

Assignment 3

P=6 bytes

$P_R = 7$ bytes

Record Length = $30+9+9+40+9+8+1+4+4+1 = R = 115$ bytes

Part Number	Q1. Block Size B=4096 bytes and File Records r=10million	Q2. Block Size B=8192 bytes and File Records r=10billion
<p>A: Suppose that the file is <i>ordered</i> by the key field SSN and we want to construct a <i>primary</i> index on SSN.</p> <p>Calculate (i) the index blocking factor b_{fri} (which is also the index fan-out fa); (ii) the number of first-level index entries and the number of first-level index blocks; (iii) the number of levels needed if we make it into a multilevel index; (iv) the total number of blocks required by the multilevel index; and (v) the number of block accesses needed to search for and retrieve a record from the file given its SSN value using the primary index.</p>	<p>Blocking factor(BFR) = B/R $= \text{floor}(4096/115) = \mathbf{35}$ Number of blocks required = $\text{ceil}(r/bfr) = 10,000,000/35 = \mathbf{285715}$</p> <p>i) Index b_{fri} Index record size = $R_i = (V_{SSN}+P) = 9+6 = 15$ bytes</p> <p>Index blocking factor = $B_{fri} = B/R_i = \mathbf{4096/15 = 273}$</p> <p>ii) Number of First Level index Entries and block: First Level Entries = $R1 = \text{ceil}(10,000,000/bfr) = \mathbf{285,715}$</p> <p>First Level Index Blocks = $B1 = R1/B_{fri} = \mathbf{285,715/273 = 1047}$</p> <p>iii) Multilevel Index</p> <p>$b1 = 1047$ $b2 = \text{ceil}(1047/273) = 4$ $b3 = \text{ceil}(8/271) = 1$ As third level has only one block, so X = 3</p> <p>iv) Total Blocks Required</p> <p>$b_i = b1+b2+b3 = 1047+ 4 + 1 = \mathbf{1052}$</p> <p>v) No. of Block Access Needed As $X = 3$, so block access = $X+1 = 3+1 = \mathbf{4}$</p>	<p>Blocking factor(BFR) = B/R $= \text{floor}(8192/115) = \mathbf{71}$ Number of blocks required = $\text{ceil}(r/bfr) = 10,000,000,000/71 = \mathbf{140845071}$</p> <p>i) Index b_{fri} Index record size = $R_i = (V_{SSN}+P) = 9+6 = 15$ bytes</p> <p>Index blocking factor = $B/R_i = B_{fri} = \mathbf{8192/15 = 546}$</p> <p>ii) Number of First Level index Entries and block: First Level Entries = $R1 = \text{ceil}(10,000,000,000/ bfr) = \mathbf{140,845,071}$</p> <p>First Level Index Blocks = $B1 = R1/B_{fri} = 140,845,071/546= \mathbf{257959}$</p> <p>iii) Multilevel Index</p> <p>$b1 = 257959$ $b2 = \text{ceil}(257959/546) = 473$ $b3 = \text{ceil}(473/546) = 1$ As third level has only one block, so X = 3</p> <p>iv) Total Blocks Required</p> <p>$b_i = b1+b2+b3 = 257959+ 473+ 1 = \mathbf{258433}$</p> <p>v) No. of Block Access Needed As $X = 3$, so block access = $X+1 = 3+1 = \mathbf{4}$</p>

<p>B: Suppose that the file is not <i>ordered</i> by the key field SSN and we want to construct a <i>secondary</i> index on SSN. Repeat the previous (part a) for the secondary index and compare with the primary index.</p>	<p>Blocking factor(BFR) = B/R $= \text{floor}(4096/115) = \mathbf{35}$</p> <p>i) Index bfr_i Index record size = $R_i = (V_{SSN} + P) = 9 + 6 = 15$ bytes</p> <p>Index blocking factor = $B_{fri} = B / R_i =$ $\mathbf{4096/15 = 273}$</p> <p>ii) Number of First Level index Entries and block: First Level Entries = $R_1 = 10,000,000$</p> <p>First Level Index Blocks = $B_1 =$ $\text{ceil}(R_1/B_{fri}) = \text{ceil}(10,000,000/273) =$ $\mathbf{36,631}$</p> <p>iii) Multilevel Index</p> <p>$b_1 = 36631$ $b_2 = \text{ceil}(36631/273) = 135$ $b_3 = \text{ceil}(135/271) = 1$ As third level has only one block, so $\mathbf{X = 3}$</p> <p>iv) Total Blocks Required</p> <p>$b_i = b_1 + b_2 + b_3 = 36631 + 135 + 1 = \mathbf{36767}$</p> <p>v) No. of Block Access Needed As $X = 3$, so block access = $X + 1 = 3 + 1 = \mathbf{4}$</p>	<p>Blocking factor(BFR) = B/R $= \text{floor}(8192/115) = \mathbf{71}$</p> <p>i) Index bfr_i Index record size = $R_i = (V_{SSN} + P) = 9 + 6 = 15$ bytes</p> <p>Index blocking factor = $B / R_i = B_{fri} =$ $\mathbf{8192/15 = 546}$</p> <p>ii) Number of First Level index Entries and block: First Level Entries = $R_1 = 10,000,000,000$</p> <p>First Level Index Blocks = $B_1 = \text{ceil}(R_1 / B_{fri})$ $= \text{ceil}(10,000,000,000/546) = \mathbf{183,150,19.}$</p> <p>iii) Multilevel Index</p> <p>$b_1 = 18315019$ $b_2 = \text{ceil}(18315019/546) = 33544$ $b_3 = \text{ceil}(33544/546) = 62$ $b_4 = \text{ceil}(62/546) = 1$ As fourth level has only one block, so $\mathbf{X = 4}$</p> <p>iv) Total Blocks Required</p> <p>$b_i = b_1 + b_2 + b_3 = 18315019 + 33544 + 62 + 1 = \mathbf{183,486,26}$</p> <p>v) No. of Block Access Needed As $X = 3$, so block access = $X + 1 = 4 + 1 = \mathbf{5}$</p>
<p>C: Suppose that the file is not <i>ordered</i> by the non-key field DEPARTMENTCODE and we want to construct a <i>secondary</i> index on DEPARTMENTCODE, with an extra level of indirection that stores record pointers. Assume there are 20000 distinct values of DEPARTMENTCODE and that the EMPLOYEE records are evenly</p>	<p>i) Index bfr_i Index record size = $R_1 = (V_{SSN} + P) = 9 + 6 = 15$ bytes</p> <p>Index blocking factor = $B_{fri} = B / R_1 =$ $\mathbf{4096/15 = 273}$</p> <p>ii) Number of blocks needed by level of indirection: As, there are 20,000 distinct values and $Pr = 7$, so total bytes to store record</p>	<p>i) Index bfr_i Index record size = $R_1 = (V_{SSN} + P) = 9 + 6 = 15$ bytes</p> <p>Index blocking factor = $B / R_1 = B_{fri} =$ $\mathbf{8192/15 = 546}$</p> <p>ii) Number of blocks needed by level of indirection: As, there are 20,000 distinct values and $Pr = 7$, so total bytes to store record pointer</p>

<p>distributed among these values. Calculate (i) the index blocking factor bfr, (which is also the index fan-out <i>fa</i>); (ii) the number of blocks needed by the level of indirection that stores record pointers; (iii) the number of first level index entries and the number of first-level index blocks; (iv) the number of levels needed if we make it into a multilevel index; (v) the total number of blocks required by the multilevel index and the blocks used in the extra level of indirection; and (vi) the approximate number of block accesses needed to search for and retrieve all records in the file that have a specific DEPARTMENTCODE value, using the index.</p>	<p>pointer for 1 department = $10,000,000/20,000 = 500$ Now, $500*7 = 3500 < 4096$, so it can fit in 1 block, so no. of blocks for level of indirection = 20,000</p> <p>iii) Number of First Level index Entries and block: $r_1 = \text{distinct departments} = 20,000$ $b_1 = \text{ceil}(r_1/BFR_i) = 20,000/273 = 74$</p> <p>iv) Multilevel Index $b_2 = \text{ceil}(74/273) = 1$ As second level has only one block, so X = 2</p> <p>v) Total Blocks Required $b_i = b_1 + b_2 + \text{level of indirection} = 74 + 1 + 20,000 = 20,075$</p> <p>vi) No. of Block Access Needed As X = 2, so block access = $X + 1 + \text{selectivity} = 2 + 1 + 500 = 503$</p>	<p>for 1 department = $10,000,000,000/20,000 = 500,000$ Now, $500,000*7 = 3,500,000 > 8192$, so it can't fit in 1 block. We will need $3,500,000/8192$ times blocks. So no. of blocks for level of indirection = $20,000*428 = 8,560,000$</p> <p>iii) Number of First Level index Entries and block: $r_1 = \text{distinct departments} = 20,000$ $b_1 = \text{ceil}(r_1/BFR_i) = 20,000/546 = 37$</p> <p>iv) Multilevel Index $b_2 = \text{ceil}(37/546) = 1$ As second level has only one block, so X = 2</p> <p>v) Total Blocks Required $b_i = b_1 + b_2 + \text{level of indirection} = 37 + 1 + 8,560,000 = 8,560,038$</p> <p>vi) No. of Block Access Needed As X = 2, so block access = $X + 1 + \text{selectivity} = 2 + 1 + 500,000 = 500,003$</p>
<p>D: Suppose that the file is <i>ordered</i> by the non-key field DEPARTMENTCODE and we want to construct a <i>clustering index</i> on DEPARTMENTCODE that uses block anchors (every new value of DEPARTMENTCODE starts at the beginning of a new block). Assume there are 20000 distinct values of DEPARTMENTCODE and that the EMPLOYEE records are evenly distributed among these values. Calculate (i) the index blocking</p>	<p>Blocking factor(BFR) = $B/R = \text{floor}(4096/115) = 35$</p> <p>i) Index bfr_i Index record size = $R_i = (V_{SSN} + P) = 9 + 6 = 15$ bytes Index blocking factor = $B_{fri} = B/R_i = 4096/15 = 273$</p> <p>ii) Number of First Level index Entries and block: First Level Entries = $R_i = 20,000$ First Level Index Blocks = $B_1 = \text{ceil}(R_i/B_{fri}) = \text{ceil}(20,000/273) = 74$</p>	<p>Blocking factor(BFR) = $B/R = \text{floor}(8192/115) = 71$</p> <p>i) Index bfr_i Index record size = $R_i = (V_{SSN} + P) = 9 + 6 = 15$ bytes Index blocking factor = $B/R_i = B_{fri} = 8192/15 = 546$</p> <p>ii) Number of First Level index Entries and block: First Level Entries = $R_i = 20,000$ First Level Index Blocks = $B_1 = \text{ceil}(R_i/B_{fri}) = \text{ceil}(20,000/546) = 37$.</p>

<p>factor bfr, (which is also the index fan-out fa); (ii) the number of first-level index entries and the number of first-level index blocks; (iii) the number of levels needed if we make it into a multilevel index; (iv) the total number of blocks required by the multilevel index; and (v) the number of block accesses needed to search for and retrieve all records in the file that have a specific DEPARTMENTCODE value, using the clustering index (assume that multiple blocks in a cluster are contiguous)</p>	<p>iii) Multilevel Index</p> <p>$b_2 = \text{ceil}(74/273) = 1$ As second level has only one block, so X = 2</p> <p>iv) Total Blocks Required</p> <p>$b_i = b_1 + b_2 = 74 + 1 = \mathbf{75}$</p> <p>v) No. of Block Access Needed</p> <p>Number of block accesses to search for the first block in the cluster of blocks = $x+1=2+1=3$</p> <p>The 74 records are clustered in $\text{ceiling}(500/bfr) = \text{ceiling}(500/35) = \mathbf{15}$ blocks.</p> <p>Hence, total block accesses needed on average to retrieve all the records with a given Department code = $x + 15 = 2 + 15 = 17$ block accesses</p>	<p>iii) Multilevel Index</p> <p>$b_2 = \text{ceil}(37/546) = 1$ As second level has only one block, so X = 2</p> <p>iv) Total Blocks Required</p> <p>$b_i = b_1 + b_2 = 37 + 1 = \mathbf{38}$</p> <p>v) No. of Block Access Needed</p> <p>Number of block accesses to search for the first block in the cluster of blocks = $x+1=2+1=3$</p> <p>The 37 records are clustered in $\text{ceiling}(37/bfr) = \text{ceiling}(500,000/71) = \mathbf{7043}$ block.</p> <p>Hence, total block accesses needed on average to retrieve all the records with a given Department code = $x + 7043 = 2 + 7043 = 7045$ block accesses</p>
<p>E: Suppose the file is not ordered by the key field Ssn and we want to construct a B⁺-tree access structure (index) on SSN. Calculate (i) the orders p and p leaf of the B⁺-tree; (ii) the number of leaf-level blocks needed if blocks are approximately 69% full (rounded up for convenience); (iii) the number of levels needed if internal nodes are also 69% full (rounded up for convenience); (iv) the total number of blocks required by the B⁺-tree; and (v) the number of block accesses needed to search for and retrieve a record from the file--given its SSN</p>	<p>i) the orders P and P_{leaf} of the B⁺-tree:</p> <p>$(p * P) + ((p-1) * VSSN) < B:$</p> <p>$(p * 6) + ((p-1) * 9) < 4096$</p> <p>15p < 4105 P = 273</p> <p>$(P_{\text{leaf}} * (VSSN + PR)) + P < B,$ $(P_{\text{leaf}} * (9+7)) + 6 < 4096$</p> <p>16 P_{leaf} < 4090 P_{leaf} = 255</p> <p>ii) the number of leaf-level blocks needed if blocks are approximately 69% full</p>	<p>i) the orders P and P_{leaf} of the B⁺-tree:</p> <p>$(p * P) + ((p-1) * VSSN) < B:$</p> <p>$(p * 6) + ((p-1) * 9) < 8192$</p> <p>15p < 8201 P = 546</p> <p>$(P_{\text{leaf}} * (VSSN + PR)) + P < B,$ $(P_{\text{leaf}} * (9+7)) + 6 < 8192$</p> <p>16 P_{leaf} < 8106 P_{leaf} = 511</p> <p>ii) the number of leaf-level blocks needed if blocks are approximately 69% full</p> <p>The average number of key values in a leaf node is $0.69 * P_{\text{leaf}} = 0.69 * 511 = 352.59$</p>

<p>value--using the B⁺-tree.</p>	<p>The average number of key values in a leaf node is $0.69 * P_{\text{leaf}} = 0.69 * 255 = 175.95$</p> <p>If we round this up for convenience, we get 176 key values (and 176 record pointers) per leaf node.</p> <p>Since the file has 10,000,000 records and hence 10,000,000 values of SSN, the number of leaf-level nodes (blocks) needed is $b1 = \text{ceiling}(10,000,000 / 176) = \mathbf{56819 \text{ blocks.}}$</p> <p>iii) If Internal nodes are 69% full:</p> <p>$Fo = \text{ceiling}(0.69 * p) = \text{ceiling}(0.69 * \mathbf{273}) = \mathbf{189}$</p> <p>$b2 = \text{ceiling}(b1 / fo) = \text{ceiling}(56819 / 189) = \mathbf{301}$ $b3 = \text{ceiling}(b2 / fo) = \text{ceiling}(301 / 189) = \mathbf{2}$ $b4 = \text{ceiling}(b3 / fo) = \text{ceiling}(2 / 189) = \mathbf{1}$</p> <p>Since the 4th level has only one block, the tree has x = 4 levels (counting the leaf level).</p> <p>$x = \text{ceiling}(\text{Log}(Fo) (b1)) + 1 = 3 + 1 = \mathbf{4 \text{ levels}}$</p> <p>iv) Total Blocks Required</p> <p>$bi = b1 + b2 + b3 + b4 = 56819 + 301 + 2 + 1 = \mathbf{57123}$</p> <p>v) No. of Block Access Needed</p> <p>$= X + 1 = 4 + 1 = \mathbf{5}$</p>	<p>If we round this up for convenience, we get 353 key values (and 353 record pointers) per leaf node.</p> <p>Since the file has 10,000,000,000 records and hence 10,000,000,000 values of SSN, the number of leaf-level nodes (blocks) needed is $b1 = \text{ceiling}(10,000,000,000 / \mathbf{353}) = \mathbf{28,328,612 \text{ blocks.}}$</p> <p>iii) If Internal nodes are 69% full:</p> <p>$Fo = \text{ceiling}(0.69 * p) = \text{ceiling}(0.69 * \mathbf{546}) = \mathbf{377}$</p> <p>$b2 = \text{ceiling}(b1 / fo) = \text{ceiling}(28,328,612 / 377) = \mathbf{75143}$</p> <p>$b3 = \text{ceiling}(b2 / fo) = \text{ceiling}(75143 / 377) = \mathbf{200}$</p> <p>$b4 = \text{ceiling}(b3 / fo) = \text{ceiling}(200 / 377) = \mathbf{1}$</p> <p>Since the 4th level has only one block, the tree has x = 4 levels (counting the leaf level).</p> <p>$x = \text{ceiling}(\text{Log}(Fo) (b1)) + 1 = 3 + 1 = \mathbf{4 \text{ levels}}$</p> <p>iv) Total Blocks Required</p> <p>$bi = b1 + b2 + b3 + b4 = \mathbf{28,328,612 + 75143 + 200 + 1 = 28,403,956}$</p> <p>v) No. of Block Access Needed</p> <p>$= X + 1 = 4 + 1 = \mathbf{5}$</p>
<p>F: Repeat (part e), but for a B-tree <i>rather than</i> for a B⁺-tree. Compare your results for the</p>	<p>i) the orders P and P_{leaf} of the B+-tree:</p> <p>$(p * P) + ((p - 1) * (VSSN + Pr)) < B:$</p>	<p>i) the orders P and P_{leaf} of the B+-tree:</p> <p>$(p * P) + ((p - 1) * (VSSN + Pr)) < B:$</p>

<p>B-tree and for the B⁺-tree.</p>	<p>$(p*6)+((p-1)*(9+7)) < 4096$</p> <p>22P < 4112 P = 186</p> <p>ii) the number of leaf-level blocks needed if blocks are approximately 69% full</p> <p>The average number of key values in a leaf node is $0.69 * P = 0.69 * 186 = 128.34$</p> <p>If we round this up for convenience, we get 129 key values (and 129 record pointers) per leaf node.</p> <p>Since the file has 10,000,000 records and hence 10,000,000 values of SSN, the number of leaf-level nodes (blocks) needed is $b1 = \text{ceiling}(10,000,000 / 129) = \mathbf{77520 \text{ blocks.}}$</p> <p>iii) If Internal nodes are 69% full:</p> <p>$Fo = \text{ceiling}(0.69 * p) = \text{ceiling}(0.69 * 186) = \mathbf{129}$</p> <p>$b2 = \text{ceiling}(b1 / fo) = \text{ceiling}(77520 / 129) = \mathbf{601}$ $b3 = \text{ceiling}(b2 / fo) = \text{ceiling}(601 / 129) = \mathbf{5}$ $b4 = \text{ceiling}(b3 / fo) = \text{ceiling}(5 / 129) = \mathbf{1}$</p> <p>Since the 4th level has only one block, the tree has x = 4 levels (counting the leaf level).</p> <p>$x = \text{ceiling}(\text{Log}(Fo) (b1)) + 1 = 3 + 1 = \mathbf{4 \text{ levels}}$</p> <p>iv) Total Blocks Required</p>	<p>$(p*6)+((p-1)*(9+7)) < 8192$</p> <p>22P < 8208 P = 373</p> <p>ii) the number of leaf-level blocks needed if blocks are approximately 69% full</p> <p>The average number of key values in a leaf node is $0.69 * P = 0.69 * 373 = 257.37$</p> <p>If we round this up for convenience, we get 258 key values (and 258 record pointers) per leaf node.</p> <p>Since the file has 10,000,000,000 records and hence 10,000,000,000 values of SSN, the number of leaf-level nodes (blocks) needed is $b1 = \text{ceiling}(10,000,000,000 / 258) = \mathbf{38,759,690 \text{ blocks.}}$</p> <p>iii) If Internal nodes are 69% full:</p> <p>$Fo = \text{ceiling}(0.69 * p) = \text{ceiling}(0.69 * 373) = \mathbf{258}$</p> <p>$b2 = \text{ceiling}(b1 / fo) = \text{ceiling}(38,759,690 / 258) = \mathbf{150232}$ $b3 = \text{ceiling}(b2 / fo) = \text{ceiling}(150232 / 258) = \mathbf{583}$ $b4 = \text{ceiling}(b3 / fo) = \text{ceiling}(583 / 258) = \mathbf{3}$ $b5 = \text{ceiling}(b4 / fo) = \text{ceiling}(3 / 258) = \mathbf{1}$</p> <p>Since the 5th level has only one block, the tree has x = 5 levels (counting the leaf level).</p> <p>$x = \text{ceiling}(\text{Log}(Fo) (b1)) + 1 = 4 + 1 = \mathbf{5 \text{ levels}}$</p>
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	$b_i = b_1 + b_2 + b_3 + b_4 = 77520 + 601 + 5 + 1 = \mathbf{78127 \text{ blocks.}}$ <p>v) No. of Block Access Needed</p> $= X + 1 = 4 + 1 = 5$	<p>iv) Total Blocks Required</p> $b_i = b_1 + b_2 + b_3 + b_4 = 38,759,690 + 150232 + 583 + 3 + 1 = \mathbf{38,910,509 \text{ blocks}}$ <p>v) No. of Block Access Needed</p> $= X + 1 = 5 + 1 = 6$ <p>As B-Tree requires more block access compared to B+ Tree, therefore for this part, it is more costly as compared to B+ Tree.</p>
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Q3. Consider a DBMS that has the following characteristics:

- 1KB fixed-size blocks
- 12-byte pointers
- 56-byte block headers

We want to build an index on a search key that is 8 bytes long. Calculate the maximum number of records we can index with a

a. 3 Level B⁺- tree index (including the root level)

Block Size = 1Kb = 1024 bytes

Pointer = 12 byte

Bhead = 56 bytes

Let us assume that each B+ Tree has n pointers and n-1 keys.

So, Formula = $(p * (n-1)) + ((n) * \text{Pointer} + \text{Bhead}) \leq B$,

Here p = 8 bytes, so

$$\Rightarrow (8 * (n-1) + (12n) + 56) \leq 1024$$

$$\Rightarrow 8n + 12n \leq 976$$

$$\Rightarrow 20n \leq 976$$

$$\Rightarrow n \leq 48.8$$

$$\Rightarrow \text{Therefore, } n \leq 48$$

$$\Rightarrow \mathbf{So \text{ } n=47}$$

Now, As leaf node of B+ Tree can hold record pointers = $47 * 48 * 48 = \mathbf{108288}$.

So maximum number of records we can index = **108,288**.

b. 3 Level B-tree index (including the root level)

Let B tree have n index pointers, n-1 keys and record pointers, therefore,

Formula = $8 * (n-1) + 12 * (2n-1) + 56 \leq 1024$

$$\Rightarrow 8n - 8 + 24n - 12 + 56 \leq 1024$$

$$\Rightarrow 32n \leq 988$$

$$\Rightarrow n \leq 30$$

so n=29.

1st Level B-Tree can hold = 29

2st Level B-Tree can hold = $29 * 30 = 870$

3st Level B-Tree can hold = $29 * 30 * 30 = 26100$

Total = $29 + 870 + 26100 = 26,999$

So maximum number of records we can index = **26,999**.

Q4. Assume a relation $R(A, B, C)$ is given; Suppose A, B, C are integer type values. Relation R is stored as an un-ordered file (un-spanned) on key field A and contains 5000 data blocks. Assume there is B^+ -tree access structure (index) on A of height $x=4$ (root, 2 intermediate layer, leaf). Moreover, one node of the B^+ -tree is stored in one block on the disk.

Estimate the number of block fetches needed to compute the following queries:

Data blocks = 5000

1. $SELECT * FROM R WHERE A = 777;$

Number of block Fetches = $X+1 = 5$

2. $SELECT C FROM R WHERE A=111 AND B=3;$

Number of block Fetches = $X+1 = 5$

3. $SELECT * FROM R WHERE A=111 OR A=3;$

Number of block Fetches = $2*(X+1) = 2(5) = 10$

4. $SELECT * FROM R WHERE A > 100;$

= $4+5000=5004$ Assuming at worst case all values are greater than 100

5. $SELECT COUNT(*) FROM R WHERE A > 100;$

Number of block Fetches = $X = 4$