Monday, July 5, 2021 9:23 AM

Introduction to quantum noise

Incoherent noise & Quantum trajectories

Qiskit Global Summer School on Quantum Machine Learning

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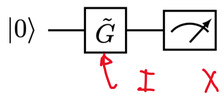
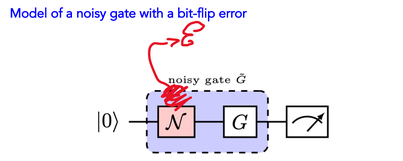
Quantum jumps: Unraveling of a quantum trajectory



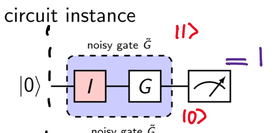
Model of a noisy gate with a bit-flip error



$G = X$

probability circuit instance

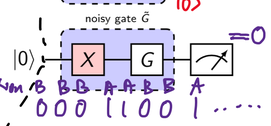
A $1-p$



$|1\rangle$ $= 1$ $|0\rangle$

$U_A = GI = G = X$ $1-p$

B p



$= 0$

$U_B = GX = XX = I$ p

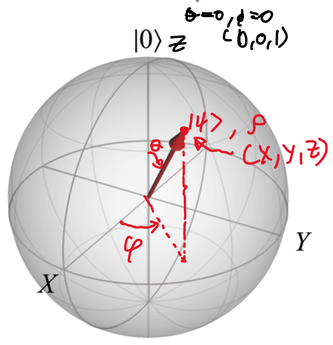
realization B B B A A B B A ...

000 100 1 .....

ideal $\langle \hat{M} \rangle = 1$

$\langle \hat{M} \rangle = (1-p)\,1 + p\,0 = 1-p$ (no-error, error)

$\rho_0 = (1-p)\,|0\rangle\langle 0| + p\,|0\rangle\langle 0| = |0\rangle\langle 0|$ $\quad \rho = |\psi\rangle\langle\psi| = \vec{\psi}\cdot\vec{\psi}^\dagger$



$|\psi\rangle = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\, e^{i\phi} \end{pmatrix}$

$\langle\psi|\psi\rangle = \left(\cos\theta/2 \quad \sin\tfrac{\theta}{2} e^{-i\phi}\right)\cdot\begin{pmatrix}\cdot\\ \cdot\end{pmatrix} = (\cos\theta/2)^2 + \left[\sin\left(\tfrac{\theta}{2}\right) e^{-i\phi}\right]^2 = \cos^2 + \sin^2 = 1$

$0 \le \theta \le \pi \qquad 0 \le \phi \le 2\pi$

$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\, e^{+i\phi} \end{pmatrix}\left(\cos\theta/2 \quad \sin\tfrac{\theta}{2} e^{-i\phi}\right) = \begin{pmatrix} (\cos\theta/2)^2 & \cos\theta/2 \sin\theta/2\, e^{-i\phi} \\ \cos\theta/2 \sin\theta/2\, e^{+i\phi} & (\sin\theta/2)^2 \end{pmatrix}$

$\left(\cos\theta/2 \sin\theta/2 = \tfrac{1}{2}\sin\theta\right) = \begin{pmatrix} \cos^2\theta/2 & \frac{1}{2}\sin\theta\left[\cos\phi - i\sin\phi\right] \\ \text{H.c.} & \sin^2\theta/2 \end{pmatrix}$

$= \frac{1}{2}\left(\hat{I} + x\hat{X} + y\hat{Y} + z\hat{Z}\right)$

$\langle\hat{X}\rangle = x = \sin\theta\cos\phi = \mathrm{Tr}[\hat{X}\rho]$ $\quad \hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\langle\hat{Y}\rangle = y = \sin\theta\sin\phi = \mathrm{Tr}[\hat{Y}\rho]$ $\quad \mathrm{Tr}[I] = 2$

$\langle\hat{Z}\rangle = z = \cos\theta = \mathrm{Tr}[\hat{Z}\rho]$ $\quad \mathrm{Tr}[X^2] = \mathrm{Tr}[I] = 2$

Case A: $|\psi_A\rangle = U_A|0\rangle = |1\rangle \rightarrow \rho_A = |\psi_A\rangle\langle\psi_A| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

Case B: $|\psi_B\rangle = U_B|0\rangle = |0\rangle \rightarrow \rho_B = |\psi_B\rangle\langle\psi_B| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Ensemble: $\rho = p_A\rho_A + p_B\rho_B = p_A U_A|0\rangle\langle 0|U_A^\dagger + p_B U_B|0\rangle\langle 0|U_B^\dagger = (1-p)\,|\psi_A\rangle\langle\psi_A| + p\,|\psi_B\rangle\langle\psi_B| = (1-p)\,U_A|0\rangle\langle 0|U_A^\dagger + p\,U_B|0\rangle\langle 0|U_B^\dagger$

$= (1-p)\,|1\rangle\langle 1| + p|0\rangle\langle 0| = (1-p)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + p\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{matrix} |0\rangle \\ |1\rangle \end{matrix}\begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}$ ($\langle 0|$, $\langle 1|$)

$|\psi_B\rangle = U_B|0\rangle$, $|\psi_A\rangle = U_A|0\rangle$

$(U|0\rangle)^\dagger = \langle 0|U^\dagger$

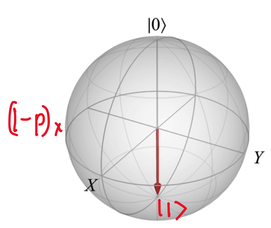
$(AB)^\dagger = B^\dagger A^\dagger$

Bloch: $x = \mathrm{Tr}(\hat{X}\rho) = 0$, $y = \mathrm{Tr}(\hat{Y}\rho) = 0$, $z = \mathrm{Tr}(\hat{Z}\rho) = \mathrm{Tr}\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}\right] = \mathrm{Tr}\begin{pmatrix} p & 0 \\ 0 & p-1 \end{pmatrix} = 2p-1$

$\mathrm{Tr}[P_\alpha P_\beta] = \delta_{2B}\,2$ $\quad \alpha, \beta \in \{X, Y, Z\}$

$\rho_{final} = \mathcal{E}(\rho_{in}) = \sum_K p_K U_K \rho_{in} U_K^\dagger \qquad K \in \{A, B\}$

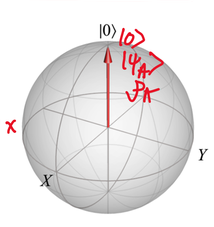
Case A



$|\psi_A\rangle$
ρ_A

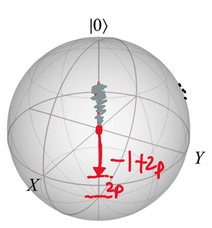
$+ \; p\times$

Case B



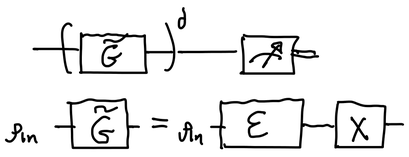
$=$

Ensemble



$\langle X\rangle^2 + \langle Y\rangle^2 + \langle Z\rangle^2 = 1$ for pure

$\langle X\rangle^2 + \langle Y\rangle^2 + \langle Z\rangle^2 = (2p-1)^2$ for our impure state



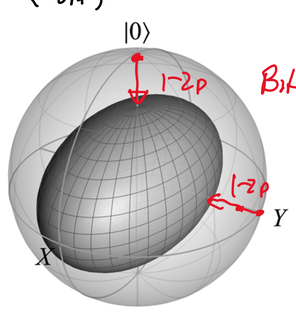
$\rho = \frac{1}{2}\left(I + x\hat{X} + y\hat{Y} + z\hat{Z}\right) = \frac{1}{2}\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & x \\ x & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iy \\ iy & 0 \end{pmatrix} + \begin{pmatrix} z & 0 \\ 0 & -z \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 1+z & x-iy \\ x+iy & 1-z \end{pmatrix}$ $\quad x, y, z \in [-1, 1]$, $x^2+y^2+z^2 \le 1$

$\rho_{in} = \frac{1}{2}\left(I + x_{in}\hat{X} + y_{in}\hat{Y} + z_{in}\hat{Z}\right)$

$\rho_{out} = \mathcal{E}(\rho_{in}) = (1-p)\,I\rho_{in}I + pX\rho_{in}X = (1-p)\begin{pmatrix} x_{in} \\ y_{in} \\ z_{in} \end{pmatrix} + p\begin{pmatrix} +x_{in} \\ -y_{in} \\ -z_{in} \end{pmatrix}$

$= \begin{pmatrix} x_{in} \\ (1-2p)\,y_{in} \\ (1-2p)\,z_{in} \end{pmatrix}$ error

$X_\beta X = X\frac{1}{2}(\hat{I} + x\hat{X} + \dots)X = \frac{1}{2}\left(\hat{X}\hat{I}\hat{X} + x\hat{X}\hat{X}\hat{X}\hat{X} + \dots\right) = \frac{1}{2}\left(\hat{I} + x\hat{X} - y\hat{Y} - z\hat{Z}\right)$



Bit-flip X

$X\rho_{in}X = \begin{pmatrix} x_{in} \\ -y_{in} \\ -z_{in} \end{pmatrix}$

$\tilde{G}^d(\rho) = \dots X\mathcal{E}(X\mathcal{E}(\rho)X)X\dots = \begin{pmatrix} x_{in} \\ (-(1-2p)^d\, y_{in} \\ (-(1-2p)^d\, z_{in} \end{pmatrix} = \rho_\mathcal{E}(d)$

$\rho_{in} = |0\rangle\langle 0| = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} \xrightarrow{\tilde{G}^d} \begin{pmatrix} 0 & & \\ & 0 & \\ & & (2p-1)^d \end{pmatrix}$

$\langle \tilde{Z} \rangle = Tr(Z \rho_{out}) = z_{out} = (2p-1)^d$ If $p=0 \Rightarrow \langle Z \rangle_{ideal} = (-1)^d$

$0 \le p \le 1$

$\langle \tilde{Z} \rangle = (2p-1)^d \simeq 1 - 2d\,p$ Linear in p

p ← error