Pulse Efficient Quantum Circuits

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Quantum Computing Application Researcher

Qiskit

Overview

- Calibrating Single and Two Qubit Gates – Rabi Oscillations and Getting a CNOT from the Cross Resonance
- Circuit Transpilation – Understanding Differences from Gate vs Pulse Perspectives
- Continuous Gate Sets – Reducing Circuit Depth by Scaling the Cross Resonance Interaction

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An X gate is Calibrated from Rabi Oscillations

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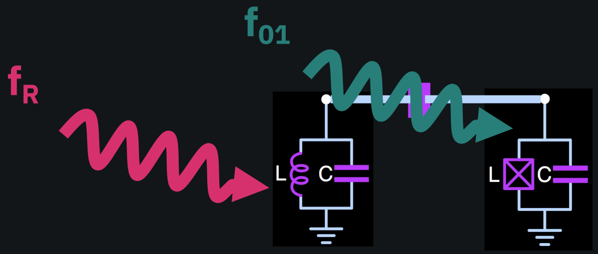
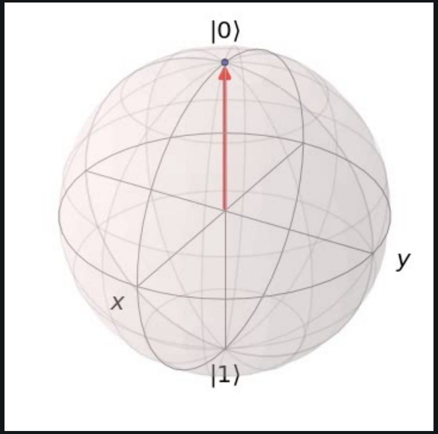
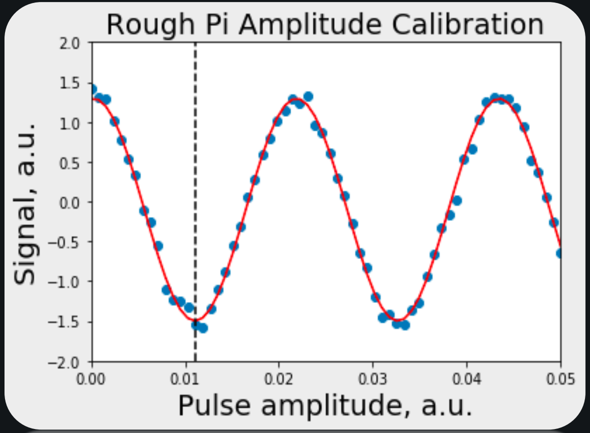
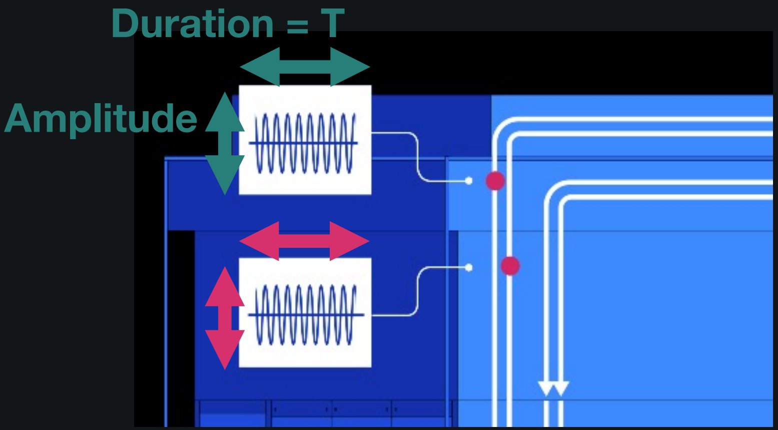


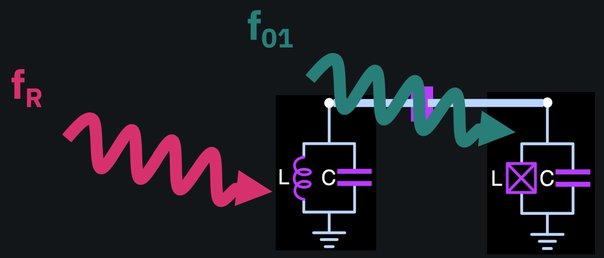


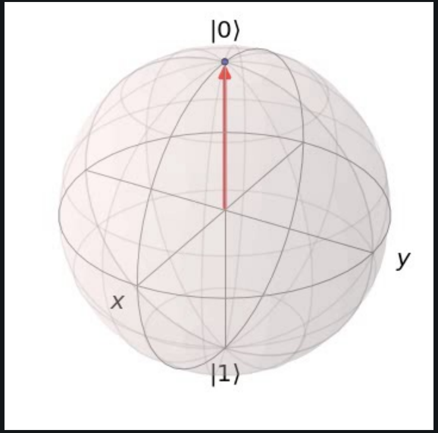


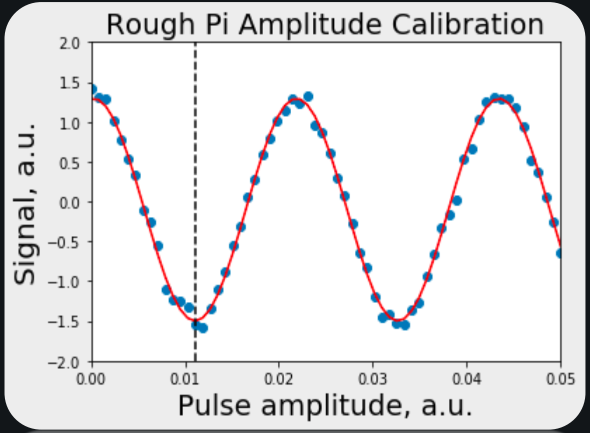


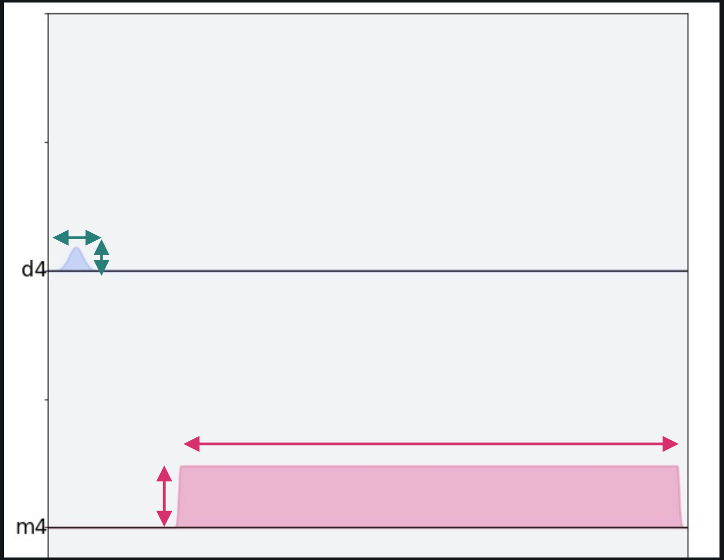
X Gate ⟷ π-pulse measured from Rabi oscillation

Control Pulses Can Have Different Shapes



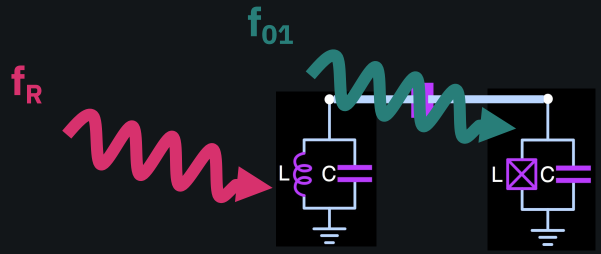


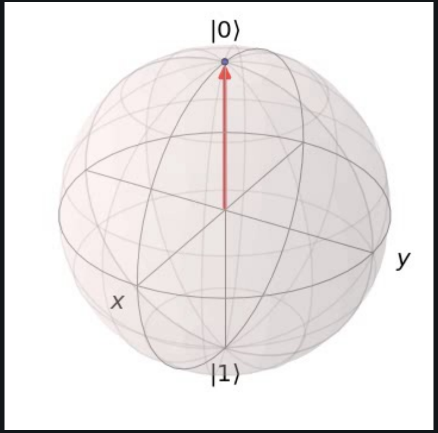


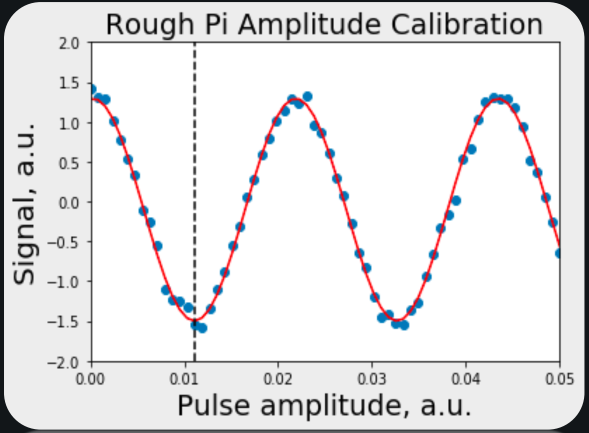


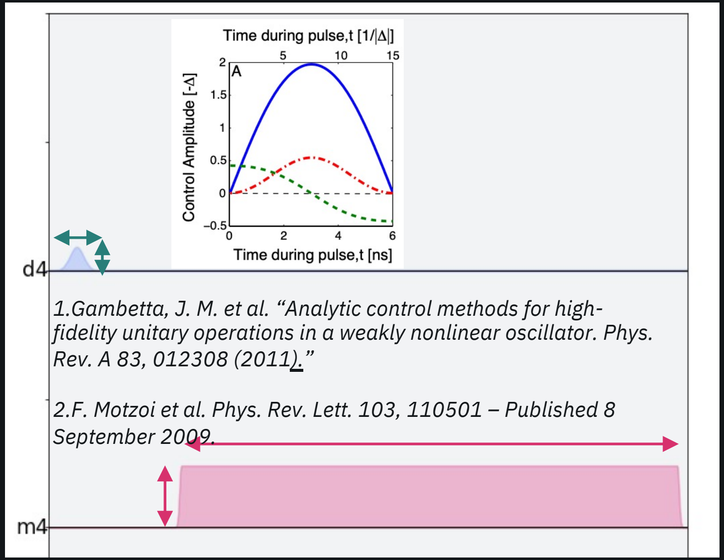
X Gate ⟷ π-pulse measured from Rabi oscillation

The Derivative Removal by Adiabatic Gate (DRAG) Reduces Leakage Errors









X Gate

π-pulse measured from Rabi oscillation

Rabi Oscillation is a Rough Amplitude Calibration: Repeated Gates to Get Amplify Error

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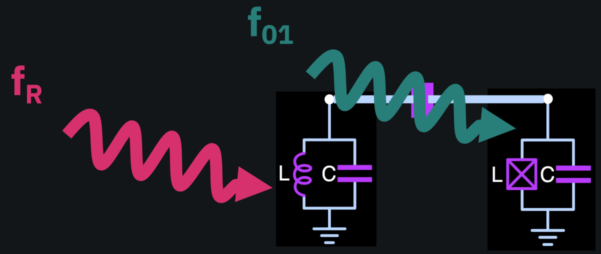

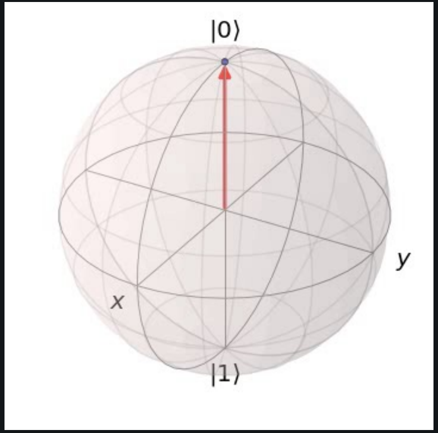

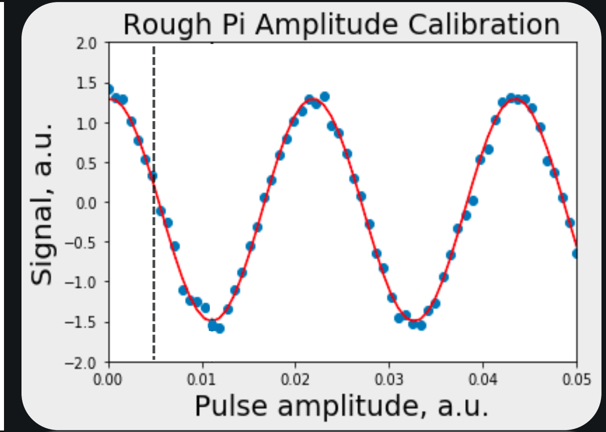

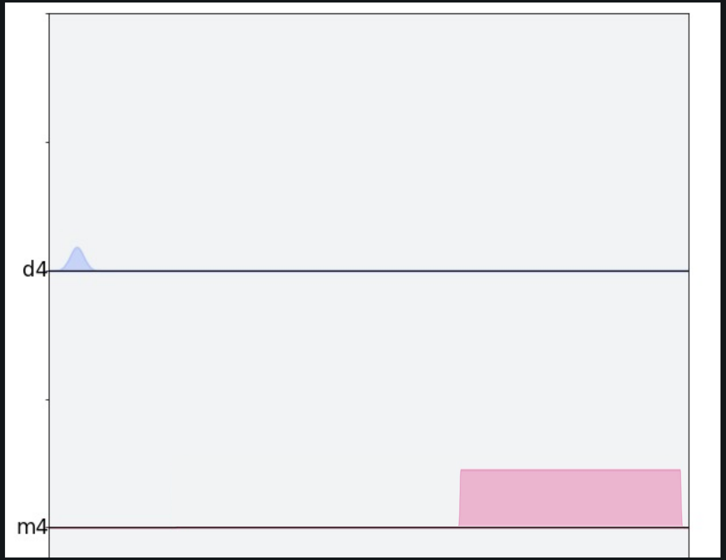

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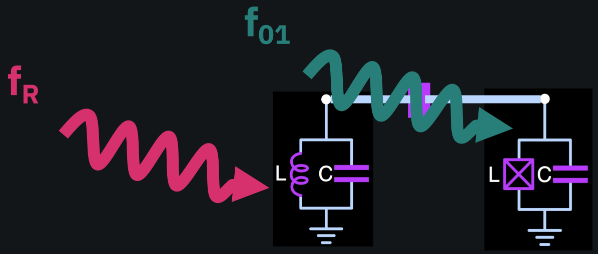

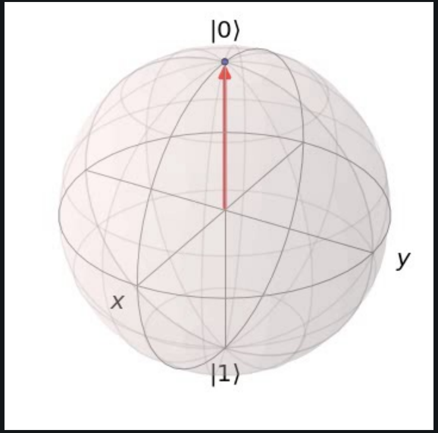

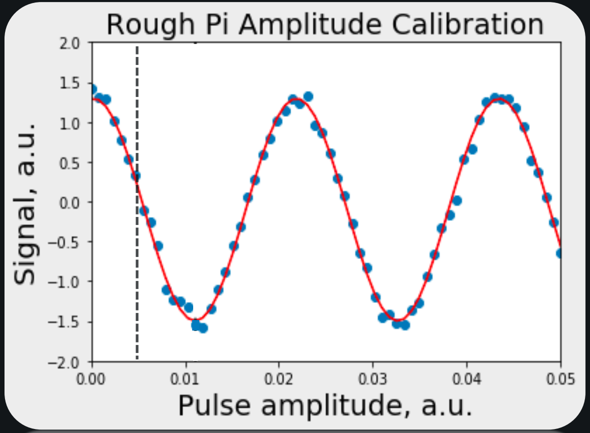

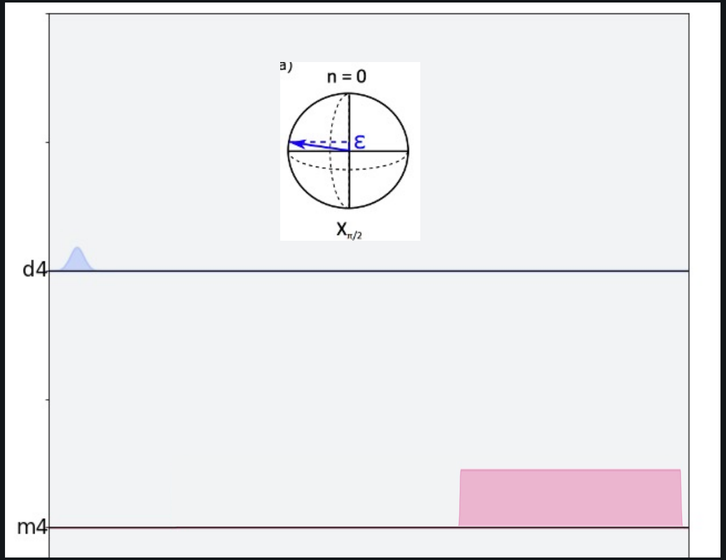

X Gate ⟷ π-pulse measured from Rabi oscillation

Sheldon, S. et al"Characterizing errors on qubit operations via iterative randomized benchmarking." *Physical Review A* 93, no. 1 (2016): 012301.

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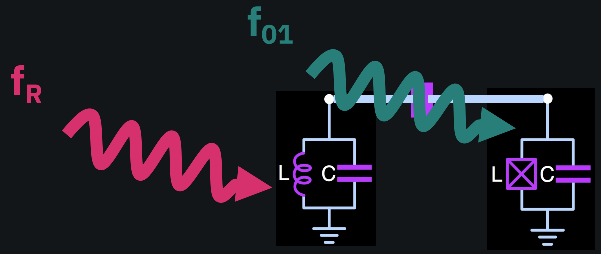

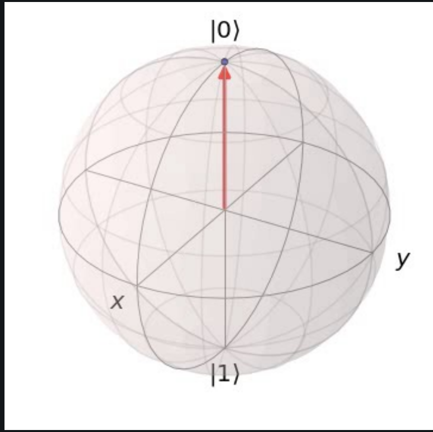

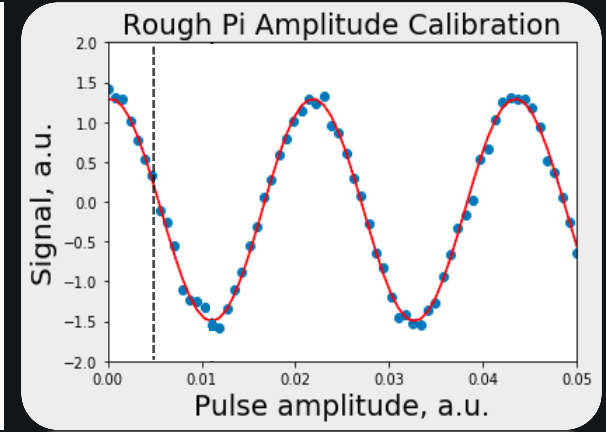

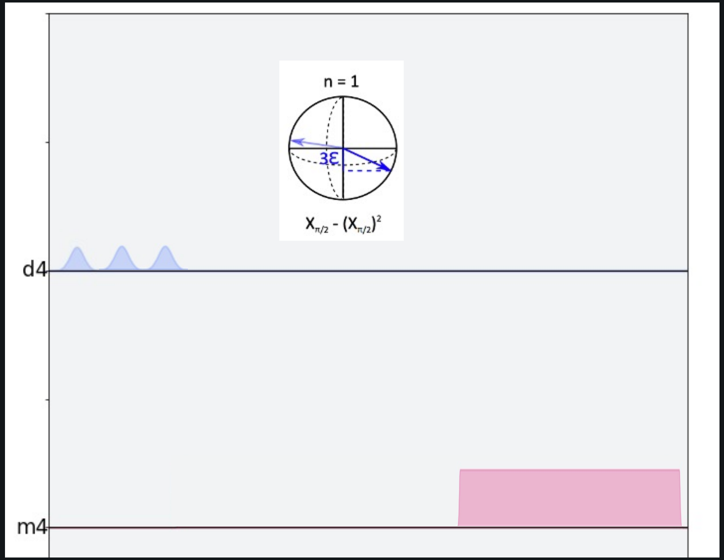

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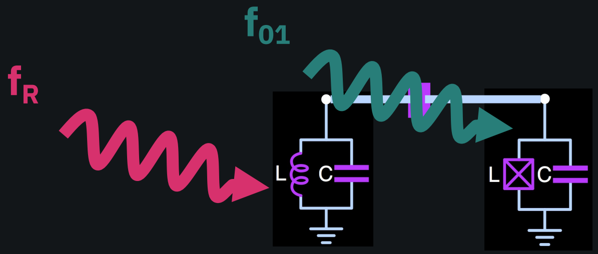

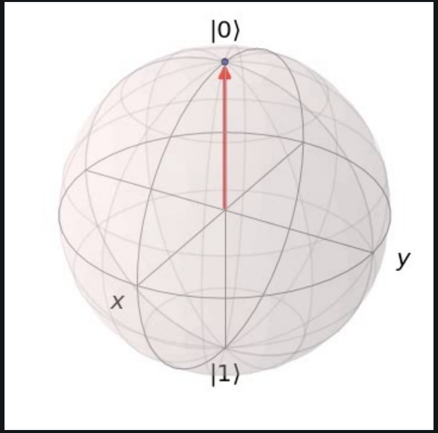

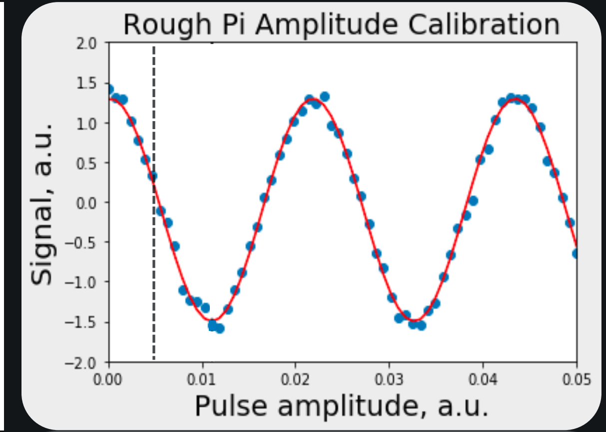

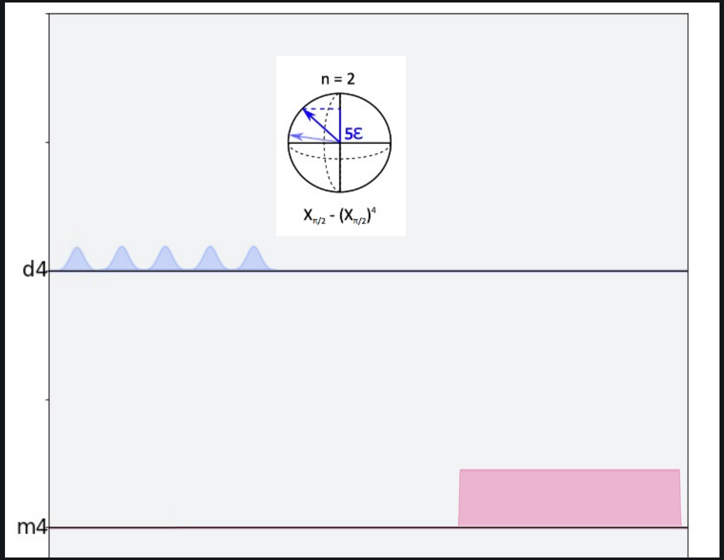

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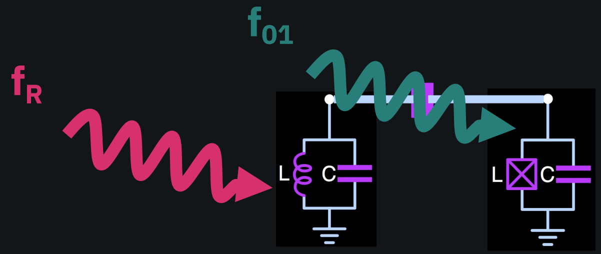

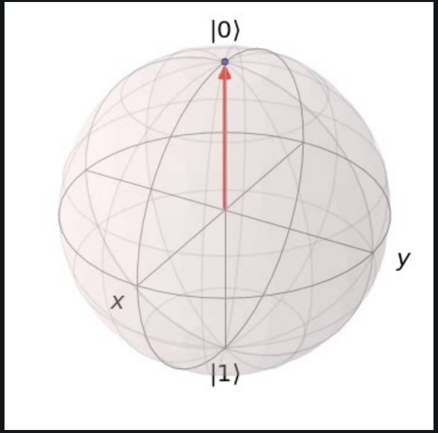

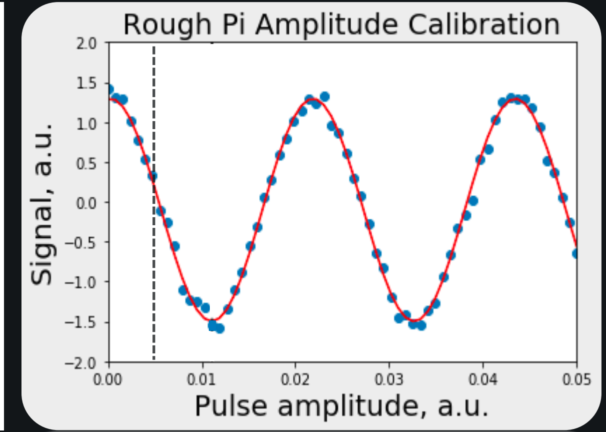

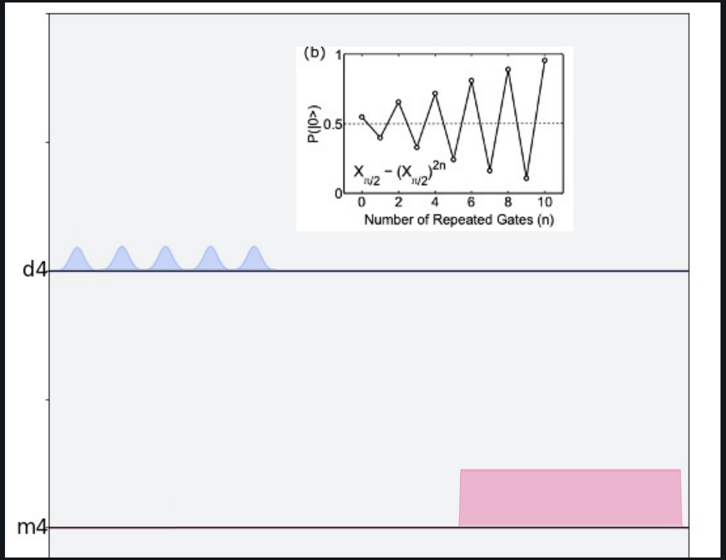

X Gate ↔ π-pulse measured from Rabi oscillation

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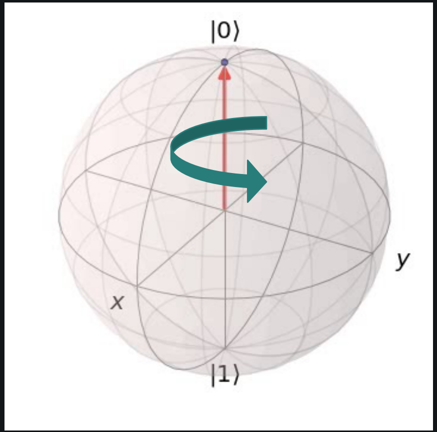
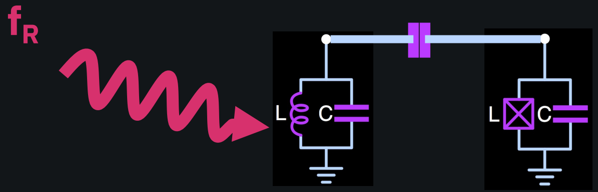
The Z Gate is Done through Control Electronic Phase

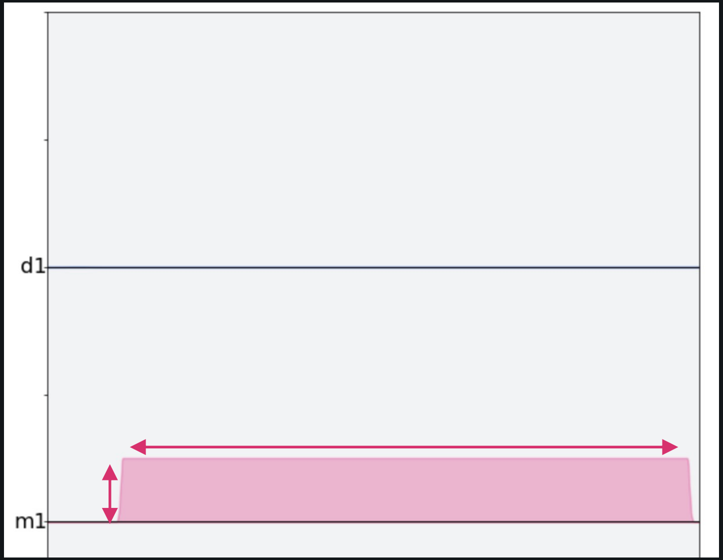
In lab frame of reference:

$$U = Exp[-i\,\Omega T(\cos(\gamma)\sigma_X + \sin(\gamma)\sigma_Y)]$$

$\gamma \longleftrightarrow \sigma_Z$







Z Gate $\longleftrightarrow$ Gate established by phase adjustment in classical electronics

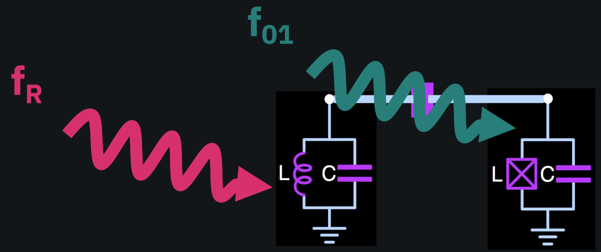
McKay, D. C. et al "Efficient Z gates for quantum computing." *Physical Review A* 96, no. 2 (2017): 022330.

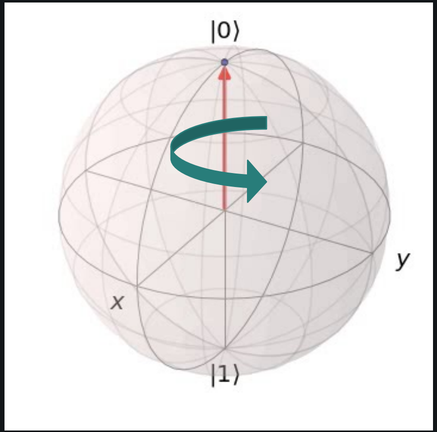
With the X and VZ gates, one Can Prepare Arbitrary 1Q States

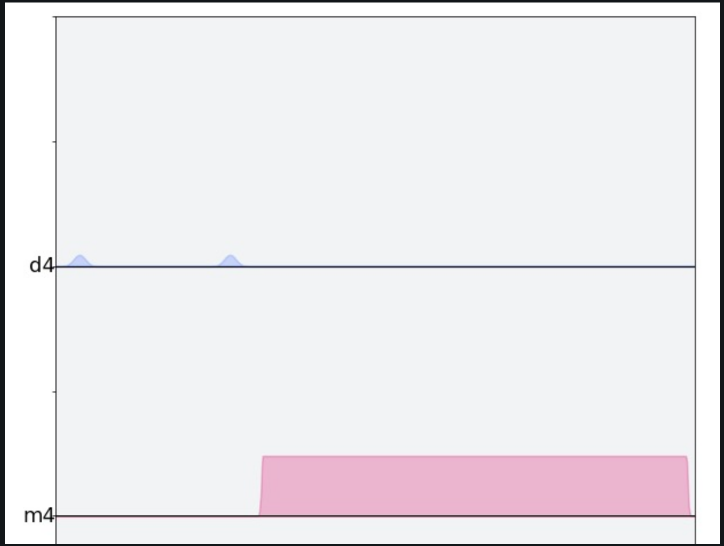
In lab frame of reference:

$$U = Exp[-i\,\Omega T(\cos(\gamma)\sigma_X + \sin(\gamma)\sigma_Y)]$$

$$\gamma \longleftrightarrow \sigma_Z$$







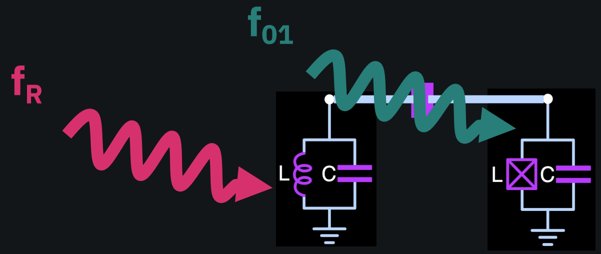
Z + X Gates allows for arbitrary single qubit rotations

McKay, D. C. et al "Efficient Z gates for quantum computing." *Physical Review A* 96, no. 2 (2017): 022330.

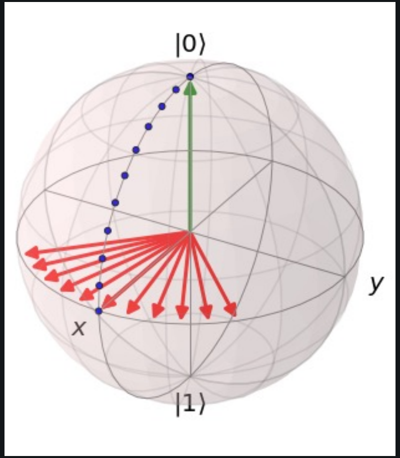
Dynamical Decoupling: Improve Qubit Coherence with Echos

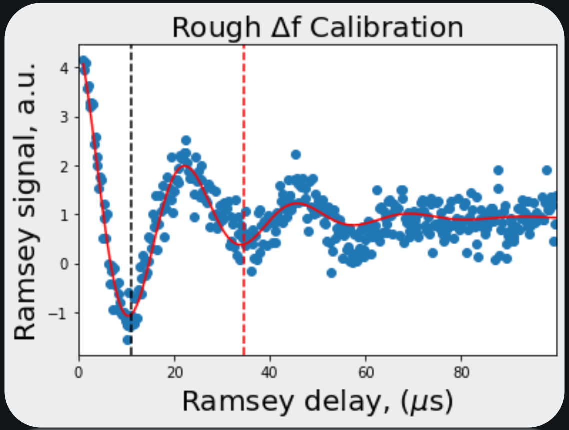
In lab frame of reference:

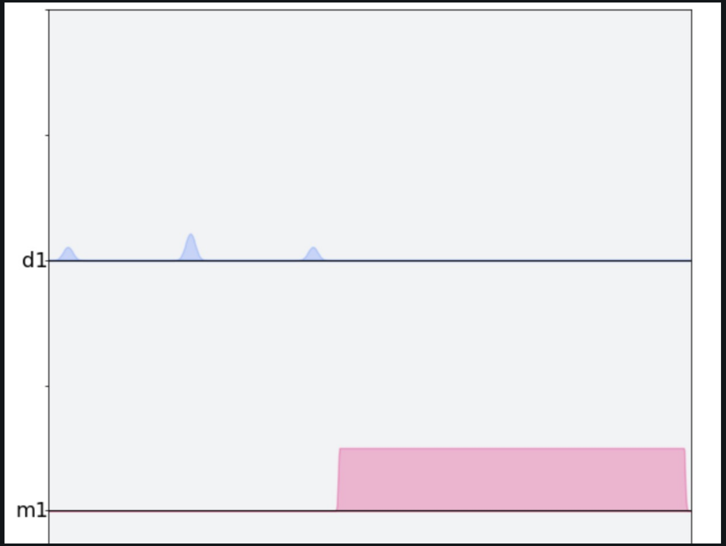
$$U = Exp[-i\,\Omega T(\cos(\gamma)\sigma_X + \sin(\gamma)\sigma_Y)]$$



$\gamma \longleftrightarrow \sigma_Z$







$\pi/2$-pulse + π-pulse result in qubit echo

McKay, D. C. et al "Efficient Z gates for quantum computing." *Physical Review A* 96, no. 2 (2017): 022330.

IBM Backends Use the Cross Resonance Gate

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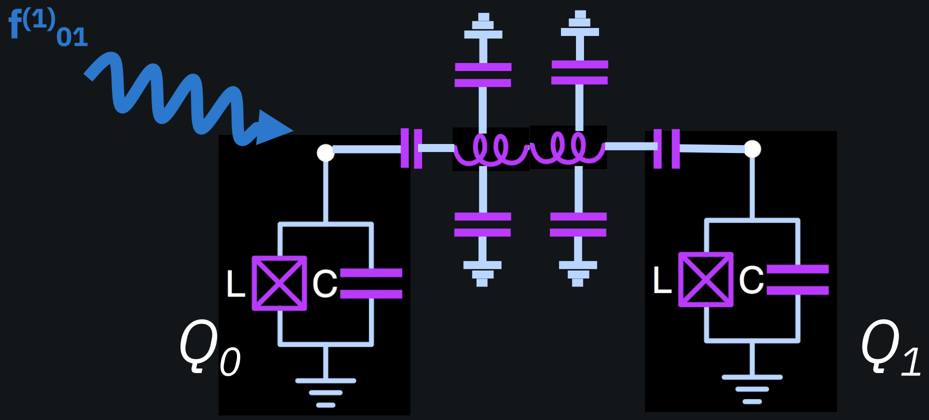
$$\psi = |00\rangle \longrightarrow CXH|\psi\rangle = |\psi'\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$H_D = \hbar\epsilon(t)\left[ZI - \nu_1 IX - \frac{J}{\Delta_{01}}ZX\right]$$



[J.M. Chow et al, *PRL* **107**, 080502 (2011)]

The Cross Resonance Interaction Is Not a CNOT Straight Out of the Box

Qiskit

Using:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

One can show:

$$U\big(ZX(\theta)\big) = e^{-i\frac{\theta}{2}ZX} = \begin{bmatrix} \begin{matrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{matrix} \end{bmatrix}$$

The Cross Resonance Interaction Is Not a CNOT Straight Out of the Box

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Using:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

One can show:

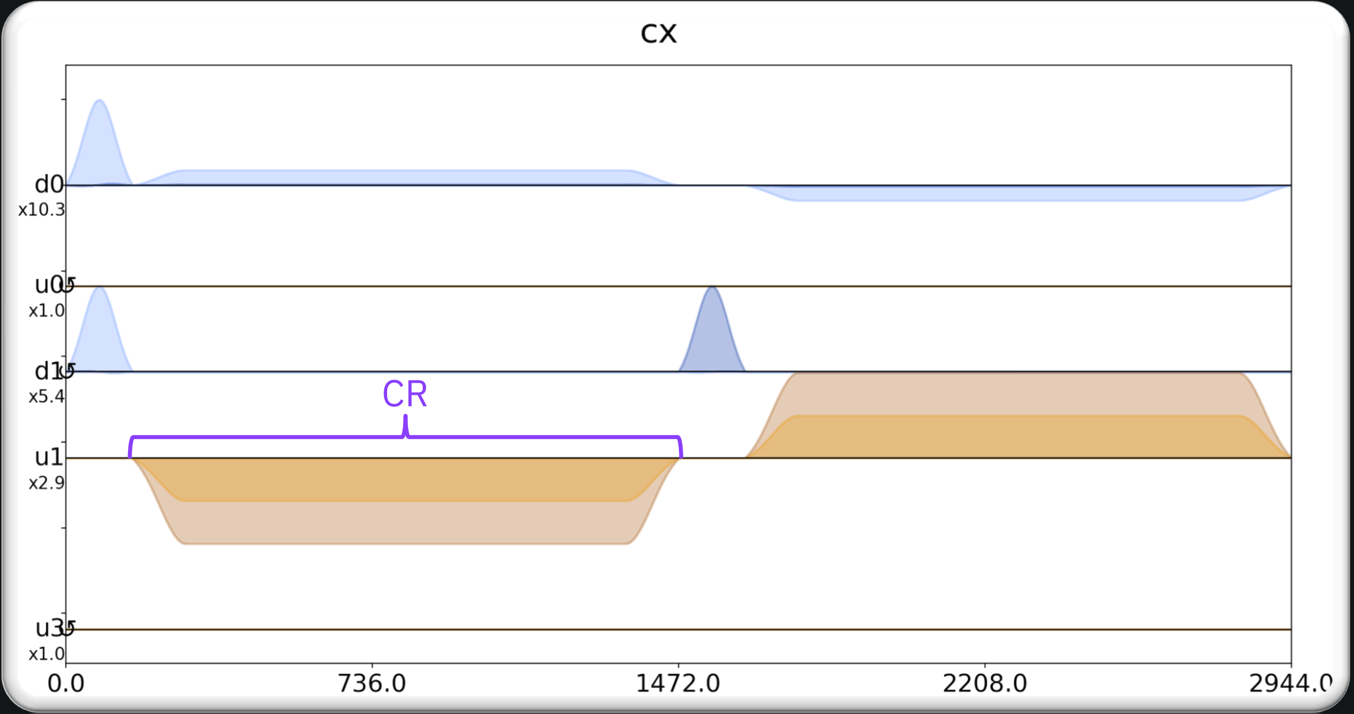
$$U\big(ZX(\pi/2)\big) = e^{-i\frac{\pi}{4}ZX} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -i\frac{\sqrt{2}}{2} & 0 & 0 \\ -i\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} & -i\frac{\sqrt{2}}{2} \\ 0 & 0 & -i\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

The CR interaction itself, does not give the desired CNOT – BUT…

The Cross Resonance Interaction Is Not a CNOT Straight Out of the Box

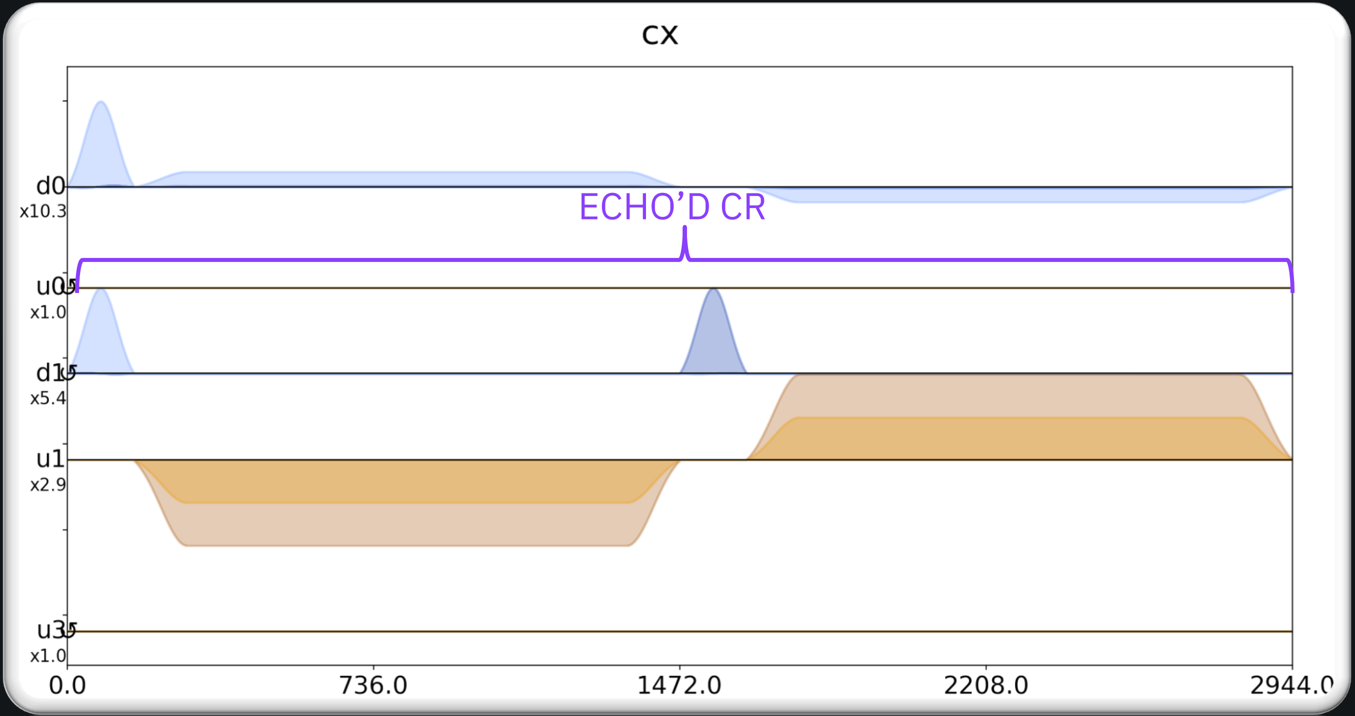
$$U\big(ZX(\pi/2)\big)U\big(ZI(-\pi/2)\big)U\big(IX(-\pi/2)\big) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Cross Resonance Interaction Alone Has More Terms than ZX



$$H_{CR} = \hbar\epsilon(t)\left[ZI - \nu_1 IX - \frac{J}{\Delta_{01}} ZX\right]$$

The Echoed Cross Resonance Interaction Removes the ZI term



$$H_{CR} = \hbar\epsilon(t)\left[\cancel{ZI} \cancel{- \nu_1 IX} - \frac{J}{\Delta_{01}} ZX\right]$$

The Rotary Tones remove the IX interaction from the CR Tone

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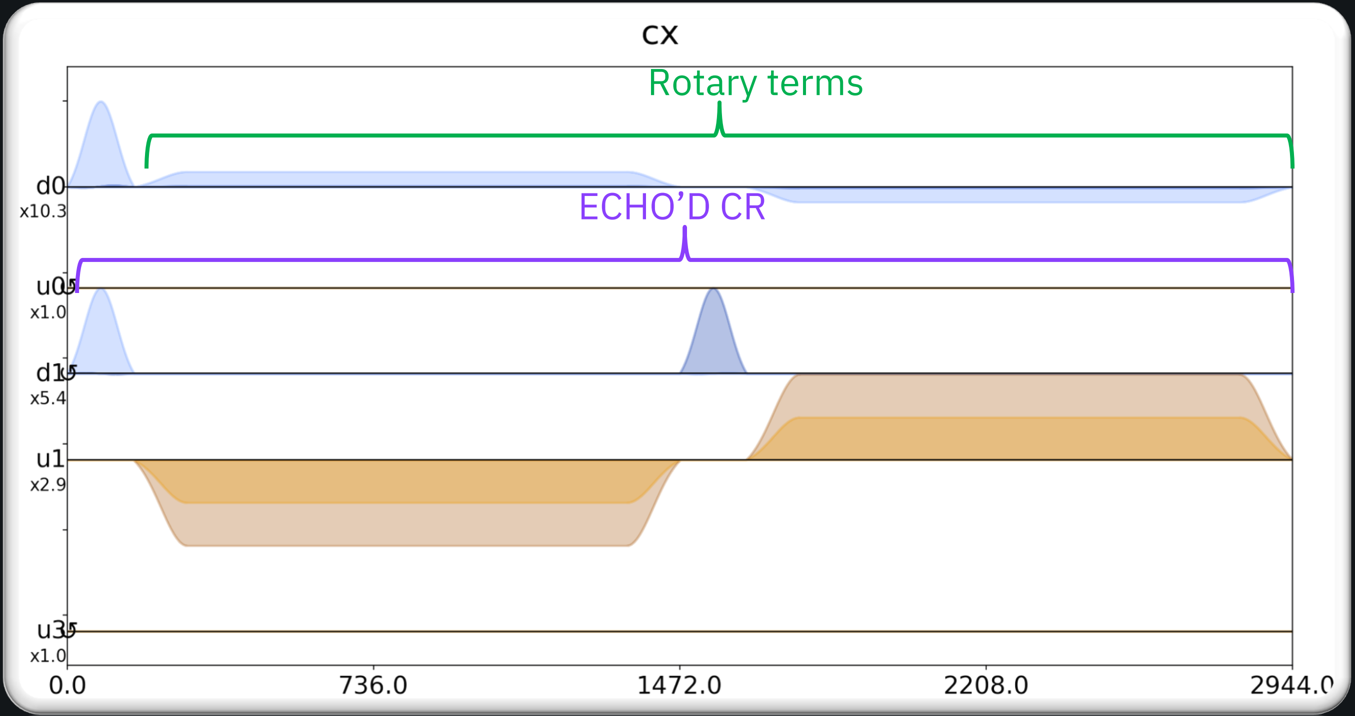
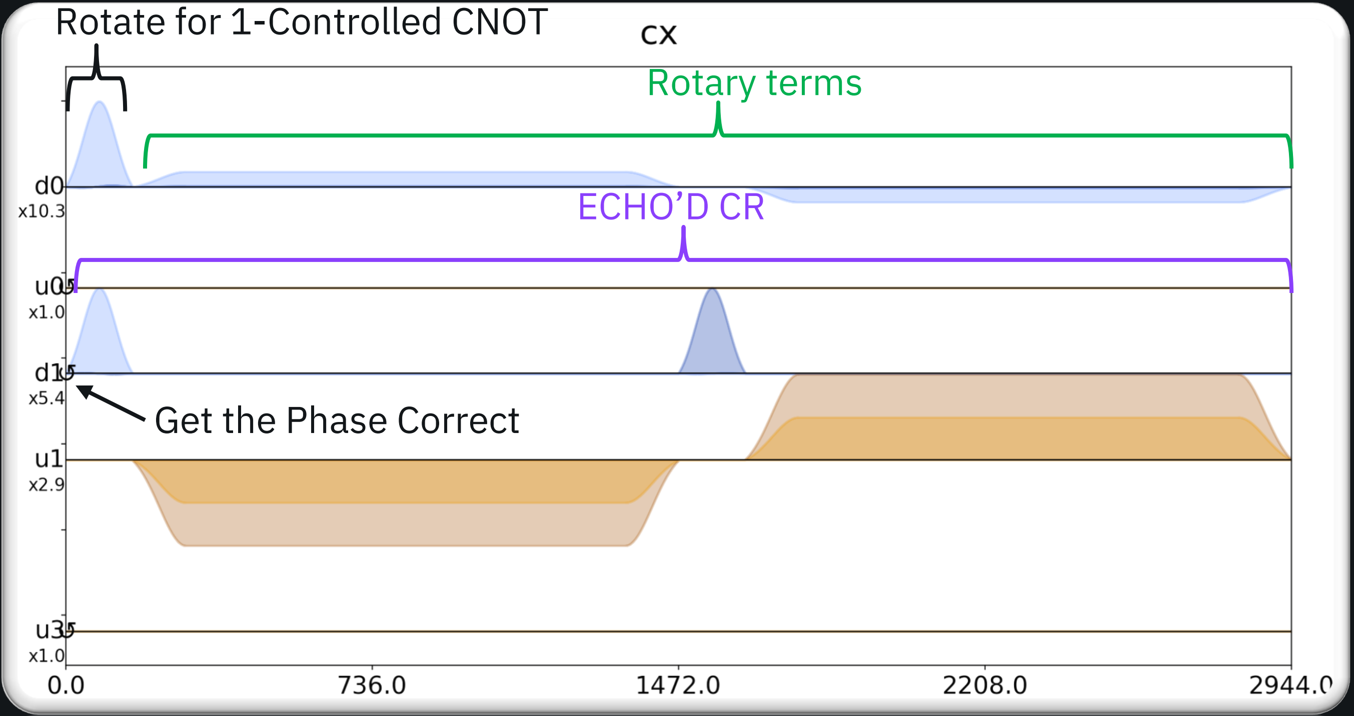


$$H_{CR} = \hbar\epsilon(t)\left[\cancel{ZI - \nu_1 IX} - \frac{J}{\Delta_{01}} ZX \cancel{- \nu_1 ZY}\right]$$

A Rotation on the Target Qubit Results in a CNOT Gate



$$H_{CR} = \hbar\epsilon(t)\left[\cancel{ZI - \nu_1 IX} - \frac{J}{\Delta_{01}} ZX \cancel{- \nu_1 ZY}\right]$$

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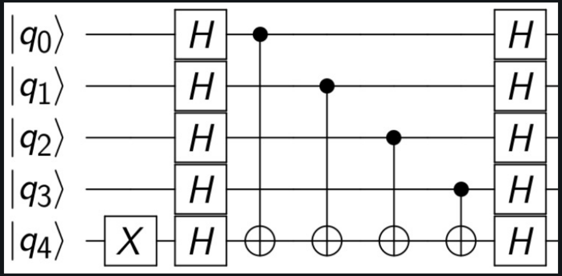
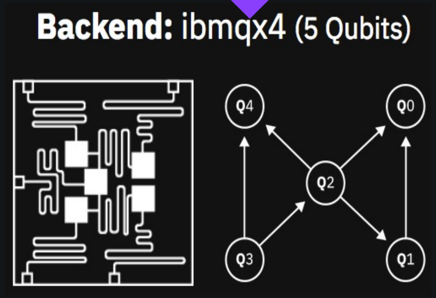
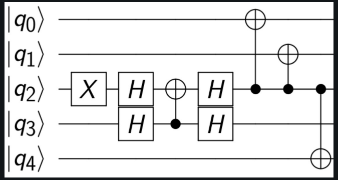
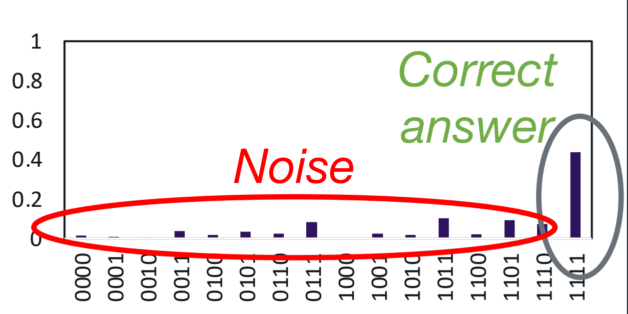
Transpilation Maps an Ideal Quantum Circuit to a Real Backend



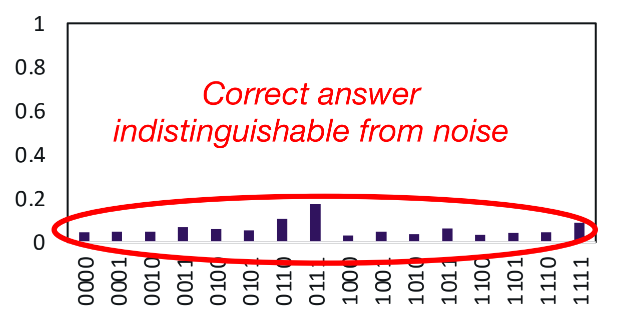


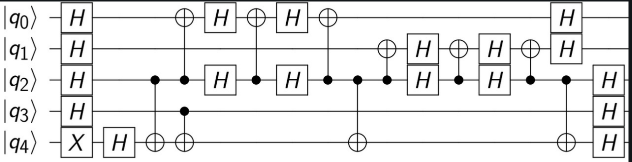
Well Compiled Circuit #1





Poorly Compiled Circuit #2





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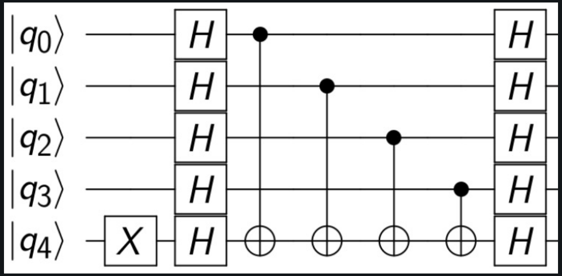

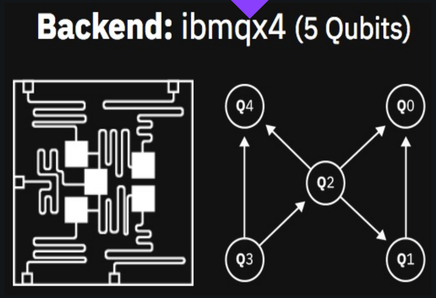

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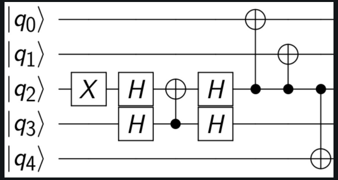
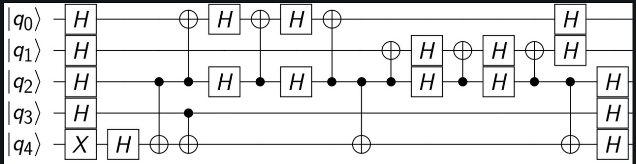
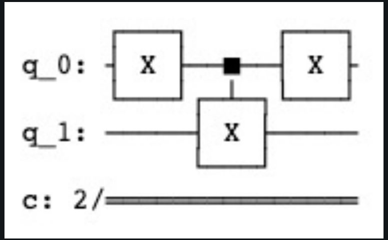


from qiskit import transpile
transpile(circ,backend,optimization_level=0,1,2,3)

Poorly Compiled Circuit #2



An Open CNOT has redundant 1Q gates



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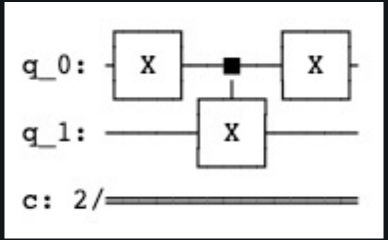

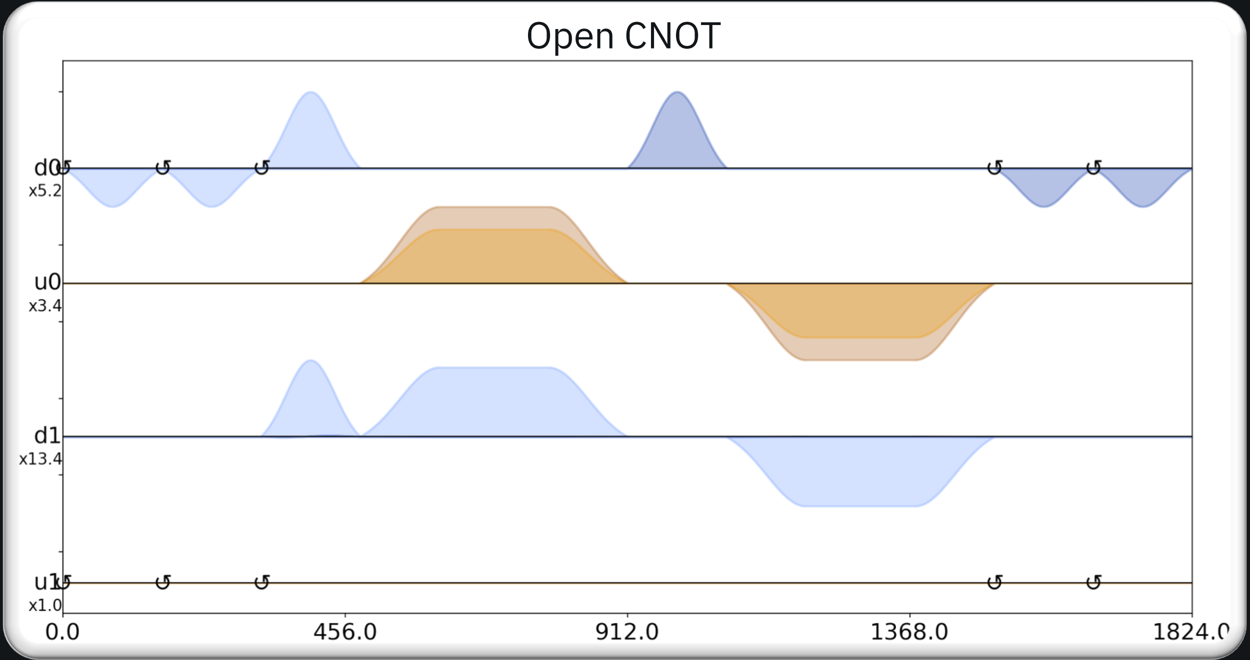

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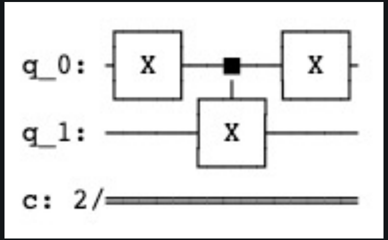
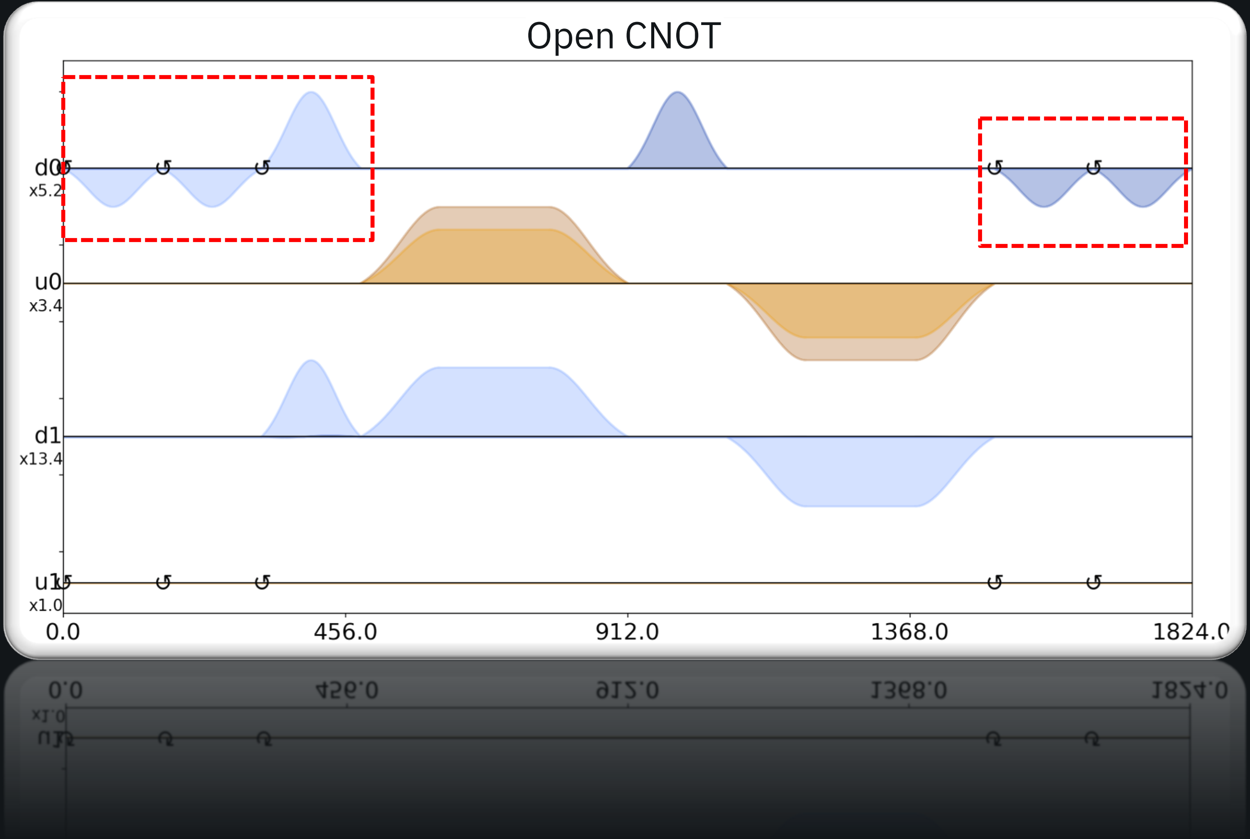
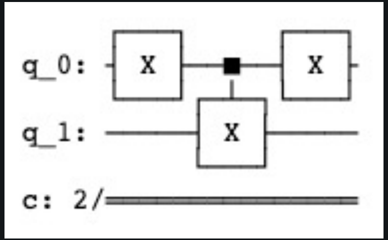
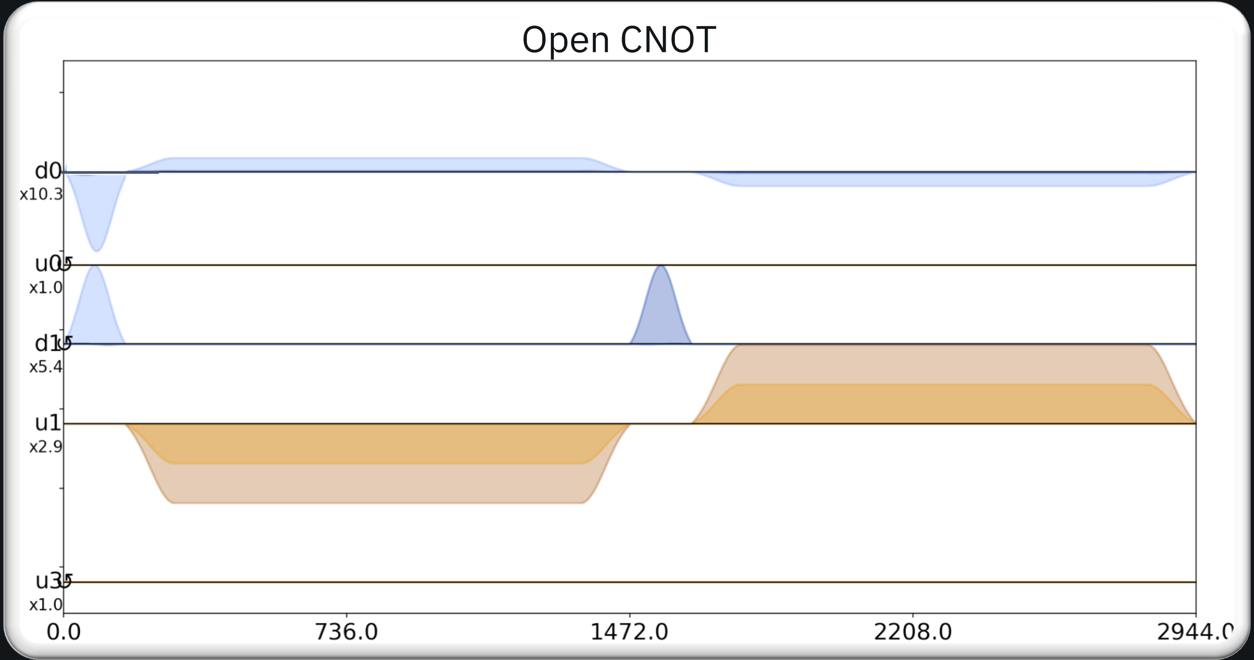




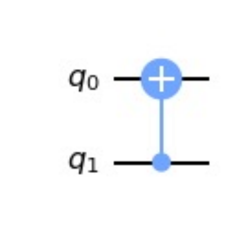
An Open CNOT can be Made from the CR Gate



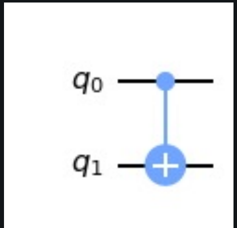


$$U\big(ZX(\pi/2)\big)U\big(ZI(-\pi/2)\big)U\big(IX(\pi/2)\big)$$

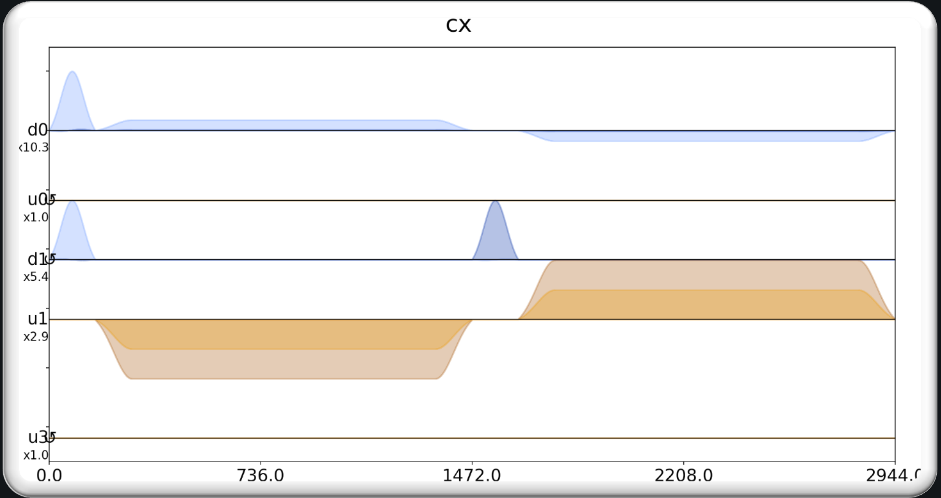
The Reversed CNOT Direction is NOT done “natively”

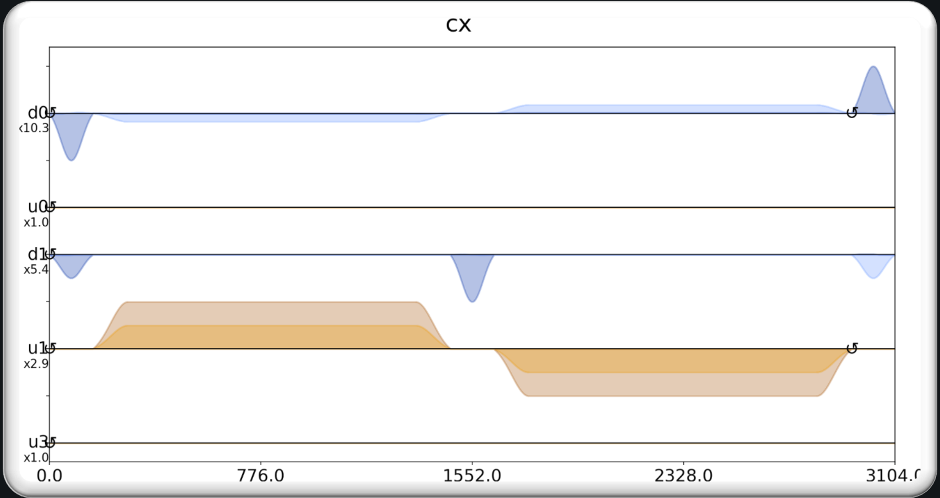


Inst_map = backend.defaults().instruction_schedule map
inst_map.get("cx",qubits=[1,0])



inst_map.get("cx",qubits=[0,1])

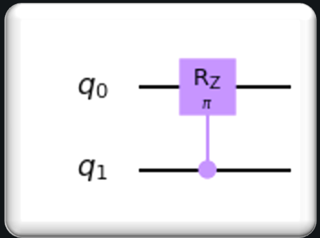




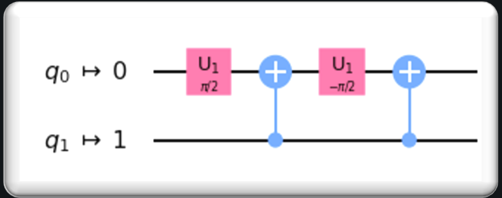
Cross Resonance is not symmetric! (One direction is slower) And we save time on daily gate calibrations!

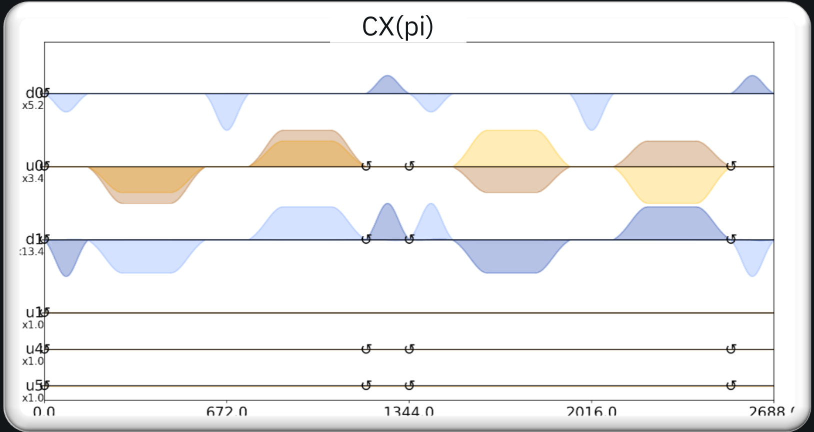
Circuit Based Optimization Can Improve Specific Gate Parameters

circ = QuantumCircuit(2,2)
circ.crz(np.pi,1,0)



transpile(circ, backend,optimization_level=0)

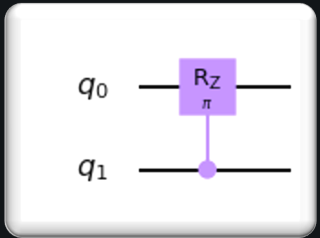




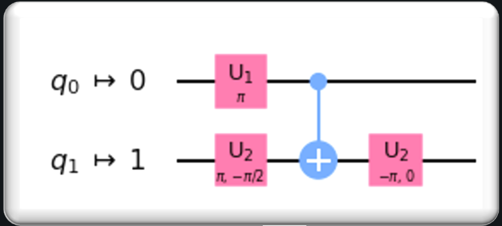
Circuit Based Optimization Can Improve Specific Gate Parameters

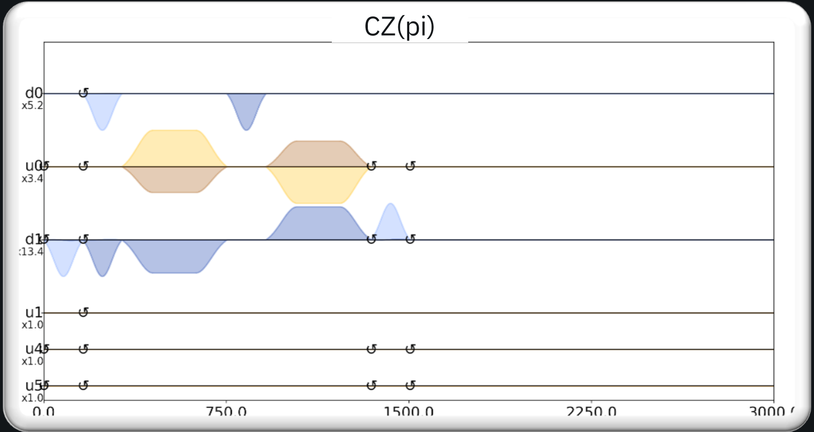
Qiskit

```
circ = QuantumCircuit(2,2)
circ.crz(np.pi,1,0)
```



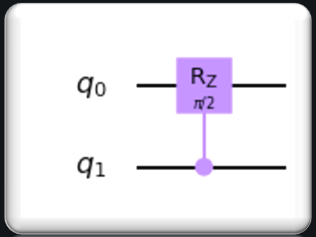

```
transpile(circ, backend,optimization_level=3)
```



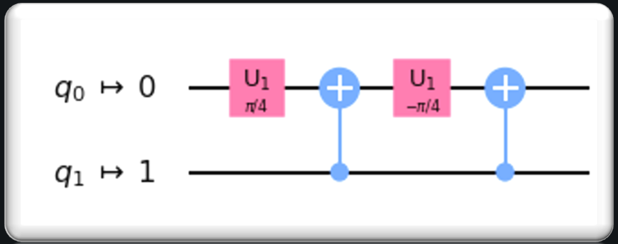


Circuit Based Optimization Cannot Improve All Gate Parameters

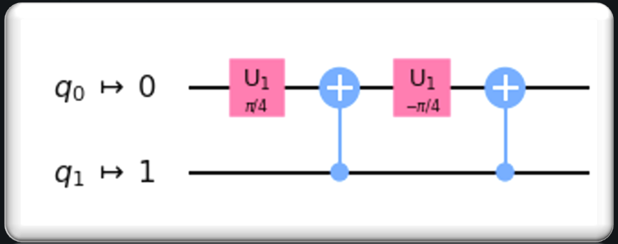
```
circ = QuantumCircuit(2,2)
circ.crz(np.pi/2,1,0)
```



transpile(circ, backend,optimization_level=0)



transpile(circ, backend,optimization_level=3)

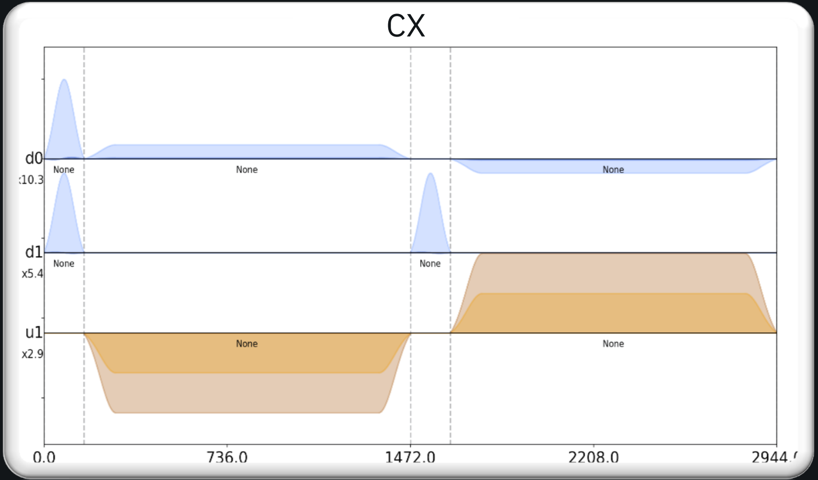


Overview

Qiskit

- Calibrating Single and Two Qubit Gates – Rabi Oscillations and Getting a CNOT from the Cross Resonance
- Circuit Transpilation – Understanding Differences from Gate vs Pulse Perspectives
- Continuous Gate Sets – Reducing Circuit Depth by Scaling the Cross Resonance Interaction

We create a Continuous Gate set with specific scaling technique – no additional calibrations required



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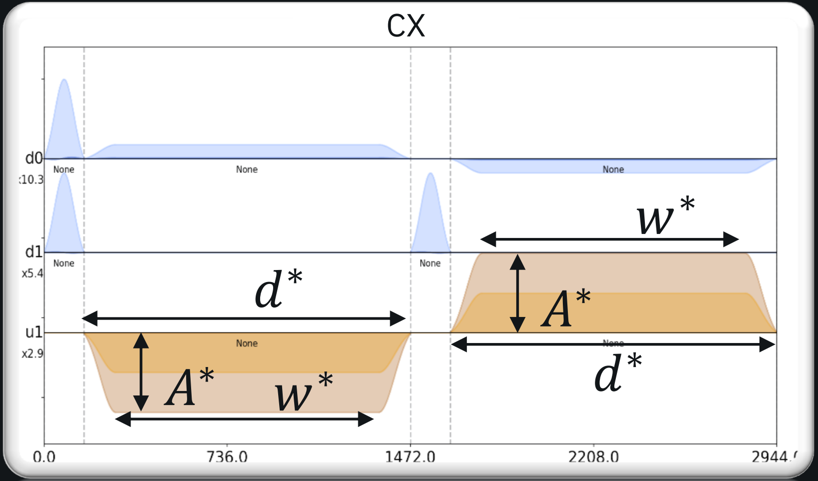

$$\alpha^* = |A^*|w^* + |A^*|\sigma\sqrt{2\pi}\mathrm{erf}(n_\sigma)$$

Stenger, John, Nicholas T. Bronn, Daniel J. Egger, and David Pekker. "Simulating the dynamics of braiding of Majorana zero modes using an IBM quantum computer." *arXiv preprint arXiv:2012.11660* (2020).

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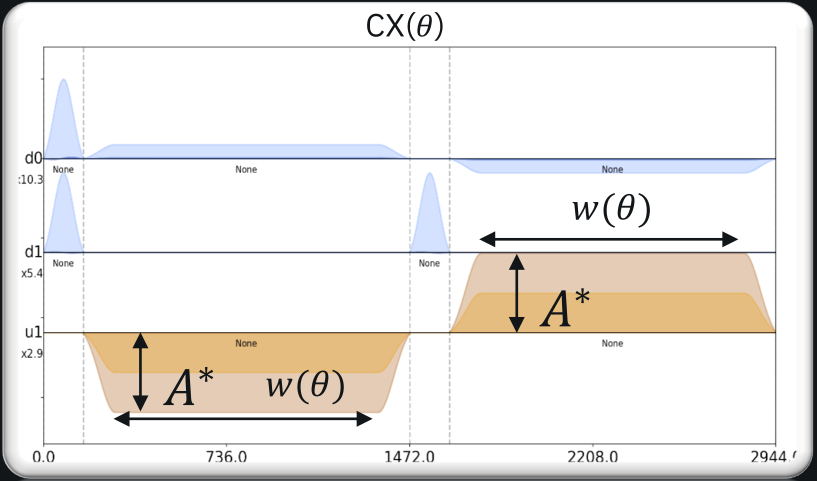

$$\alpha^* = |A^*|w^* + |A^*|\sigma\sqrt{2\pi}\mathrm{erf}(n_\sigma)$$

When there is a non-zero width w:

$$\alpha(\theta) = \frac{\theta}{\pi/2}\alpha^*$$

$$w(\theta) = \frac{\alpha(\theta)}{|A^*|} - \sigma\sqrt{2\pi}\mathrm{erf}(n_\sigma)$$

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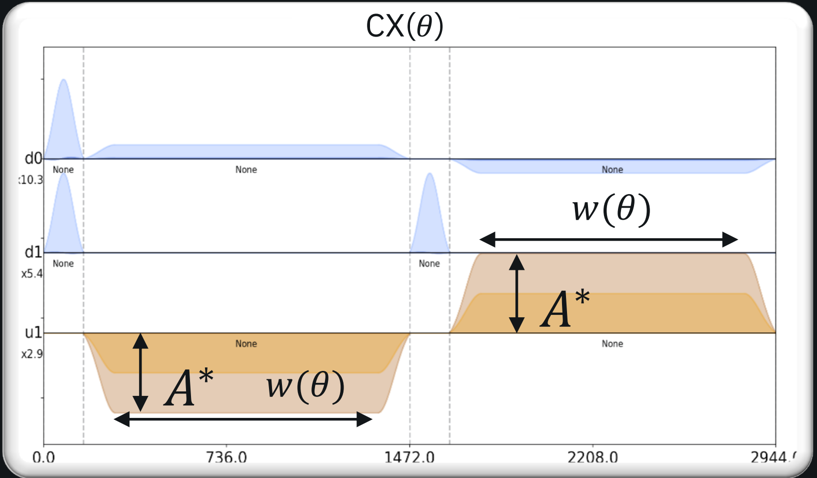
Qiskit



$$\alpha^* = |A^*|w^* + |A^*|\sigma\sqrt{2\pi}\mathrm{erf}(n_\sigma)$$

When there is a non-zero width w:

$$\alpha(\theta) = \frac{\theta}{\pi/2}\alpha^*$$

$$w(\theta) = \frac{\alpha(\theta)}{|A^*|} - \sigma\sqrt{2\pi}\mathrm{erf}(n_\sigma)$$

When the width w is zero:

$$|A(\theta)| = \frac{\alpha(\theta)}{\sigma\sqrt{2\pi}\mathrm{erf}(n_\sigma)}$$

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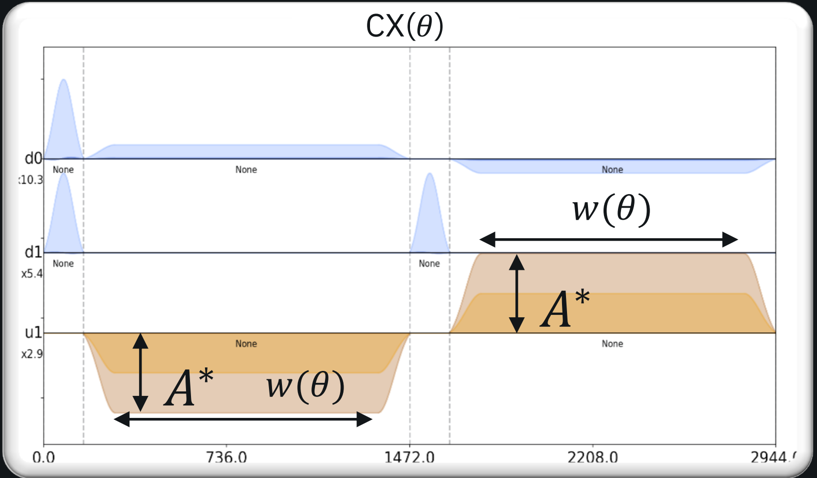
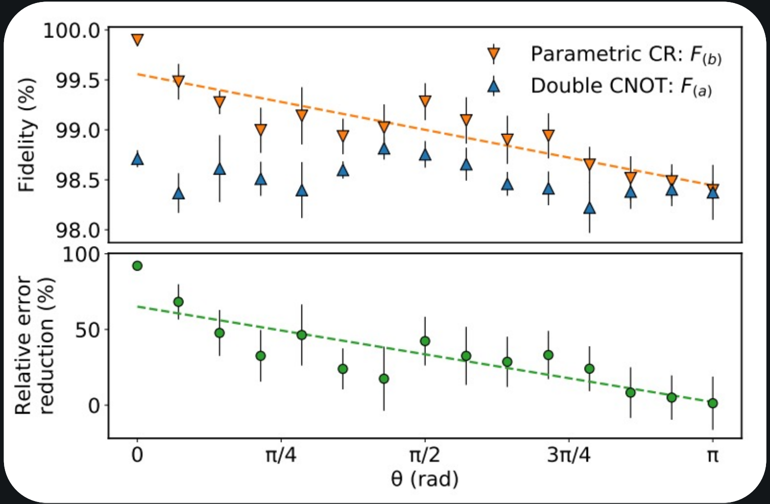
We create a Continuous Gate set with specific scaling technique – no additional calibrations required





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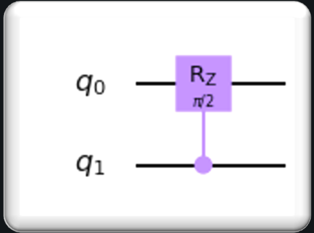
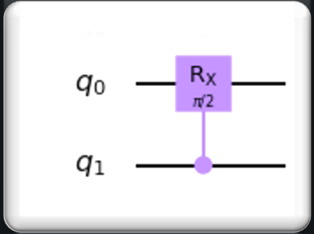
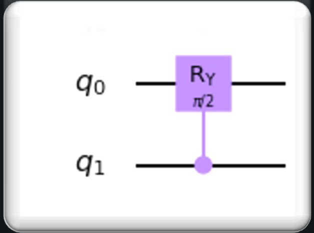
Applications Using CR___ Gates

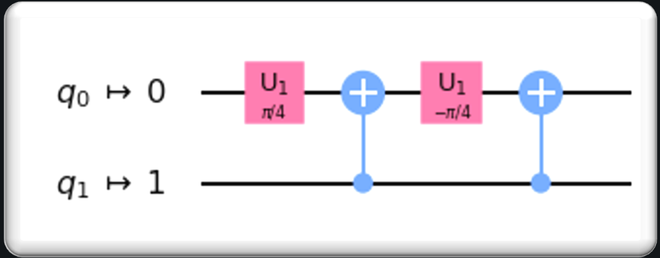


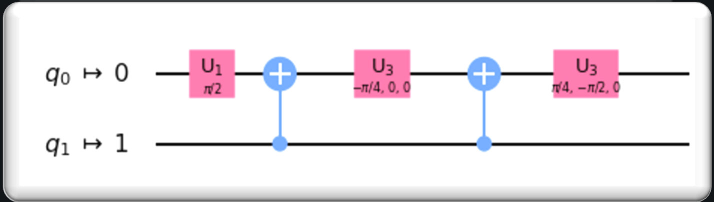


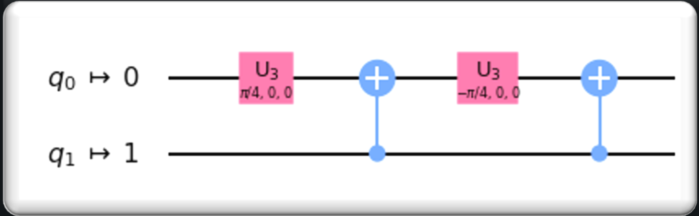


Transpile

(OptimizationLevel=3)



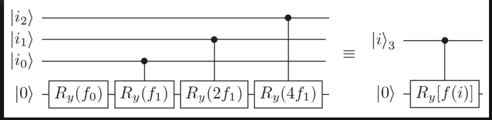




Useful in QAOA

Pauli Feature map

for QML



Approximates

$$|i\rangle_n|0\rangle \rightarrow |i\rangle_n|f(i)\rangle$$

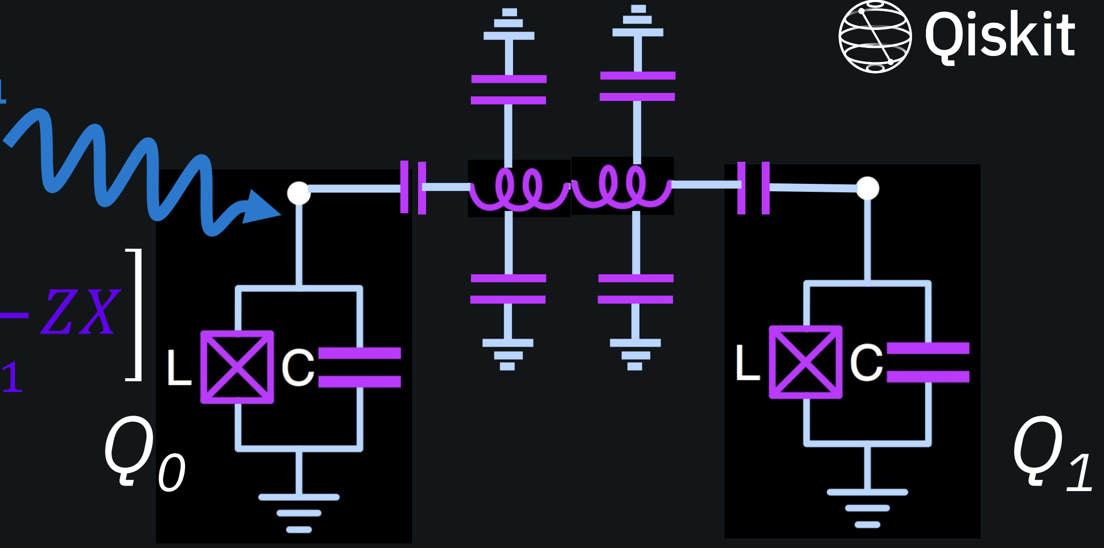
Useful for portfolio optimization

Stamatopoulos, N. et al "Option pricing using quantum computers." *Quantum* 4 (2020): 291.

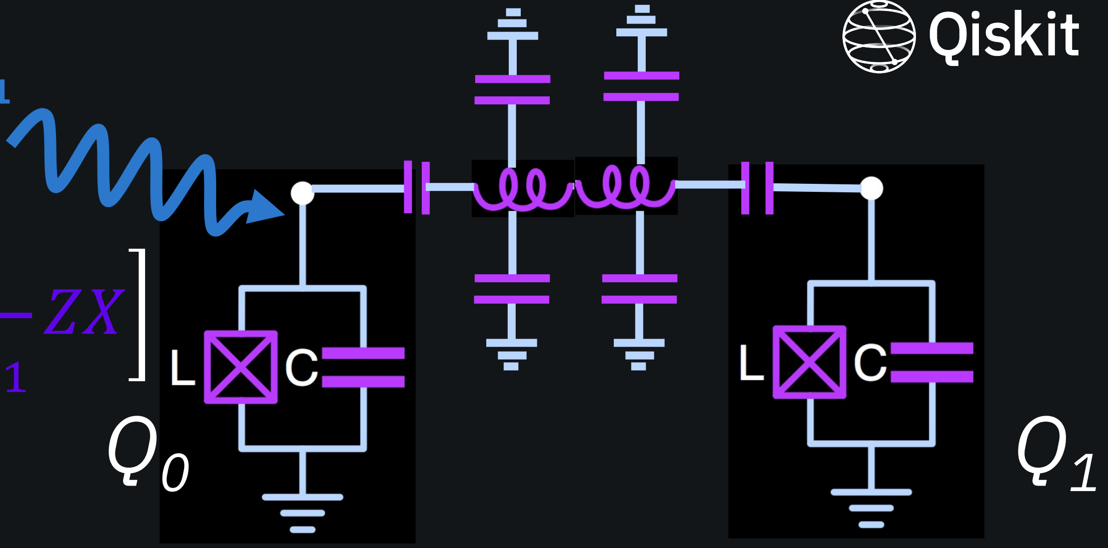
CX vs CY vs CZ

$$H_D = \hbar\epsilon(t)\left[ZI - \nu_1 IX - \frac{J}{\Delta_{01}}ZX\right]$$

$f^{(1)}_{01}$

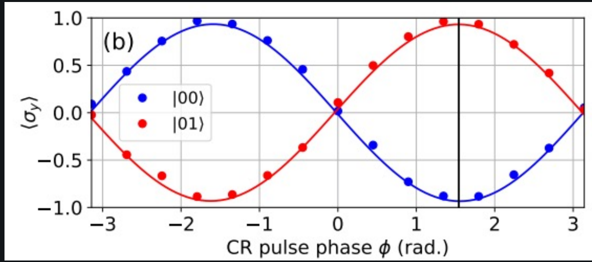


CX vs CY vs CZ



$$H_D = \hbar\epsilon(t)\left[ZI - \nu_1 IX - \frac{J}{\Delta_{01}}ZX\right]$$

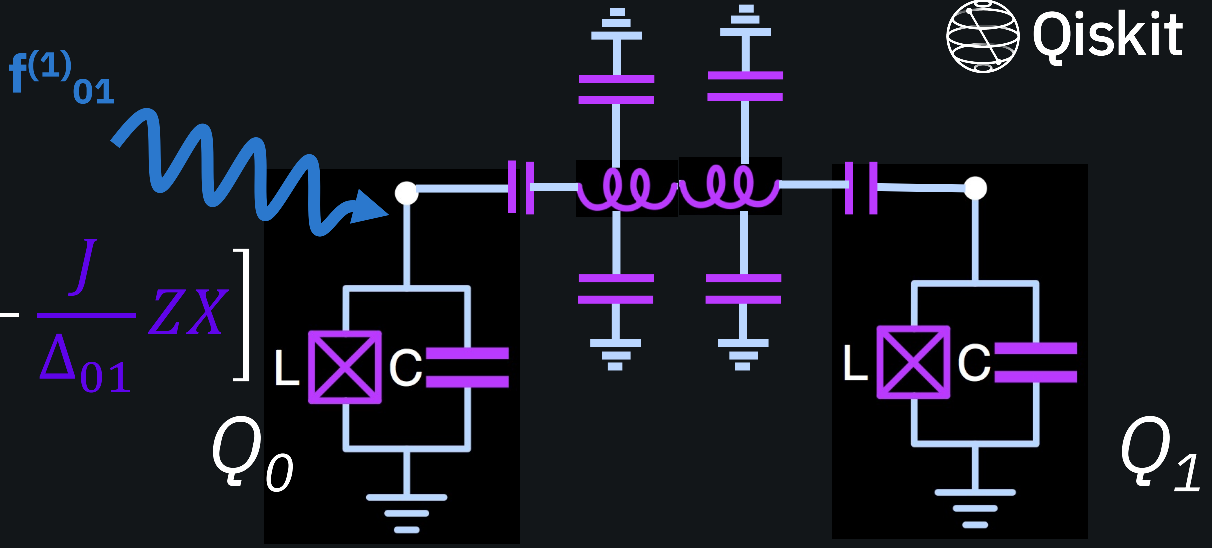
Shift the CR phase:



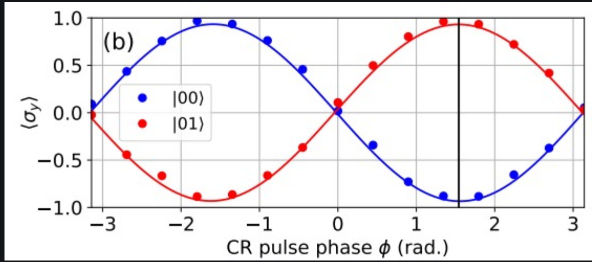
Garion, S. et al *arXiv:2007.08532* (2020).

CX vs CY vs CZ

$$H_D = \hbar\epsilon(t)\left[ZI - \nu_1 IX - \frac{J}{\Delta_{01}} ZX\right]$$

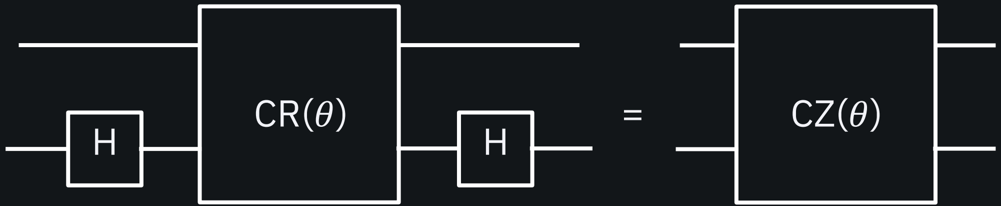


Shift the CR phase:



Garion, S. et al *arXiv:2007.08532* (2020).

Append Hadamards to the target:



Generalizing to SU4 gates with Cartan's KAK Decompostion

Qiskit

Cartan's KAK Decompostion

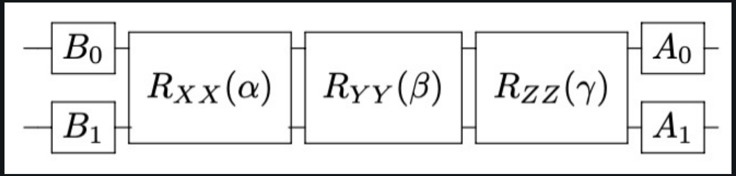
$$U = k_1 A k_2$$

$k_{1,2} \in SU(2) \otimes SU(2)$

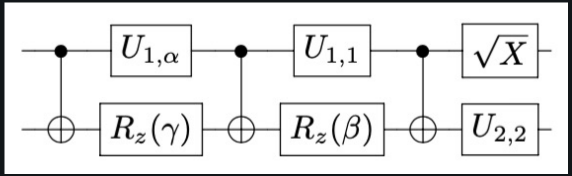
$A = e^{ik\,T\cdot\Sigma/2} \in SU(4) \setminus SU(2) \otimes SU(2)$

Where Σ T = (XX, Y Y, ZZ)

Continuous CR Based SU4:



CNOT Based SU4:



Earnest, Nathan, Caroline Tornow, and Daniel J. Egger. "Pulse-efficient circuit transpilation for quantum applications on cross-resonance-based hardware." *arXiv preprint arXiv:2105.01063* (2021).

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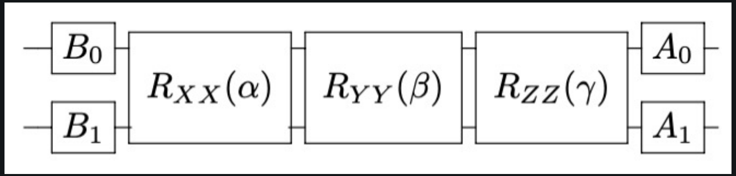
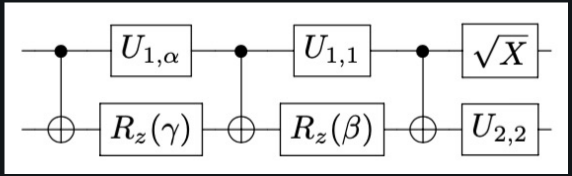
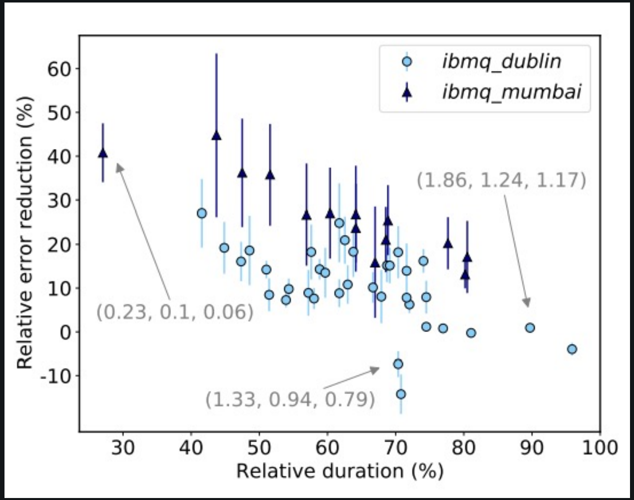


CNOT Based SU4:





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Qiskit

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$k_{1,2} \in SU(2) \otimes SU(2)$

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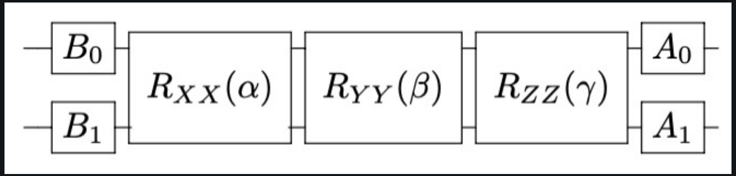
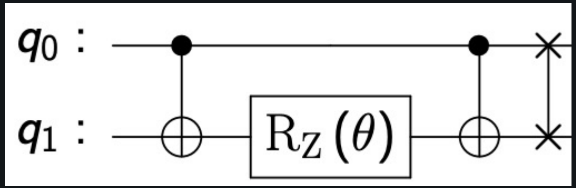
Where Σ T = (XX, Y Y, ZZ)

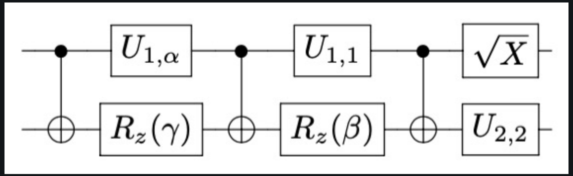
Continuous CR Based SU4:



SWAP(θ) decomposition gives k T = (ηπ/2, ηπ/2, θ + ηπ/2))
where η = −1 if θ > 0 and 1 otherwise.



CNOT Based SU4:



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Lab 5!

In Lab 5 we will:

- Produce the ZZ Pauli Feature Map
- Simulate QPT Circuits With and Without Noise
- Study the Impact of SPAM Errors on QPT circuits
- Do an Analogous Study of the Pulse Scaling Technique