SOLUTION KEY

National University of Computer and Emerging Sciences, Lahore Campus

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Exam Type:	Mid-2 Exam		

Student : Name:____________________ **Roll No.**______________ **Section:**______

Instruction/Notes: Attempt all questions. Programmable calculators are not allowed. The answer sheet is NOT required, Paper must be solved in the given space on the question paper.

QUESTION # 1: Given the LP problem: **(10)**

Maximize: $Z = 2X_1 + 1.5X_2$

Subject to:

$3X_1 + 4X_2 \leq 1000$ (Cast Irons Constraint)

$6X_1 + 3X_2 \leq 1200$ (Labor Hours Const.)

$X_1 \leq 180$ (Model-A Production Cont.)

$X_1, X_2 \geq 0$

and the following Excel Solver output

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
G6	Quantity Produced: Model-A	120	0	2	1	0.875
H6	Quantity Produced: Model-B	160	0	1.5	1.166666667	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I9	Cast Irons: TOTAL	1000	0.2	1000	600	300
I10	Labor Hour: TOTAL	1200	0.233333333	1200	225	450
I11	Model-A Production: TOTAL	120	0	180	1E+30	60

Answer the following questions:

1. What is the optimal solution and optimal value for the problem?

Optimal Solution:
$x_1 = 120$
$x_2 = 160$

optimal Value:
$Z = 2*x_1 + 1.5*x_2 = 2*120 + 1.5*160$
$= 240 + 240 = 480$

2. What is the impact on the optimal solution and optimal value if we decrease the cost coefficient C_1=2 to 1.5? Why?

As allowable decrease in C_1 is 0.875, we can decrease it upto $(2-0.875=1.125)$ 1.125. So 2 to 1.5 decrease is feasible.

1. optimum solution will be same (still producing same amount of x_1 & x_2).
2. optimum value is changed because coefficient of x_1 is decreased from 2 to 1.5.

New optimal value:

$Z = 1.5*120 + 1.5*160$
$= 180 + 240 = 420$

3. What is the shadow price for the Cast Iron (RHS # 1)? How do you interpret it?

Shadow Price of Cast Iron = 0.2

Shadow price is the unit worth of resource (Constraint 1 in this Case)
If we increase/decrease in R.H.S by 1 unit, Z will increase/decrease by 0.20.

4. What is the shadow price for the Model–A Production (RHS # 3)? How do you interpret it?

Shadow Price for Model-A production = 0

means resource 3 is not fully utilized, some slack amount of this resource is available (Associated constraint is non-binding.
If we want to add some additional amount, we donot need to pay for it.
(we already have this resource

5. What is the impact on the optimal value if we decrease the right-hand side of constraint–1 (Cast Iron) by 30?

R.H.S of resource 1 = 1000
allowable decrease = 300] we can decrease it upto (1000 − 300) 700.

As shadow price/unit worth of resource 1 is 0.2.
If we decrease it by 30 units.

So, $30 * 0.2 = 6$

New optimal value will be :
$Z = 480 - 6 = 474$

QUESTION # 2: Consider the transportation problem shown below. **(10)**

Initial Start: Use the "Northwest Corner Rule" to form an initial basis for this problem.

	Moscow	Cape Town	Sydney	
Hamburg	4 (100)	10	6	~~100~~ 0
Minneapolis	8 (100)	16 (200)	6 (0)	~~300~~ ~~200~~ 0 0
Tokyo	14	18	10 (200)	~~200~~ 0
	~~200~~ ~~100~~ 0	~~200~~ 0	~~200~~ 0	

educed Shipping Costs:

a. Calculate the value of v_1 below.

b. Calculate the reduced cost for cell Minneapolis-Cape Town.
(Note that reduced costs for other non-basic cells are shown in brackets [ ] below.)

$u_1 + v_1 = 4$
$0 + v_1 = 4$
$\boxed{v_1 = 4}$

$C_{22} - u_2 - v_2 =$
$16 - 4 - 10 = 2$

	Moscow	Cape Town	Sydney	
Hamburg	4 100	10 [0]	6 [4]	$u_1 = 0$
Minneapolis	8 100	16 [2]	6 200	$u_2 = 4$
Tokyo	14 [2]	18 200	10 0	$u_3 = 8$
	$v_1 =$ 4	$v_2 = 10$	$v_3 = 2$	

c. Assuming that this is a cost minimization problem, which non-basic route would you next bring into the basis (ship through) to further reduce costs? Briefly explain.

~~As $m + n - 1 = 3 + 3 - 1 = 5$~~
~~and there are basic cells are 4.~~
~~the above solution is degenerate.~~
~~non-basic cell as basic with zero solution value and then~~

As reduced costs of all non-basic cells are positive,
so above solution is optimal.

$x_{11} = 100$
$x_{21} = 100$
$x_{23} = 200$
$x_{32} = 200$
$x_{33} = 0$

optimal z :

$z = 4 \times 100 + 8 \times 100 + 6 * 200 + 18 * 200 + 0 * 10$
$= 400 + 800 + 1200 + 3600$
$= 6000$