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**National University of Computer and Emerging Sciences, Lahore Campus**

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| **Course:** | **Numerical Computing** | **Course Code:** | **CS2008** |
| **Program:** | **BSCS/BSR/BSSE** | **Semester:** | **Fall 2023** |
| **Duration:** | **3 hours** | **Total Marks:** | **100** |
| **Paper Date:** | **December 22, 2023** | **Weight** | **50%** |
| **Section:** | **All** | **Page(s):** | **02** |
| **Exam:** | **Final Term** | **Roll No:** | |

**Instruction/Notes:** Attempt all questions on the answer book. Don't write anything on a question paper except your name and roll number.

**Q1.** Let $P_2(x)$ be the interpolating polynomials for the data $(0.5, y), (1, 3)$, and $(2, 2)$. The coefficient of $x^2$ in $P_2(x)$ is 6. Find $y$. **Points (10)**

**Q2.** In a circuit with impressed voltage $v(t)$ and inductance $L$, Kirchhoff's first law gives the relationship **Points (10)**

$$v(t) = L\frac{di}{dt} + R\,i,$$

where $R$ is the resistance in the circuit and $i$ is the current. Suppose we measure the current for several values of $t$ and obtain

| | | | | | |
|---|---|---|---|---|---|
| X | $T$ | 1.00 | 1.01 | 1.02 | 1.03 |
| y | $i$ | 3.10 | 3.12 | 3.14 | 3.18 |

Where $t$ is measured in seconds, $i$ is in amperes, the inductance $L$ is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage $v(t)$ when $t = 1.03$.

**Hint:** Choose numerical differentiation formula based on time $t$ when $v(t)$ is required.

**Q3.** The solid of revolution obtained by rotating the region under the curve $y = g(x)$ over the interval $a \leq x \leq b$ about the axis has surface area is given by the following formula: **Points (10)**

$$Surface\ Area = \int_a^b 2\pi\, g(x)\sqrt{1 + [g'(x)]^2}\; dx$$

Approximate the surface area if $g(x) = e^{-x}$ for $0 \leq x \leq 1$, using composite Trapezoidal rule with $h = 0.2$.
Note: Throughout the computations, use at least five decimal approximations.

**Q4.** Using Newton Raphson's method, establish the formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ to calculate the cube root of $N$. Hence find cube root of 15 correct to three decimals places. **Points (10)**

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**Q5.** Solve the following linear system using $LU$ decomposition algorithm with $l_{ii} = 1$ for all $i$. **Points (10)**

$$4x + y - z = 5$$
$$-x + 3y + z = -4$$
$$2x + 2y + 5z = 1$$

**Q6.** The concentration of salt $y$ in a homemade soap maker is given as a function of time $t$ by

$$\frac{dy}{dt} = 1 - 0.5\,y - 0.1\,y\,t,$$

at the initial time $t = 0$, the salt concentration in the tank is $5g/L$. Use Picard method to find the thid approximate solution. Also, find residual function in this case. Moreover, compute salt concentration and corresponding residual errors at
$t = 0.25, 0.5$ and $0.75$? **Points (15)**

**Q7.** Use obtained solutions from **Q6** evaluate $y(1)$ through Milne's Predictor-Corrector scheme. **Points (10)**
where

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$$

**Q8.** Transform the following second order IVP **Points (15)**

$$y'' - 2y' + 2y - e^{2x}\,Sinx, \quad with\; y(0) = -0.4,\; y'(0) = -0.6$$

in to a system of first order IVPs and solve the obtain system at $x = 0.2$ using Runge-Kutta method of order 4 (RK4).

**Q9.** Use the finite difference algorithm, solve the following BVP **Points (10)**

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0, \;\text{with}\; y(0) = 0,\; y(1) = 1.$$

Divide the domain interval in to five subintervals.

$$\frac{y_{i+1} + y_{i-1} - 2i}{h^2} + 2\left(\frac{y_{i+1} - y_{i-1}}{\cancel{2}h}\right) + y_i = 0 \quad \times h^2$$

$$2y_{i+1} + 2y_{i-1} - 4y_i + 2hy_{i+1} - 2hy_{i-1} + 2h^2 y_i = 0$$



$$(2+2h)y_{i+1} + (2-2h)y_{i-1} + 2h^2 y_i = 0$$

2.4 1.6.