

The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 9

1. Classify the following PDE as linear, semi-linear, quasi-linear or fully nonlinear:

(a) $u_t + xu_x = 0$, (b) $u_t + au_x = u^2$, (c) $xu_t + u_x = u$,
(d) $u_t + uu_x = 0$, (Inviscid Burgers' Eqn.) (e) $-\Delta u(x) = f(x, u(x))$, $x \in \mathbb{R}^n$, (Poisson Eqn.)
(f) $\Delta^2 u(x, y) = 0$, $(x, y) \in \mathbb{R}^2$, (Biharmonic Equation) (g) $u_t = u^3 u_{xxx}$, (Dym Equation)

2. Find the first order PDE by eliminating the arbitrary function f , satisfied by z :

(a) $z(x, y) = xy + f(x^2 + y^2)$, (b) $z(x, y) = f(x/y)$, (c) $f(x - z, y - z) = 0$.

3. Find the first order PDE by eliminating the arbitrary constants a and b , satisfied by z :

(a) $z(x, y) = (x + a)(y + b)$, (b) $z(x, y) = ax + by$, (c) $z^2(1 + a^3) = 8(x + ay + b)^3$.

4. Find the first order PDE whose solution is the given function (or surface) $z(x, y)$:

- (a) The family of all spheres in \mathbb{R}^3 whose centres lie on the z -axis with radius a , i.e., $x^2 + y^2 + (z - c)^2 - a^2 = 0$.
(b) The family of all right circular cones with vertex at $(0, 0, c)$, whose axes coincides the z -axis with inclination θ , i.e., $(x^2 + y^2) \cos^2 \theta - (z - c)^2 \sin^2 \theta = 0$. Compare the PDE obtained with the one obtained in (a). Further, show that any surface $z(x, y) = f(x^2 + y^2)$ satisfies the same PDE as obtained in (a).

Note: All surfaces of revolution with z -axis as the axis of revolution are governed by the equation $z(x, y) = f(x^2 + y^2)$.

5. (a) Let $v(x, y)$ be a given function of x, y and $f : \mathbb{R} \rightarrow \mathbb{R}$ is an arbitrary (or unknown) one variable function. Find the first order PDE whose solution is $u(x, y) = f(v)$.
(b) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two arbitrary (or unknown) one variable functions. Find the second order PDE whose solution is $u(x, y) = f(x - ay) + g(x + ay)$, for some given $a \in \mathbb{R}$.
6. In the following, find general integral i.e., solution containing arbitrary function:

(a) $xp + yq = z$, (b) $x^2p + y^2q = (x + y)z$, (c) $yzp + xzq = xy$, (d) $(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z)$.

7. Discuss the existence of integral surface of $2p + 3q + 8z = 0$, which contains the curve: (a) $\Gamma : z = 1 - 3x$ and the line $y = 0$; (b) $\Gamma : z = x^2$ on the line $2y = 1 + 3x$, (c) $\Gamma : z = e^{-4x}$ on the line $2y = 3x$.
8. Find the integral surface for the following Cauchy's problem: (a) $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$, $\Gamma : x_0(s) = 1, y_0(s) = 0, z_0(s) = s$; (b) $x^3p + y(3x^2 + y)q = z(2x^2 + y)$, $\Gamma : x_0(s) = 1, y_0(s) = s, z_0(s) = s(1 + s)$.