MATHEMATICS

Roll. No. .... (Pages 1)

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Department of Mathematics

FOURTH SEMESTER B.TECH. DEGREE EXAMINATIONS, INTERIM TEST-I JANUARY 2014

MA 2002 MATHEMATICS IV

Time: 1 Hour 15 Minutes **Max. Marks: 25**

Answer **ALL** Questions

1. Using Frobenius method, obtain two linearly independent solutions for the differential equation $2x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-3)y=0$ near the regular singular point x = 0. (3.5)
2. State and prove Rodrigue's formula for Legendre polynomials. (3)
3. Show that $J_{n+1}(x)=\frac{2n}{x}J_n(x)-J_{n-1}(x)$. Hence express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. (2)
4. Prove the following : (i) $\int_{-1}^{1} P_n(x)P_m(x)dx=0, \quad for \quad m\neq n$ (2)

   (ii) $\cos(x\sin\theta)=J_0+2(J_2\cos 2\theta+J_4\cos 4\theta+........)$ (2)

   (iii) If n is an integer, prove that $J_{-n}(x)=(-1)^n J_n(x)$. (1)
5. If $f(x)=\begin{cases} x, & -1<x<0 \\ 1, & 0<x<1 \end{cases}$, expand $f(x)$ as a Fourier-Legendre series on (-1, 1) by finding three nonzero coefficients. (3)
6. Find the eigen values and eigen functions of the Sturm-Liouville problem $\frac{d^2y}{dx^2}+\lambda y=0$ with $y(0)=0, y'(\pi)=0$ and $\lambda\in\mathbf{R}$. Also verify the orthogonality property of eigen functions. (3.5)
7. Form the PDE by eliminating the function ϕ from $\phi(x+y+z, x^2+y^2+z^2)=0$. (2)
8. Solve the PDE: $(y^2+z^2)p-xy\,q=-xz$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. (3)

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