The LNM Institute of Information Technology
Jaipur, Rajsthan

Math-II (2016-17), Quiz-2:

Name: Roll No:

Section: Tutorial Batch:

Time: 15 Minutes Maximum Marks: 10

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1. If y_1 and y_2 are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then prove that their Wronskian is either identically zero or never zero on $[a, b]$. [5]

   Sol. Let y_1 and y_2 are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$.
   Then
   $$y_1'' + P(x)y_1' + Q(x)y_1 = 0$$
   $$y_2'' + P(x)y_2' + Q(x)y_2 = 0$$
   Multiply y_2 in first equation and y_1 in second equation and then subtracting we get
   $$\frac{dW}{dx} + P(x)W = 0$$
   By solving we get $W = Ce^{-\int P(x)dx}$.
   Since exponentiation never zero,
   $$W \equiv 0 \text{ if } C = 0$$
   $$W \neq 0 \text{ if } C \neq 0$$
   Hence the proof follows.

2. Work out first three coefficients of Lagrange polynomial expansion of x^4. What would be the coefficient of x^6 in that expansion, give justification. [5]
   (Note: $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2-1)^n]$, is called the Rodrigues' formula for Legendre polynomial P_n of degree n.)

   Sol. We can write down
   $$x^4 = \sum_{n=0}^{\infty} a_n P_n(x)$$
   where,
   $$a_n = \frac{2n+1}{2}\int_{-1}^{1} x^4 P_n(x)\, dx$$
   Therefore,
   $$a_0 = 1/5, a_1 = 0$$
   $$a_2 = 4/7$$
   Coefficient of x^6 is
   $$a_6 = \frac{2 \cdot 6 + 1}{2}\int_{-1}^{1} x^4 P_6(x)\, dx.$$
   By using Rodrigue's formula for $P_6(x)$ and integrating by parts repeatedly we can observe that $a_6 = 0$.