The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-I ■ Assignment #7

(Calculus of functions of several variables, Directional derivatives, Max/Min and Lagrange Multipliers)

Q1. Examine the following functions for continuity at the point $(0,0)$ where $f(0,0)=0$ and $f(x,y)$ for $(x,y)\neq(0,0)$ is given by
(a) $|x|+|y|$, (b) $\frac{-x}{\sqrt{x^2+y^2}}$, (c) $\frac{2x}{x^2+x+y^2}$, (d) $\frac{x^4-y^2}{x^4+y^2}$, (e) $\frac{x^4}{x^4+y^2}$.

Q2. Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by

$$f(x,y)=\begin{cases}1, & \text{if } x=0 \text{ or if } y=0\\ 0, & \text{otherwise.}\end{cases}$$

Show that the function satisfy the following:
(a) The iterated limits $\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right]$ and $\lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right]$ exist and equals 0,
(b) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist,
(c) $f(x,y)$ is not continuous at $(0,0)$,
(d) the partial derivatives exist at $(0,0)$.

Q3. Let

$$f(x,y)=\begin{cases}xy\left(\frac{x^2-y^2}{x^2+y^2}\right), & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0).\end{cases}$$

Prove that
(a) $f_x(0,y)=-y$ and $f_y(x,0)=x$ for all x and y,
(b) $f_{xy}(0,0)=-1$ and $f_{yx}(0,0)=1$ and
(c) $f(x,y)$ is differentiable at $(0,0)$.

Q4. Suppose f is a function with $f_x(x,y)=f_y(x,y)=0$ for all (x,y). Then show that f is constant.

Q5. Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by

$$f(x,y)=\begin{cases}\frac{y^3}{x^2+y^2}, & \text{if } (x,y)\neq 0\\ 0, & \text{otherwise.}\end{cases}$$

Show that f is continuous at $(0,0)$, it has all directional derivatives at $(0,0)$ but not differentiable at $(0,0)$.

Q6. Examine the following functions for local maxima, local minima and saddle points:
(i) $4xy-x^4-y^4$, (ii) x^3-3xy, (iii) $(x^2+y^2)\exp^{-(x^2+y^2)}$.

Q7. Let $f(x,y)=3x^4-4x^2y+y^2$. Show that f has a local minimum at $(0,0)$ along every line through $(0,0)$. Does f have a minimum at $(0,0)$? Is $(0,0)$ a saddle point for f?

Q8. Find the absolute maxima of $f(x,y)=xy$ on the unit disc $\{f(x,y): x^2+y^2\leq 1\}$.

Q9. Find the equation of the surface generated by the normals to the surface $x+2yz+xyz^2=0$ at all points on the $z-$axis.

Q10. Given n positive numbers $a_1,a_2,\ldots,a_n$, find the maximum value of the expression the function $a_1x_1+a_2x_2+\ldots+a_nx_n$ where $x_1^2+x_2^2+\ldots+x_n^2=1$.

Q11. Assume that among all rectangular boxes with fixed surface area of 20 square meters, there is a box of largest possible volume. Find its dimensions.

Q12. Minimize the function $x^2+y^2+z^2$ subject to the constraints $x+2y+3z=6$ and $x+3y+9z=9$.