Non-Inertial Frames.

Here we are dealing with two kinds of non-inertial frames. The first one is linearly accelerated frams and the second one is rotating frams. In both these frams we deal with fictitious forces. In the rotating frames we get Centrifugal force & Coriolis force. Since earth is also a rotating body we do experience these forces on the surface of the earth.

I Consulted mainly three books & some other sources to prepare my nots. They are.

1.) Kleppner

2) Engineering Mechanics — Harbola.

3) Intermediate Dynamics — Patrick Hamill.

Please go through it carefully & let me know if you have doubt somewhere or if you find any error in the nots so that it can be corrected & posted again.

Aneogi

(AMIT NEOGI)

18 october, 2016.

Non Inertial Frame / Systems & Fictitious Forces.

We studied earlier that Newton's Second Law $F=ma$ holds true only in inertial coordinate system. Any coordinate system moving uniformly with respect to an inertial system is also Inertial. This is given by Galilean transformation, the fact that acceleration remains same in all inertial system is known as Galilean invariance.

Galilean Transformation.

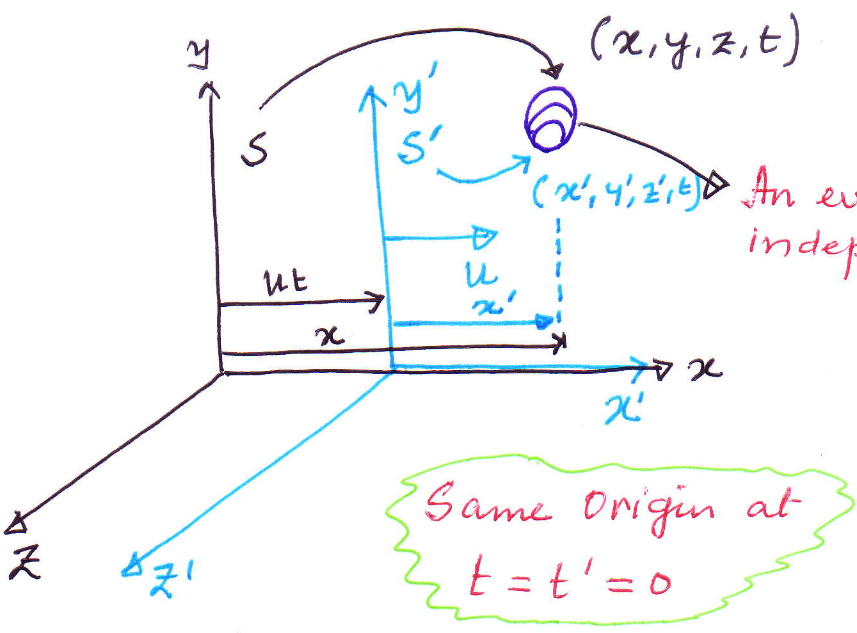
This relate the observations of a single event by two different observers. The three major points of this transformation

(1) An observer S at rest in one inertial frame.

(2) Another observer S' at rest in a different inertial frame that is in motion at const velocity wrt to S.

(3) A single event that is observed by both S and S'.

So a single event occurs in space, irrespective of any reference frame. It is being observed by two different observers in two different inertial frame one moving wrt to other at velocity v. Let us relate the coordinates (x, y, z, t) of the first with the second (x', y', z, t')



An event occured in space independent of any ref frame.

$$x' = x - ut$$

Two observers whose frames of reference are represented by S and S', observe the same event. S' moves relative to S with velocity $\bar{u}$ along the common xx' direction. S measures the coordinate (x, y, z, t) of the event while S' measures the coordinate (x', y', z', t') of the same event.

Galilean transformations eqns are.

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

With motion along x axis one can write eqns. as.

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

The above set of equations shows Galilean Invariance. Galilean Invariance usually refers to this principle as applied to Newtonian Mechanics. That is Newton's Laws hold good in all inertial frams

Differentiating the first equation. Let us write $r(t)$ in place of x, y, z, t

$$r'(t) = r(t) - ut.$$

~~$v'(t) =$~~

$$v'(t) = \frac{dr'(t)}{dt} = \frac{d}{dt}(r(t)) - u = v(t) - u.$$

Differentiating further

$$a'(t) = a(t).$$

Assuming that mass is invariant in all inertial frame.

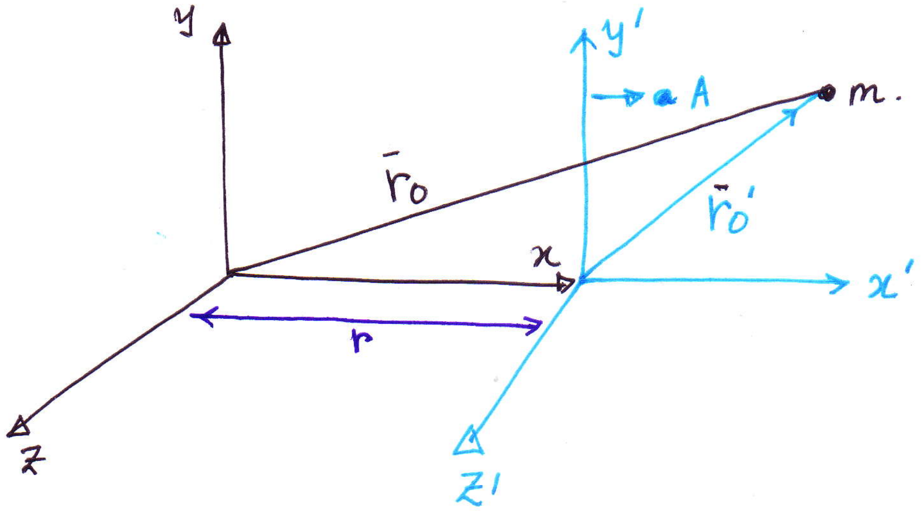
$$m\,a'(t) = m\,a(t).$$

$$F'(t) = F(t).$$

The above equation shows that Newton's Laws if valid in one frame must be valid in all other inertial frames.

Accelerated Reference frame.

How to express Newton's second Law in a non-inertial frame.



This can be done in different ways like $x = x' - \frac{1}{2}at^2$. & differentiating. But let us derive here by Vectors.

At time $t=0$ both the frames coincide and after that $x'y'z'$ coordinate frame accelerate towards the right with acceleration $\bar{A}$ to the right. Suppose mass m accelerats. We can relate by Vector addition of position Vectors in two reference frame.

$$\bar{r}_0 = \bar{r} + \bar{r}_0'$$

Differentiating twice wrt to time.

$$\ddot{\bar{r}}_0 = \ddot{\bar{r}} + \ddot{\bar{r}}_0' \quad \text{multiplying by } m \text{ through out}$$

$$F = m\ddot{\bar{r}}_0 = m\ddot{\bar{r}} + m\ddot{\bar{r}}_0'$$

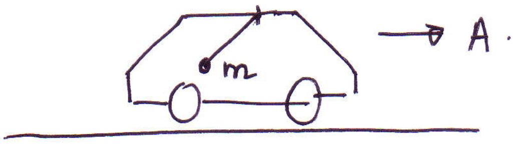
$$\Rightarrow \quad m\ddot{\bar{r}}_0' = F - m\ddot{\vec{r}} = \bar{F} - m\bar{A}$$

$$\boxed{\bar{F}' = \bar{F} - m\bar{A}}$$

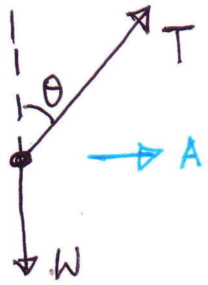
$$\boxed{\bar{a}' = \bar{a} - A}$$

See a Simple problem from ⟨ Inertial frame / Accelerating frame.

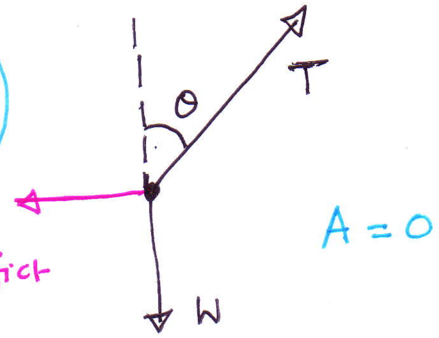
\# A small weight of mass m hangs from a string in an automobile which accelerates at rate A. What is the static angle of the string from the vertical, and what is its tension?



Inertial System.	System Accelerating with Auto.



Real Forces are same in both the frames.



Inertial System:

$$T\cos\theta - W = 0$$

$$T\sin\theta = MA$$

$$\underline{\tan\theta} = \frac{MA}{W} = \frac{A}{g}.$$

$$\underline{T} = M(g^2 + A^2)^{1/2}$$

System Accelerating with Auto:

$$T\cos\theta - W = 0$$

$$T\sin\theta - F_{fict} = 0$$

$$F_{fict} = -MA$$

$$\underline{\tan\theta} = \frac{A}{g}.$$

$$\underline{T} = M(g^2 + A^2)^{1/2}$$

F_{fict} is applied in a direction opposite to the accelerating system.

Both Answers are Same.

This is also written as.

$$\bar{F}' = \bar{F} - F_{fict} \quad , \quad F_{fict} = -MA.$$

F_{fict} is called a fictitious force – They originate in the acceleration of the coordinate system, not in the interaction between bodies.

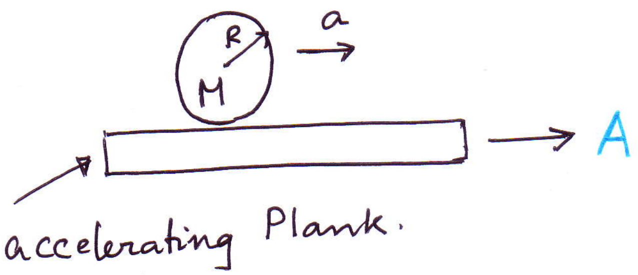
The Principle of Equivalence.

The laws of physics in a uniformly accelerating system are identical to those in an inertial system provided that we introduce a fictitious force on each particle,

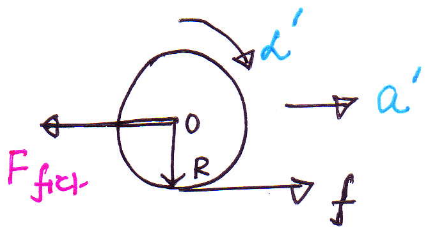
$$F_{fict} = -mA.$$

So in order to apply the Newtons Laws of motion that is $F = mA$ in non-inertial system, the systems which are linearly accelerating, we need to add an extra fictitious force F_{fict} which is equal to $-mA$ that is in a direction opposite to which the frame/system is accelerating. Let us see few examples.

A cylinder of Mass M and radius R rolls without Slipping on a pank which is accelerated at the rate A. Find the acceleration of the cylinder.



Let us solve the problem in accelerating frame that is accelerating Plank.
First fix a frame to the accelerating Plank. The situation in that accelerating frame is as follows.



- Here f is a real force which is there in an inertial / noninertial frame
- α', a' are angular acc & linear acceleration in dash frame — accelerating frame
- F_{fict} = Fictitious force – opp to that of A = MA.

1. Force Eqns : $f - F_{fict} = Ma'$
2. Torque Eqns : $Rf = -I_0\alpha'$ (about COM, O)
3. No Slip Cond : $\alpha' R = a'$

Solving $\Rightarrow$ $Ma' = -I_0 \dfrac{a'}{R^2} - F_{fict}$

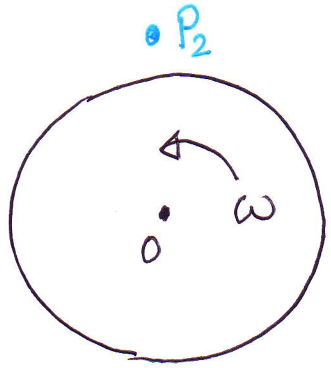
$$\Rightarrow \underline{a'} = -\frac{F_{fict}}{M + I_0/R^2} = \frac{MA}{M + \frac{MR^2}{2}\times\frac{1}{R^2}} = \underline{-\frac{2}{3}A}.$$

The acc of the System in Inertial system =

$$a = A + a' = A - \frac{2}{3}A = A/3.$$

Rotating Coordinate System.

Before we go into the formal mathematics we must develop a rough view about what we are going to do in this chapter. Let me clarify it by an example.



Suppose there is a platform which is rotating about an axis passing through O with angular velocity ω. Now there are two questions.

(a) How will an observer in the rotating frame see any man who is at rest standing on earth. (P_2)

(b) How will that observer in rotating frame justify mathematically.

Now if we want to find the motion of P_2 wrt the rotating frame — the way is to add a opposite vector $-\bar{\omega}$ to both $\bar{\omega}$ (Rotating frame) and P_2. Then the rotating frame will be at rest and the man standing outside will be rotating at angular velocity $-\bar{\omega}$.

But according to the inertial frame, net force acting on P_2 is zero. His weight balances his reaction. So we need to invoke two forces acting on P_2 which are not real but real to an observer in a rotating frame. These force are 1) Centrifugal force going outward from the Center of rotation & other 2) Coriolis force towards inside — the net total force will be towards inside justifying his (observer in rotating frame) observance of a rotating person (P_2) in a circle — Similar to Centripetal force.

So There are few important points about this Chapter:

(a) Concept of an observer & frame is V. Imp- How will an observer in a rotating frame observe something.

Observer in Rotating Frame
- Observe Something Physically
- Justify That mathematically.

(b) Something
- That something may occur outside the rotating frame like previous case.
- That something may occur within the frame like in the earth itself - earth being a rotating frame — Ex - Foucault pendulum, deflection of a falling body under gravity, deflection of air in North/south hemisphere because of Coriolis effect.

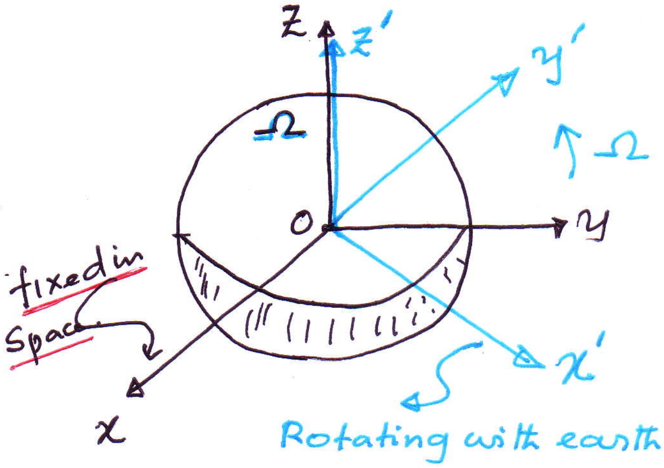
So Something is happening either in rotating frame or in inertial frame - infact that something/event is independent of any Coordinate system. It is about an Observer who is in rotating Coordinate system - What will he/she observe & justify mathematically.

Rotating Coordinate System. (Mathematics)

In the last section of accelerated Reference frame it is found that if we want to analyze motion in accelerated reference frame, then along with the real forces that we find in inertial reference frame we need to add certain pseudo forces to apply $F = ma$ – that is Newton's second Law of motion.

So Rotating Coordinate System is also an accelerated reference frame – we need to add certain pseudo forces. We will find out the relations of Velocity and acceleration in Rotating Coordinate system with inertial Coordinate systems.

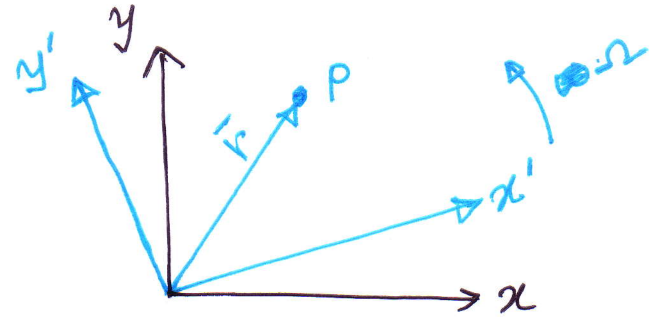
For all practical purposes we take earth as inertial frame but strictly Speaking it is not. Earth is a rotating frame of reference.



Suppose a rotating Coordinate System (x', y', z') fixed in the earth and an inertial system (x, y, z) at rest wrt to the fixed stars.

Z, Z' are common to both frame along $\hat{k}$.

The Common Origin of these two Coordinate system is the Center of earth. The X, Y and X', Y' axes all lie in the equatorial plane & Z, Z' along North pole.



Suppose a person standing still at point P on the surface of the earth — is at rest wrt to rotating coordinate system. The position vector of this person is shown by position vector $\bar{r}$. We can write the components of this vector in both rotating and fixed frame of reference.

(a) Since P is at rest in the rotating frame (Earth or primed frame).

$$\left[\frac{d\bar{r}}{dt}\right]_{rot} = 0$$

Point P stationary in dash frame.

rot

According to an observer in an inertial system the point P is moving with angular velocity Ω. The linear velocity of the point in the inertial system is

$$\left[\frac{d\bar{r}}{dt}\right]_{inertial} = \bar{\Omega} \times \bar{r}$$

(b) Next assume the person has a velocity $\bar{v}_r$ wrt to Earth.

$$\left[\frac{dr}{dt}\right]_{rot} = \bar{v}_r$$

Pt P is moving with $\bar{v}_r$ in dash frame.

rot

In Inertial Frame

$$\left[\frac{dr}{dt}\right]_{inertial} = \bar{v}_r + \bar{\Omega} \times \bar{r}$$

Inertial.

We may not feel very comfortable with the last step of inertial frame. Many other books have given rigorous mathematical ~~group~~ proof. But we need to understand physics. I want to make it in such a way so that it contains less of mathematics so that we get a feel of it. Imagine a train and an object O at rest. In the platform a person will observe O moving with velocity of the train. Now if the object O moves with some velocity in the train then we ~~as~~ need to add vectorially that velocity with the velocity of the train — in order for the velocity to be observed from Platform. We are doing the same thing here.

The last equation can be written as

$$\boxed{\left(\frac{d\bar{r}}{dt}\right)_{inertial} = \Big[\underbrace{\frac{d\bar{r}}{dt}}_{V_r}\Big]_{rot} + \bar{\Omega}\times\bar{r}} \qquad (1)$$

One can generalize this relationship to the time derivative of <u>any vector u</u>.

$$\boxed{\left[\frac{d\bar{u}}{dt}\right]_{inertial} = \left[\frac{d\bar{u}}{dt}\right]_{rot} + \bar{\Omega}\times\bar{u}} \qquad (2)$$

It can also be expressed as <u>operator</u> eq

$$\left[\frac{d}{dt}\right]_{inertial} = \left[\frac{d}{dt}\right]_{rot} + \bar{\Omega}\times$$

When we go from equation (1) to (2) in the last page it looks quite simple. But it is not. The rigorous derivation of eq (2) is omitted here. It can be included in some appendix.

$$\left[\frac{d\bar{u}}{dt}\right]_{inertial} = \left[\frac{d\bar{u}}{dt}\right]_{rot} + \bar{\Omega} \times \bar{u}$$

The Important point to note: We need to note that $\bar{u}$ vector is same through out. Its' meaning will be clear only when we derive the expression & relation for acceleration. Here we try to understand this fact by an example.

\# An old fashioned record player is rotating with an angular velocity $\Omega_0 = \Omega_0 \hat{k}$. A bug is crawling with constant speed v_b relative to the record along a groove in the record, a distance R from the axis of rotation. Assume the bug is moving in a circular path, so there is no radial component to its velocity.

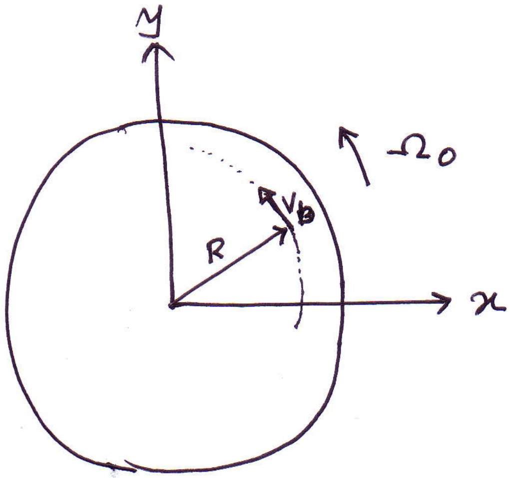
(a) Find the velocity of the bug relative to the room. (That is inertial space)

(b) Determine the acceleration of the bug by elementary means using the definition of centripetal acceleration.

(c) Determine the acceleration of bug by eqn (1).

$\Rightarrow$

(a)



$$\left[\frac{d\bar{r}}{dt}\right]_{rot} = v_b\,\hat{\phi}$$

$$\left[\frac{d\bar{r}}{dt}\right]_{inertial} = \left[\frac{d\bar{r}}{dt}\right]_{rot} + \bar{\Omega}_0 \times \bar{r}$$

$$V_i = V_{rot} + \bar{\Omega}_0 \times \bar{r}$$
$$= v_b\,\hat{\phi} + \bar{\Omega}_0 R\,\hat{\phi}$$
$$= (v_b + \Omega_0 R)\,\hat{\phi}$$
$$= (\Omega_b R + \Omega_0 R)\,\hat{\phi} = (\Omega_b + \Omega_0) R\,\hat{\phi}.$$

(b) By definition of Centripetal Acceleration.

$$a_i = \frac{v^2}{r}(-\hat{r}) = -\frac{\hat{r}}{R}\left(\Omega_b + \Omega_0\right)R\hat{\phi}\cdot\left(\Omega_b + \Omega_0\right)R\hat{\phi}$$
$$= -\hat{r}\,R\,(\Omega_b + \Omega_0)^2$$

(c) $$\left[\frac{d\bar{V}_i}{dt}\right]_{inertial} = \left[\frac{d\bar{V}_i}{dt}\right]_{rot} + \bar{\Omega} \times \bar{v}_i$$

Now $V_i = V_b\,\hat{\phi}$

$$\left(\frac{dv_i}{dt}\right)_{rot} = v_b\,\frac{d\hat{\phi}}{dt} = V_b(-\dot{\phi}\hat{r}) = V_b(-\Omega_b\hat{r})$$

X

This will not give the right answer.

If you are Calculating $\left[\frac{dv_i}{dt}\right]_{\text{inertial}}$ you get the wrong answer if you put $\left[\frac{dV_{rot}}{dt}\right]_{rot}$ on the other side. It is a common error.

NOTE $\quad a_i = \left[\frac{dv_i}{dt}\right]_{\text{inertial}}$.

$$a_i = \left[\frac{d\bar{v}_i}{dt}\right]_{\text{inertial}} = \left[\frac{d\bar{V}_i}{dt}\right]_{rot} + \Omega_0 \times \bar{v}_i$$

$$= \left[\frac{d}{dt}\left((\Omega_b + \Omega_0) R\hat{\phi}\right)\right]_{rot} + \Omega_0 \hat{k} \times (\Omega_b + \Omega_0) R\hat{\phi}.$$

$$= \left[\frac{d}{dt}(\Omega_b + \Omega_0) R\hat{\phi}\right]_{rot} + \Omega_0 (\Omega_b + \Omega_0) R (\hat{k} \times \hat{\phi})$$

$$= (\Omega_b + \Omega_0) R \underline{\left[\frac{d\hat{\phi}}{dt}\right]_{rot}} + \Omega_0 (\Omega_b + \Omega_0) R(-\hat{r})$$

The rate of change of $\hat{\phi}$ in the rotating system is due only to the motion of the bug. So

$$\frac{d\hat{\phi}}{dt} = -\Omega_b \hat{r}$$

$$a_i = -(\Omega_b + \Omega_0) R(\Omega_b)\hat{r} + \Omega_0(\Omega_b + \Omega_0) R(-\hat{r})$$

$$= -\hat{r} R[(\Omega_b + \Omega_0)(\Omega_b) + \Omega_0(\Omega_b + \Omega_0)]$$

$$= -\hat{r} R[\Omega_b^2 + \Omega_b\Omega_0 + \Omega_0^2 + \Omega_b\Omega_0]$$

$$= -\hat{r} R (\Omega_b + \Omega_0)^2 \quad \text{as before.}$$

Fictitious Forces.

$$\left[\frac{d\bar{r}}{dt}\right]_{\text{Inertial}} = \left[\frac{d\bar{r}}{dt}\right]_{\text{rot}} + \bar{\Omega}\times\bar{r}$$

$\left[\frac{dr}{dt}\right]_{\text{inertial}}$ = The velocity of the particle wrt to inertial space, call it v_i

$\left[\frac{dr}{dt}\right]_{\text{rot}}$ = The Velocity of the particle wrt to Rotating frame — Call it v_r

$$v_i = v_r + \Omega \times \bar{r}$$

Dont Confuse V_r as some kind of radial velocity in rotating frame. It is the net velocity observed in Rotating frame – r stands for rotating frame.

This equation indicates that if a person on Earth perceives a particle to be at rest, an observer in inertial space will perceive the particle to have velocity. $\bar{v}_i = \bar{\Omega}\times\bar{r}$.

Now. acceleration in inertial frame is

$$a_i = \left[\frac{dv_i}{dt}\right]_i \quad \text{and.}$$

$$\bar{a}_i = \left[\frac{d\bar{v}_i}{dt}\right]_{\text{inertial}} = \left[\frac{d\bar{v}_i}{dt}\right]_{\text{rot}} + \bar{\Omega}\times\bar{v}_i$$

$$\bar{a}_i = \left[\frac{d}{dt}(\bar{v}_r + \bar{\Omega}\times\bar{r})\right]_i = \left[\frac{d}{dt}(\bar{v}_r + \bar{\Omega}\times\bar{r})\right]_{\text{rot}} + \bar{\Omega}\times(\bar{v}_r + \bar{\Omega}\times\bar{r})$$

$$= \left[\frac{d v_r}{dt}\right]_r + \left[\frac{d}{dt}(\Omega\times r)\right]_r + \bar{\Omega}\times\bar{v}_r + \bar{\Omega}\times(\Omega\times\bar{r})$$

$$= a_r + \frac{d\bar{\Omega}}{dt}\times\bar{r} + \left[\bar{\Omega}\times\frac{d\bar{r}}{dt}\right]_r + \bar{\Omega}\times\bar{v}_r + \bar{\Omega}\times(\bar{\Omega}\times\bar{r})$$

$$\Rightarrow \bar{a}_i = \bar{a}_r + \underbrace{\frac{d\bar{\Omega}}{dt}}_{0} \times \bar{r} + \underbrace{\left[\bar{\Omega} \times \frac{d\bar{r}}{dt}\right]_{rot}}_{\bar{v}_r} + \bar{\Omega} \times \bar{v}_r + \bar{\Omega} \times (\bar{\Omega} \times \bar{r})$$

Earths Rotation is constant

$$\bar{a}_i = \bar{a}_r + 2\bar{\Omega} \times \bar{v}_r + \bar{\Omega} \times (\bar{\Omega} \times \bar{r})$$

Newton's Second Law is valid in the inertial frame, so $a_i = F/m$ where $\bar{F}$ is the net external (physical) force acting on the particle. Rearranging

$$a_r = F/m - 2\bar{\Omega} \times \bar{v}_r - \bar{\Omega} \times (\bar{\Omega} \times \bar{r})$$

This equation shows that the acceleration as measured in the rotating reference frame will depend not only on the force acting on the particle, but also on other terms. These additional terms are a consequence of considering the motion in the non-inertial reference frame.

We often write the above eqns. as:

$$\boxed{\bar{F} - 2m\bar{\Omega} \times \bar{v}_r - m(\bar{\Omega} \times (\bar{\Omega} \times \bar{r})) = m\bar{a}_r}$$

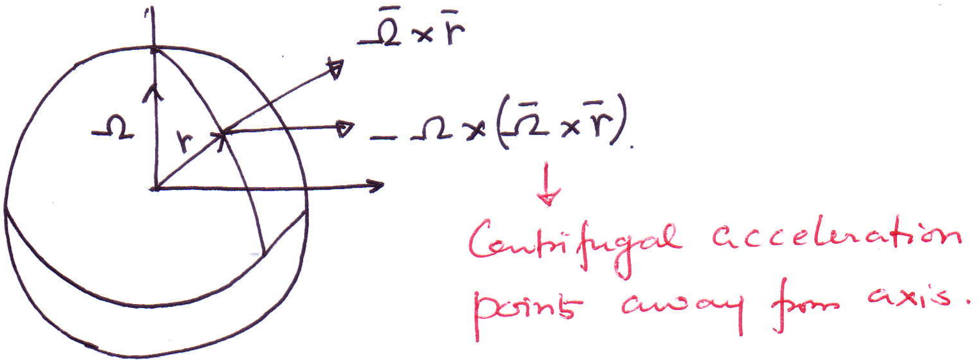
This equation looks like $F = ma$. The additional terms on the left hand side of the equation have the units of force and they are referred to as fictitious forces. You should be aware, however, that they really are just mass times acceleration terms that have been brought over from the other side of the equation.

A "real" force is an interaction between bodies.

Imp | A "fictitious" force is a consequence of the acceleration of the coordinate system. The term.

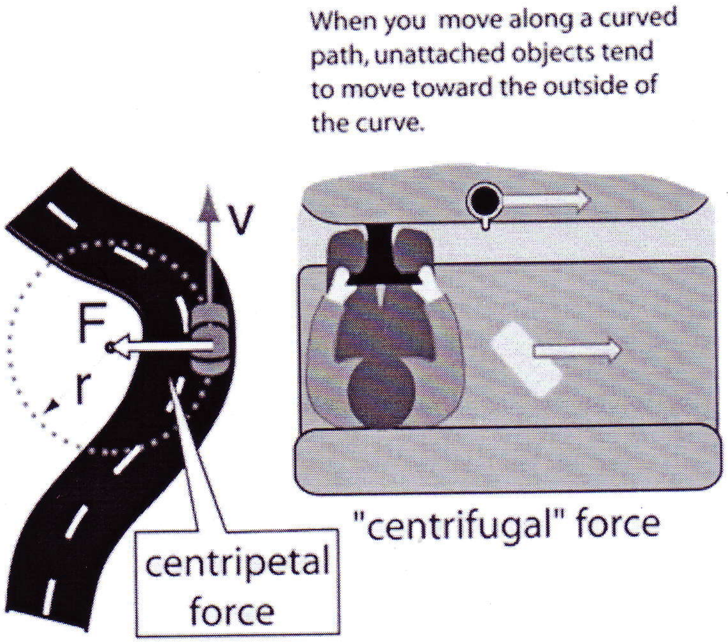
$-2m\bar{\Omega}\times\bar{v}_r \Rightarrow$ Coriolis force. [$\bar{v}_r$ = velocity wrt to Rotating frame]

$-m\bar{\Omega}\times(\bar{\Omega}\times\bar{r}) \Rightarrow$ Centrifugal force.

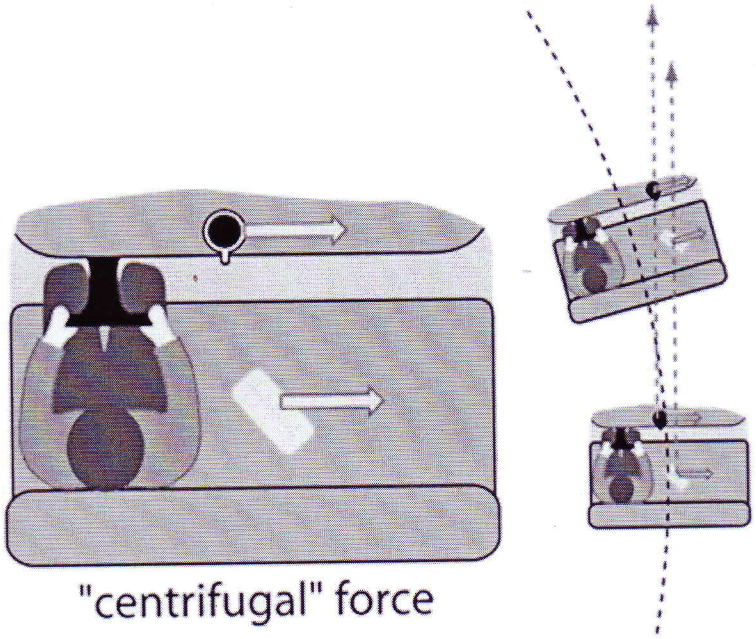


- Centrifugal force arises purely because of the rotation of the frame. So, even if the object is at rest wrt to rotating frame, Centrifugal force is there.
- Coriolis force arises because of its motion wrt to rotating frame.
- As mentioned earlier the event to be viewed may be inside the rotating frame or ~~at~~ outside in the inertial frame.
- If the body to be viewed in within the rotating frame then the effect of Coriolis & Centrifugal forces appears to be quite real from the rotating system as Foucault pendulum
- Included some examples.

(3) Centrifugal Force.



The driver of a car on a curve is in a rotating reference frame and he could invoke a "centrifugal" force to explain why his coffee cup and the carton of eggs he has on the seat beside him tend to slide sideways. The friction of the seat or dashboard may not be sufficient to accelerate these objects in the curved path.



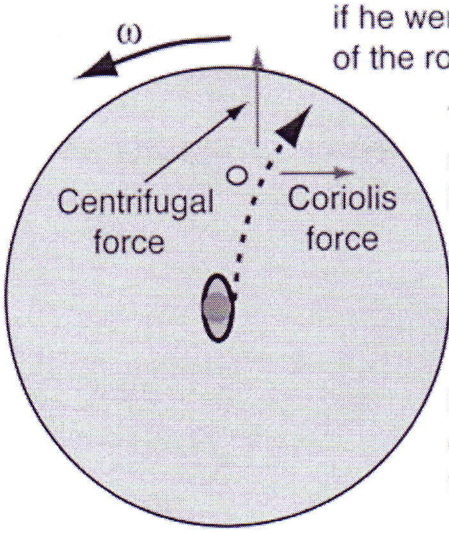
A person in a hovering helicopter above the car could describe the movement of the cup and the egg carton as just going straight while the car travels in a curved path. It is similar to the "broken string" example.

The centrifugal force is a useful concept when the most convenient reference frame is one which is moving in a curved path, and therefore experiencing a centripetal acceleration. Since the car above will be experiencing a centripetal acceleration v^2/r, then an object of mass m on the seat will require a force mv^2/r toward the center of the circle to stay at the same spot on the seat. From the reference frame of a person in the car, there seems to be an outward centrifugal force mv^2/r acting to move the mass radially outward. In practical descriptive terms, you would say that your carton of eggs is more likely to slide outward if you have a higher speed around the curve (the velocity squared factor) and more likely to slide outward if you go around a sharper curve (the inverse dependence upon r).

— HyperPhysics

(1) Coriolis Force

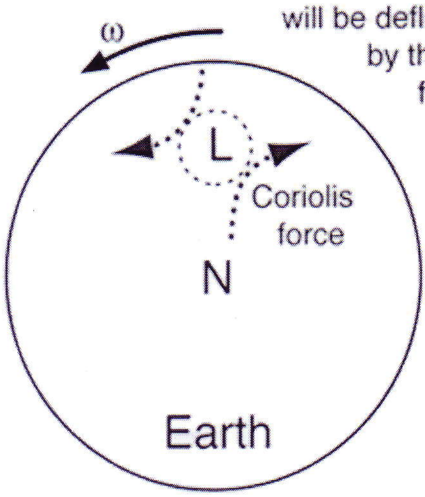
A golfer who is putting the ball from the center of a rotating platform would tend to miss to the right and overshoot the hole if he were unaware of the rotation.



The rightward miss he could blame on the coriolis force and the overshoot he could blame on centrifugal force.

"HyperPhysics" website.

Air which moves from the North Polar region toward a low pressure area in the northern hemisphere will be deflected west by the coriolis force.



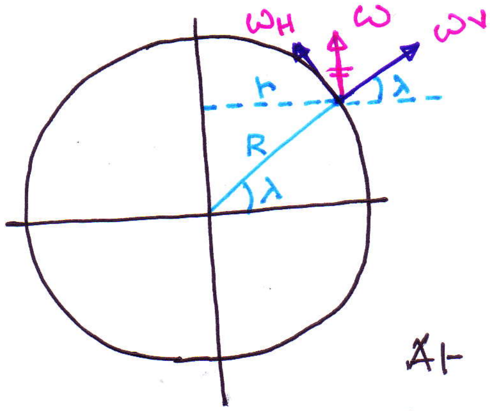
Air approaching the low pressure area from near the equator will be deflected eastward by the coriolis force. This gives the cyclonic counterclockwise rotation of air around lows in the northern hemisphere.

$\Downarrow$

At the equator Coriolis Force $= 0$

Why?

$\bar{\omega}$ and $\bar{v}$ are in the same direction at the equator. How?



$\bar{\omega}$ = angular velocity of earth usually written as Ω at other places of the notes.

ω_V = Vertical Component $= \omega \sin\lambda$

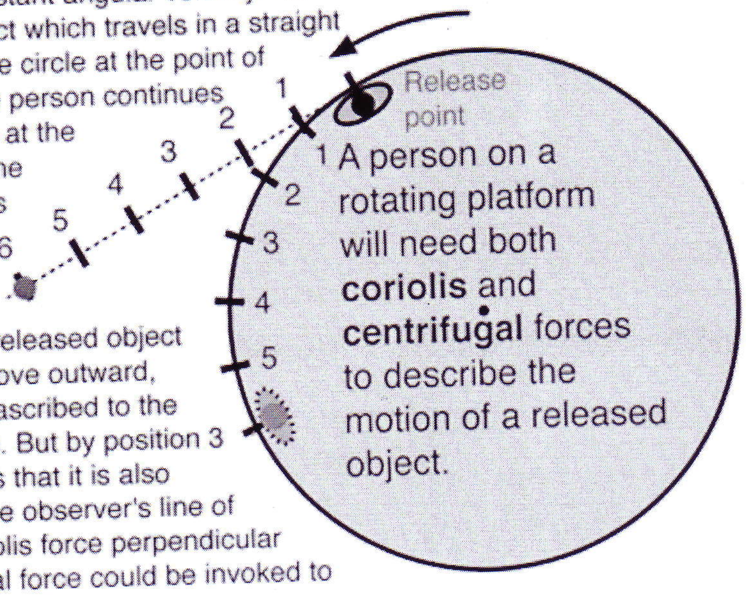
ω_H = Horizontal " $= \omega \cos\lambda$

At Equator, $\Rightarrow \lambda = 0$ $\quad \omega_V = 0$

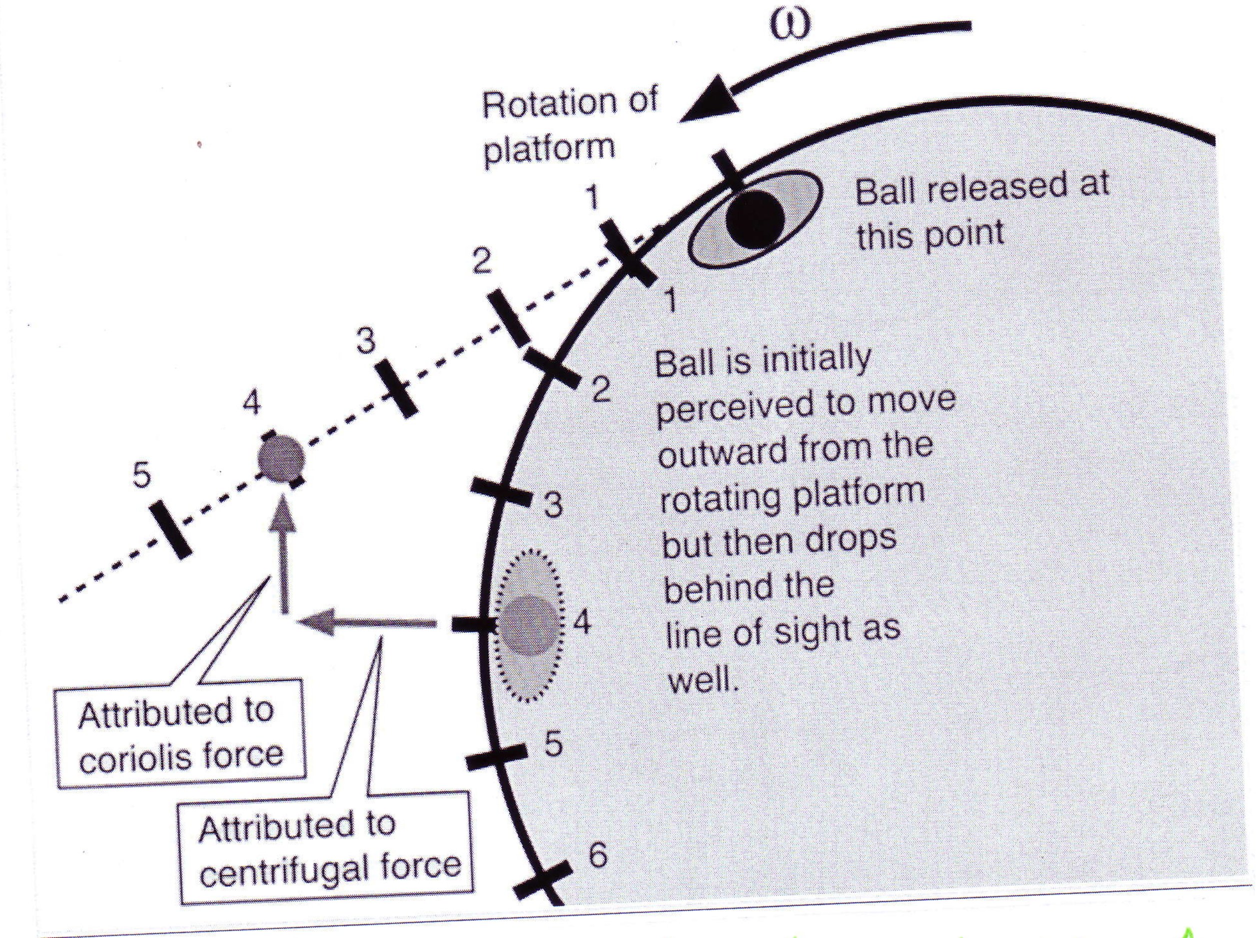
ω_H and v at the equator are $\parallel$ for the winds blowing North, So Coriolis Force $= -2m\,\bar{\omega} \times \bar{v} = 0$.

(2) Centrifugal & Coriolis Forces.

A person stands at the edge of a platform which is rotating at constant angular velocity. He releases an object which travels in a straight line tangent to the circle at the point of release while the person continues in a circular path at the same speed. If the person continues to look straight outward in a radial direction, then at first the released object will appear to move outward, which could be ascribed to the centrifugal force. But by position 3 it will be obvious that it is also falling behind the observer's line of sight, so a Coriolis force perpendicular to the centrifugal force could be invoked to explain that behavior.



An observer with a "bird's eye view" as shown here would clearly see that the released object is simply traveling in a straight line in the absence of any centripetal force to constrain it into the circular motion. The rotating observer sees it as a more complicated motion and invokes both centrifugal and coriolis forces to explain it.

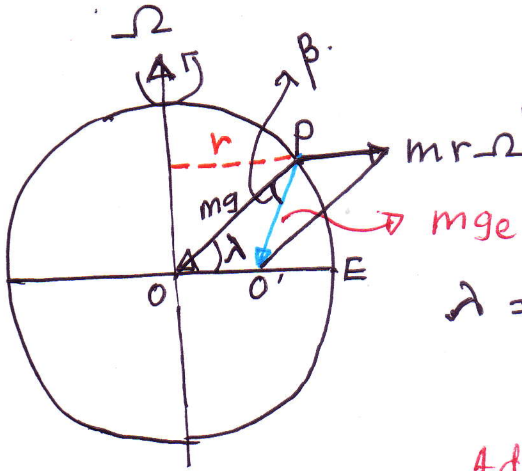


These Pages are taken from hyperphysics — A Very good internet Resource of Physics — must See for every student.

Effect of Fictitious Forces due to Rotation of Earth

Centrifugal force reduces the value of acceleration due to gravity

Let us Calculate the effective value of $g \Rightarrow g_e$



λ = latitude.

Addition of two vectors – Cosine Rule

$$mg_e = \left[(mg)^2 + (mr\Omega^2)^2 + 2\, mg\, mr\Omega^2 \cos(180-\lambda)\right]^{1/2}$$

$$g_e = \left(g^2 + r^2\Omega^4 - 2gr\Omega^2\cos\lambda\right)^{1/2}$$

Ω is very small $= 7.3 \times 10^{-5}$ rad/s.

One can neglect ω^4 term.

$$g_e = \left(g^2 - 2gr\Omega^2\cos\lambda\right)^{1/2}$$

$$= g\left(1 - \frac{2r\Omega^2\cos\lambda}{g}\right)^{1/2} \simeq g\left(1 - \frac{r\Omega^2}{g}\cos\lambda\right)$$

$$g_e = g - r\Omega^2\cos\lambda$$

from fig $r = R\cos\lambda$

$$\boxed{g_e = g - R\Omega^2\cos^2\lambda}$$

Effective value of g at any latitude λ, at equator $\lambda = 0$, at pole $\lambda = 90^\circ$.

$$\tan\beta = \frac{m r \Omega^2 \sin(\pi - \lambda)}{mg + m r \Omega^2 \cos(\pi - \lambda)}$$

for $r = R\cos\lambda$

$$\tan\beta = \frac{m R \Omega^2 \cos\lambda \sin(\pi - \lambda)}{mg + m R \Omega^2 \cos\lambda \cos(\pi - \lambda)}$$

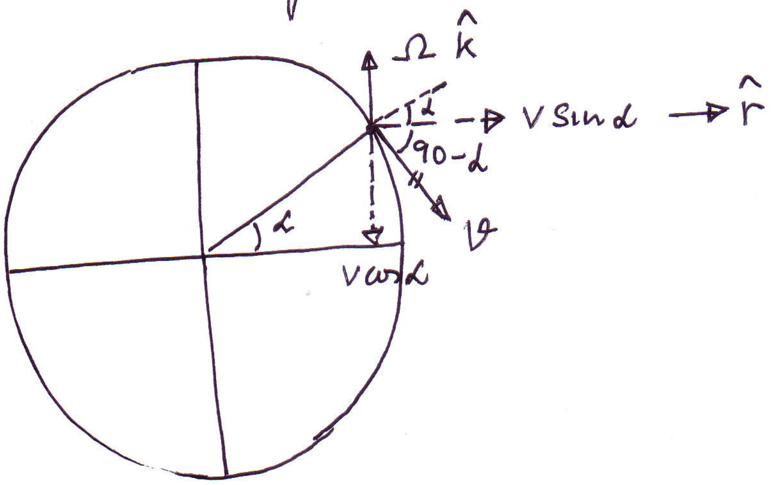
$$\tan\beta = \frac{m R \Omega^2 \cos\lambda \sin\lambda}{mg - \cancel{m} R \Omega^2 \cos^2\lambda} = \frac{1}{2}\frac{R\Omega^2 \sin 2\lambda}{(g - R\Omega^2\cos^2\lambda)}$$

(Not that Important)

This gives the deflected angle for gravity.

A 400 ton train runs South at a speed of 60 mi/h at a latitude of 60° north. Find the horizontal Force on the raib.

$\Rightarrow$ In all these problems we have to define a Coordinate System. $\Rightarrow$ Follow Cylindrical Coord.



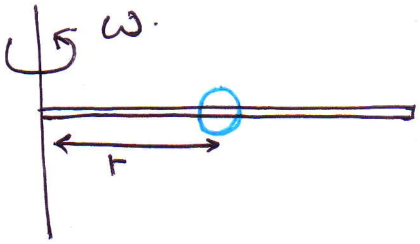
$$F_{coriolis} = -2m\,\bar{\Omega}\times\bar{v}$$
$$= -2m(\Omega\,\hat{k})\times\left(v\cos\alpha\,(-\hat{k}) + v\sin\alpha\,(\hat{r})\right)$$
$$= -2m\Omega v\sin\alpha\,\hat{\theta} = 2m\Omega v\sin\alpha\,(-\hat{\theta})$$
$$\Rightarrow \text{Westward}$$

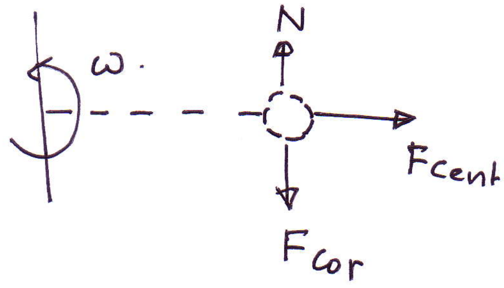
- Other Forces = normal direction cancel each other
- Friction neglected.
- In problems connected with the earth one has to define a Coordinate System – (1) Cartesian Coord.
  (2) Cylindrical Polar

V. Imp

- The Coordinate System – Origin – Take the Origin/Position of the particle.
- Here it is Cylindrical polar – $\hat{r}, \hat{k}, \hat{\theta}$ .. – you may choose other Coordinate.
- Here Ω (or ω) is not resolved in to $\parallel$ & $\perp$ components but v is resolved $\parallel$ & $\perp$ to $\hat{K}$ – depends what you choose.

A bead slides without friction on a rigid wire rotating at constant angular speed ω. Find the force exerted by the wire on the bead.





Fix a coordinate system rotating with the wire. Free body diagram in rotating coordinate frame is shown above.

$$F_{cent} = F_{centrifugal} \Rightarrow m\omega^2 r = m\ddot{r}$$
$$\Rightarrow m\ddot{r} - m\omega^2 r = 0 \qquad (1)$$

Solution to this is $r = Ae^{\omega t} + Be^{-\omega t}$

where A & B are const depending on initial cond.

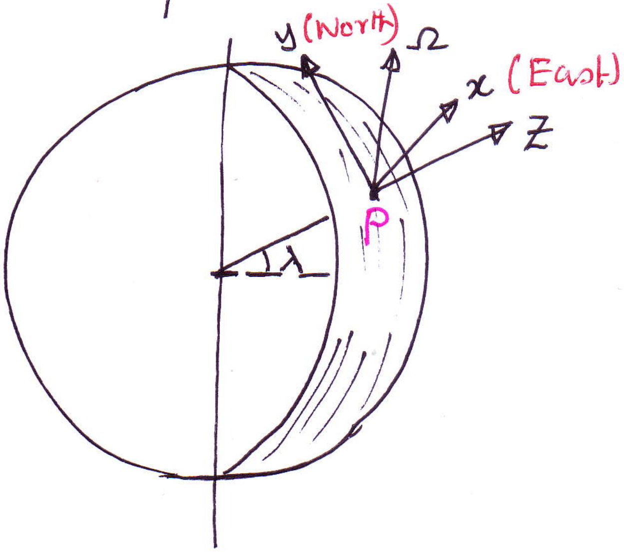
$$N = F_{cor} = F_{coriolis} = 2m\dot{r}\omega \quad - \quad (2)$$
$$N = 2m\omega^2 (Ae^{\omega t} - Be^{-\omega t})$$

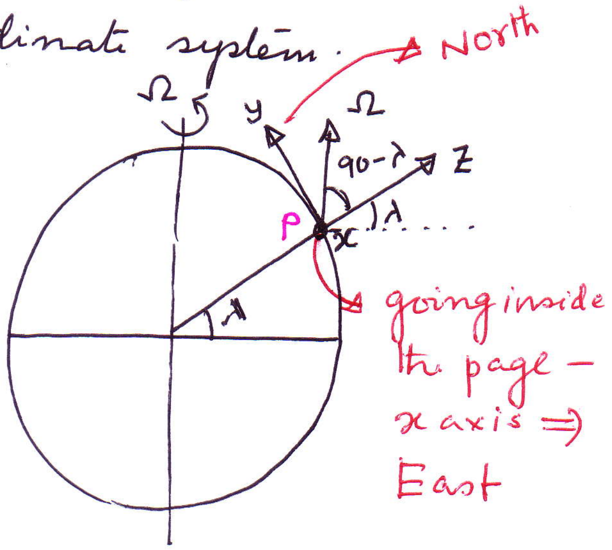
To complete the solution we must be given the initial conditions which specify A & B.

- This is another type of problem usually asked in rotating coordinate.
- This kind of problem – much easier to solve using accelerating rotating frame than from inertial frame.

Deflection of a freely Falling Body

The first step when we need to develop the theory behind the motion on the surface of the Earth is to develop - Sketch a coordinate system.





There are different ways to deal with this problem.

Method - 1. (Gives a rough estimate, approximate sol in the first order)

- First try to understand the coordinate system - why x axis is east, y axis is north. ? And the direction of Ω.
- When a body is falling under gravity, Coriolis force acts because of its velocity. Centrifugal force is much lower
- Why the y axis shown above is North & x axis going into the page is east?

⇒ If you are standing at P and would like to go towards North pole you have to walk along y axis. Once you define y axis as North, x axis becomes East automatically

Method 1 gives an approximate solution. (why).

If h is the height of the point from where the body starts falling down & v is its velocity on reaching the ground at time t.

$$v = gt \qquad h = \frac{1}{2}gt^2$$

If z axis is vertically upward and x axis is along the east (into the paper) then Coriolis acceleration is

$$\bar{a} = -2\,\bar{\Omega}\times\bar{v} = 2\,\Omega v \sin(90+\lambda)\,\hat{i}$$

$$\bar{a} = 2\,\Omega v \cos\lambda\,\hat{i}.$$

[$-\bar{\Omega}\times\bar{v}$ acts eastwards in both the cases : North and South Hemisphere]

$$|\bar{a}| = 2\Omega v \cos\lambda$$

This is first order deflection. Higher order can be ~~def~~ Calculated. But Second order is Ω^2 term which at present is neglected.

$$\frac{d^2x}{dt^2} = 2\Omega g t\cos\lambda$$

$$\frac{dx}{dt} = \cancel{2}\Omega g\frac{t^2}{\cancel{2}}\cos\lambda + c$$

at $t=0$ $v=0$ $c=0$.

When eastward velocity further interact with Ω we call that second order term.

$$\boxed{x = \Omega g\frac{t^3}{3}\cos\lambda}$$

Now $h = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2h}{g}}$$

$$= \frac{\Omega g}{3}\left(\frac{2h}{g}\right)^{3/2}\cos\lambda$$

First Order.

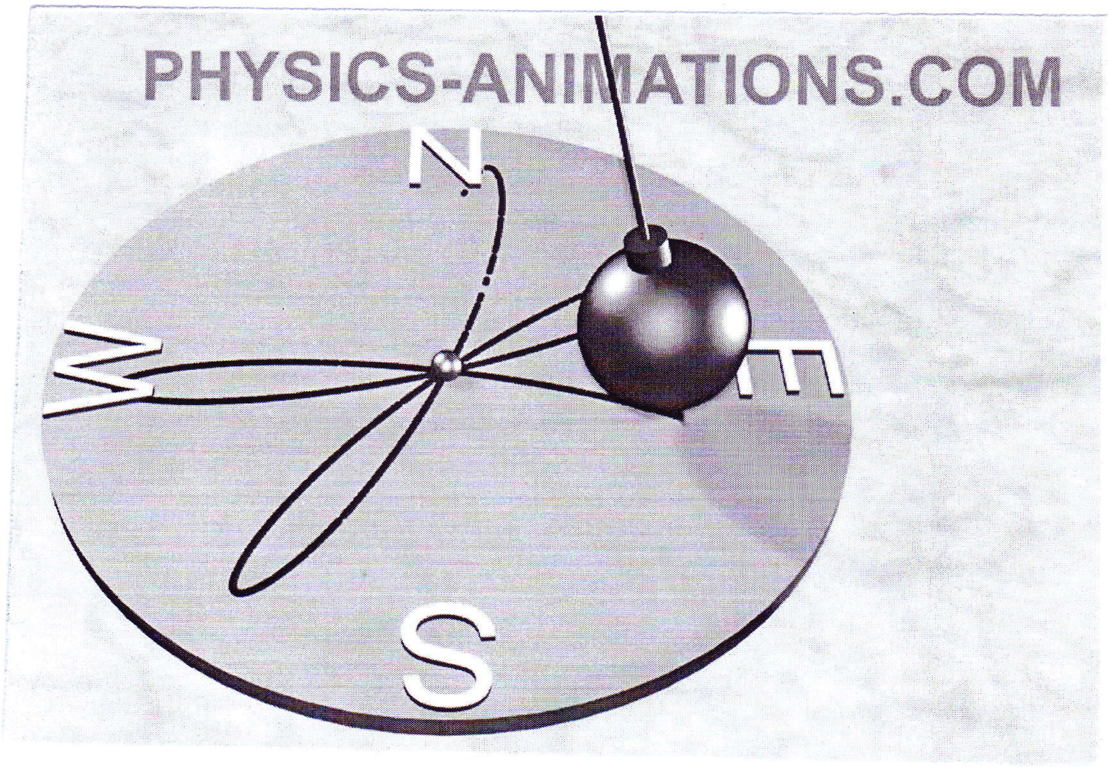
$$= \frac{\Omega g}{3}\left(\frac{8h^3}{g^3}\right)^{1/2}\cos\lambda = \boxed{\frac{\Omega}{3}\left(\frac{8h^3}{g}\right)^{1/2}\cos\lambda}$$

$g \to g_{eff}$

h can be replaced by v, v is the ground velocity in vertical direction. [$v^2 = 2gh$ ... $h = v^2/2g$]

Centrifugal force can be coupled with g — so instead of g one can write g_{eff}.

Foucault's Pendulum.

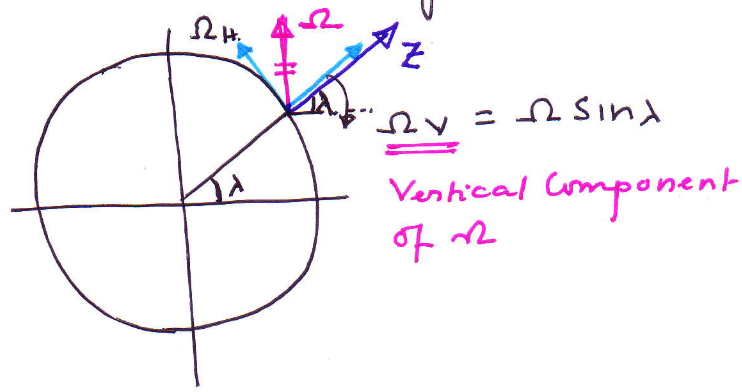


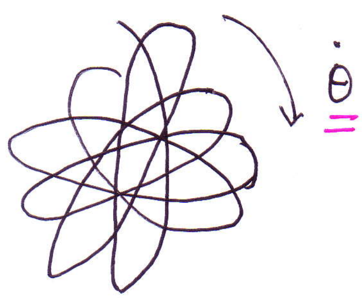
What is Foucault's Pendulum?

This pendulum consists of a massive bob suspended by a very long string or thin wire. Foucault pendulums range from ten to sixty meters in length. When set in motion the bob swings back & forth and the plane of motion of the pendulum slowly precesses. The precession rate is $\Omega \sin\lambda$ where Ω is the angular velocity of the earth (2π rad/day) and λ is the colatitude. A long pendulum goes through many oscillations before the motion is damped out, and the precession is quite noticeable. The behavior of the Foucault pendulum is easy to understand if you imagine a pendulum suspended above the north pole. The earth rotates under the pendulum and an observer standing on the surface sees the plane of motion precessing at a rate of one revolution per day. On the other hand at the equator, a pendulum does not precess at all. The motion at an arbitrary latitude requires an analysis to understand.

Different books have taken different approaches to study its behavior & calculating the time period of one complete revolution. The treatment given by Kleppner is quite simple & short.

The two real forces which are acting on the bob is Tension and its weight mg. We very well know the physics because of these two forces. Now the Coriolis force - which is a fictitious force which arises only because of rotating frame acts on the bob which results the pendulum to precess. So let us work only with the Coriolis force.

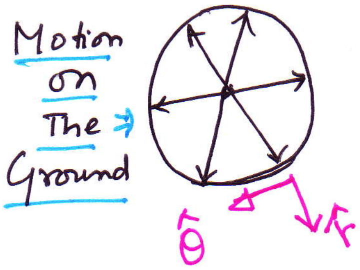




If there is any motion along the surface of the earth then Vertical Component of Ω gives a force parallel to the surface. — This is Coriolis force — force arising because of rotating frame.

$$F_{Coriolis} = -2m\,\bar{\Omega} \times \bar{v}$$

Since motion is along the surface of the earth we take only the Vertical Component of Ω which will give a force along the surface.



If we look at the motion of the bob the best way to describe its motion is by r, θ Coordinate.

$$F_{Coriolis} = -2m\,\bar{\Omega} \times \bar{v} = -2m\,\Omega_v \hat{k} \times \dot{r}\,\hat{r} = -2m\,\Omega_v \dot{r}\,\hat{\theta}$$

$$(\hat{k} \times \hat{r} = \hat{\theta}).$$

In ~~radial~~ plane polar coordinate the $\hat{\theta}$ component of acceleration $= (r\ddot{\theta} + 2\dot{r}\dot{\theta})$ [see the notes of 2nd chapter if you have forgotten]

So $\quad -2\cancel{m}\,\Omega_v \dot{r} = \cancel{m}\,(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

The simplest solution to this equation is found by taking $\dot{\theta} = \text{const}$.

So $\quad r\ddot{\theta} = 0. \qquad \& \quad \Omega_v = \Omega \sin\lambda.$

$$-\cancel{2}\,\Omega \sin\lambda\, \cancel{\dot{r}} = \cancel{2}\,\cancel{\dot{r}}\,\dot{\theta}$$

$$\boxed{\dot{\theta} = -\Omega \sin\lambda}$$

The pendulum precesses uniformly in a clockwise direction. The time for plane of oscillation to rotate once is

$$T = \frac{2\pi}{\dot{\theta}} = \frac{2\pi}{\Omega \sin\lambda} = \frac{24h}{\sin\lambda}.$$

Thus, at a latitude of 45°, the Foucault pendulum rotats once in 34h.

Project:

Find out latitude of Jaipur & from there find T.

Go to Science & Technology park & find out the time period of pendulum there.