National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Calculus and Analytical Geometry	Course Code:	MT1003
Program:	BCS, BDS, BSE	Semester:	Fall 2023
Duration:	60 Minutes	Total Marks:	40
Paper Date:	30-09-2023	Weight	15
Section:	ALL	Page(s):	6
Exam Type:	Midterm-I		

Student : Name: [redacted] Roll No [redacted] Section [redacted]

Instruction/Notes: Answer all questions neatly on the space provided. Answer sheet may be used for the rough work only. Exchange of calculators or programmable calculators are not allowed at all.

Q#1 ([CLO-1]): Solve the given inequality and show the solution set on real line. (10)

$(x-5)(x-2)>0$

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$$x^2 - 7x + 10 > 0$$

Using Middle term break:-

$x^2 - 5x - 2x + 10 > 0$

$x(x-5) - 2(x-5) > 0$

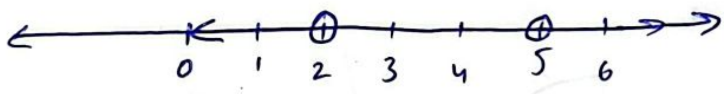
$(x-2)(x-5) > 0$

when $y=0$.

10

$\boxed{x=2,\ x=5}$ => critical points

$\boxed{x>5 \text{ or } x<2}$ $\boxed{Df(x) = (-\infty, 2) \cup (5, \infty)}$



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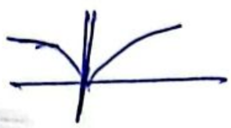
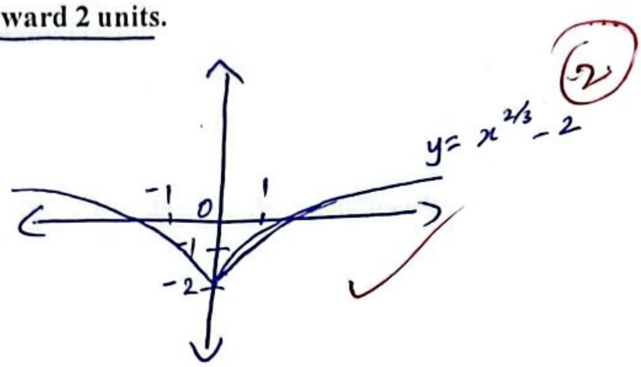
Q#1 ([CLO-2]): Write the equation and plot the graph of each of the following for the given function.

$$f(x) = x^{\frac{2}{3}}$$



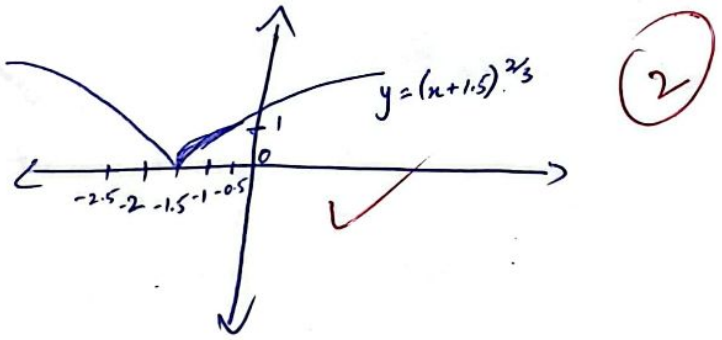
I. Shift the graph of $f(x)$ downward 2 units. (2)

$f(x) = x^{2/3} - 2$



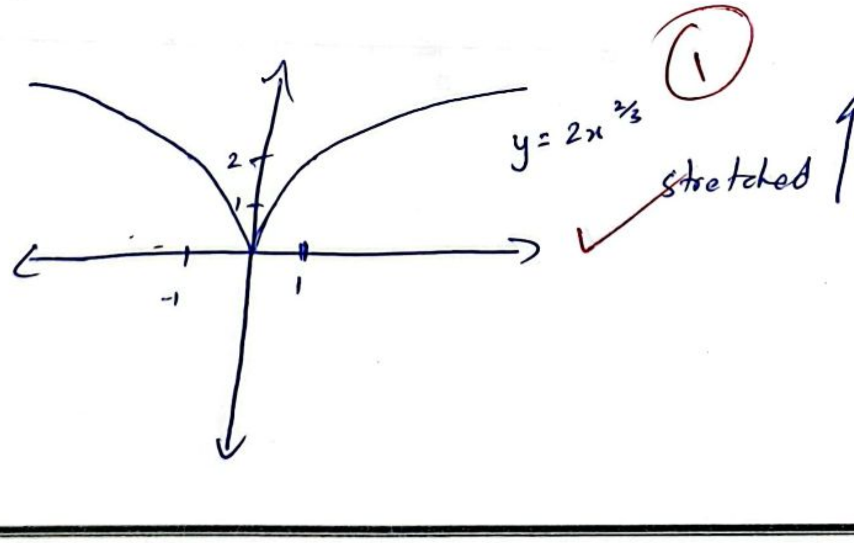
II. Shift the graph of $f(x)$ left 1.5 units. (2)

$f(x) = (x+1.5)^{2/3}$



III. Stretch vertically by the factor of 2 units. (1)

$f(x) = 2x^{2/3}$



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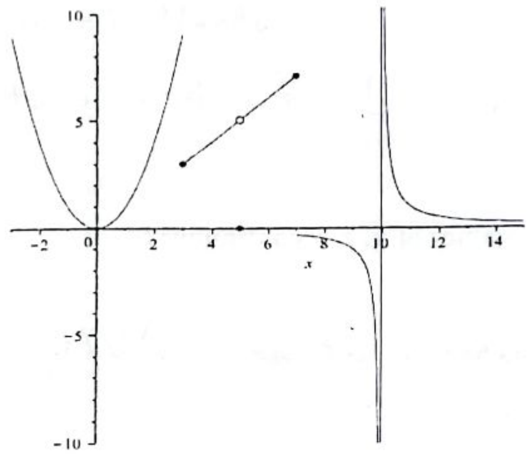
Q#1 ([CLO-3]): Refer to the following function

$$f(x) = \begin{cases} x^2 & -3 < x < 3 \\ x & 3 \le x < 5 \\ 0 & x = 5 \\ x & 5 < x \le 7 \\ \frac{1}{x-10} & x > 7 \end{cases}$$

$(-3, 3)$
$(3, 5)$
$(5, 7)$
$(7, \infty)$

Graphed in the accompanying figure



Answers the following questions. Give reasons for your answers.

I. Does $f(3)$ exists? (1)

Yes, since $f(3) = 3$.

1

II. Does $\lim_{x \to 3} f(x)$ exists? (1)

Since $\lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$, it does not exist.

1

III. Is f continuous at $x = 3$? (1)

1

No, since at $x = 3$, limit does not exist, hence discontinuous.

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IV. Does $f(5)$ exists? (

$f(5)$ exists since $f(5) = 0$ 1

V. Is f continuous at $x = 5$? If not, what value should be assigned to $f(5)$ to make the extended function continuous at $x = 5$? (2)

Not continuous. it should be assigned $f(5) = 5$ to make the extended function continuous at $x = 5$. 2

VI. At what values of x, f is continuous? (2)

f is continuous at ~~x=~~ $(-3,3) \cup (3,5) \cup (5,7) \cup (7,\infty)$ 2

VII. Is f continuous at $x = 7, 10$? (2)

It is not continous at $x=7$, however it is continuous at $x=10$. 1

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Q#1 ([CLO-3]): For what values of a and b

$$\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = x+2.$$

$$g(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ ax^2 - bx + 3 & 2 \le x < 3 \\ 2x - a + b & x \ge 3 \end{cases}$$

Is continuous at every x? (10)

At $x = 2$,

$\cancel{\frac{(2)^2-4}{(2)-2}} \Rightarrow \cancel{a(2)^2 - 2b(2) + 3}$

$x + 2 = ax^2 - bx + 3$

$2 + 2 = 4a - 2b + 3$

$4a - 2b = 1$ — ①

At $x = 3$,

$9a - 3b + 3 = 6 - a + b$

$10a - 4b = 3$ — ②

10

Using Elimination:-

$10a - 4b = 3$

$-8a + 4b = -2$

$2a = 1$

$\boxed{a = 1/2}$

Put $a = 1/2$ in ①:-

$\frac{4}{2} - 2b = 1$

$\boxed{b = \frac{1}{2}}$

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Q#1 ([CLO-3], 5 marks): For the following function

$$k(x) = \frac{-3x^2+2}{x-1}$$

Use the limit to determine all asymptotes of $k(x)$. (5)

$$\begin{array}{r} -3x-3 \\ x-1 \overline{\big)\, -3x^2 + 2} \\ +\ 3x^2 \mp 3x \\ \hline -3x+2 \\ +\ 3x-3 \\ \hline -1 \end{array}$$

$k(x) = -3x-3 - \frac{1}{x-1}$

- At $\lim_{x\to 1} k(x) = \pm\infty$ (undefined) (Vertical)

$y = -3x-3$ (oblique asymptote)

5

Two asymptotes:-

$$\boxed{\begin{array}{l} x = 1 \ \text{(Vertical)} \\ y = -3x-3 \ \text{(oblique)} \end{array}}$$