# Sensitivity Analysis for Balancing Weights

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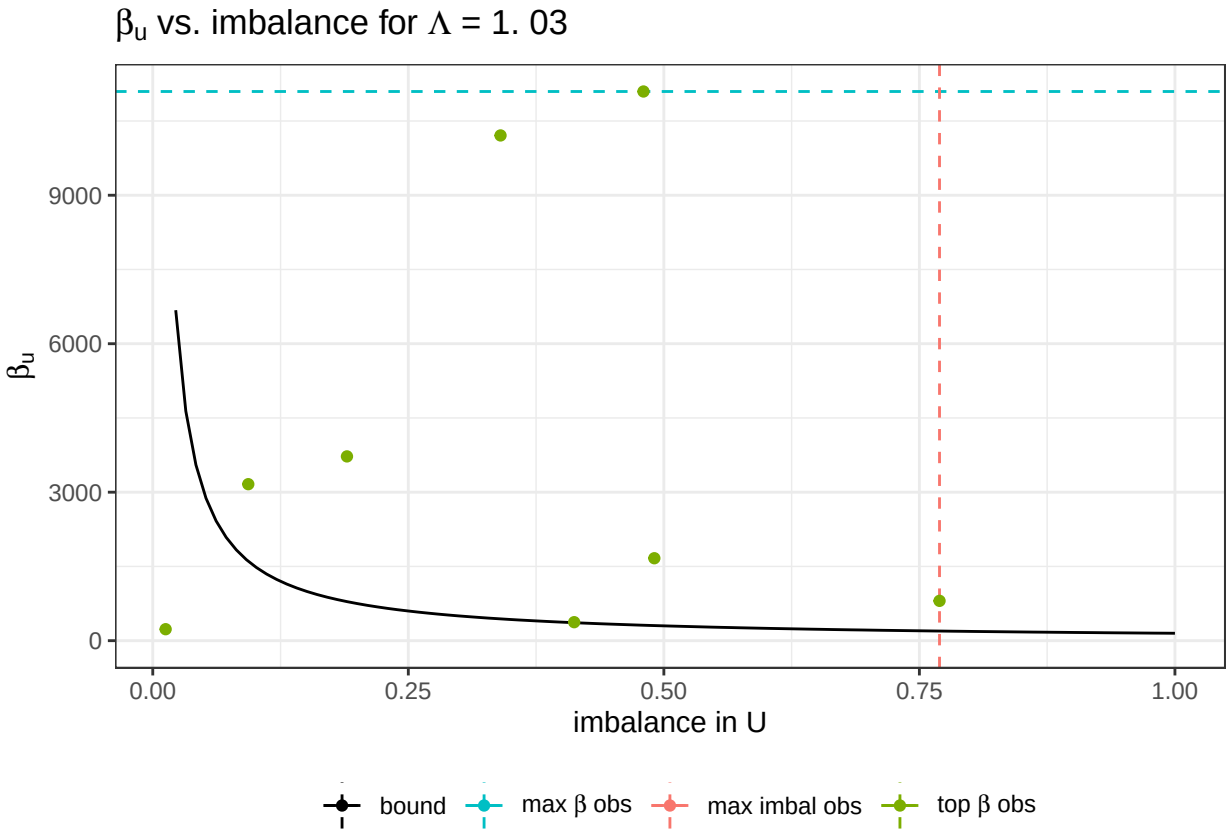
## Amplification

Amplification of bias = imbalance in $U \times \beta_u$



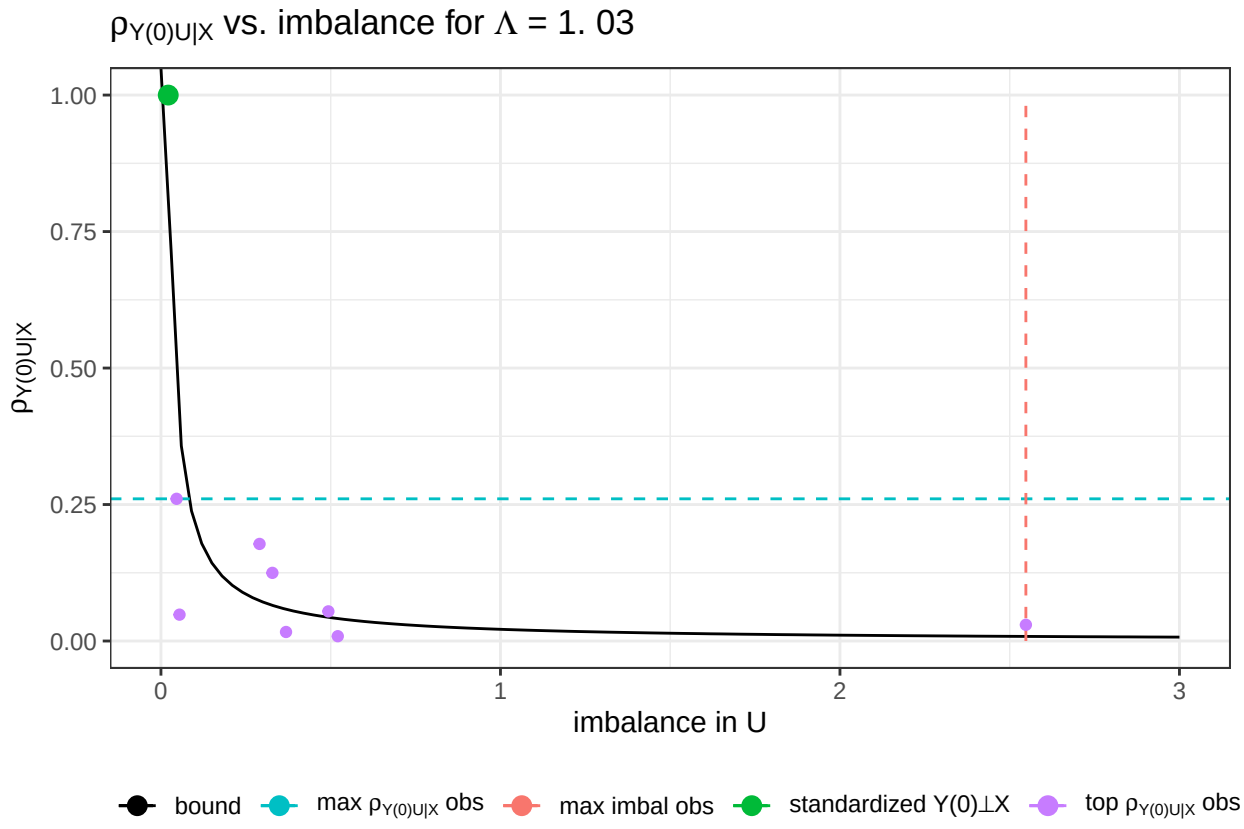

- **bound:**
  - if estimated ATT is positive, bound $= \left(\sup_{h \in \mathcal{H}(\Lambda)} \hat{\mu}_0^{(h)}\right) - \hat{\mu}_0$
  - if estimated ATT is negative, bound $= \left(\inf_{h \in \mathcal{H}(\Lambda)} \hat{\mu}_0^{(h)}\right) - \hat{\mu}_0$
- We consider $U \in [0, 1]$, so we transform each observed covariate as follows:
  - Make min $= 0$: subtract min value of covariate

  - Make max = 1: divide by max of shifted covariate

- **max $\beta$ obs:** max absolute value of coefficients of transformed covariates from OLS of Y on transformed covariates for control units.
- **max imbal obs:** max absolute value of difference in means of transformed covariates before weighting between treatment and control.
- **top $\beta$ obs:** coefficient and imbalance for specified number of observed covariates sorted by descending coefficient value

| covar | coeff | imbal |
|---|---|---|
| re75 | 11095.990 | 0.480 |
| re74 | 10208.155 | 0.340 |
| age | 3723.355 | 0.190 |
| education | 3162.144 | 0.093 |
| unemployed_75 | 1666.033 | 0.491 |
| black | 804.809 | 0.770 |
| nodegree | 373.719 | 0.412 |
| hispanic | 232.325 | 0.013 |
| unemployed_74 | 204.942 | 0.588 |
| married | 159.745 | 0.523 |

Amplification of bias = imbalance in $U \times \rho_{Y(0),U|X} \times \text{sd}(Y(0)^{\perp X})$




- **bound:**

  - if estimated ATT is positive, bound = $\frac{\left(\sup_{h \in \mathcal{H}(\Lambda)} \hat{\mu}_0^{(h)}\right) - \hat{\mu}_0}{\text{sd}(Y(0)^{\perp X})}$
  - if estimated ATT is negative, bound = $\frac{\left(\inf_{h \in \mathcal{H}(\Lambda)} \hat{\mu}_0^{(h)}\right) - \hat{\mu}_0}{\text{sd}(Y(0)^{\perp X})}$
- We consider $U \perp X$ with $\text{sd}(U) = 1$, so we transform each observed covariate as follows:
  - Residualize: residualized covarites = residuals from OLS of a covariate on the other covariates
  - Standardize: divide the residualized covariate by the standard deviation of the residualize covariate
- **max $\rho_{Y(0)U|X}$ obs:** max absolute value of the partial correlations of $Y(0)$ and a covariate given the other covariates for control units.
- **max imbal obs:** max absolute value of difference in means of transformed covariates before weighting between treatment and control.
- **standardized $Y(0)^{\perp X}$:** point for strongest possible confounder with partial correlation = 1.
- **top $\rho_{Y(0)U|X}$ obs:** partial correlation and imbalance for specified number of observed covariates sorted by descending partial correlation

| covar | pcorr | imbal |
|---|---|---|
| re75 | 0.260 | 0.046 |
| re74 | 0.178 | 0.290 |
| age | 0.125 | 0.328 |
| unemployed_75 | 0.054 | 0.493 |
| education | 0.048 | 0.055 |
| black | 0.030 | 2.548 |
| nodegree | 0.017 | 0.368 |
| married | 0.009 | 0.521 |
| hispanic | 0.008 | 0.142 |
| unemployed_74 | 0.007 | 0.892 |