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MA 2001 Mathematics III

(Common to all branches)

Statistical tables are permitted

Time: 3 hours

Max. Marks: 50

NOTE: ANSWER ALL QUESTIONS

1. The number of customers who visit a car show-room on Sunday morning in a metro city is a random variable with mean 20 and variance of 24. With what probability, we can assert that half of the number of customers visiting will be lying between 5 and 15? (2)
2. With necessary assumptions, obtain Poisson distribution as the limit of Binomial distribution. (3)
3. What is the maximum error one can expect to make with probability 0.95 when using the mean of a random sample of size 49 to estimate the mean of a population with variance of 2.56? (2)
4. A large number of automobile batteries have an average life of 24 months. 34% of them lasted between 22 months and 26 months, and 188 of them lasted longer than 30 months. If the life of such batteries follows Normal distribution, estimate the number of batteries that were tested. (2)
5. Given a continuous random variable having probability density function

$$f(x) = \begin{cases} \frac{2}{\theta^2}(\theta - x), & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

Find an MLE of the parameter θ based on a sample of size one. Is this estimator unbiased for θ ? (3)

6. Suppose that we have two brands of soft drinks with different sugar contents. The following are the data obtained from the samples.

Brand 1: $n_1 = 50, \bar{x}_1 = 2.65, s_1 = 0.13$.

Brand 2: $n_2 = 39, \bar{x}_2 = 2.40, s_2 = 0.15$.

Test $H_0: \mu_1 - \mu_2 = 0.2$ against $H_1: \mu_1 - \mu_2 \neq 0.2$ at $\alpha = 0.05$, where μ_1, μ_2 are mean sugar contents of respective population of soft drinks.

(3)

7. Given the following data pertaining to the interests in Statistics and ability in Mathematics of a group of 360 people.

		Ability in Mathematics		
		Low	Average	High
Interest in Statistics	Low	63	42	15
	Average	58	61	31
	High	14	47	29

Test at 1% level of significance, whether a person's ability in Mathematics is independent of his or her interest in Statistics.

(4)

8. A machine puts out 16 imperfect articles in a sample of 500 articles. After the machine is overhauled it puts out 3 defective articles in a sample of 100. Has the machine improved? Draw appropriate conclusions at 5% level of significance.

(3)

9. Two different types of electrical cable insulation have been tested to determine the voltage level at which failures tend to occur. When specimens were subjected to an increasing voltage stress in a laboratory experiment, failures for the two types of cable insulation occurred at the following voltages:

Type I: 36, 44, 41, 53, 38, 36, 34, 54, 52, 37

Type II: 52, 64, 38, 68, 66, 52, 60, 44, 48, 46

Suppose that it is known that the amount of voltage that cables having Type I insulation can withstand is normally distributed with unknown mean μ_1 and known variance 39, whereas the corresponding distribution for Type II insulation is normal with unknown mean μ_2 and known variance 98. Construct a 95% two sided confidence interval for $\mu_1 - \mu_2$.

(3)

10. The number of pounds of steam used per month by a chemical plant is thought to be related to the average ambient temperature for that month. The past 12 month's (Jan-Dec) usage and temperature are shown in the following table.

Month	Jan	Feb	Mar	Apr	May	Jun
Temperature	21	24	32	47	58	69
Usage/1000 lbs	185.79	214.47	288.03	424.84	414.88	539.05

Month	July	Aug	Sep	Oct	Nov	Dec
Temperature	68	74	62	50	41	30
Usage/1000 lbs	621.55	675.06	562.03	452.93	365.95	273.98

- Fit a linear regression model to the data.
- Test the hypothesis that $\beta=10$ against $\beta \neq 10$, at 5% level of significance.
- Construct a 99% confidence interval for β .

(4)

11. The weight and systolic blood pressure (BP) of 12 randomly selected males in the age group of 25 to 30 are shown in the following table.

Subject	1	2	3	4	5	6
Weight (in lbs)	165	167	180	155	212	175
Systolic BP	130	133	150	128	151	146

Subject	7	8	9	10	11	12
Weight (in lbs)	190	218	200	149	158	169
Systolic BP	150	140	148	125	133	135

- Estimate the correlation coefficient between weight and systolic BP.
- Test the hypothesis for $\rho = 0$ against $\rho \neq 0$, at 5% level of significance.
- Construct 95% confidence interval for ρ .

(4)

12. Given the yields per 10 hectare of five different seeds varieties grown on lots treated with three different types of fertilizers. Test at 5% level of significance whether there is significant difference in yields per hectare (a) due to fertilizers (b) due to crops.

		Crops				
Fertilizers		C1	C2	C3	C4	C5
	F1	4.5	6.4	7.2	6.7	7.0
	F2	8.8	7.8	9.6	7.0	7.5
	F3	5.9	6.8	5.7	5.2	6.4

(5)

13. Given the following data obtained through 3x3 Latin square design, perform an analysis of variance and draw your conclusions at 5% level of significance.

B	A	C
10.7	10.3	11.2
A	C	B
11.3	10.5	12.0
C	B	A
11.8	10.9	10.5

A	B	C
10.9	10.5	14
C	A	B
11.5	10.3	13
B	C	A
11.3	7.5	12.7

14. State and prove the identity for the analysis of variance under one-way classification.

(6)

15. Briefly describe with necessary assumptions, the method of analysis of covariance.

(3)

(3)
