National University of Computer and Emerging Sciences, Lahore Campus

	Course Name:	Calculus and Analytical Geometry	Course Code:	MT1003
	Program:	BCS, BDS, BSE	Semester:	Fall 2023
	Duration:	60 Minutes	Total Marks:	30
	Paper Date:	10-11-2023	Weight	15
	Section:	ALL	Page(s):	6
	Exam Type:	Midterm-II		

Student : Name: [redacted] **Roll No** [redacted] **Section** [redacted]

Instruction/Notes: **Answer all questions neatly on the space provided. Answer sheet may be used for the rough work only. Exchange of calculators or programmable calculators are not allowed at all.**

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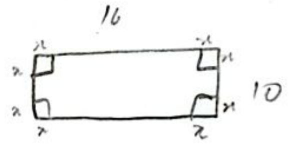
[CLO4] Q1. On An open-top rectangular box is constructed from a 10-in.-by-16-in. ~~piece of~~ cardboard by cutting squares of equal side length from the corners and folding up the sides. Find the dimensions of the box of largest volume. $V(x)$ **(10)**

$V(x) = x(16-2x)(10-2x)$ $\quad V = h.l.w$



$V(x) = 4x^3 - 52x^2 + 160x$

$h = x$ $\quad V = x(16-2x)(10-2x)$
$L = 16-2x$
$w = 10-2x$
$(16x - 2x^2)(10-2x)$
$160x - 32x^2 - 20x^2 + 4x^3$

$V'(x) = 12x^2 - 104x + 160$ $\quad | \quad V''(x) = 24x - 104$

When $V'(x) = 0$;

when $x = \frac{20}{3}$, +ve Min

$12x^2 - 104x + 160 = 0$

$x = 2$, -ve. Max.

$x = \frac{20}{3}$, $\boxed{x = 2}$

10

Largest Volume Box dimensions => $2 \times 12 \times 6$

where:-

$h = 2$ in

$L = 12$ in

$w = 6$ in

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[CLO5] Q2. For the following function **(10)**

$$f(x) = x\sqrt{4 - x^2}$$

Find

Domain = $[-2, 2]$

4) **Symmetry (if any).** **(2)**

~~When f(3)~~, ~~f(x)~~ When $x = \pm 1$, $f(x) = \sqrt{3}$

$f(x) = -\sqrt{3}$

The function is odd.

$x = \pm 2$, $f(x) = 0$

$f(x) = 0$.

1

Symmetric about origin .? or y-axis

5) **Interval of increase and decrease** **(2)**

$x = \pm 2$, $x = \pm\sqrt{2}$

$f'(x) = \infty$ $f'(x) = 0$

$f'(x) = \sqrt{4-x^2} - \frac{x \cdot x}{\sqrt{4-x^2}}$ x^2

$f'(x) = (-2 +++ 1.56 --- 2)$

When $f'(x) = 0$.

$4 - 2x^2$

$\frac{4 - x^2 - x}{\sqrt{4-x^2}} = 0$

1

$\boxed{x = \frac{-1 \pm \sqrt{17}}{2}}$ and $\boxed{x = \pm 2}$ ⇓ undefined.

$\boxed{1.56}$, -2.56 reject out of domain.

Interval of increase:-

$\left(-2, \frac{-1+\sqrt{17}}{2}\right)$ ∝

Interval of decrease:-

$\left(\frac{-1+\sqrt{17}}{2}, 2\right)$ ∝

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8) Intervals of concave up or down

$$f''(x) = \frac{-x^3 + x^2 - 5x - 4}{4 - x^2\sqrt{4-x^2}}$$ α

$$f''(x) = \frac{2x^3 - 12x}{(4-x^2)^{3/2}}$$

When $f''(x) = 0$,

$\boxed{x = -0.657}$

Infl point $x = 0 \rightarrow f''(x) = 0$

$x = \pm 2 \rightarrow f''(x) = \infty$

$f''(x) = [-2 \cdots (-0.657) \cdots 2]$

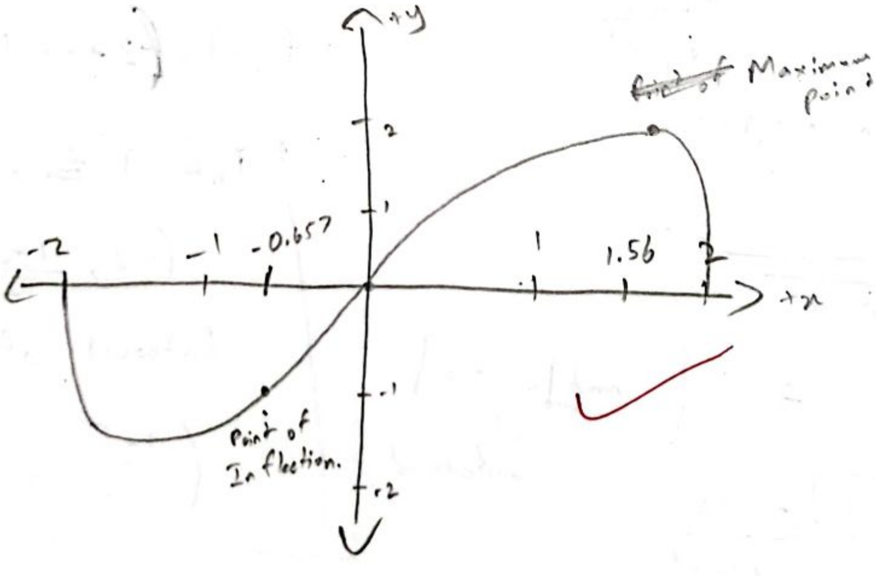
The function is concave down in intervals:-

$[-2, 2]$ α $(-2, 0)$ $(0, 2)$

+ve up   -ve down

1

9) Use above information to sketch the curve of function. (3)



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[CLO5] Q3. Check whether the following function satisfy the hypotheses of the Mean Value Theorem on the given interval? Give reason for your answer. (5)

Continuity, derivative.

$$g(x) = x\sqrt{1-x}, \qquad [0,1]$$

$g'(x) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} = \frac{2-2x-x}{2\sqrt{1-x}}$

$= \frac{2-3x}{2\sqrt{1-x}}$

Function is continuous. $[0,1]$.

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Since function is not differentiable on $x=1$, it does not ~~satisf~~ satisfy the theorem.

$g'(0) = 1$

$g'(1) =$ undefined

Diff in $(0,1)$.

- Value of c?
- $g'(c) = \frac{g(b)-g(a)}{b-a}$.

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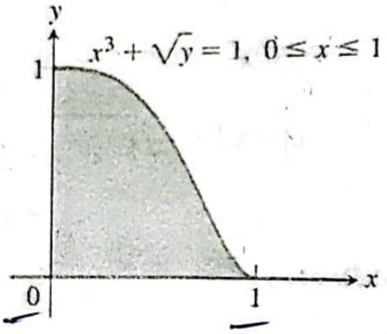
[CLO6] Q4. Find the total area of the shaded region. (5)



$$x^3 + \sqrt{y} = 1$$

$$y = (1 - x^3)^2$$

$$= (1-x^3)(1-x^3)$$

$$= 1 - 2x^3 + x^6$$

$$A = \int_0^1 (1-x^3)^2 \, dx$$

$$A = \int_0^1 1 - 2x^3 + x^6 \, dx$$

$$= x - \frac{x^4}{2} + \frac{x^7}{7} \Big]_0^1$$

$$= 1 - \frac{1}{2} + \frac{1}{7}$$

$$\boxed{A = \frac{9}{14} \text{ unit}^2}$$

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