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MA 1001 - MATHEMATICS I

Duration: 75 minutes **Max. Marks: 25**

1. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, find the Jacobian $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (2)

2. Let $w = \ln(e^x + e^y)$. Then prove that $-\dfrac{1}{4} \leq \dfrac{\partial^2 w}{\partial x \partial y} < 0$ (3)

3. (a) Determine whether the following series is convergent or divergent. Find the sum if it is convergent. (1½)

$$\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + \cdots$$

(b) Find the Maclaurin series expansion of the function $f(x) = 1 + 2x - 4x^2$. Find the radius of convergence of this series. (1½)

4. Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty}\left[n^4\sin^2\left(\frac{3n}{2n^3 - 2n^2 + 5}\right)\right]^n$. (2)

5. Test the convergence of the series (3)

$$x^2 + \frac{2^2}{3.4}x^4 + \frac{2^2.4^2}{3.4.5.6}x^6 + \frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8 + \cdots$$

6. Discuss the convergence of the series $\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{1}{n}\right)$. (2)

7. Obtain the interval of convergence of the power series (3)

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots$$

8. Show that the critical points of the function $\sin x + \sin y + \sin(x + y)$ are $(\pm\pi, \pm\pi), (\pm\frac{\pi}{3}, \pm\frac{\pi}{3})$. Examine for maximum and minimum at $(\pm\frac{\pi}{3}, \pm\frac{\pi}{3})$. (3)

9. Find the current at time t in the following electrical circuit if $R = 40\Omega, L = 1H$, $C = \dfrac{1}{625}\text{F}, E = 100\cos(10t)$ volts. At time $t = 0$, both the charge and current are zero. (4)