Table A.1 Continuous distributions

Distribution	Notation	Parameters	Density function	Mean, variance, and mode
Uniform	$\theta \sim \mathrm{U}(\alpha,\beta)$ $p(\theta)=\mathrm{U}(\theta\mid\alpha,\beta)$	boundaries α,β with $\beta>\alpha$	$p(\theta)=\frac{1}{\beta-\alpha},\ \ \theta\in[\alpha,\beta]$	$\mathrm{E}(\theta)=\frac{\alpha+\beta}{2}$ $\mathrm{var}(\theta)=\frac{(\beta-\alpha)^2}{12}$ no mode
Normal	$\theta\sim\mathrm{N}(\mu,\sigma^2)$ $p(\theta)=\mathrm{N}(\theta\mid\mu,\sigma^2)$	location μ scale $\sigma>0$	$p(\theta)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(\theta-\mu)^2\right)$	$\mathrm{E}(\theta)=\mu$ $\mathrm{var}(\theta)=\sigma^2$ $\mathrm{mode}(\theta)=\mu$
Lognormal	$\theta\sim\mathrm{lognormal}(\mu,\sigma^2)$ $p(\theta)=\mathrm{lognormal}(\theta\mid\mu,\sigma^2)$	location μ log-scale $\sigma>0$	$p(\theta)=(\sqrt{2\pi}\sigma\theta)^{-1}\exp(-\frac{1}{2\sigma^2}(\log\theta-\mu)^2)$	$\mathrm{E}(\theta)=\exp(\mu+\frac{1}{2}\sigma^2)$, $\mathrm{var}(\theta)=\exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)$ $\mathrm{mode}(\theta)=\exp(\mu-\sigma^2)$
Multivariate normal	$\theta\sim\mathrm{N}(\mu,\Sigma)$ $p(\theta)=\mathrm{N}(\theta\mid\mu,\Sigma)$ (implicit dimension d)	symmetric, pos. definite, $d\times d$ variance matrix Σ	$p(\theta)=(2\pi)^{-d/2}\lvert\Sigma\rvert^{-1/2}$ $\times\exp\left(-\frac{1}{2}(\theta-\mu)^T\Sigma^{-1}(\theta-\mu)\right)$	$\mathrm{E}(\theta)=\mu$ $\mathrm{var}(\theta)=\Sigma$ $\mathrm{mode}(\theta)=\mu$
Gamma	$\theta\sim\mathrm{Gamma}(\alpha,\beta)$ $p(\theta)=\mathrm{Gamma}(\theta\mid\alpha,\beta)$	shape $\alpha>0$ inverse scale $\beta>0$	$p(\theta)=\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta},\ \ \theta>0$	$\mathrm{E}(\theta)=\frac{\alpha}{\beta}$ $\mathrm{var}(\theta)=\frac{\alpha}{\beta^2}$ $\mathrm{mode}(\theta)=\frac{\alpha-1}{\beta}$, for $\alpha\geq 1$
Inverse-gamma	$\theta\sim\mathrm{Inv\text{-}gamma}(\alpha,\beta)$ $p(\theta)=\mathrm{Inv\text{-}gamma}(\theta\mid\alpha,\beta)$	shape $\alpha>0$ scale $\beta>0$	$p(\theta)=\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{-(\alpha+1)}e^{-\beta/\theta},\ \ \theta>0$	$\mathrm{E}(\theta)=\frac{\beta}{\alpha-1}$, for $\alpha>1$ $\mathrm{var}(\theta)=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)},\alpha>2$ $\mathrm{mode}(\theta)=\frac{\beta}{\alpha+1}$
Chi-square	$\theta\sim\chi^2_\nu$ $p(\theta)=\chi^2_\nu(\theta)$	degrees of freedom $\nu>0$	$p(\theta)=\frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{\nu/2-1}e^{-\theta/2},\ \ \theta>0$ same as $\mathrm{Gamma}(\alpha=\frac{\nu}{2},\beta=\frac{1}{2})$	$\mathrm{E}(\theta)=\nu$ $\mathrm{var}(\theta)=2\nu$ $\mathrm{mode}(\theta)=\nu-2$, for $\nu\geq 2$
Inverse-chi-square	$\theta\sim\mathrm{Inv\text{-}}\chi^2_\nu$ $p(\theta)=\mathrm{Inv\text{-}}\chi^2_\nu(\theta)$	degrees of freedom $\nu>0$	$p(\theta)=\frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{-(\nu/2+1)}e^{-1/(2\theta)},\ \ \theta>0$ same as $\mathrm{Inv\text{-}gamma}(\alpha=\frac{\nu}{2},\beta=\frac{1}{2})$	$\mathrm{E}(\theta)=\frac{1}{\nu-2}$, for $\nu>2$ $\mathrm{var}(\theta)=\frac{2}{(\nu-2)^2(\nu-4)},\nu>4$ $\mathrm{mode}(\theta)=\frac{1}{\nu+2}$
Scaled inverse-chi-square	$\theta\sim\mathrm{Inv\text{-}}\chi^2(\nu,s^2)$ $p(\theta)=\mathrm{Inv\text{-}}\chi^2(\theta\mid\nu,s^2)$	degrees of freedom $\nu>0$ scale $s>0$	$p(\theta)=\frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)}s^\nu\theta^{-(\nu/2+1)}e^{-\nu s^2/(2\theta)},\ \ \theta>0$ same as $\mathrm{Inv\text{-}gamma}(\alpha=\frac{\nu}{2},\beta=\frac{\nu}{2}s^2)$	$\mathrm{E}(\theta)=\frac{\nu}{\nu-2}s^2$ $\mathrm{var}(\theta)=\frac{2\nu^2}{(\nu-2)^2(\nu-4)}s^4$ $\mathrm{mode}(\theta)=\frac{\nu}{\nu+2}s^2$
Exponential	$\theta\sim\mathrm{Expon}(\beta)$ $p(\theta)=\mathrm{Expon}(\theta\mid\beta)$	inverse scale $\beta>0$	$p(\theta)=\beta e^{-\beta\theta},\ \ \theta>0$ same as $\mathrm{Gamma}(\alpha=1,\beta)$	$\mathrm{E}(\theta)=\frac{1}{\beta}$ $\mathrm{var}(\theta)=\frac{1}{\beta^2}$ $\mathrm{mode}(\theta)=0$
Laplace (double-exponential)	$\theta\sim\mathrm{Laplace}(\mu,\sigma)$ $p(\theta)=\mathrm{Laplace}(\theta\mid\mu,\sigma)$	location μ scale $\sigma>0$	$p(\theta)=\frac{1}{2\sigma}\exp\left(-\frac{\lvert x-\mu\rvert}{\sigma}\right)$	$\mathrm{E}(\theta)=\mu$ $\mathrm{var}(\theta)=2\sigma^2$ $\mathrm{mode}(\theta)=\mu$
Weibull	$\theta\sim\mathrm{Weibull}(\alpha,\beta)$ $p(\theta)=\mathrm{Weibull}(\theta\mid\alpha,\beta)$	shape $\alpha>0$ scale $\beta>0$	$p(\theta)=\frac{\alpha}{\beta^\alpha}\theta^{\alpha-1}\exp(-(\theta/\beta)^\alpha),\ \ \theta>0$	$\mathrm{E}(\theta)=\beta\Gamma(1+\frac{1}{\alpha})$ $\mathrm{var}(\theta)=\beta^2[\Gamma(1+\frac{2}{\alpha})-(\Gamma(1+\frac{1}{\alpha}))^2]$ $\mathrm{mode}(\theta)=\beta(1-\frac{1}{\alpha})^{1/\alpha}$
Wishart	$W\sim\mathrm{Wishart}_\nu(S)$ $p(W)=\mathrm{Wishart}_\nu(W\mid S)$ (implicit dimension $k\times k$)	degrees of freedom ν symmetric, pos. definite $k\times k$ scale matrix S	$p(W)=\left(2^{\nu k/2}\pi^{k(k-1)/4}\prod_{i=1}^k\Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1}$ $\times\lvert S\rvert^{-\nu/2}\lvert W\rvert^{(\nu-k-1)/2}$ $\times\exp\left(-\frac{1}{2}\mathrm{tr}(S^{-1}W)\right)$, W pos. definite	$\mathrm{E}(W)=\nu S$
Inverse-Wishart	$W\sim\mathrm{Inv\text{-}Wishart}_\nu(S^{-1})$ $p(W)=\mathrm{Inv\text{-}Wishart}_\nu(W\mid S^{-1})$ (implicit dimension $k\times k$)	degrees of freedom ν symmetric, pos. definite $k\times k$ scale matrix S	$p(W)=\left(2^{\nu k/2}\pi^{k(k-1)/4}\prod_{i=1}^k\Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1}$ $\times\lvert S\rvert^{\nu/2}\lvert W\rvert^{-(\nu+k+1)/2}$ $\times\exp\left(-\frac{1}{2}\mathrm{tr}(SW^{-1})\right)$, W pos. definite	$\mathrm{E}(W)=(\nu-k-1)^{-1}S$
LKJ correlation	$\Sigma\sim\mathrm{LkjCorr}(\eta)$ $p(\Sigma)=\mathrm{LkjCorr}(\Sigma\mid\eta)$ (implicit dimension $k\times k$)	shape $\eta>0$	$p(\Sigma)=\det(\Sigma)^{\eta-1}$ $\times 2^{\sum_{i=1}^k(2\eta-2+k-i)(k-i)}$ $\times\prod_{i=1}^k\left(\mathrm{B}\left(\frac{i+1}{2},\frac{i+1}{2}\right)\right)^k$	$\mathrm{E}(\Sigma)=\mathrm{I}_k$,

Table A.1 Continuous distributions *continued*

Distribution	Notation	Parameters	Density function	Mean, variance, and mode
t	$\theta\sim t_\nu(\mu,\sigma^2)$ $p(\theta)=t_\nu(\theta\mid\mu,\sigma^2)$ t_ν is short for $t_\nu(0,1)$	degrees of freedom $\nu>0$ location μ scale $\sigma>0$	$p(\theta)=\frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\nu\pi}\sigma}(1+\frac{1}{\nu}(\frac{\theta-\mu}{\sigma})^2)^{-(\nu+1)/2}$	$\mathrm{E}(\theta)=\mu$, for $\nu>1$ $\mathrm{var}(\theta)=\frac{\nu}{\nu-2}\sigma^2$, for $\nu>2$ $\mathrm{mode}(\theta)=\mu$
Multivariate t	$\theta\sim t_\nu(\mu,\Sigma)$ $p(\theta)=t_\nu(\theta\mid\mu,\Sigma)$ (implicit dimension d)	degrees of freedom $\nu>0$ location $\mu=(\mu_1,\ldots,\mu_d)$ symmetric, pos. definite $d\times d$ scale matrix Σ	$p(\theta)=\frac{\Gamma((\nu+d)/2)}{\Gamma(\nu/2)\nu^{d/2}\pi^{d/2}}\lvert\Sigma\rvert^{-1/2}$ $\times(1+\frac{1}{\nu}(\theta-\mu)^T\Sigma^{-1}(\theta-\mu))^{-(\nu+d)/2}$	$\mathrm{E}(\theta)=\mu$, for $\nu>1$ $\mathrm{var}(\theta)=\frac{\nu}{\nu-2}\Sigma$, for $\nu>2$ $\mathrm{mode}(\theta)=\mu$
Beta	$\theta\sim\mathrm{Beta}(\alpha,\beta)$ $p(\theta)=\mathrm{Beta}(\theta\mid\alpha,\beta)$	'prior sample sizes' $\alpha>0,\beta>0$	$p(\theta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$ $\theta\in[0,1]$	$\mathrm{E}(\theta)=\frac{\alpha}{\alpha+\beta}$ $\mathrm{var}(\theta)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ $\mathrm{mode}(\theta)=\frac{\alpha-1}{\alpha+\beta-2}$
Dirichlet	$\theta\sim\mathrm{Dirichlet}(\alpha_1,\ldots,\alpha_k)$ $p(\theta)=\mathrm{Dirichlet}(\theta\mid\alpha_1,\ldots,\alpha_k)$	'prior sample sizes' $\alpha_j>0;\ \alpha_0\equiv\sum_{j=1}^k\alpha_j$	$p(\theta)=\frac{\Gamma(\alpha_1+\cdots+\alpha_k)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)}\theta_1^{\alpha_1-1}\cdots\theta_k^{\alpha_k-1}$ $\theta_1,\ldots,\theta_k\geq 0;\sum_{j=1}^k\theta_j=1$	$\mathrm{E}(\theta_j)=\frac{\alpha_j}{\alpha_0}$ $\mathrm{var}(\theta_j)=\frac{\alpha_j(\alpha_0-\alpha_j)}{\alpha_0^2(\alpha_0+1)}$ $\mathrm{cov}(\theta_i,\theta_j)=-\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$ $\mathrm{mode}(\theta_j)=\frac{\alpha_j-1}{\alpha_0-k}$
Logistic	$\theta\sim\mathrm{Logistic}(\mu,\sigma)$ $p(\theta)=\mathrm{Logistic}(\theta\mid\mu,\sigma)$	location μ scale $\sigma>0$	$p(\theta)=\frac{\exp(-\frac{x-\mu}{\sigma})}{\sigma(1+\exp(-\frac{x-\mu}{\sigma}))}$	$\mathrm{E}(\theta)=\mu$ $\mathrm{var}(\theta)=\frac{1}{3}\sigma^2\pi^2$ $\mathrm{mode}(\theta)=\mu$
Log-logistic	$\theta\sim\mathrm{Log\text{-}logistic}(\alpha,\beta)$ $p(\theta)=\mathrm{Log\text{-}logistic}(\theta\mid\alpha,\beta)$	scale $\alpha>0$ shape $\beta>0$	$p(\theta)=\frac{\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left[1+\left(\frac{x}{\alpha}\right)^\beta\right]^2},\ \ \theta>0$	$\mathrm{E}(\theta)=\frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}$ $\mathrm{var}(\theta)=\alpha^2\frac{2\pi/\beta}{\sin(2\pi/\beta)},\ \ \beta>2$ $\mathrm{mode}(\theta)=\alpha\left(\frac{\beta-1}{\beta+1}\right)^{\frac{1}{\beta}},\ \ \beta>1$

Table A.2 Discrete distributions

Distribution	Notation	Parameters	Density function	Mean, variance, and mode
Poisson	$\theta\sim\mathrm{Poisson}(\lambda)$ $p(\theta)=\mathrm{Poisson}(\theta\mid\lambda)$	'rate' $\lambda>0$	$p(\theta)=\frac{1}{\theta!}\lambda^\theta\exp(-\lambda)$ $\theta=0,1,2,\ldots$	$\mathrm{E}(\theta)=\lambda$, $\mathrm{var}(\theta)=\lambda$ $\mathrm{mode}(\theta)=\lfloor\lambda\rfloor$
Binomial	$\theta\sim\mathrm{Bin}(n,p)$ $p(\theta)=\mathrm{Bin}(\theta\mid n,p)$	'sample size' n (positive integer) 'probability' $p\in[0,1]$	$p(\theta)=\binom{n}{\theta}p^\theta(1-p)^{n-\theta}$ $\theta=0,1,2,\ldots,n$	$\mathrm{E}(\theta)=np$ $\mathrm{var}(\theta)=np(1-p)$ $\mathrm{mode}(\theta)=\lfloor(n+1)p\rfloor$
Multinomial	$\theta\sim\mathrm{Multin}(n;p_1,\ldots,p_k)$ $p(\theta)=\mathrm{Multin}(\theta\mid n;p_1,\ldots,p_k)$	'sample size' n (positive integer) 'probabilities' $p_j\in[0,1]$; $\sum_{j=1}^k p_j=1$	$p(\theta)=\binom{n}{\theta_1\ \theta_2\cdots\theta_k}p_1^{\theta_1}\cdots p_k^{\theta_k}$ $\theta_j=0,1,2,\ldots,n;\sum_{j=1}^k\theta_j=n$	$\mathrm{E}(\theta_j)=np_j$ $\mathrm{var}(\theta_j)=np_j(1-p_j)$ $\mathrm{cov}(\theta_i,\theta_j)=-np_ip_j$
Negative binomial	$\theta\sim\mathrm{Neg\text{-}bin}(\alpha,\beta)$ $p(\theta)=\mathrm{Neg\text{-}bin}(\theta\mid\alpha,\beta)$	shape $\alpha>0$ inverse scale $\beta>0$	$p(\theta)=\binom{\theta+\alpha-1}{\alpha-1}\left(\frac{\beta}{\beta+1}\right)^\alpha\left(\frac{1}{\beta+1}\right)^\theta$ $\theta=0,1,2,\ldots$	$\mathrm{E}(\theta)=\frac{\alpha}{\beta}$ $\mathrm{var}(\theta)=\frac{\alpha}{\beta^2}(\beta+1)$
Beta-binomial	$\theta\sim\mathrm{Beta\text{-}bin}(n,\alpha,\beta)$ $p(\theta)=\mathrm{Beta\text{-}bin}(\theta\mid n,\alpha,\beta)$	'sample size' n (positive integer) 'prior sample sizes' $\alpha>0,\beta>0$	$p(\theta)=\frac{\Gamma(n+1)}{\Gamma(\theta+1)\Gamma(n-\theta+1)}\frac{\Gamma(\alpha+\theta)\Gamma(n+\beta-\theta)}{\Gamma(\alpha+\beta+n)}$ $\times\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},\ \ \theta=0,1,2,\ldots,n$	$\mathrm{E}(\theta)=n\frac{\alpha}{\alpha+\beta}$ $\mathrm{var}(\theta)=n\frac{\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$