National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Calculus and Analytical Geometry	Course Code:	MT 1003
Degree Program:	BS (CS, SE & DS)	Semester:	Fall 2023
Exam Duration:	3 hours	Total Marks:	80
Paper Date:	29- 12- 23	Weight	50%
Section:	ALL	Page(s):	2
Exam Type:	Final Exam		

Instruction/Notes: Attempt all questions. Programmable calculators are not allowed.

Question#1: CLO-2, 3 [3+3+4]

a) Explain how $g(x)$ and $h(x)$ are related to $f(x)$ where $f(x) = |x|$, $g(x) = 2|x|$, and $h(x) = -2|x + 2| - 1$.

b) Using limit properties and the definition of continuity, determine if the following function is continuous or not at $x = -6$ and $x = 1$.

$$g(x) = \begin{cases} 1 - 3x, & < -6, \\ 7, & x = -6, \\ x^3, & -6 < x < 1, \\ 1, & x = 1, \\ 2 - x, & x > 1. \end{cases}$$

c) Find all possible asymptotes of the graph of $f(x) = \frac{x^2-4x+1}{x^2+7x+6}$.

Question#2: CLO-4[10]. Consider a right triangle which is changing shape in the following way. The base of the right triangle is decreasing at the rate of 5in./sec and the altitude of the right triangle is increasing at the rate of 7in./sec. At what rate is the triangle's

i. Hypotenuse changing?
ii. Perimeter changing?
iii. Area changing?
when the base is 8in.and altitude is 6in.

Question#3: CLO-5 [10+10]

a) A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given 100 ft. of wire fencing, determine the dimensions that would create a garden of maximum area. Also, what is the maximum area?

b) For the following function

$$f(x) = x^{\frac{4}{3}}(x-2)$$

Answer each of the following questions:

i. Identify the critical points of the function.
ii. Determine the intervals on which the function increases and decreases.
iii. Classify the critical points as relative maxima, relative minima or neither.
iv. Determine the intervals on which the function is concave up and concave down.
v. Determine the inflection points of the function.
vi. Use the information from steps (i)-(v) to sketch the graph of the function.

Question#4: CLO-6 [5+5+5]

a) Use any method of your choice to determine the volume of the solid obtained by rotating the region bounded by the curve $x = y^2 - 6y + 10$ and the line $x = 5$ about the y-axis

b) Find the length of the curve $y = log_e(secx), \quad o \le x \le \frac{\pi}{3}$

Question#5: CLO-6 [5+5]. Evaluate the following integrals.

a) $\int \frac{(3+\ln x)^2(2-\ln x)}{4x}\,dx.$

b) $\int \frac{\sqrt{9-x^2}}{x^2}\,dx.$

Question#6: CLO-6 [5+10]

a) Use L' Hopital's rule to find the following limit:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

b) Evaluate the following improper integrals and determine whether the given improper integral converges or diverges

i. $\int_{-\infty}^{0} e^{-|x|}\,dx$ ii. $\int_{-\infty}^{0}(1+2x)e^{-x}\,dx$