: Show the occupation of the first and second band as a function of temperature. Show that for N=1, N=2, N=3, the system behaves exactly the same. And show that there exists a range of temperatures that illustrate that there is a small gap present at those low temperature.

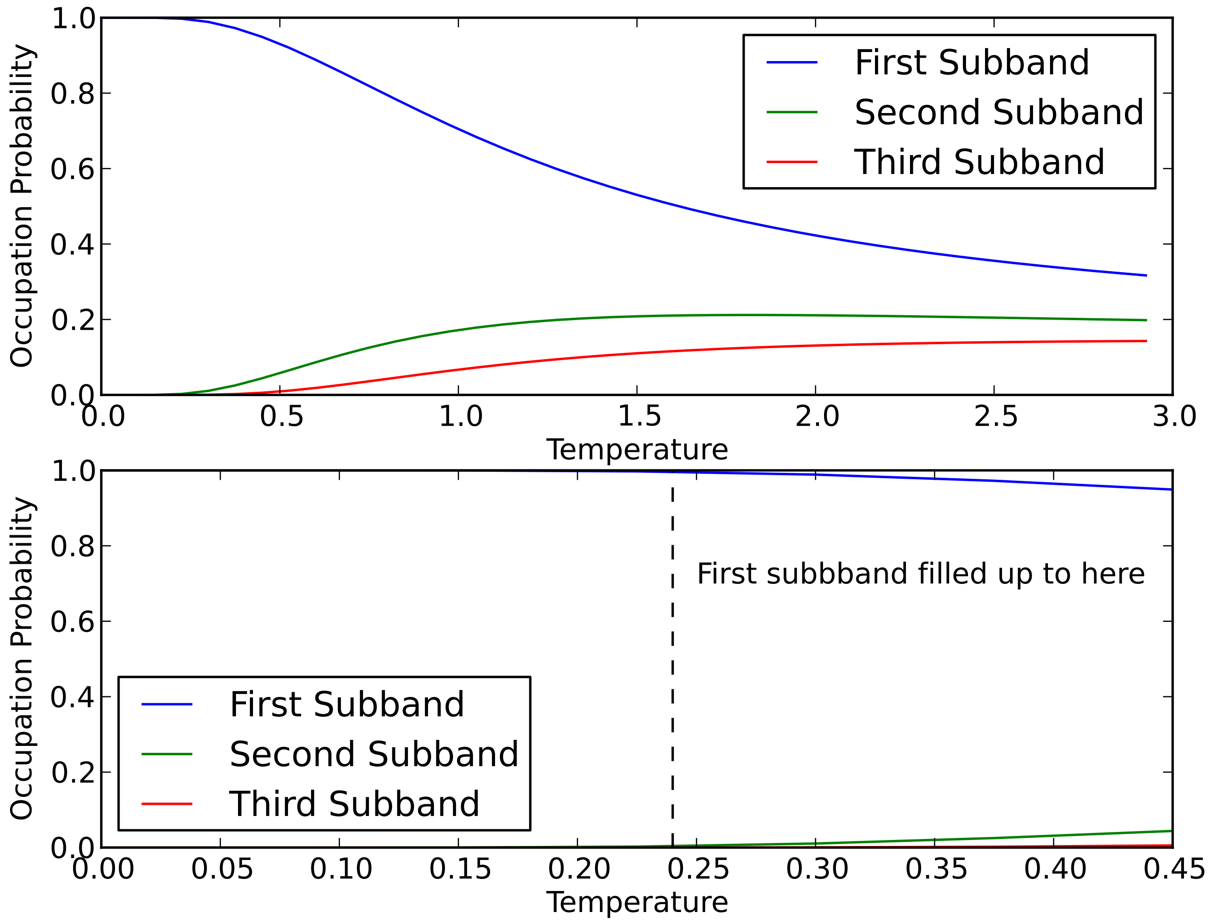

Figure 2: The occupation as a function of temperature. Note how the first band dominates the occupation factor until the dashed line. This is thought to be because the gap is large enough that the bosons cannot become thermally excited enough to occupy the second subband, but they have more then enough energy to equally thermally occupy the first band.

2.2 Problems

P:I am explicitly using a fixed number N of particles, but the bose distribution (which I use for the statistics) requires a grand canonical ensemble. A: Valid problem. Need to see how the occupation of the levels works in the grand canonical system. See if there are platueas in the average energy of the system when the temperature is increased. The specific heat of the system will also give indicition of energy gaps.

So, I have know shown evidence that a gap is present at low temperatures. Also, talk about how if interactions are off that a gap must always be present at low temperatures, just because of the way the single particles combine. However, the subbands will get bigger.

3 Hall Conductance for Fermions

I need to at least replicate the results for the fermion system with the code that I had generate. These results are shown below, as well as the scaling with different parameters in my system. These results correspond to the following formula:

$$\sigma_F = \frac{2\pi}{L^2 Q} f(\alpha) \sum_{\alpha} \sum_{\beta \neq \alpha} \frac{< \alpha|j_x|\beta >< \beta|j_y|\alpha > - < \alpha|j_y|\beta >< \beta|j_x|\alpha >}{[E_\beta(t) - E_\alpha(t)]^2} \tag{3}$$

with the $\vec{j}$ equal to:

$$j_x = \sum_{h=0}^{Q-1} -2\sin(k_x^0 + 2\pi \frac{P}{Q} h) a(k_x^0, k_y) a^\dagger(k_x^0, k_y)$$
$$j_y = \sum_{h=0}^{Q-1} -i\exp(k_y) a(k_x - 2\pi \frac{P}{Q}(h-1), k_y) a^\dagger(k_x^0, k_y) + i\exp(ik_y) a(k_x + 2\pi \frac{P}{Q}(h+1), k_y) a^\dagger(k_x, k_y)$$

and where $f(\alpha$ is the fermi occupation. Note, I wrote σ_F istead of the Hall current because I have several unaccounted for prefactors. This is the raw equation I used to generate the graphs.

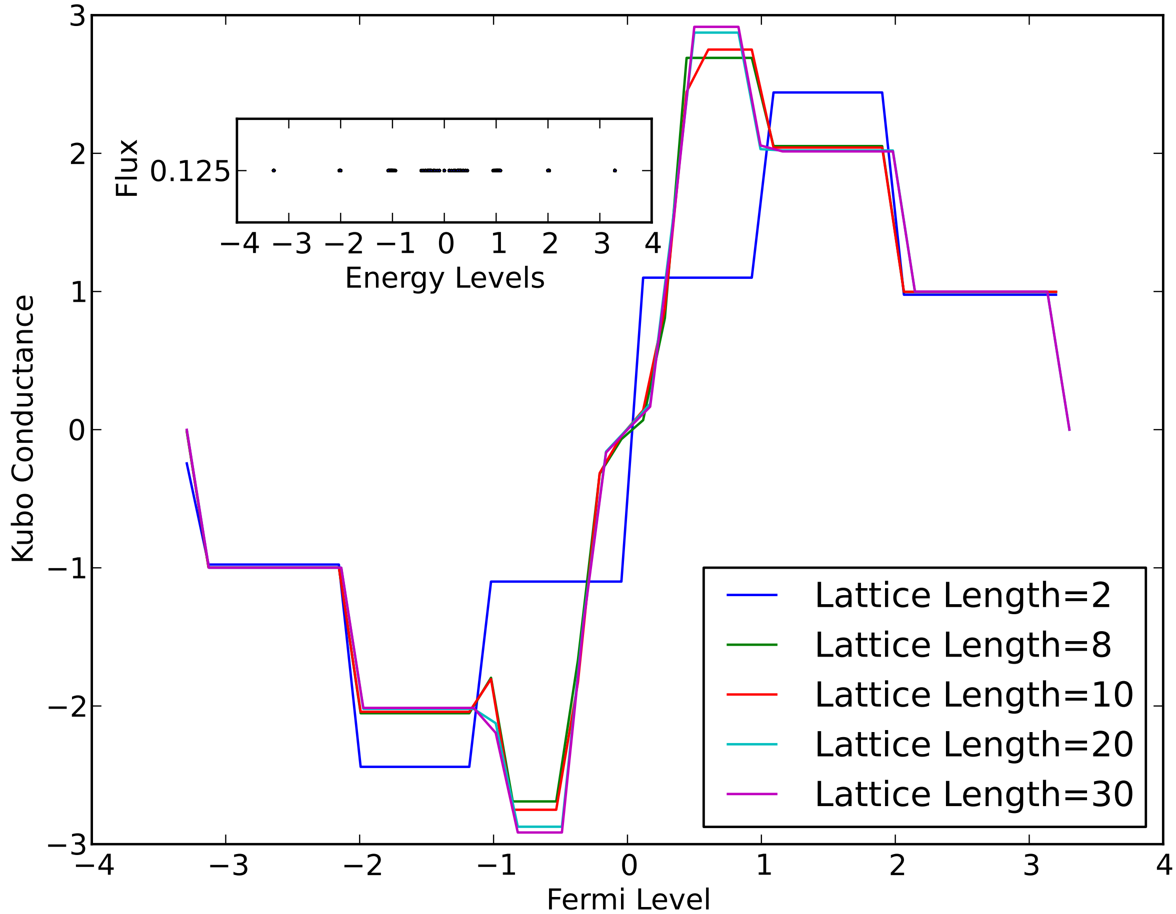

Figure 3: As the size of the lattice is increased, it seems that the system is converging to the quantum hall effect. The predicted values should be platuaus seen at (-1,-2,-3,3,2,1). The scaling behaviour seen can be explained by noting that the kubo formula is a cumulative sum over the contributions. Each subband should contribute -1 (at least for the first three platueas). Therefore, if the calculation of the kubo for each subband is independant, the third platuea should have 3*(K). If we treat K as a measured value, the error will be: $\Delta K_3 \approx 3K_1$. So we can predict that the program will need more precision in the subbands to obtain a value that converges to -3, simply because the error propogation involved in adding the contributions from the subbands together

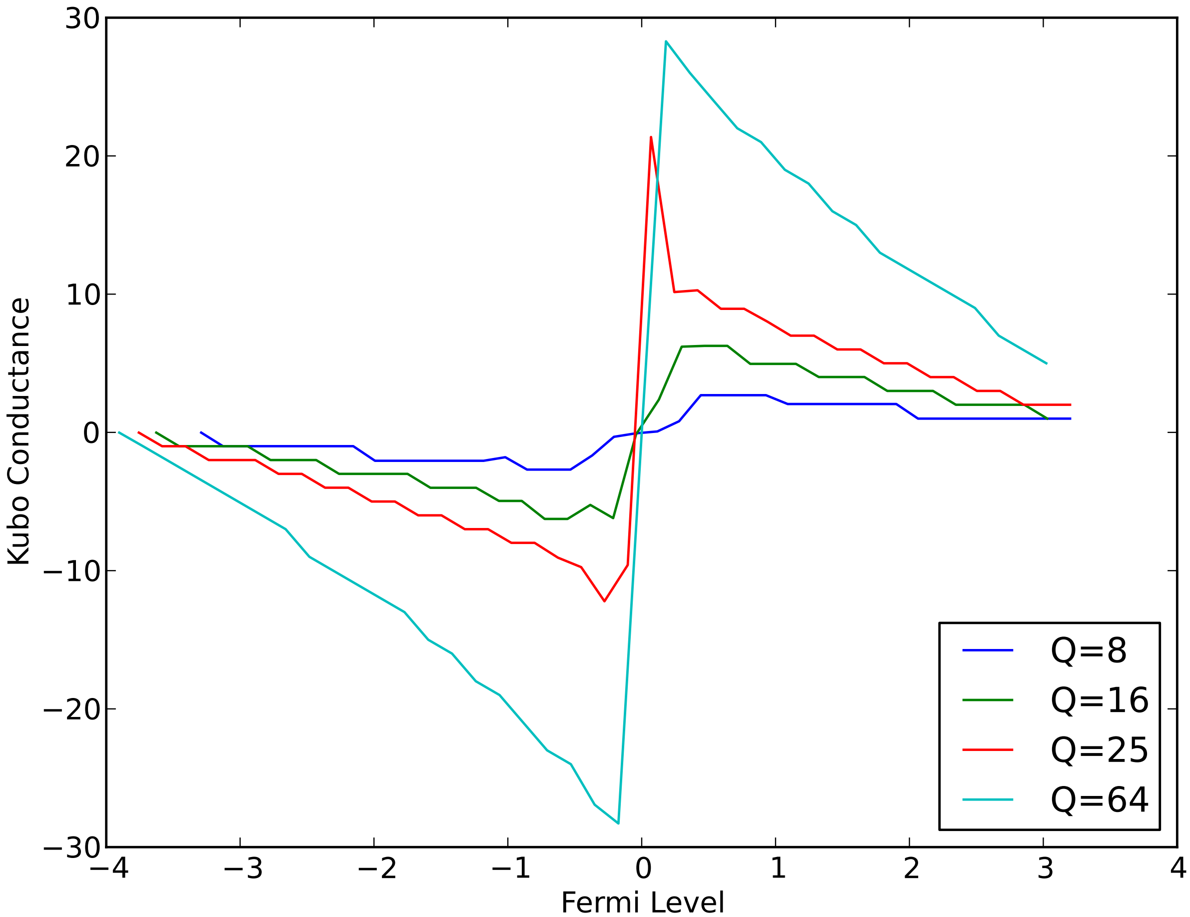

Figure 4: This figure isn't as important as Figure ??, but it is interesting to see what happens for different Q's. I tried out several different ones, and one can see quantization (platueas) for several other values. This seems promising, as being able to see quantization at lower magnetic fields is always good. Future work would be to apply the bose case to these lower magnetic fields and explore them more.

4 Bose QHE

After confirming that my code worked by replicating the QHE shown in Figures 4 and ??, I moved onto trying to replicate the conditions of the Fermi case, using bosons.

The idea is to have Boson's at finite temperature. Boson's at high temperature essentially act like Boltzmanon's: they will smear out over the energy levels equally. However, if the energy levels being smeared over are actually a subband, and if there is a high enough energy level gap to the next subband, then the bosons will smear out equally over the lwest subband, while note occupying the second one. This essentially mimics a Fermion system where each energy level in the lowest subband is filled, but no other energy level is occupied; ie. the fermi energy is a little above the first subband. Note, that to get a direct analogy, the bose density ($\frac{\text{N of boson's}}{N of states}$) has to be approximately equal to 1.

However, we see exactly the conditions described above in Figure 2. With this system, I used the kubo formula to calculate the hall conductance.

$$\sigma_B = \frac{2\pi}{L^2 Q}\frac{b(\alpha)}{N}\sum_{\alpha}\sum_{\beta\neq\alpha}\frac{<\alpha|j_x|\beta><\beta|j_y|\alpha> - <\alpha|j_y|\beta><\beta|j_x|\alpha>}{[E_\beta(t)-E_\alpha(t)]^2} \tag{4}$$

with the $\vec{j}$ equal to:

$$j_x = \sum_{h=0}^{Q-1} -2\sin(k_x^0 + 2\pi\frac{P}{Q}h)a(k_x^0,k_y)a^\dagger(k_x^0,k_y)$$
$$j_y = \sum_{h=0}^{Q-1} -i\exp(k_y)a(k_x - 2\pi\frac{P}{Q}(h-1),k_y)a^\dagger(k_x^0,k_y) + i\exp(ik_y)a(k_x + 2\pi\frac{P}{Q}(h+1),k_y)a^\dagger(k_x,k_y)$$

and where $b(\alpha$ is the bose occupation, and N is the number of particles. Note, I wrote σ_B istead of the Hall current because I have several unaccounted for prefactors. This is the raw equation I used to generate the graphs.

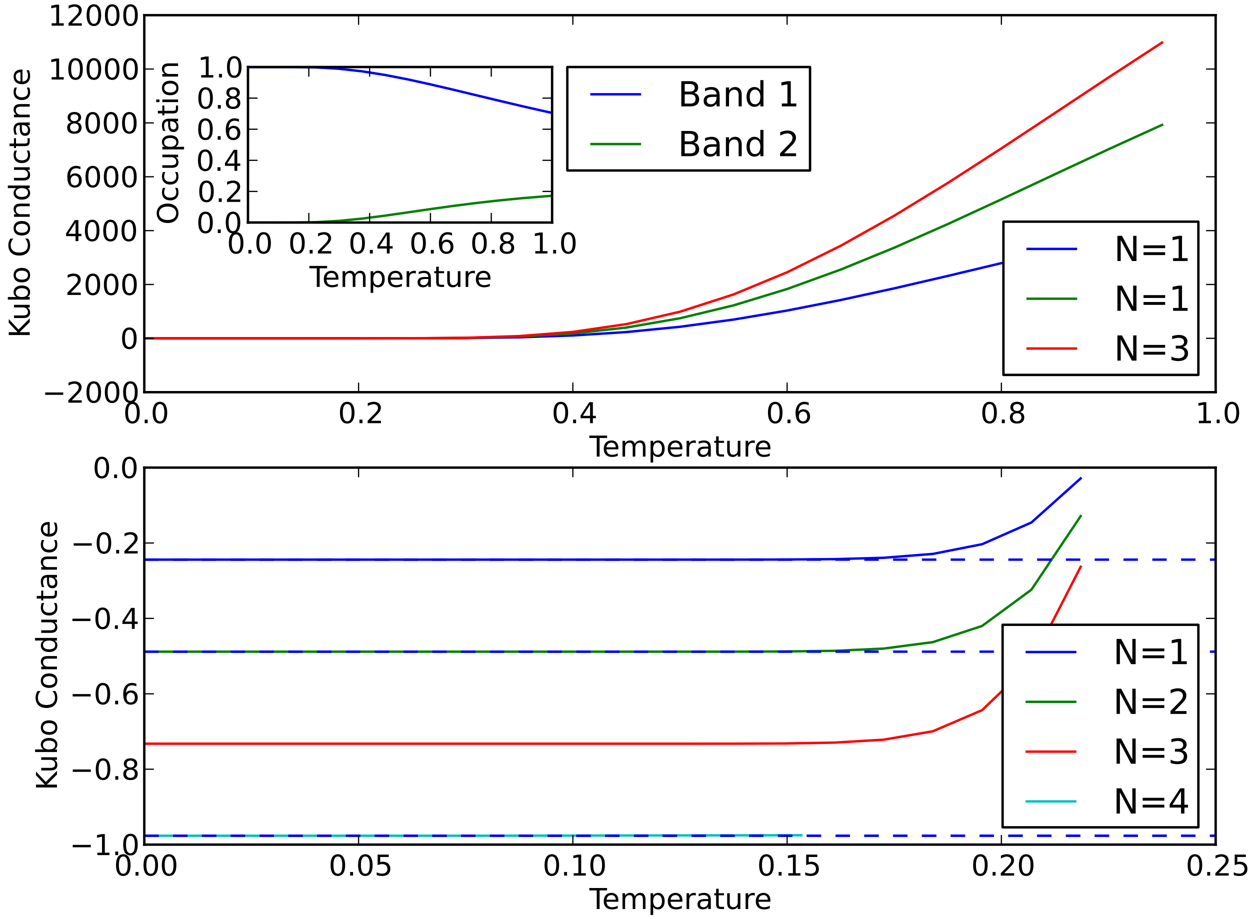

Figure 5: The first graph shows what happens at a large temperature range. The second graph zooms into low temperature, and, as you can see the hall effect is quantized. The dashed lines represent the hall conductance calculated for an equivalent N number of fermions. So the $N = 1$ boson and fermi line up, and so on. Because of a degeneracy in the energy levels, my code jumped from 2 fermions to 4, which is why there is no $N = 3$ fermion to compare to the boson. The inset shows the occupation of the first two subbands.

5 Derivations

5.1 Kubo Derivation (1)

The first way I derived the kubo formula was by using the argument from Mahan. First calculate the polarization operator, and then the particle current operator is the commutator of the polarization operator with Hamiltonian, multiplied by i.

$$\hat{j} = i[H, P]$$

P is given as:

$$\hat{P} = \vec{r}\hat{n}(x, y) \tag{5}$$

$$\hat{P} = \vec{r}a(\hat{x, y})a(\hat{x, y})^{\dagger} \tag{6}$$

and the Hamiltonian is:

$$\hat{H} = -\tau \sum_{i} \sum_{j=0}^{3} \hat{a}_i \hat{a}_{i+\delta_j} e^{i\Theta_j} \tag{7}$$

$$= -\tau \sum_{x} \sum_{y} \sum_{j=0}^{3} \hat{a}_{x,y} \hat{a}_{x+T_j, y+P_j} e^{iS(x)P_j} \quad \exists T = (-d, d, 0, 0), \exists P = (0, 0, -d, d), \exists S(x) = 2\pi \frac{P}{Q} x \tag{8}$$

The above notation is a bit confusing, but the rational is as follows. The j index indicts a hopping term. So the T vector is a hopping xvector, d is the lattice spacing. It can hop in the -1x, 1x directionsx. Now, the P vector is the same, but the hopping is in the y direction. (Sorry for the confusion, but $\hat{P} \neq P$, I wanted to be consistant with my labbook. However, the particle will gain phase only if hops in the y directions. So the P vector is also the phase vector. You gain phase only when you are hopping in the plus-y/minus-y directions.

Θ was also expanded out in terms of the prefactor $S(x)$ and the phase vector P, ie. $\Theta_j = S(x)P_j$.

Using these expressions for the Hamiltonian and the Polarization operator, Eq. (5.1) takes the form:

$$\hat{j} = i\left[\hat{H}, \hat{P}\right] \tag{9}$$

$$= -\tau \sum_{x,y,x',y'} \sum_{j=0}^{3} \left[\hat{a}^{\dagger}_{x,y}\hat{a}_{x+T_j,y+P_j}e^{iS(x)P_j}, \vec{r'}\hat{a}^{\dagger}_{x',y'}\hat{a}_{x',y'}\right] \tag{10}$$

Now using the following commutator identities:

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}], \qquad [\hat{A}\hat{B}, \hat{C}] = [\hat{A}, \hat{C}]\hat{B} + \hat{A}[\hat{C}, \hat{B}] \tag{11}$$

$$\tag{12}$$

we can expand out the current operator equation to obtain:

$$\hat{j} = -\tau \sum_{x,y,x',y'} \sum_{j=0}^{3} \sum_{x'} [\hat{a}^{\dagger}_{x,y}, \vec{r}\hat{a}^{\dagger}_{x',y'}\hat{a}_{x',y'}]\hat{a}_{x+T_j,y+T_j}e^{iS(x)P_j} + \hat{a}^{\dagger}_{x,y}[\hat{a}_{x+T_j,y+T_j}e^{iS(x)P_j}, \vec{r}\hat{a}^{\dagger}_{x',y'}\hat{a}_{x',y'}] \tag{13}$$

$$= -\tau \sum_{x,y,x',y'} \sum_{j=0}^{3} [\hat{a}^{\dagger}_{x,y}, \vec{r'}\hat{a}^{\dagger}_{x',y'}]\hat{a}_{x',y'}\hat{a}_{x+T_j,y+T_j}e^{iS(x)P_j} + \vec{r'}\hat{a}^{\dagger}_{x',y'}[\hat{a}_{x,y}, \hat{a}_{x',y'}]\hat{a}_{x+T_j,y+P_j}e^{iS(x)P_j} \tag{14}$$

$$+ \hat{a}^{\dagger}_{x,y}[\hat{a}_{x+T_j,y+P_j}e^{iS(x)P_j}, \vec{r'}\hat{a}^{\dagger}_{x',y'}]\hat{a}_{x',y'} + \hat{a}_{x,y}\vec{r'}\hat{a}^{\dagger}_{x',y'}[\hat{a}_{x+T_j,y+P_j}e^{iS(x)P_j}, \hat{a}_{x',y'}] \tag{15}$$

$$= 0 + \vec{r'}\hat{a}_{x',y'}(-\delta_{x,x'}\delta_{y,y'})\hat{a}_{x+T_j,y+P_j}e^{iiS(x)P_j} + \hat{a}_{x,y}(\delta_{x+T_j,x'}\delta_{y+P_j,y'})\hat{a}_{x',y'}e^{iS(x)P_j}\vec{r} + 0 \tag{16}$$

Summing over x' and y' gets rid of the deltas and leaves us with:

$$\hat{j} = -\tau \sum_{x,y} \sum_{j=0}^{3} \left(- \begin{pmatrix} x \\ y \end{pmatrix} \hat{a}^\dagger_{x,y} \hat{a}_{x+T_j,y+P_j} e^{iS(x)P_j} + \begin{pmatrix} x+T_j \\ y+P_j \end{pmatrix} \hat{a}^\dagger_{x,y} \hat{a}_{x+T_j,y+P_j} e^{iS(x)P_j} \right) \tag{17}$$

$$= -\tau \sum_{x,y} \sum_{j=0}^{3} \begin{pmatrix} T_j \\ P_j \end{pmatrix} \hat{a}^\dagger_{x,y} \hat{a}_{x+T_j,y+T_j} e^{iiS(x)P_j} \tag{18}$$

NOTE: I think I have made a mistake with the sign. I think the it is supposed to be a $+\tau$ when you originally solve for the j operator as the commutator. I think that is what happens in Mahan, it switches sign from the way the Hamiltonian is. Now, we use the fourier series to transform to k-space.

Anyway, the j current operator, using this setup, becomes:

$$\hat{j} = -i\tau \sum_{x,y} \sum_{j=0}^{3} \begin{pmatrix} T_j \\ P_j \end{pmatrix} \hat{a}i^\dagger_{x,y} \hat{a}_{x+T_j,y+T_j} e^{iS(x)P_j} \tag{19}$$

$$\tag{20}$$

Current Problems: The prefactor doesn't seem to work. I seem to be grossly overcounting it. If I am just using the Fourier transform, as I was originally going to do, then I will only have the $\sqrt{2\pi}$ which isn't what I want either.

Then we can use the discrete fourier transform. This is the key point!

The following are definitions of the terms that I will be using:

$$d := \text{spacing of crystal sites in position-space}$$

$$2N := \text{The number of crystal sites in position-space}$$

$$L := \text{The length of the lattice in position-space}$$

$$\therefore L = (2N)a$$

$$h := \text{rows of crystal lattice sites in k-space}$$

$$d := \text{columns of crystal lattice sites in k-space}$$

With these definitions, and using the discrete Fourier transform, I can write the anihilation operator at lattice point (n, m) in position-space:

$$c_{nm} = \frac{1}{(2N)^2} \sum_{h=0}^{2N-1} \sum_{b=0}^{2N-1} \exp\left[-\frac{2\pi i}{2N}(bm + hn)\right] c_{h,b}$$

$$c^{\dagger}_{n+T_j/d,m+P_j/d} = \frac{1}{(2N)^2} \sum_{h=0}^{2N-1} \sum_{b=0}^{2N-1} \exp\left[-\frac{2\pi i}{2N}(b(m + P_j/d) + h(n + T_j/d))\right] c^{\dagger}_{h,b}$$

The current operator becomes:

$$\hat{j} = i\tau sum_{n,m=0}^{2N-1} \sum_{j=0}^{3} \begin{pmatrix} T_j \\ P_j \end{pmatrix} \frac{1}{(2N)^4} \sum_{h,h'=0}^{2N-1} \sum_{b,b'=0}^{2N-1} a^{\dagger}_{h,b} e^{-\frac{2\pi i}{2N} bm} e^{-\frac{2\pi i}{2N} hn} a_{h',b'} e^{\frac{2\pi i}{2N} b'(m+P_j/d)} e^{\frac{2\pi i}{2N} h'(n+T_j/d)} e^{iS(n)P_j} \tag{21}$$

$$\hat{j} = -i\tau \sum_{m,n}^{2N-1} \sum_{j=0}^{3} \begin{pmatrix} T_j \\ P_j \end{pmatrix} \frac{1}{(2N)^4} \sum_{h,h'=0}^{2N-1} \sum_{b,b'=0}^{2N-1} a_{h,b} e^{\frac{2\pi i}{2N} b' P_j / d} e^{\frac{2\pi i}{2N} h' T_j / d} e^{-\frac{2\pi i}{2N}(b-b')m} e^{-\frac{2\pi i}{2N}(h-h'-S(n)P_j n)n} a^{\dagger}_{h',b'} \tag{22}$$

We now expand out the sum over j, as there will be multiple times when the orthogonality condition used by the discrete Fourier transform is met:

$$\hat{j} = -i\tau \sum_{m,n}^{2N-1} \frac{1}{(2N)^4} \sum_{h,h'=0}^{2N-1} \sum_{b,b'=0}^{2N-1} \left(\begin{pmatrix} -d \\ 0 \end{pmatrix} e^{-h' \frac{2\pi i}{2N}} e^{-\frac{2\pi i}{2N}(b-b')m} e^{-\frac{2\pi i}{2N}(h-h')n} a^{\dagger}_{h,b} a_{h',b'} \right.$$
$$+ \begin{pmatrix} d \\ 0 \end{pmatrix} e^{h' \frac{2\pi i}{2N}} e^{-\frac{2\pi i}{2N}(b-b')m} e^{-\frac{2\pi i}{2N}(h-h')n} a^{\dagger}_{h,b} a_{h',b'} + \begin{pmatrix} 0 \\ -d \end{pmatrix} e^{-b' \frac{2\pi i}{2N}} e^{-\frac{2\pi i}{2N}(b-b')m} e^{-\frac{2\pi i}{2N}(h-h'+2\pi P/Q)n} a^{\dagger}_{h,b} a_{h',b'}$$
$$\left. + \begin{pmatrix} 0 \\ d \end{pmatrix} e^{b' frac2\pi i2N'} e^{-\frac{2\pi i}{2N}(b-b')m} e^{-\frac{2\pi i}{2N}(h-h'-2\pi P/Q)n} a^{\dagger}_{h,b} a_{h',b'} \right) \tag{23}$$

Summing over the m (y direction) will present no problem. Realizing that $\sum_{m}^{2N-1} e^{\frac{2\pi i}{2N}(b-b')m}$ will only be equal to one if $b = b' \bmod 2\pi$, shown below:

Let r be defined as follows:

$$r = \exp[\frac{2\pi i}{2N}(b-b')]$$

If b=b', then r =1 and the sum over r will give: $\sum_{m=0}^{2N-1} r^m = 2N$

If b $\neq$ b' then, using induction:

$$p = \sum_{m=0}^{2N-1} r^m = 1 + r + r^2 + ...r^{2N-1}$$

$$rp = r + r^2 + ... + r^{2N} = p + r^{2N} - 1$$

$$\therefore rp - p = r^{2N} - 1, \qquad \text{factoring and dividing gives:}$$

$$p = \frac{r^{2N}-1}{r-1}\text{But, } r^{2N} = \exp[2\pi i(b-b')m] = 1, \ \therefore p = 0$$

So this gives the unscaled orthogonality condition.

We can now use this orthogonality condition and sum over n and m to reduce our sum: first summing over m gives:

$$\hat{j} = -i\tau \frac{1}{(2N)^2} \sum_{h,h'=0}^{2N-1} \sum_{b,b'=0}^{2N-1} \left(\begin{pmatrix} -d \\ 0 \end{pmatrix} e^{-h' \frac{2\pi i}{2N}'} e^{\dagger}_{h,b} \delta_{b,b'} \delta_{h,h'} a_{h',b'} + \begin{pmatrix} d \\ 0 \end{pmatrix} e^{h' \frac{2\pi i}{2N}'} a^{\dagger}_{h,b} \delta_{b,b'} \delta h, h' a_{h',b'} \right.$$
$$\left. + \begin{pmatrix} 0 \\ -d \end{pmatrix} e^{-b' \frac{2\pi i}{2N}'} a^{\dagger}_{h,b} \delta_{b,b'} \delta_{h,h'+2\pi P/Q} a_{h',b'} + \begin{pmatrix} 0 \\ d \end{pmatrix} e^{b' \frac{2\pi i}{2N}'} a^{\dagger}_{h,b} \delta_{b-b'} \delta_{h,h'-2\pi P/Q)n} a_{h',b'} \right) \quad (24)$$

Therefore, we can transform the current operator to be in terms of k's

$$\hat{j} = -\tau \frac{1}{(2N)^2} \sum_{h=0}^{2N-1} \sum_{b=0}^{2N-1} \left(\begin{pmatrix} d \\ 0 \end{pmatrix} - 2\sin(-h'\frac{2\pi i}{2N}) a^{\dagger}_{h,b} a_{h,b} + \begin{pmatrix} 0 \\ -d \end{pmatrix} ie^{-b'\frac{2\pi i}{2N}} a^{\dagger}_{h-2\pi P/Q,b} a_{h,b} \right.$$
$$\left. + \begin{pmatrix} 0 \\ d \end{pmatrix} ie^{b'\frac{2\pi i}{2N}} a^{\dagger}_{h+2\pi P/Q,b} a_{h,b} \right) \quad (25)$$

Note, I summed over h in the above, so I was left over with h'. You can rename the dummy variable to h', but I wanted to be pedantic. We can further transform this into a sum over k, by realizing that we can define k in terms of the integers:

$$k_h = \frac{2\pi h}{QL}, \qquad k_b = \frac{2\pi b}{L}$$

Therefore:

$$h = \frac{k_h QL}{2\pi}, \qquad b = \frac{k_b L}{2\pi}$$

Also,

$$2N * d = L$$

$$\hat{j} = -\tau \frac{1}{L^2} \sum_{k_h=0}^{\frac{2\pi(2N-1)}{QL}} \sum_{k_b=0}^{\frac{2\pi(2N-1)}{L}} \left(\begin{pmatrix} d \\ 0 \end{pmatrix} - 2sin(k_x) a^{\dagger}_{h,b} a_{h,b} \begin{pmatrix} 0 \\ -d \end{pmatrix} ie^{-k_b} a^{\dagger}_{k_h-2\pi P/Q,k_b} a_{k_h,k_b} \right.$$
$$\left. + \begin{pmatrix} 0 \\ d \end{pmatrix} ie^{k_b} a^{\dagger}_{k_h+2\pi P/Q,k_b} a_{k_h,k_b} \right) \quad (26)$$

Changing the bounds of summation, this becomes more familiar:

$$\hat{j} = -\tau \frac{1}{L^2}\frac{1}{L^2}\sum_{k_h=\frac{-\pi}{Q}}^{\frac{\pi}{Q}} \left(\begin{pmatrix} d \\ 0 \end{pmatrix} - 2sin(k_x) a^{\dagger}_{h,b} a_{h,b} \begin{pmatrix} 0 \\ -d \end{pmatrix} ie^{-k_b} a^{\dagger}_{k_h-2\pi P/Q,k_b} a_{k_h,k_b} \right.$$
$$\left. + \begin{pmatrix} 0 \\ d \end{pmatrix} ie^{k_b} a^{\dagger}_{k_h+2\pi P/Q,k_b} a_{k_h,k_b} \right) \quad (27)$$

NOTE: I think I have made a mistake with the sign. I think the it is supposed to be a $+\tau$ when you originally solve for the j operator as the commutator. I think that is what happens in Mahan, it switches sign from the way the Hamiltonian is.

However, while this is my best shot at the derivation, it still doesn't give the right prefactors, so it is probably wrong.

6 Conclusion

: Show that regardless of prefactor, the bose case resembles the fermi case at low temperatures. Think about why it is that N=2 is the best one. Talk about how the scaling has to be right. You need the bose density to be right. This means having a high particle number and a low system size. You want the number of bosons in an energy level to be about the number of energy levels present in the subband. This ensures that the band 'looks' all filled up.

You can see a Quantum Hall effect using boson's at finite temperature.