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MA 2001 MATHEMATICS III (Probability and Statistics)

(Common to all branches)

Statistical tables are permitted

Time: 1 hr. 15 min Max. Marks: 25

Note: Answer all questions

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1. The probability mass function of a random variable X, is given by

$$f(x) = \begin{cases} k, & if\ x = 0, \\ 2k, & if\ x = 1, \\ 3k, & if\ x = 2, \\ 0, & otherwise, \end{cases}$$

where k is a real number.

(a) Determine the value of k so that $f(x)$ is indeed a probability mass function.

(b) What is the smallest value of c such that $P(X \leq c) > 0.5$. (2)

2. The moment generating function of a discrete random variable X is given to be $M_X(t) = (0.3+0.7e^t)^5$. Specify the probability distribution of X, and obtain its probability mass function. Also find the mean and variance of X using $M_X(t)$. (3)

3. Twenty percent of IC chips produced in a certain plant are defective. Use normal distribution to find the approximate probability that at most 13 are defective in a random sample of 100 IC chips produced by this plant. (2)

4. The working life of light bulb is normally distributed with a mean of 500 hours and a standard deviation of 60 hours. If a light bulb is still working after 440 hours of operation, what is the conditional probability that its lifetime exceeds 560 hours? (2)

5. (a) State Chebyshev's theorem.

(b) The mean time taken by all participants to run a road race was found to be 220 mins ~~meters~~ with a standard deviation of 20 meters. Using Chebyshev's theorem, find the percentage of runners who ran the road race in

(i) 180 to 260 mins ~~meters~~ (ii) 160 to 280 mins ~~meters~~. (3)

6. Two computers are needed for an experiment. Of the five available, two have electronic defects, another one has a defect in memory, and two are in good working condition.

Two computers are selected at random. Let X_1 denote the number of computers having electronic defects, and X_2 denote the number of computers having defects in memory. Find the joint probability mass function of (X_1, X_2). (2)

7. Suppose that X and Y have joint probability density function given by

$$f(x,y) = \begin{cases} 2e^{-x-2y}, 0 < x < \infty, 0 < y < \infty \\ 0, \quad otherwise \end{cases}$$

(a) Find the marginal density functions of X and Y.

(b) Find $P(X>Y)$. (3)

8. Let $X_1, X_2, \ldots X_n$ be a random sample from a distribution given by

$$f(x) = p(1-p)^{x-1}, x = 1,2, \ldots \infty.$$

Obtain a maximum likelihood estimator of p. (3)

9. (a) The mean and variance of a Uniform distribution are 5 and 3. Obtain the Uniform distribution, and hence find $P(X>5)$. (3)

(b) The random variable X, the time to failure (in thousands of miles) of signal light on an automobile has a Weibull distribution with $\alpha = 0.05$, $\beta = 2$. What is the probability that the light will fail during the first 3000 miles driven? (2)

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