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Tutorial 5

1) a) $L = \{a^n b^m c^k \mid n + 2m = k\} \quad n \geq 0, m \geq 0, k \geq 0$

$$S \rightarrow aSc \mid A \mid \epsilon$$

$$A \rightarrow bAcc \mid \epsilon$$

b) $L = \{a^n b^m c^k \mid k = |n - m|\} \quad n \geq 0, m \geq 0, k \geq 0$

$k = |n - m|$ can be written as

$$k = n - m \text{ or } k = m - n$$

$$\Rightarrow n = m + k \Rightarrow L_1 \rightarrow aL_1c \mid A \mid \epsilon$$

$$A \rightarrow aAb \mid \epsilon$$

or

$$\Rightarrow m = n + k \Rightarrow L_2 \rightarrow AB \mid \epsilon$$

$$A \rightarrow aAb \mid \epsilon$$

$$B \rightarrow bBc \mid \epsilon$$

$$L = L_1 \cup L_2 \Rightarrow L \rightarrow L_1 \mid L_2$$

2) $S \rightarrow aAB$

$$A \rightarrow bBb$$

$$B \rightarrow A \mid \epsilon$$

String - abbbb

a) Leftmost derivation

$$S \rightarrow aAB$$

$$aAB \rightarrow abBbB$$

$$abBbB \rightarrow abAbbB$$

$$abAbbB \rightarrow abbbBbbB$$

$$abbbBbbB \rightarrow abbbbbb$$

b) Rightmost derivation

$$S \rightarrow aAB$$

$$aAB \rightarrow aA\epsilon$$

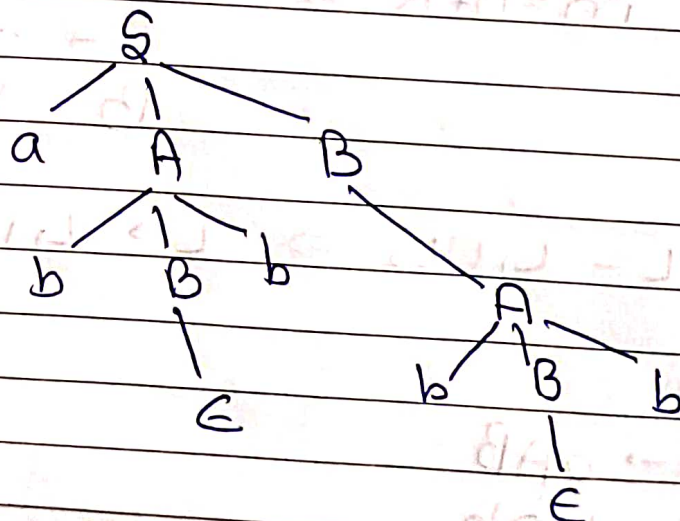
$$aA \rightarrow abBb$$

$$abBb \rightarrow abAb$$

$$abAb \rightarrow abbbBbb$$

$$abbbBbb \rightarrow abbbbbb$$

c) Parse tree



3) Show grammar is ambiguous

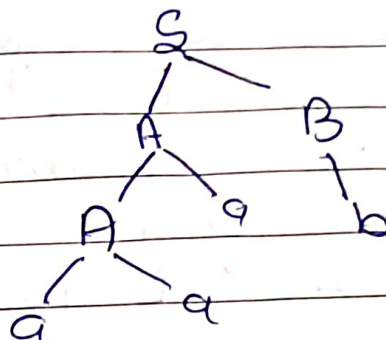
$$S \rightarrow AB \mid aaab$$

$$A \rightarrow a \mid Ab$$

$$B \rightarrow b$$

Taking String - aaab

S
aaab



As we can see string aaab have more than 1 parse tree, thus it is ambiguous

4) Show grammar is ambiguous
 $S \rightarrow aSbS \mid bSaS \mid \epsilon$

Taking String abob

$S \rightarrow aSbS$

$aSbS \rightarrow abSaSbS$

$abSaSbS \rightarrow ab\epsilon a\epsilon b\epsilon$

$ab\epsilon a\epsilon b\epsilon \rightarrow abab$

$S \rightarrow aSbS$

$aSbS \rightarrow a\epsilon bS$

$a\epsilon bS \rightarrow ab\epsilon aSbS$

$ab\epsilon aSbS \rightarrow abab$

as string abob have 2LD, thus grammar is ambiguous

5) $L = \{a^n b^n c^m\} \cup \{a^n b^m c^m\}$ is an inherently ambiguous CFG

for string 'aaabbbccc', it can be generated by both parts of the language

As we can see strings of form $a^n b^n$ can be generated by both parts thus having more than 1 derivations indicating that grammar is ambiguous. As ambiguity arises from nature of language itself, we can say L is inherently ambiguous. No CFL can generate this language unambiguously.

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Tutorial 6

1) Eliminate useless production

$$S \rightarrow a|aA|B|C$$

$$A \rightarrow aB|E$$

$$B \rightarrow Aa$$

$$C \rightarrow c|cD$$

$$D \rightarrow d|d|Cd$$

from derivability

$$T = \{a, c, d, e\} \quad D$$

$$W_1 = \{a, c, d, e, S, A\}$$

$$W_2 = \{a, c, d, e, S, B, A, D\}$$

$$S \rightarrow a|aA|B$$

$$A \rightarrow aB|E$$

$$B \rightarrow Aa$$

$$D \rightarrow d|d|d$$

from Reachability

$$S \rightarrow a|aA|B$$

$$A \rightarrow aB|E$$

$$B \rightarrow Aa$$

2) Eliminate ϵ production

$$S \rightarrow AaB|a|aB$$

$$A \rightarrow \epsilon$$

$$B \rightarrow b|bA|E$$

$A \rightarrow \epsilon$ } Nullable
 $B \rightarrow \epsilon$ } Variables

Remaining rule production from A

$S \rightarrow AaB \mid aaB \mid aB$

$A \rightarrow \epsilon$

$B \rightarrow bbA \mid bb$

for $B \rightarrow \epsilon$

$S \rightarrow AaB \mid aaB \mid aB \mid Aa \mid aa \mid a$

$A \rightarrow \epsilon$

$B \rightarrow bbA \mid bb$

final

$S \rightarrow aB \mid aaB \mid aB \mid aa \mid a$

$B \rightarrow bb$

3) Remove unit production

$S \rightarrow Aa \mid B$

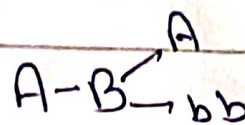
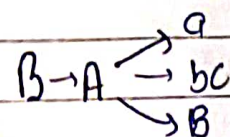
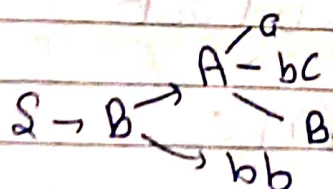
$B \rightarrow A \mid bb$

$A \rightarrow a \mid bc \mid B$

Unit productions $\Rightarrow S \rightarrow B$

$B \rightarrow A$

$A \rightarrow B$



$$S \rightarrow Aa | bb | a | bc$$

$$B \rightarrow bb | a | bc$$

$$A \rightarrow a | bc | bb$$

4) Simplify the following grammar

$$S \rightarrow aA | aBB$$

$$A \rightarrow aaA | \epsilon$$

$$B \rightarrow bB | bbc$$

$$C \rightarrow B$$

(i) Removing useless production

derivability

$$T = \{a, b, \epsilon\}$$

$$W_1 = \{a, b, \epsilon, A\}$$

$$W_2 = \{a, b, \epsilon, A, S\}$$

$$W_3 = \{a, b, \epsilon, A, S\}$$

Reachability both A and S can be reached

$$S \rightarrow aA$$

$$A \rightarrow aaA | \epsilon$$

(ii) Removing ϵ production

$$S \rightarrow aA | a$$

$$A \rightarrow aaA | aa$$

} Reduced
grammar

Language is $\{a^{2n+1} \mid n \in \{0, 1, 2, \dots\}\}$

5) $L = \{a^{4n} \mid n > 0\}$
write grammar in CNF

CFG of L

$S \rightarrow aaaa \mid aaaaS$

Conversion to CNF

$S \rightarrow AAAA \mid AAAAS$

$A \rightarrow a$

$S \rightarrow BB \mid BBS$

$B \rightarrow AA$

$A \rightarrow a$

$S \rightarrow BB \mid CS$

$B \rightarrow AA$

$C \rightarrow BB$

$A \rightarrow a$

} final
CNF

6) Convert to CNF

$S \rightarrow aSaaA \mid A^2$

$A \rightarrow abA \mid bb$

$S \rightarrow PSPPA \mid A$

$A \rightarrow PQA \mid QQ$

$Q \rightarrow b$

$P \rightarrow a$

$S \rightarrow \cancel{S} VTA 1A$
 $A \rightarrow \cancel{PA} 1d$
 $R \rightarrow Pd$
 $P \rightarrow a$
 $d \rightarrow b$
 $V \rightarrow P\$$
 $\cancel{U} \rightarrow P \quad T \rightarrow PP$

$S \rightarrow W A I R A 1 d$
 $A \rightarrow R A 1 d$
 $R \rightarrow P d$
 $W \rightarrow V T$
 $V \rightarrow P S$
 $T \rightarrow P P$
 $P \rightarrow a$
 $d \rightarrow b$

final
 CNF