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NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

Department of Mathematics

FOURTH SEMESTER B. TECH. DEGREE EXAMINATIONS, INTERIM TEST-II, MARCH 2014

MA 2002 MATHEMATICS IV

Time: 1 Hour 15 Minutes **Max. Marks: 25**

Answer ALL Questions

1. Using Charpit's method obtain the complete integral of $p^2 + q^2 - 5z = 0$. (3)

2. Using the method of separation of variables, solve the initial value problem

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \quad u(x,0) = 6e^{-3x}.$$ (2)

3. Find the D'Alembert's solution of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$,

satisfying the initial conditions $u(x,0) = f(x)$ and $\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0.$ (3)

4. Find the temperature $u(x, t)$ at any time t and at a distance x from one end of a homogeneous rod of heat conducting material of length L with its ends kept at zero temperature with initial temperature given by $\frac{k\,x\,(L-x)}{L^2}$ for some constant k. (4)

5. Derive the Cauchy-Riemann equations in polar form. Hence prove that the real part $u(r,\theta)$ of the analytic function $f(z) = u(r,\theta) + i\,v(r,\theta)$ satisfies

$$r^2 \frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0.$$ (4)

6. Can $u(x,y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ be the real part of an analytic function? Justify. If so, find the analytic function having $u(x,y)$ as the real part. (3)

7. Show that the map $w = \frac{1}{z}$ carries the circle $|z-2| = 7$ to the circle $\left|w + \frac{2}{45}\right| = \frac{7}{45}$. (3)

8. Discuss the mapping $w = \sin z$. (3)

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