# DMS ASSIGNMENT 1 SOLUTIONS

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ROLL NO: 16UCS004

Sol: 1) It is given that $A = \{1\}$ and $B = \{1, \{1\}\}$

$\because$ $P(B) = \{\{1\}, \{\{1\}\}, \{1, \{1\}\}, \phi\}$

(i) Clearly $A \in P(B) \Rightarrow A \subseteq B$ (proved)

(ii) Also $A \neq B$ $\therefore$ A is proper subset of B

Sol 2) Let $a = b-1$ which gives $A = B$ as it's given that $A = \{m \in Z \mid m = 2a$ for some integer $a\}$

$B = \{n \in Z \mid n = 2b-2$, for some $b \in Z\}$

Let x be unique but any arbitrary chosen member of A. By definition there's an integer a such that $x = 2a$.

Let $b = a+1$, then b is an integer because it is a sum of integer. Also $2b-2 = 2(a+1)-2 = 2a$ thus, by definition of B, x is an element of B.

Now to prove $B \subseteq A$

Let y is a unique but arbitrary chosen element of B. By B's definition, there is an integer b such that $y = 2b-2$

Let $a = b-1$, a is an integer because it is difference of integers. Also $2a = 2a+2-2 = 2(a+1)-2 = 2b-2 = y$.

Thus by definition of A, y is an element of A.

Clearly from above proofs we can say that $A = B$.

Sol: 3) $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{b, c, e\}$

(i) $A \cup (B \cap C) = \{a, b, c\}$

$(A \cup B) \cap C = \{b, c\}$

and $(A \cup B) \cap (A \cup C) = \{a, b, c\}$

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From above it is clear that

$$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$$

(ii) $A\cap(B\cup C) = \{b, c\}$

$(A\cap B)\cup C = \{b, c, e\}$

$(A\cap B)\cup(A\cap C) = \{b, c\}$

From above it's clear that

$$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$$

(iii) $(A-B)-C = (A\cap B')-C = A\cap B'\cap C' = A\cap(B\cup C)' = \{a\}$

$A-(B-C) = A-(B\cap C') = A\cap(B'\cup C) = (A\cap B')\cup(A\cap C)$

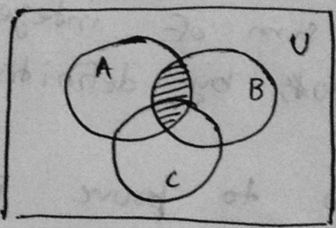
$= \{a, b, c\}$

We can see that

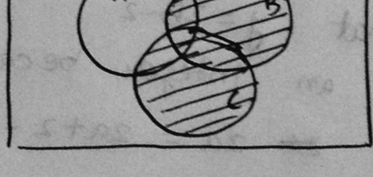
$(A-B)-C \neq (A-(B-C))$

since $[(A-B)-C] \not\subseteq [A-(B-C)]$

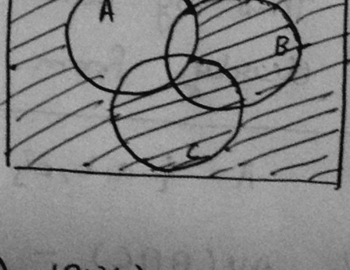
Sol : 4) (a) $A\cap B$



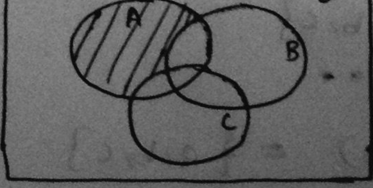

(b) $B\cup C$




(c) $A^c$




(d) $A-(B\cup C)$



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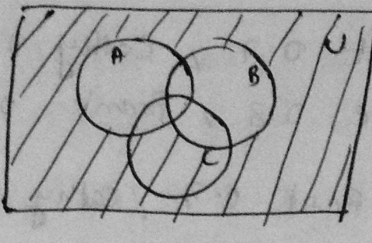
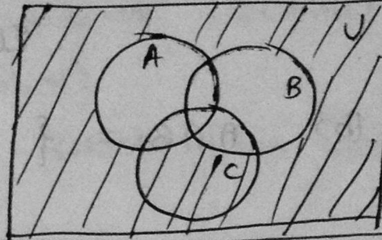
(e)




$(A \cup B)^c$

(f)

$A^c \cap B^c$




Sol: 5) To prove : $A-(A\cap B) = A-B$

$LHS = A-(A\cap B)$

$= A\cap(A\cap B)' = A\cap(A'\cup B')$

$= \phi \cup (A\cap B') = A\cap B'$

$= A-B = RHS$. (proved).

Sol 6:) $LHS = (X-Y)-Z = (X\cap Y')-Z$

$= (X\cap Y')\cap Z' = X\cap(Y\cup Z)'$

$RHS = X-(Y\cap Z')^{\neq} = X\cap(Y\cap Z')' = X\cap(Y'\cup Z)$

$= (X\cap Y')\cup(X\cap Z) \neq LHS$.

$=/ \cancel{X\cap(Y'\cup Z)}$

$\therefore LHS \neq RHS$

Sol: 7) $(A\cap B')\cup(A'\cap B)\cup(A'\cap B')$

$= (A'\cap B)\cup(A'\cap B')\cup(A\cap B')$

$= (A'\cap(B\cup B'))\cup(A\cap B')$

$= (A'\cap U)\cup(A\cap B')$

$= A'\cup(A\cap B')$

$= (A'\cup A)\cap(A'\cup B')$

$= A'\cup B'$

$= U-(A\cap B)$

$= (A\cap B)'$

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Sol: 8)

$A = \{$ Angelo 0.4, Bart 0.7, Cathy 0.6 $\}$

$B = \{$ Dan 0.3, Elsie 0.8, Frank 0.4 $\}$

(a) $A \cup B = \{$ Angelo 0.4, Bart 0.7, Cathy 0.6, Dan 0.3, Elsie 0.8, Frank 0.4 $\}$

(b) $A \cap B = \{$ Angelo 0, Bart 0, Cathy 0, Dan 0, Elsie 0, Frank 0 $\} = \{ \}$

(c) $A' = \{$ Angelo 0.6, Bart 0.3, Cathy 0.4 $\}$

(d) $A \cup B' = \{$ Angelo 0.4, Bart 0.7, Cathy 0.6, Dan 0.7, Elsie 0.2, Frank 0.6 $\}$

(e) $A \cap B' = \{$ Angelo 0, Bart 0, Cathy 0, Dan 0, Elsie 0, Frank 0 $\}$

(f) $A \cap A' = \{$ Angelo 0.4, Bart 0.3, Cathy 0.4 $\}$

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Sol: 9) Let $A = \{a, a_2, \dots, a_n\}$

$P(n) = 2^n$ subsets for $n$ elements.

Basic step $(x=0)$

$\Rightarrow A = \phi \Rightarrow$ No. of subset $= 1$

Also, $2^0 = 1 \Rightarrow P(0)$ is true

Inductive step

let $P(k)$ be true $\Rightarrow$ set containing $k$ elements have $2^k$ subsets suppose $|B| = k$, $2^k$

$|A| = k+1$

Let $x \in A$ and $x \notin B \Rightarrow A = B \cup \{x\}$

All the subsets of B are subset of $A = 2^k$.

Now adding $x$ element to all these subsets. So, new subsets having $x$ or not having $x$ are $2^k$ & $2^k$

$\Rightarrow$ Total subsets $= 2^k + 2^k = 2 \cdot 2^k = 2^{k+1}$

$\Rightarrow P(k+1)$ is also true.

By P.M.I $\Rightarrow P(n)$ is true

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Sol: 10) (a) $n(S) = 26$; $S = \{a, b, c, \ldots, z\}$

(b) $n(L) = 5$; $L = \{T, W, E, D, L\}$

(c) $n(M) = 7$; $M = \{$ January, March, May, July, August, October, December $\}$

(d) 0 $\because$ identifiers cannot start from numeric value

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Sol: 11) (a) No. of positive integers $< 500$ divisible by 2 or 3

$\left[\frac{499}{2}\right] = 249$, $\left[\frac{499}{3}\right] = 166$, $\left[\frac{499}{6}\right] = 83$

~~Ans~~ $249 + 166 - 83 = 332$

(b) ~~No~~ ~~2 or 5~~: No. of divisibilities by 2 or 5 $= 40$

No. of divisibilities by 3 or 5 $= 33$

$249 + 166 + 99 - 83 - 40 - 33 = 358$

(c) $332 - 83 = 249$

(d) $499 - 358 = 141$

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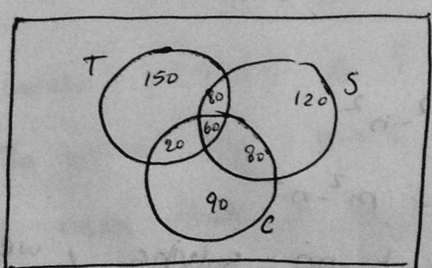
Sol: 12)




$220 \rightarrow T, S$
$200 \rightarrow S, C$
$170 \rightarrow C, T$
$80 \rightarrow T, S, C$
$80 \rightarrow S, C, T$
$70 \rightarrow C, T, S$
Total $= 700$

(a) No. of families with exactly one of the items $= 360$

(b) " with exactly 2 of the items $= 230$

(c) " at least one of the items $= 650$

(d) " all of the items $= 60$

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Sol : 13) $2 \in S$ if $x \in S$ then $x^2 \in S$
set by the listing methode $= \{2, 2^2, 2^{2^2}, 2^{2^{2^2}}, \dots\}$

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Sol : 14) Let $D_f$ and $R_f$ denote domain and range of the functions

(a) $D_f = (Z, Z)$ , $R_f = Z$

(b) $D_f = Z^+$ , $R_f = \{x \in Z^+ \mid x < 10\} = R$

(c) $D_f =$ a set of all bit strings , ~~Ra~~ $R_f = Z$

(d) $D_f = Z^+$ , $R_f = Z^+$

(e) $D_f =$ a set of all bit strings
$R_f =$ bit strings containing only 1's and the empty string

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Sol : 15) (a) $f(m,n) = 2m-n$
let $m = k$ where $k \in R$
$f(k,n) = \begin{cases} 2k & \text{if } n=0 \\ 2k-1 & \text{if } n=1 \\ \dots & \text{otherwise} \end{cases}$

all even no's + all odd no's = integers
$\therefore$ onto

(b) $f(m,n) = m^2 - n^2$
$f(m,n) = 2 = m^2 - n^2$
$\because$ $2 \in Z$ but on solving , we are unable to get integers $m$ & $n$ $\Rightarrow$ not onto

(c) $f(m,n) = m+n+1$
$\because$ $m$ is integer , $n$ & $1$ are also
2 integers add to give ~~of~~ integer $\Rightarrow f(m,n) \in Z_0$ = onto

(d) $f(m,n) = |m| - |n|$
$\because$ $m, n \in Z$ and all integers can be created using $m, n$ . so, $f(m,n)$ is onto .

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(e) $f(m,n) = m^2 - 4$

$\because m^2 \geq 0 \Rightarrow f(m,n) \geq -4$

$-5 = f(m,n)$ is not possible for any integer value $m, n \Rightarrow$ not onto.

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Sol : 16) (a) $f(x) = -3x + 4$

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ $\therefore$ one-one

$x = \frac{y-4}{-3}$ $\therefore$ Range $= R$ $\therefore$ onto

$\Rightarrow$ bijective

(b) $f(x) = -3x^2 + 7$

$f(x_1) = f(x_2) \Rightarrow x_1 x_2 = 0$ or $x_1 - x_2 = 0$

$\therefore$ not one-one

Not onto

$\Rightarrow$ not bijective

(c) $f(x) = \frac{x+1}{x+2}$

$\Rightarrow y = \frac{x+1}{x+2} \Rightarrow x = \frac{1-2y}{y-1}$ & $y \neq 1$

$\therefore$ not bijective

(d) $f(x) = x^5 + 1$

$x_1^5 + 1 = x_2^5 + 1 \Rightarrow x_1 = x_2$ $\therefore$ one-one

$y = x^5 + 1 \Rightarrow x = (y-1)^{1/5}$ $\therefore$ onto

$\Rightarrow$ bijective

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Sol : 17) Given $g : A \to B$ & $f : B \to C$

(a) To prove $fog : A \to C$ is one-one (injective)

we need that if $fog(x) = fog(y)$ then $x = y$

Suppose $fog(x) = fog(y) = c \in C$

This means that $f(g(x)) = f(g(y))$

let $f(x) = a$, $g(y) = b$ ; so $f(a) = f(b)$

$\because f : B \to C$ is injective and $f(a) = f(b)$, we know that $a = b$. This means that $g(a) = g(b)$

$\because g : A \to B$ is injective and $g(x) = g(y)$, we know that $x = y$, we have shown that if $fog(x) = fog(y)$ then $x = y$.

Hence, proved.

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(b) $g: A \to B$ & $f: B \to C$ are onto

To prove $fog: A \to C$ is onto, we need to prove that $\forall\, c \in C\ \exists\, a \in A \ni fog(a) = c$

Let $c$ be any element of $C$.

$\because f: B \to C$ is onto, $\exists\, b \in B \ni f(b) = c$

Since, $f: A \to B$ is onto, $\exists\, a \in A \ni g(a) = b$

So, $(f \circ g)(a) = g(f(a)) = g(b) = c$

~~Hence~~ Hence, proved.

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Sol: 18) Let $f, g, h: Z \to Z$

$f(x) = x-1$, $g(x) = 3x$, $h(x) = \begin{cases} 1, & x \text{ is odd} \\ 0, & x \text{ is even} \end{cases}$

(a) $fog(x) = g(x) - 1 = 3x - 1$, $fo(goh) = \begin{cases} 2 & x \text{ is odd} \\ -1 & x \text{ is even} \end{cases}$

$gof(x) = 3x - 3$

$goh(x) = \begin{cases} 3 & x \text{ is odd} \\ 0 & x \text{ is even} \end{cases}$

$hog(x) = \begin{cases} 1 & x \text{ is odd} \\ 0 & x \text{ is even} \end{cases}$

(b) $f^2 = (x-1)^2$

$f^3 = (x-1)^3$, $g^2 = (3x)^2$

$g^3 = (3x)^3$,

$h^2 = h^3 = h^{500} = \begin{cases} 1, & x \text{ is odd} \\ 0, & x \text{ is even} \end{cases}$

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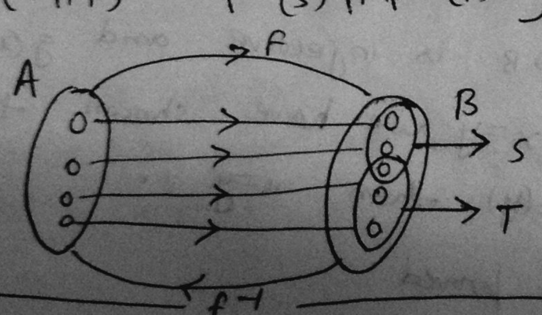
Sol: 19) Let $f$ be a function from $A$ to $B$. Let $S$ and $T$ be subsets of $B$ i.e. $f: A \to B$

(a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$

(b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

$\Rightarrow$ By using the below function diagram we get the result



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Sol : 20) $S \subseteq U$, $f_S : U \to \{0,1\}$

$$f_S(x) = \begin{cases} 1, & \text{if } x \in S \\ 0, & \text{if } x \notin S \end{cases}$$

(a) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$, as both A and B are independent

(b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_{A \cap B}(x)$

$= f_A(x) + f_B(x) - f_A(x) \cdot f_B(x)$

(c) $f_{\bar{A}}(x) = 1 - f_A(x)$

(d) $f_{A \otimes B}(x) = f_A(x) + f_B(x) - 2 f_{A \cap B}(x)$

$= f_A(x) + f_B(x) - 2 f_A(x) \cdot f_B(x)$

[ Using various basic set laws like intersection and union laws, like we know,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

and $n(A \cap B) = n(A) \cdot n(B)$ for A and B are independent events or sets ]

—x— —x—