NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
DEPARTMENT OF MATHEMATICS
Fourth Semester B.Tech. First interim test - February 2015
MA 2002 MATHEMATICS IV

Time: 75 minutes | Answer all questions | Max. marks: 20 T + 5 A

PART – A (5 marks)

1. Obtain power series solution of $(1+x^2)y''+xy'-y=0$.

(3)

2. Reduce into Bessel equation and hence write solution: $4x^2y''+4xy'+(64x^2-9)y=0$.

(2)

PART – B (20 marks)

3. Obtain the Rodrigue's formula for Legendre polynomials.

(3)

4. With usual notations prove that

(i) $P_{2n}(0)=\dfrac{(-1)^n(2n)!}{2^{2n}(n!)^2}$ and (ii) $P_{2n+1}(0)=0$.

(3)

5. Obtain the Fourier – Legendre series expansion (with at least two non-zero terms) of the function $f(x)=\begin{cases}1+x, & -1<x<0\\ 1-x, & 0<x<1\end{cases}$.

(3)

6. Prove that the Bessel functions of the first kind $J_n(x)$ and $J_{-n}(x)$ are linearly dependent, when n is an integer.

(2)

7. Prove that $\dfrac{2p}{x}J_p(x)=J_{p+1}(x)+J_{p-1}(x)$. Hence obtain $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(3)

8. Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx=0$, if α and β are distinct positive roots of the equation $J_n(x)=0$.

(3)

9. Obtain the eigenvalues and eigenfunctions for the periodic Sturm-Liouville problem $y''+\lambda^2y=0;\ \ y(0)=y(2L)$ and $y'(0)=y'(2L)$. Verify orthogonality property of eigenfunctions by direct calculations.

(3)