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Second Semester B.Tech End Semester Examination, Winter Semester 2012-13

MA 1002 - MATHEMATICS II

Part A

This part contains 10 questions each of 4 marks. Answer all questions in the answer book provided. Part B of the question paper is attached with the main answer book. The duration of the examination (for both parts together) is 3 hours and the total marks is 50.

1. (a) Find the scalar potential of the function $(2x + yz)\hat{\imath} + (4y + \overset{xz}{\cancel{2x}})\hat{\jmath} - (6z - xy)\hat{k}$.

   (b) Find the circulation of the function $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ around the circle $C : x^2 + y^2 = 1; z = 0$.

2. Verify Green's theorem for the plane for $\oint_C xy\,dx + x^2\,dy$ where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

3. Verify Stokes' theorem for the function $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ over the region defined by the upper half of the sphere $x^2 + y^2 + z^2 = 1$.

4. Let $S_1 = \{x_1, x_2, x_3\}$ be a linearly independent set in a vector space V. Let $y_1 = x_1 + x_2 + 2x_3$; $y_2 = x_1 + 2x_2 + x_3$; $y_3 = x_1 + x_2 + 3x_3$. Then is the set $S_2 = \{y_1, y_2, y_3\}$ linearly independent in V? Justify your answer.

5. Find an orthogonal basis for the space spanned by the vectors $(0, 1, 2), (2, 0, -2), (2, 1, 0)$ in the vector space $\mathbb{R}^3$ over $\mathbb{R}$.

6. Let T be a linear transformation from a vector space V into another vector space W. Then prove that T is one-one if and only if kernel(T) is $\{0\}$. Use this result to check whether $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, y - z, z - x)$ is one-one.

7. Let V be the real vector space of all even polynomials with real coefficients and degree at most 6. Let $T\colon V \to V$ be defined by $T(p(x)) = \frac{d^2}{dx^2}(p(x))$. Verify rank-nully theorem for T.

8. Find a basis and the dimension of the null-space (kernel) of the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x - 2y + z, 2x - 4y + 2z)$. Extend the basis you obtained for the null-space to a basis of the vector space $\mathbb{R}^3$.

9. The eigen vectors corresponding to the eigen values $2, 3, 6$ of a 3×3 matrix A are $[-1\ 0\ 1]^T, [1\ 1\ 1]^T, [1\ \ -2\ 1]^T$ respectively. Find the matrix A.

10. Reduce the quadratic form $Q(x_1, x_2, x_3) = 7x_1^2 + 6x_2^2 + 5x_3^2 - 4x_1x_2 - 4x_2x_3$ to its canonical form using an orthogonal transformation. Specify the matrix of transformation. Also find the nature of this quadratic form; that is whether it is indefinite or positive/negative (semi) definite.