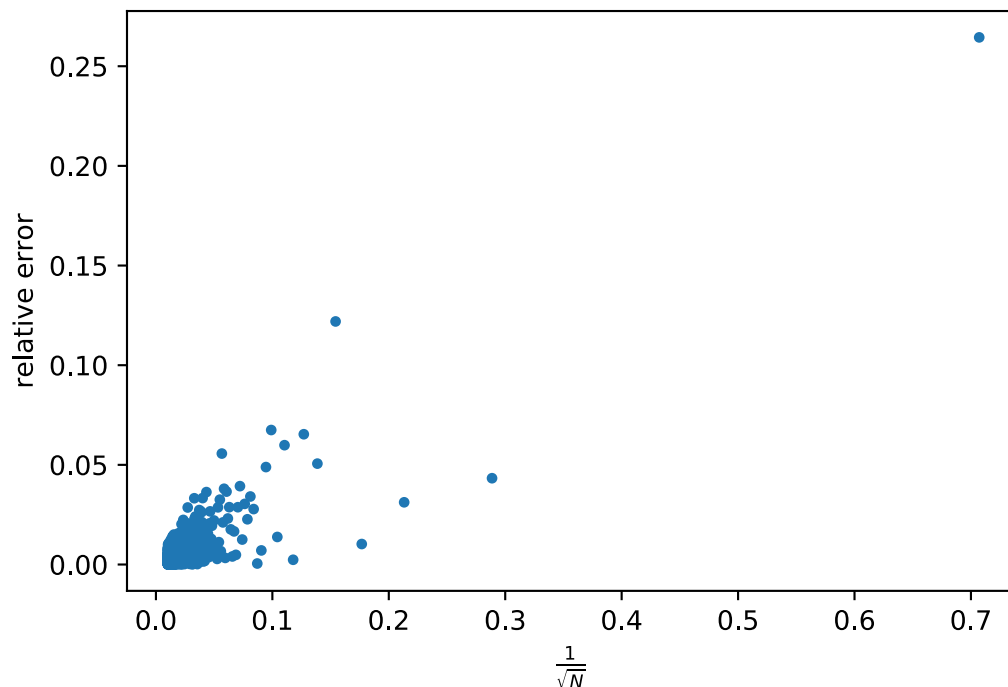


- (1) Calculating the 10-dimensional integral using the limit definition of an integral would require  $100^{10}$  calculations for 100 segments, which would require  $10^{14}$  seconds or  $\sim 10^7$  years. Using 1000 segments would require  $1000^{10}$  calculations, and  $10^{23}$  seconds, or  $\sim 10^{16}$  years to complete.

$$(2) I_{n-D} = (b-a)(d-c) \dots (\alpha-\gamma) \frac{\sum_i^N f(x_1, x_2, \dots x_n)}{N}$$

- (3) For 1000000 samples, Numeric Approximation = 25.819041034064217, Analytic Solution = 25.833333333333332.

- (4) Looking at the plot we can see as  $\frac{1}{\sqrt{N}}$  gets small, or the sample rate approaches infinity, the relative error goes to zero. Also, the relative error decreases quite rapidly with increasing sample size.



- (5) At a sampling size of 10000 or 10000 calculations, the numeric solution is accurate to the 1st decimal place, and the computation time is  $< 1$  second, or  $10^{14}$  orders of magnitude less time than the limit method would take to compute the same integral.
- (6) Assuming a computation rate of  $10^6$  calculations per second, and 100 intervals per dimension, a 1D integral would take  $\sim 0.0001$  seconds, and is more efficient than the MC method. A 2D integral would take  $\sim 0.01$  seconds, and would be equal to the computation time required for 10000 samples of the MC method. A 3D integral would require  $\sim 1$  second to complete, at which point a MC method of 10000 samples would be more efficient than the limit method.