State prep [bonus]

Saturday, July 3, 2021 2:00 PM

Introduction to quantum noise

State preparation noise

Qiskit Global Summer School on Quantum Machine Learning
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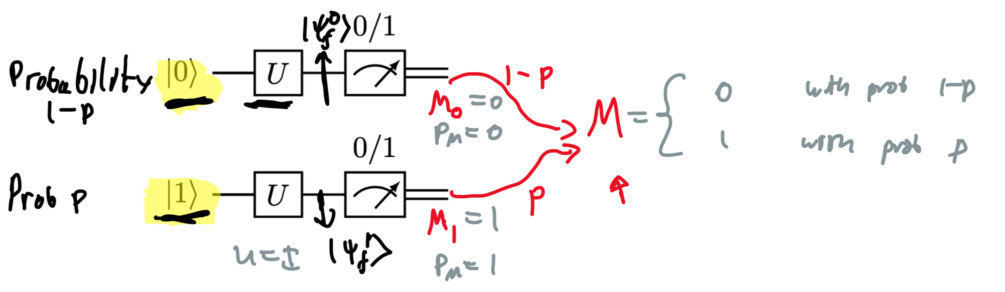


<u>Density operator</u>

$$\rho = (1-p)\,\overset{|0\rangle}{|0\rangle\langle 0|} + p\,\overset{|1\rangle}{|1\rangle\langle 1|}$$
$$= P_{\text{no error}}\,\rho_{\text{ideal}} + P_{\text{error}}\,\rho_{\text{error}}$$

$$\rho_f = P_{\text{no error}}\,\rho^{\text{final}}_{\text{ideal}} + P_{\text{error}}\,\rho^{\text{final}}_{\text{error}}$$
$$= (1-p)\,|\psi_f^0\rangle\langle\psi_f^0| + p\,|\psi_f^1\rangle\langle\psi_f^1|$$
$$= (1-p)\,U|0\rangle\langle 0|U^\dagger + p\,U|1\rangle\langle 1|U^\dagger$$
$$= U\left[(1-p)|0\rangle\langle 0| + p|1\rangle\langle 1|\right]U^\dagger$$
$$= U\overset{\text{initial}}{\rho}U^\dagger$$

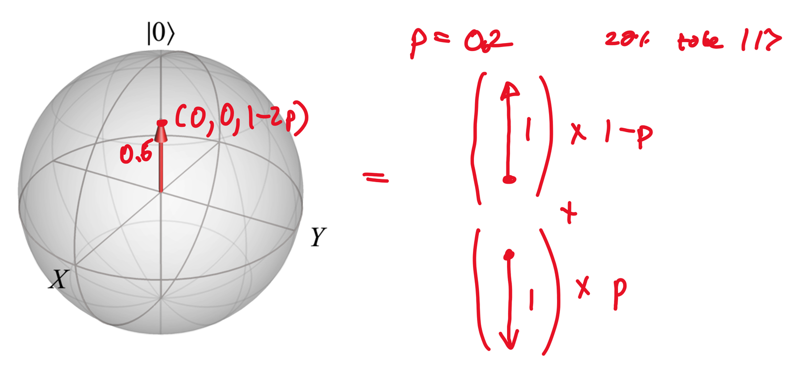
$$(U|\psi\rangle)^\dagger = (|\psi_f\rangle)^\dagger$$
$$\langle\psi_f| = \langle\psi|U^\dagger$$

$$\rho_U = U\rho U^\dagger$$

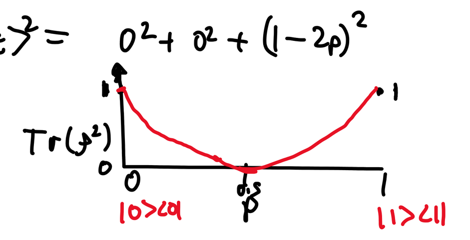
$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\rho = \begin{array}{c|cc} & \langle 0| & \langle 1| \\ \hline |0\rangle & 1-p & 0 \\ |1\rangle & 0 & p \end{array} = \frac{1}{2}\left(\hat{I} + (1-2p)\hat{Z}\right)$$

$$\begin{cases} \langle X\rangle = \mathrm{Tr}[X\rho] = 0 \\ \langle Y\rangle = \mathrm{Tr}[Y\rho] = 0 \\ \langle Z\rangle = \mathrm{Tr}[Z\rho] = 1-2p \end{cases}$$

$\mathrm{Tr}[P_\alpha P_\beta] = 2\delta_{\alpha\beta}$ for $\alpha, \beta \in \{X, Y, Z\}$

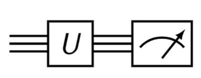


$$\mathrm{Tr}(\rho^2) = \langle X\rangle^2 + \langle Y\rangle^2 + \langle Z\rangle^2 = 0^2 + 0^2 + (1-2p)^2$$



Scaling to larger number of qubits

$[(1-p)\,|0\rangle\langle 0| + p\,|1\rangle\langle 1|]^{\otimes n}$



$$\rho_0 = \left[(1-p)|0\rangle\langle 0| + p|1\rangle\langle 1|\right]^{\otimes n}$$
$$= (1-p)^n|000\cdots 0\rangle\langle 0\cdots 0| + \cdots |0100\cdots\rangle\langle\cdot 1\cdot\cdots$$

$$P(M = 00000\cdots 0) = \begin{cases} 1 & \text{if } p=0 \quad (\text{ideal case}) \\ (1-p)^n \simeq 1 - np + O(p^2) & \text{for } p>0 \end{cases}$$

$$\langle Z_k\rangle = \langle 1111\overset{\text{at }k}{Z}111\cdots\rangle = 1-2p \quad \xrightarrow{\text{much faster}}$$
$$\langle ZZZ\cdots\cdots Z\rangle = \mathrm{Tr}[Z^{\otimes n}\rho] = (1-2p)^n$$
$$\rho_0 = \prod_{k=1}^{n}\otimes\frac{1}{2}\left(\hat{I}_k + (1-2p)\hat{Z}_k\right)$$
$$= \frac{1}{2^n}(1+\cdots Z)\otimes(1+\cdots Z)\otimes\cdots\cdots$$
$$= \frac{1}{2^n}\left[(1-2p)^n Z^{\otimes n} + \cdots\cdots\right]$$
$$\mathrm{Tr}[\rho_0^2] = \prod_{k=1}^{n}\mathrm{Tr}(\rho_{0,k}^2) = (1-2p)^{2n}$$