Coherent error: ZZ [bonus]

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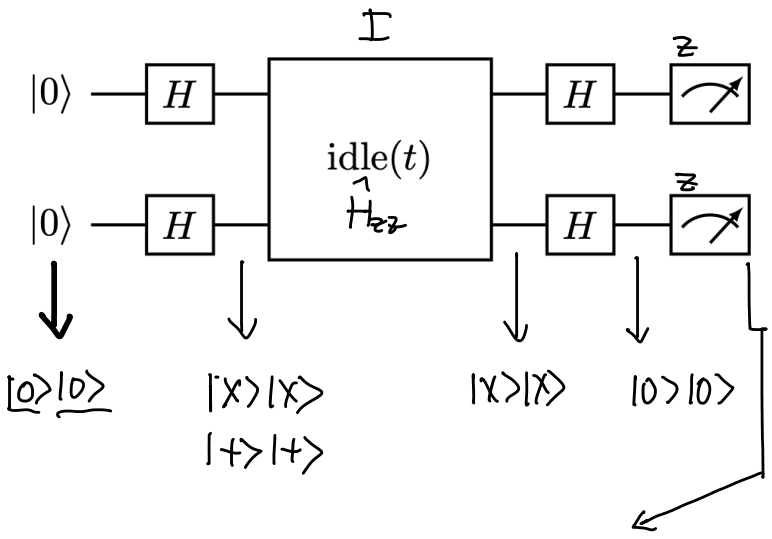
Introduction to quantum noise

Coherent errors

Qiskit Global Summer School on Quantum Machine Learning

Zlatko K. Minev



Hadamard gate

H

$$H = \begin{array}{c} \\ |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \begin{array}{cc} \langle 0| & \langle 1| \end{array} \\ \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \end{array}$$

$$\begin{cases} H|0\rangle = |+x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\ H|1\rangle = |-x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \end{cases}$$

$X|+x\rangle = +1|+x\rangle$

$X|-x\rangle = -1|-x\rangle$

$|+x\rangle := |+\rangle$

$|-x\rangle := |-\rangle$

$$\begin{cases} Z|0\rangle = +1|0\rangle \\ Z|1\rangle = -1|1\rangle \end{cases}$$

$\langle ZI\rangle = +1$
$\langle IZ\rangle = +1$
$\langle ZZ\rangle = \langle 01|\langle 01|Z|0\rangle|0\rangle$
$= \langle 0|Z|0\rangle\langle 0|Z|0\rangle$
$= \langle +|-\rangle$
$= +1$

NOISY ZZ Interaction

$$\hat{H} = \frac{1}{2}\hbar\omega\,\hat{Z}\hat{Z}$$

$$\hat{U}(t) = \exp\left(-i\hbar^{-1}\hat{H}t\right)$$
$$= \exp\left(-i\frac{\omega t}{2}\hat{Z}\hat{Z}\right)$$
$$= \cos\left(\frac{\omega t}{2}\right)\hat{I} - i\sin\left(\frac{\omega t}{2}\right)\hat{Z}\hat{Z}$$
$$= \hat{R}_{ZZ}(\theta = \omega t)$$

$$\hat{R}_X(\theta) = \exp\left(-i\frac{\theta}{2}X\right) \qquad X^2 = I$$
$$= \cos\left(\frac{\theta}{2}\right)I - i\sin\left(\frac{\theta}{2}\right)X$$

$$(\hat{Z}\hat{Z})^2 = Z^2Z^2 = I_2 \otimes I_2 = I_4$$

$|0\rangle|0\rangle \xrightarrow{HH} |+\rangle|+\rangle \xrightarrow{\text{idle}} R_{ZZ}(\theta)|+\rangle|+\rangle = \cos\frac{\theta}{2}|+\rangle|+\rangle - i\sin\frac{\theta}{2}|-\rangle|-\rangle \xrightarrow{HH} \cos\theta/2\,|0\rangle|0\rangle - i\sin\theta/2\,|1\rangle|1\rangle$

$R_{ZZ}(\theta)|+\rangle|+\rangle = \cos\frac{\theta}{2}|+\rangle|+\rangle - i\sin\frac{\theta}{2}(Z|+\rangle)_1 \otimes (Z|+\rangle)_2$

$Z|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = |-\rangle$

$Z|+\rangle = |-\rangle$

$Z|-\rangle = |+\rangle$

$\langle ZI\rangle = \cos\omega t$

$\langle IZ\rangle = \cos\omega t$

$\langle ZZ\rangle = 1$