MATHEMATICS

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MA 1002 - MATHEMATICS-II

Duration: 1 Hour 15 Minutes

Max. Marks: 25

1. Find the volume included between $x^2+y^2+z^2=a^2$ and the cone $x^2+y^2=z^2$ and lying above the xy plane ($z \geq 0$). 3

2. Use Green's theorem to compute $I = \oint_C (2y+\sqrt{1+x^5})dx + (5x - e^{y^2})dy$ where C is the circle $x^2+y^2=4$. 2

3. Evaluate the flux of $\vec{F} = y\hat{i} + 2x\hat{j} - z\hat{k}$ over the surface S, where S is the surface of the plane $2x+y=6$ in the first octant cut by $z=4$. 3

4. Use Stokes theorem to evaluate $\oint_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0)$ and $(0,0,6)$. 3

5. Use Gauss divergence theorem to find the outward flux of the vector field $\vec{F} = (x^2+y)\hat{i} + z^2\hat{j} + (e^y - z)\hat{k}$, across the surface of the region enclosed by the coordinate planes and the planes $x=3, y=1$ and $z=2$. 3

6. A large apartment building is to be constructed using modular construction techniques. The arrangement of the apartment on any particular floor is to be chosen from one of the 3 basic floor plans. Each floor plan of A includes 3 three bedroom units, 7 two bedroom units and 8 one bedroom units. Each floor plan of B includes 4 three bedroom units, 4 two bedroom units and 8 one bedroom units. Each floor plan of C includes 5 three bedroom units, 3 two bedroom units and 9 one bedroom units. If the building should have exactly 66 three bedroom units, 74 two bedroom units and 136 one bedroom units, determine the number of floors of each type to be constructed. 3

7. Determine whether the set of all ordered triples of real numbers (x,y,z) with the operations $(x,y,z) \oplus (x',y',z') = (x+x', y+y', z+z')$ and $c \odot (x,y,z) = (cx, y, z)$ is a vector space or not. Justify your answer. 2

8. Let V be a vector space over the field R. Let $v_1, v_2, ..., v_k, w$ be vectors of V. If $S = \{v_1, v_2, ..., v_k\}$ is linearly independent and if $v_1+w, v_2+w, ..., v_k+w$ are linearly dependent vectors, then prove that $w \in Span(S)$. Also prove that $v_1+v_2, v_2+v_3, ..., v_{k-1}+v_k$ are linearly independent. 3

9. Let V be the vector space of all real polynomials of degree at most three. Let W be the subset of V defined by $W = \{p(x) \in V : p'(1) = 0 = p'(2)\}$, where p' denotes the derivative of p. Is W a subspace of V? If so find a basis of W and hence find its dimension. 3