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NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Department of Mathematics

First Semester B.Tech End Semester Examination, November 2013

MA 1001 - MATHEMATICS I

Part A

This part contains 10 questions each of 4 marks. Answer all questions in the answer book provided. Part B of the question paper is attached with the main answer book. The duration of the examination (for both parts together) is 3 hours and the total marks is 50.

1. (a) Let $y_1(t)$ and $y_2(t)$ be solutions of the differential equation $y''(t) + f(t)y'(t) + g(t)y(t) = 0$ and $W(y_1, y_2)$ their Wronskian. Then prove that

$$f(t).W(y_1, y_2)(t) = y_1(t)y_2''(t) - y_2(t)y_1''(t).$$

   (b) Determine $P(x)$ and $Q(x)$ so that $y_1(x) = 1 + x$ and $y_2(x) = e^x$ are solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$.

2. (a) Let $f(x, y) = \ln(x^2 + y^2 + xy)$. Then use Euler's theorem for homogeneous functions to prove that $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = 2$.

   (b) Using the method of Lagrange multiplier, find the points on the circle $x^2 + y^2 = 18$ where the function $f(x, y) = xy + 14$ assumes maximum and also the points where f attains minimum.

3. Expand $f(x) = x \sin x$ as a half-range Fourier cosine series in $0 \leq x < \pi$ and find the sum of this series at $x = -\frac{\pi}{2}$.

4. Find the Fourier series expansion of the function $f(x) = \frac{x^2}{4}, -\pi \leq x < \pi$. Use this to find the sum of the following infinite series

   (i) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$.

   (ii) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$.

5. Find the Fourier cosine transform of the function $f(x) = e^{-x^2}, x \geq 0$.

6. Find the Fourier cosine integral representation of the function $f(x) = \begin{cases} \frac{\pi}{2} & \text{if } 0 < x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$

and hence evaluate $\displaystyle\int_0^\infty \frac{\sin y \cos y}{y}\, dy$.

7. (a) Using convolution, find the inverse Laplace transform of $F(s) = \dfrac{1}{(s-2)(s+2)^2}$.

(b) Evaluate $\displaystyle\int_0^1 \left(x \ln \frac{1}{x}\right)^{1/2} dx$.

8. (a) Find the inverse Laplace transform of the function $F(s) = \ln \dfrac{s+1}{s-1}$.

(b) Express the following function in terms of the unit step functions and hence find its Laplace transform.

$$f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 1, \\ 2t - 1 & \text{if } 1 \leq t < 2 \\ 3 & \text{if } t \geq 2. \end{cases}$$

9. (a) Find the Laplace transform of the function $f(t) = te^{-t}\sin^2 t, t \geq 0$.

(b) Let $f(t) = 1, t \geq 0$. Then find $\underbrace{f * f * \cdots * f}_{n \text{ times}}$, where $*$ denotes the convolution operation.

10. Solve the following initial value problem using Laplace transforms:

$$ty''(t) - ty'(t) + y(t) = 2; \; y(0) = 2, y'(0) = -4.$$

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