(1) The equations I used to describe the system are as follows:

$$f_1 = L_1 cos\theta_1 + L_2 cos\theta_2 + L_3 cos\theta_3 - L$$
$$f_2 = L_1 sin\theta_1 + L_2 sin\theta_2 - L_3 sin\theta_3$$
$$f_3 = sin^2\theta_1 + cos^2\theta_1 - 1$$
$$f_4 = sin^2\theta_2 + cos^2\theta_2 - 1$$
$$f_5 = sin^2\theta_3 + cos^2\theta_3 - 1$$
$$f_6 = - T_1 cos\theta_1 + T_2 cos\theta_2$$
$$f_7 = - T_1 sin\theta_1 + T_2 sin\theta_2 + W_1$$
$$f_8 = - T_2 cos\theta_2 + T_3 cos\theta_3$$
$$f_9 = - T_2 sin\theta_2 - T_3 sin\theta_3 + W_2$$

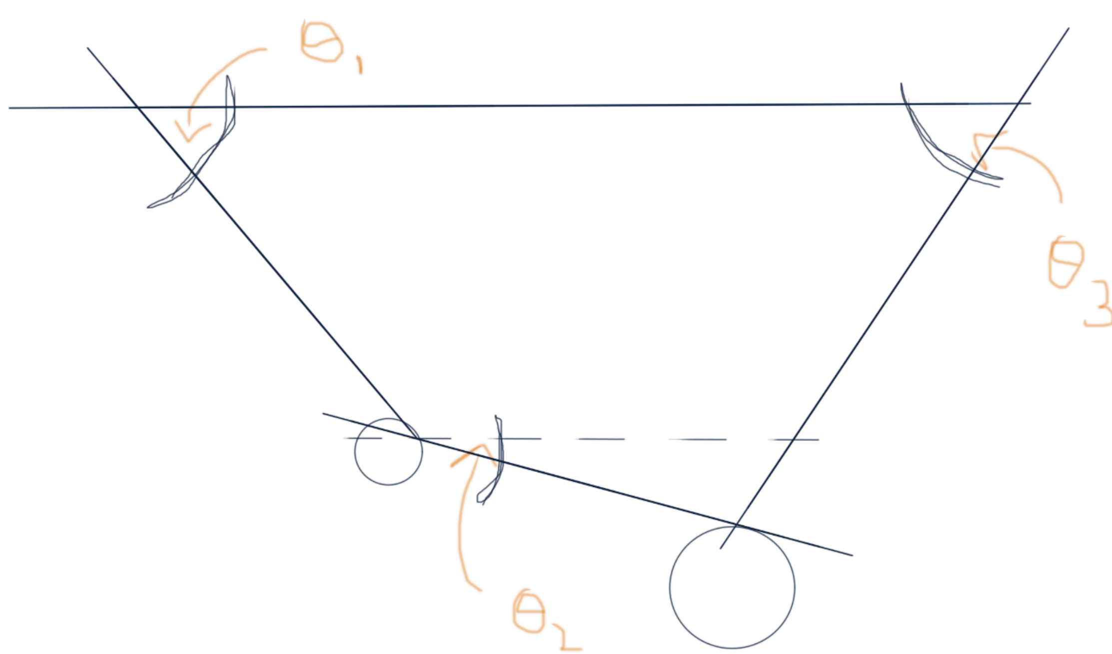
Where T_1, T_2, T_3, $sin\theta_1$, $cos\theta_1$, $sin\theta_2$, $cos\theta_2$, $sin\theta_3$, $cos\theta_3$ are unknown and L_1, , L_2, L_3, W_1, W_2 are given.

(2) As an initial initial guess, I set the tension variables to some value between 10 and 20, and the angle variables to some value between 0 and 1. However, it was easier to set the initial guess for all of the variables equal to 1, and the program was still able to solve.

(3) I find

$$T_1 = 17.16$$
$$T_2 = 11.55$$
$$T_3 = 20.27$$
$$\theta_1 = 49.55^\circ$$
$$\theta_2 = 15.36^\circ$$
$$\theta_3 = 56.69^\circ$$

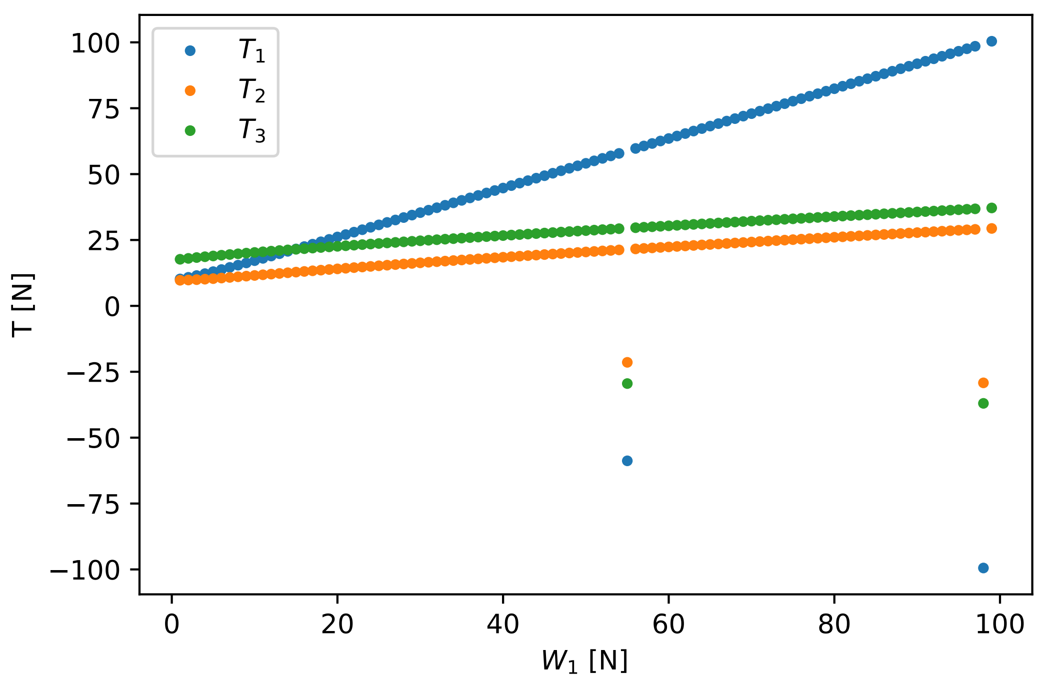
(4) All of my tension forces are positive, and the angles look reasonable when drawn out:

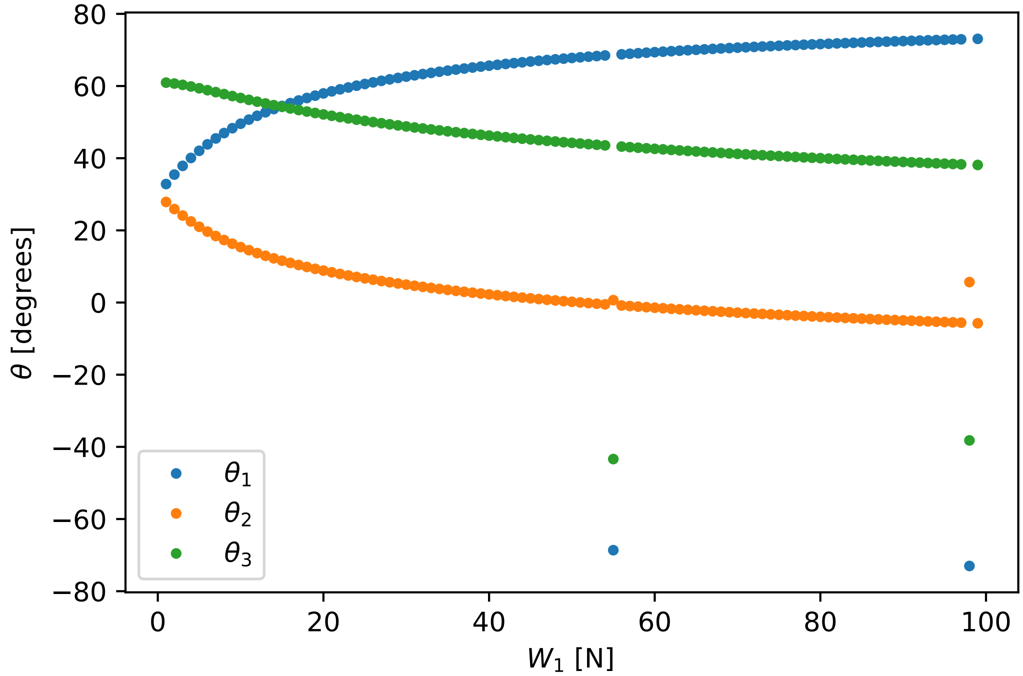


If we look at each tension force individually, we can see qualitatively how this answer makes physical sense. T_1 has some force acting upon it in the x and y-directions because of weight one. With forces in two directions, the magnitude of T_1 should be greater than W_1 and T_2. T_2 is mostly in the x-direction, with the a fraction of W_2 contributing to T_2 in the y-direction. Given the relatively small force in the y, it is reasonable to assume T_2 would have the smallest magnitude of the tension forces. T_3 has the greatest force in the y-direction, caused by W_2 and some component in the x-direction. Because of the contribution in the x-direction, T_3 should have a greater magnitude than W_2, as we find.

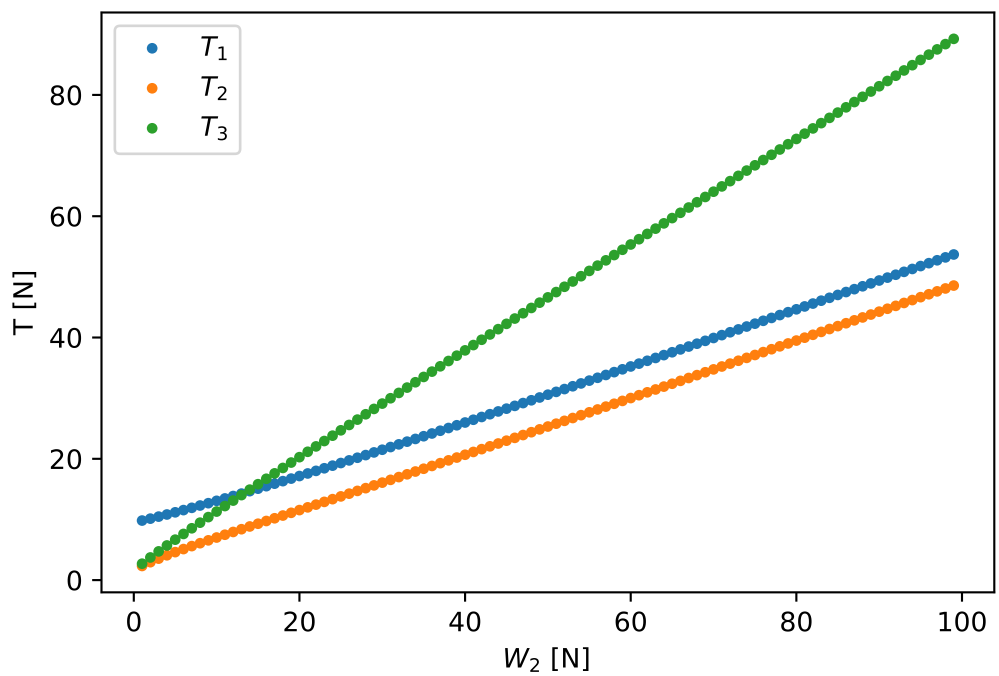
(5) When I set all of the variables to equal to 10000, the algorithm does not derive the solution. The values for tension go to an order of magnitude ~0.1, and the angles all approach 90 degrees. I'm skeptical of this result I'm not entirely sure what's happening, but the program seems a bit too robust.

(6)

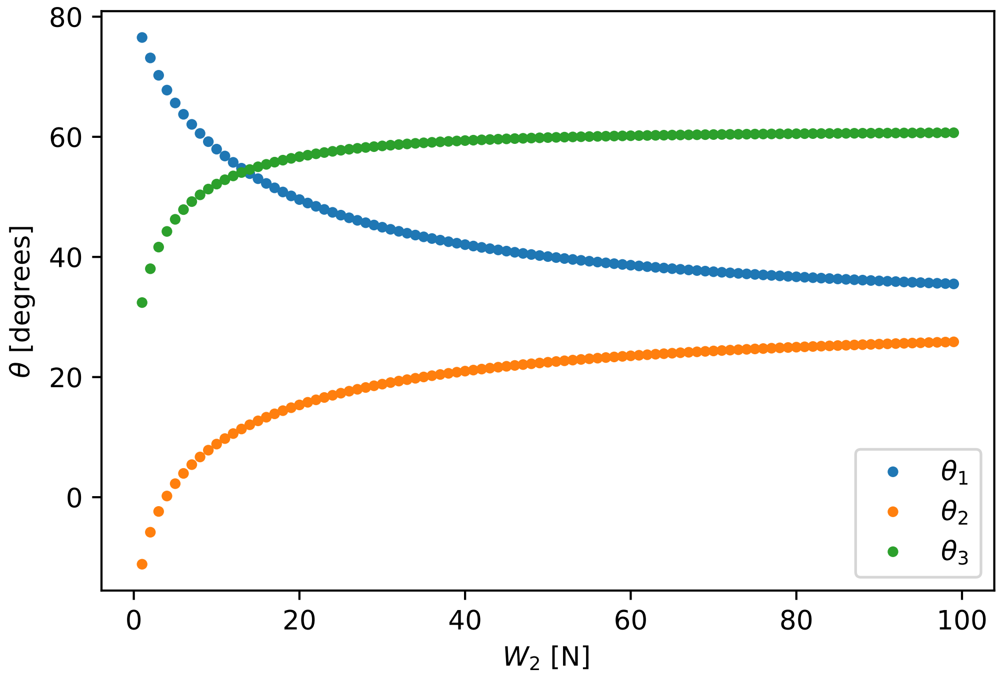




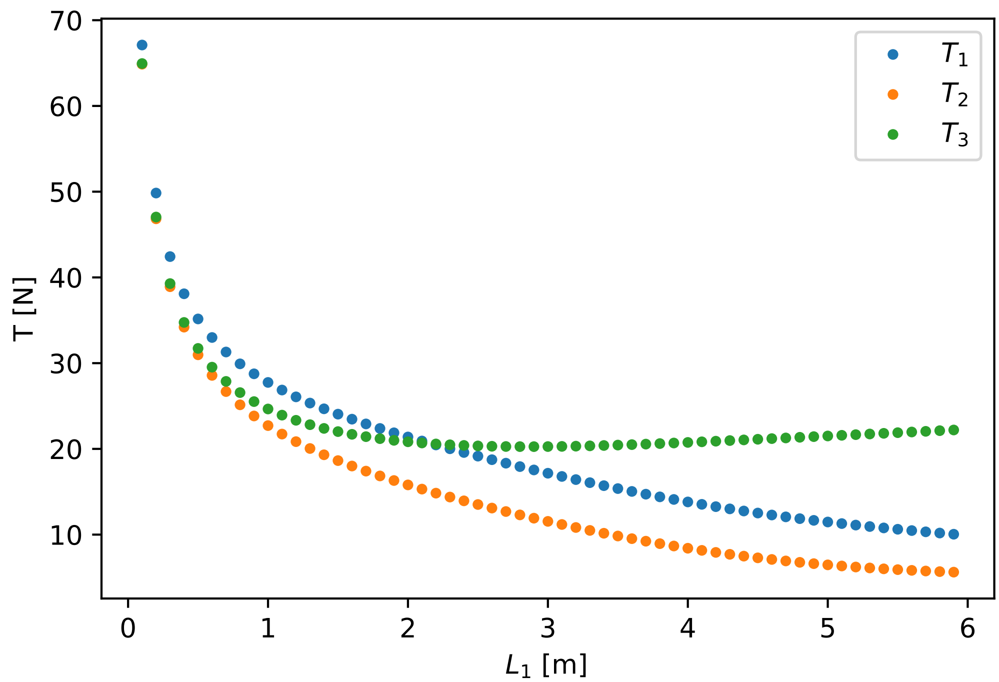
80
60
40
20
0
−20
−40
−60
−80
θ [degrees]
0
20
40
60
80
100
W_1 [N]
θ_1
θ_2
θ_3



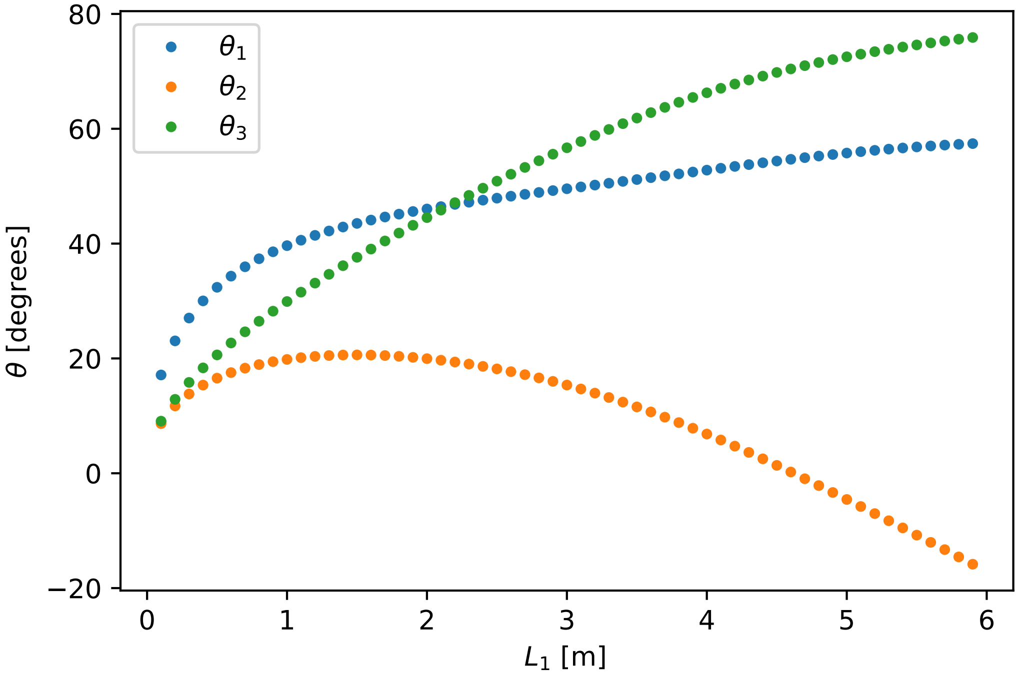
80
60
40
20
0
T [N]
0
20
40
60
80
100
W_2 [N]
T_1
T_2
T_3



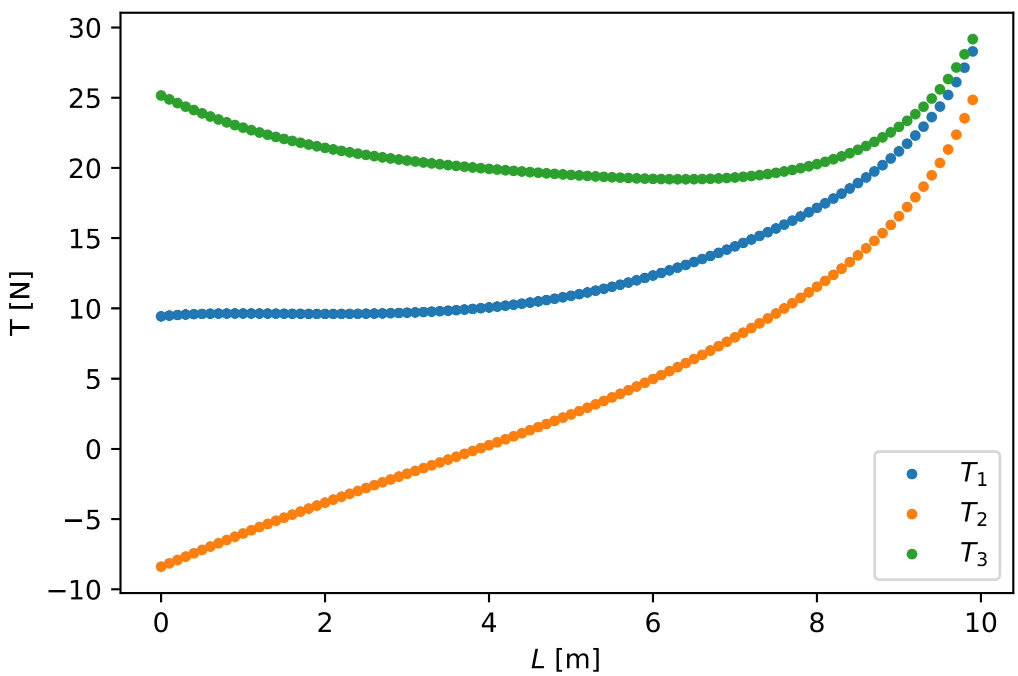
80
60
40
20
0
θ [degrees]
0
20
40
60
80
100
W_2 [N]
θ_1
θ_2
θ_3



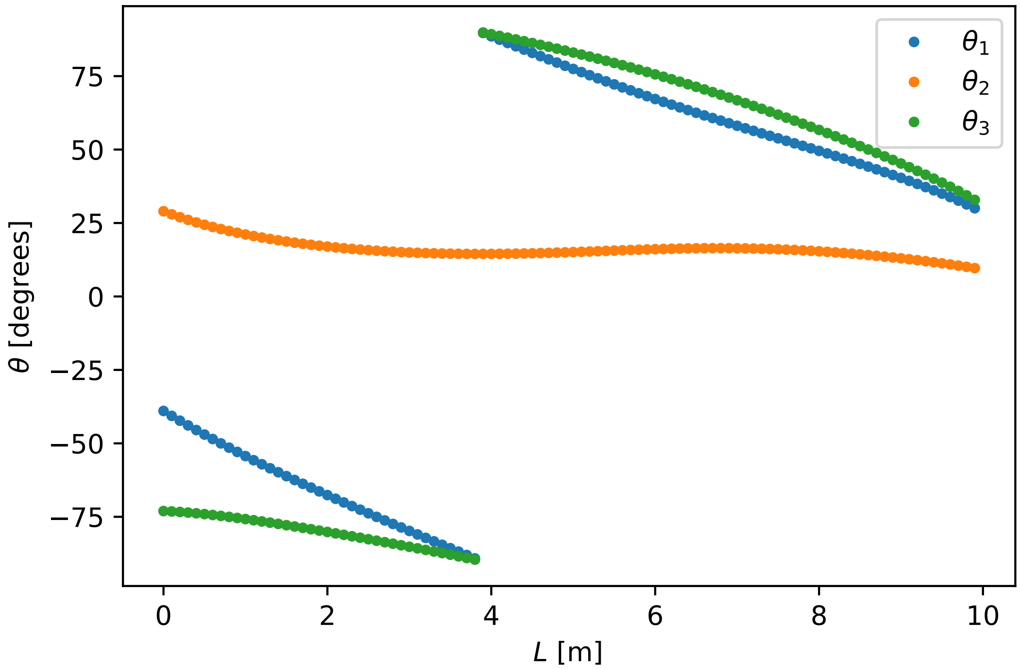
70
60
50
40
30
20
10
T [N]
0
1
2
3
4
5
6
L_1 [m]
T_1
T_2
T_3



θ [degrees]
L₁ [m]
θ₁
θ₂
θ₃
80
60
40
20
0
−20
0
1
2
3
4
5
6



T [N]
L [m]
T₁
T₂
T₃
30
25
20
15
10
5
0
−5
−10
0
2
4
6
8
10



Overall, the relationships between the weights of the two masses and the tensions and angles aren't too shocking. When the weights increase, the tensions increase at different rates depending on their relationship to the weights, as we would expect. The relationship between angle and weight is bit less apparent, but looking at a changing W_1, we see that as W_1 increases, θ_1 approaches ~90˚, or L_1 approaches the vertical. This makes sense, since W_1 will pull down on the string and dominate in the x-direction as W_1 becomes very large. Consequently, θ_2 approaches 0˚ as W_1 becomes very large, or L_2 approaches the horizontal, given the geometry of the situation. W_2 will follow a similar trend, only with θ_2 and θ_3.

Perhaps not surprising given the system of equations causes all of the variables to be related to one another, but seeing the tension forces change depending on the length of a segment of the string and the total length of the string isn't intuitive. At first glance, I would expect the length of the string to not have an impact on the tension forces, but looking at a changing L_1 and L, we see this is not the case. As L_1 approaches zero, all of the tension forces approach infinity. However, as L approaches zero, T_1, T_2, and T_3 behave differently, with T_1 becoming negative (pulling down on the mass?) and T_2, and T_3 remaining positive. As L becomes larger, the tension forces increase.