NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
DEPARTMENT OF MATHEMATICS
Fourth Semester B.Tech. Second interim test - April 2015

MA 2002 MATHEMATICS IV

Time: **75** minutes | Answer all questions | Max. marks: **20** T + **5** A

PART – A (5 marks)

1. If f(z) is analytic prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

(3)

2. Find the image of the circle $|\ z-3i\ |=3$ under the inversion map $w=\frac{1}{z}$.

(2)

PART – B (20 marks)

3. When do we say that a function $f(z)$ is analytic at a point? Show that Cauchy-Riemann equations are necessary for the function $f(z)=u+iv$ to be analytic at a point.

(3)

4. Determine the analytic function $f(z)$ such that the real part of $f'(z)$ is $3x^2-4y-3y^2$ and $f(0)=1, f'(0)=i.$

(3)

5. Find the bilinear transformation that maps the points $z=1, i, -1$ to $w=-i, 0, i$ respectively. What is the image of the real axis under this map?

(3)

6. Evaluate $\int_C f(z)dz$ where $f(z)=\begin{cases}4y, \text{when } y>0\\ 1, \text{when } y<0\end{cases}$ and C is the arc from $z=1-i$ to $z=1+i$ of the cubical curve $y=x^3$.

(2)

7. State and prove Cauchy's integral theorem. Using this theorem find the value of $\int_C \frac{1}{z-2}dz$, where C is the circle $|z|=1$.

(3)

8. Using Cauchy's integral formula, evaluate $\int_C \frac{(3z^2-2z)}{(z+1)^2(z-2)}dz$, where C is the circle $|z-1|=3$. **(3)**

9. Find the Laurent series expansions of $f(z)=\frac{1}{z^2+1}$ about its singular points. Also state the regions of convergence.

(3)

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