National University of Computer and Emerging Sciences, Lahore Campus

Course:	Linear Algebra	Course Code:	MT-1004
Program:	BS(CS)/BS(DS)/ BS(SE)	Semester:	Fall 2022
Duration:	60 mins	Total Marks:	30
Date:	10-11-2022	Weight:	12.5%
Section:	All	Page(s):	1
Exam:	MID-2	Roll No:	
Name:			

Instruction/Notes: Attempt all questions. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem. Use of programmable calculators is not allowed. Attempt all questions in order. Best of luck!

$(-2,0,0)$. 20

Question no. 1: (5+5 marks) (CLO #02)

$(6,0,3)$
$(4,6,3)$

a) Find the vector and parametric equation of the plane in R^3 that passes through the origin and is orthogonal to vector $v = (3,1,-6)$.

b) Find the distance between the given parallel planes

$$7x - 3y - 6 = 5z \text{ and } x = \frac{1}{3}z + \frac{1}{6}y - 2.$$

$\frac{20}{\sqrt{83}}$ $-\frac{6}{5} = 7.$

$\frac{-\frac{1}{3}\left(-\frac{6}{5}\right)+2}{\sqrt{1^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{6}\right)^2}}$ $\frac{\frac{12}{5}}{\frac{\sqrt{41}}{6}}$

Question no. 2: (6+4 marks) (CLO #02)

a) Check that whether the set of all ordered pairs of real numbers with the standard vector addition but with scalar multiplication defined by $k(x,y) = (2kx, 2ky)$ form the vector space or not. And if the given set is not a vector space, then identify the vector space axioms that fail with complete working. fails

b) Consider the vector space R^3 under usual addition and scalar multiplication. Check that whether the set $S = \{(a,b,c),\ where\ c = a - b\}$ forms a subspace in R^3 or not?

Question no. 3: (5+5 marks) (CLO #02)

2.25
2.2

For the following vectors in R^3

$$v_1 = (3,1,-4), \quad v_2 = (2,5,6), \quad v_3 = (1,4,8)$$

a) Check that given vectors are linearly independent or not by using definition.

b) Determine whether the given vectors span R^3.

det. 26

Note: Any other method used to find the Linear independence will not be considered for marking. Show complete working.

 $4(3)\ 1(6)\ -6(3)$

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Course:	Linear Algebra	Course Code:	MT-1004
Program:	BS (CS, DS, SE)	Semester:	Fall 2022
Duration:	3 Hours	Total Marks:	100
Date:	31-12-2022	Weight	50%
Section:	All	Page(s):	3
Exam:	Final	Roll No:	

Name:

Instruction/Notes:

Use of programmable calculators is not allowed. Exchange of stationary is strictly prohibited. Show complete working in all questions. Attempt all question parts together. Question attempted in separate parts will not be marked.

Q#8 is a BONUS Question. (You will not lose marks if unable to attempt).

Question#1: [5+5 +5+5 marks, CLO#1, 5]

a) Use **Inversion Algorithm** to find the Inverse of matrix $A = \begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$.

b) Write down A^{-1} as a product of elementary matrices $A^{-1} = E_k E_{k-1} \dots.. E_3 E_2 E_1$.

c) Verify that $A = E_1^{-1}\, E_2^{-1}\, E_3^{-1} \dots\, E_{k-1}^{-1}\, E_k^{-1}$ for some k.

d) Use the decomposition done in part (c) to discuss the Geometric Effect on the Unit Square of the elementary matrices.

Do the following steps for part (d):

i) Show the effect of $E_1^{-1}\, E_2^{-1}\, E_3^{-1} \dots\, E_{k-1}^{-1}\, E_k^{-1}$ on the unit square step by step.

ii) Show the action of each elementary matrix mathematically and graphically.

Question#2: [5+5 marks, CLO #2]

a) Find the area of the triangle in 3-space that has the given vertices.

$$P = (1, -1,\ 2),\ Q = (0, 3,\ 4),\quad R = (6,\ 1,\ 8).$$

b) Find the distance between the point $(2, 1, -3)$ and the plane $2x - y - 2z = 6$.

Question#3: [10 marks, CLO #3]

no. of vectors, degree.

Find the geometric and algebraic multiplicity of each eigenvalue of the matrix A,

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A and find $P^{-1}AP$.

3 3 0 1/4 1/8 1 1 1/3 2/3 1/2 3/4

Question#4: [10 marks, CLO #03]

Suppose you are designing a simple video game where the player controls a shooter to hit a moving targets. If the shooter is an arrowhead whose vertices at any point is in the span of row vectors of A where

$$A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

(**Hint:** Take Transpose of given matrix A)

a) Find the basis for the row space of A consisting entirely of the row vectors from A.

b) Use Dimension theorem to find the rank and nullity of A and A^t.

Question#5: [20 marks, CLO #04]

Suppose $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ define the column vectors u_1, u_2 and u_3 as

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \ \& \ u_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

a) Use Gram – Schmidt process to find the orthogonal set of vectors $\{v_1, v_2, v_3\}$ and then find orthonormal set of vectors $\{q_1, q_2, q_3\}$ by considering standard inner product between vectors.

b) Considering standard inner product between the vectors find QR- decomposition of the given matrix. Also verify that $A = QR$ where,

$Q = [\ q_1 \mid q_2 \mid q_3\ ]$ and R is given below

$$R = \begin{bmatrix} \langle u_1, q_1\rangle & \langle u_2, q_1\rangle & \langle u_3, q_1\rangle \\ 0 & \langle u_2, q_2\rangle & \langle u_3, q_2\rangle \\ 0 & 0 & \langle u_3, q_3\rangle \end{bmatrix}.$$

Question#6: [5+5 marks, CLO #04]

a) Find $\|u\|$ and $d\,(u, v)$ relative to the weighted inner product defined as $\langle u, v\rangle = 2u_1v_1 + 3u_2v_2$ on R^2 where, $u = (-3, 2)$ and $v = (1, 7)$.

b) Considering the weighted inner product defined in part (a) check that whether the vectors are orthogonal.

Question#7: [10 marks, CLO #05]

Let $T: R^2 \to R^3$ be the linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -x_1 \\ -3x_1 + 2x_2 \\ -4x_1 + 3x_2 \end{bmatrix}$. Find the matrix for the transformation T i.e. $[T]_{B',B} = [\ [T(u_1)]_{B'} \mid [T(u_2)]_{B'}]$ relative to the basis $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2, v_3\}$, where

$$u_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

$\frac{\sqrt{6}}{3}$

$\frac{1}{3\sqrt{6}}$

$\frac{\sqrt{6}}{6}$ $\frac{2\sqrt{6}}{6}$

$\frac{\sqrt{3}}{3}$ $\frac{2}{\sqrt{6}}$

Question#8: [5+5 marks, CLO-3, 5] (Bonus Question)

$$\text{If } C = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \text{ and } D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \text{ then}$$

Do the following steps:

a. Find a matrix P (consisting of the Eigen vectors of the matrix C) using Eigenvalues of C and show that $P^{-1}CP = D$. Also, find the dimension of Eigen Spaces associated with each Eigen value.

b. Show that C and D represents same linear operator $T : R^2 \to R^2$ by showing $P^{-1}CP = D$, where $P = P_{B' \to B} = [[u_1']_B \ [u_2']_B]$ and $P^{-1} = P_{B \to B'}$, $B' = \{u_1', u_2'\}$, $B = \{e_1, e_2\}$, $u_1' = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $u_2' = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Here P & P^{-1} shows the transition matrices.

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Date:	26-09-2022	Weight	12.5%
Section:	All	Page(s):	2
Exam:	Mid 1	Roll No:	

Name:

Instruction/Notes:

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Question no. 1: (5+5+5+3+2 marks) (CLO #1)

a) Suppose that the augmented matrix of a system of linear equations has been partially reduced using elementary row operations to

$\boxed{-4}$

$$\left[\begin{array}{ccc|c} 1 & -1 & a-2 & 3 \\ 0 & 1 & 2b+1 & -1 \\ 0 & 0 & a-1 & b+3 \end{array}\right]$$

Find all values of a and b for which the given system has no solution, exactly one solution and infinitely many solutions.

b) Find the matrix B by using the given information

$$(5B^T)^{-1} = \begin{bmatrix} 1 & 5 \\ 3 & -4 \end{bmatrix}$$

c) Evaluate the given determinant by using row reduction method.

$$\begin{vmatrix} 1 & -1 & 5 & 5 \\ 3 & 1 & 2 & 4 \\ -1 & -3 & 8 & 0 \\ 1 & 1 & 2 & -1 \end{vmatrix}$$

d) For the following linear system of equation, find the general solution in parametric form and also give the geometric representation.

$$x + 3y = 3$$
$$2x + 6y = 6$$

-1

e) Identify the row operation corresponding to E and verify that the product EC results from applying the row operation to C.

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$$

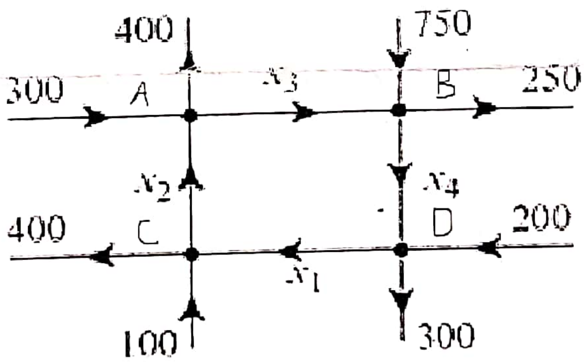
Question no. 2: (10 marks) (CLO #1)

Use inversion algorithm to find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 4 & 0 \\ -2 & 2 & 3 \\ 3 & 8 & 7 \end{bmatrix}$$

Question no. 3: (10 marks) (CLO #1)

The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour



Set up a linear system whose solution provides the unknown flow rates and solve the system for the unknown flow rates using Gauss Elimination.