

Ninety-nine per cent of the restaurants in Kentish Town practise good hygiene. Each time you eat in a clean restaurant, there is a 1 per cent chance that you will get sick, independent of your previous visits. On the other hand, each time you eat in a restaurant that does not practise good hygiene, there is a 50 per cent chance that you will get sick, independent of your previous visits.

(A) You eat at a restaurant in Kentish Town and you get sick. What is the probability that the restaurant practices good hygiene?

Key:

S = Sick

C = Clean

NC = Not Clean

NS = Not Sick

$$P(S|C) = 1/100$$

$$P(NS|C) = 99/100$$

$$P(C) = 99/100$$

$$P(NC) = 1/100$$

$$P(S|NC) = 0.5$$

To Calculate: $P(C|S)$

$$P(C|S) = P(S|C) * P(C) / P(S)$$

By Law of total probability,

$$P(S) = P(S|C) * P(C) + P(S|NC) * P(NC)$$

$$= 1/100 * 99/100 + 50/100 * 1/100$$

$$= 99+50/(10^4)$$

$$= 149/10^4$$

$$P(C|S) = 1/100 * 99/100 * 10^4 / 149$$

$$= 99/149$$

$$= 0.664$$

There is a 66% chance that the restaurant is clean even if a person has fallen sick.

(B) You go to the same restaurant for a second time and you get sick again. What is the probability of the restaurant practicing good hygiene now?

Key:

ST = Sick Twice

S = Sick

C = Clean

NC = Not Clean
NS = Not Sick

To Calculate: $P(C|ST)$

Given:

$$P(ST|C) = P(S|C)^2$$

$$P(ST|NC) = P(S|NC)^2$$

By law of total probability,

$$\begin{aligned} P(ST) &= P(ST|C) * P(C) + P(ST|NC) * P(NC) \\ &= (1/100)^2 * 99/100 + (50/100)^2 * 1/100 \\ &= 99 + 2500 / (10^6) \\ &= 2599 / 10^6 \end{aligned}$$

$$\begin{aligned} P(C|ST) &= P(ST|C) * P(C) / P(ST) \\ &= ((1/100)^2 * 99/100) * 10^6 / 2599 \\ &= 99 / 2599 \\ &= 0.0381 \end{aligned}$$

The probability of the restaurant being hygienic when someone has fallen sick twice in a row is very low as expected.