The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment #1

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1. Classify each of the following differential equations as ordinary, partial, linear, nonlinear and specify the order

$$(i)\ y'' + y\sin x = 0 \quad (ii)\ y'' + x\sin y = 0 \quad (iii)\ u_x u_{xy} + x^2 u = y^2$$
$$(iv)\ y'' + (y')^2 + y = x \quad (v)\ y'' + xy' = \cos y' \quad (vi)\ (xy')' = xy$$

2. Find the differential equation of each of the following families of plane curves:

   (i) $xy^2 - 1 = cy$ $\quad$ (ii) $cy = c^2 x + 5$ $\quad$ (iii) $y = ax^2 + be^{2x}$
   (iv) Circles of unit radius with centers on y-axis $\quad$ (v) $y = a\sin x + b\cos x + b$,
   where a, b and c are arbitrary constants.

3. (a) Verify that $x^3 + y^3 = 3cxy$ is solution of the first order differential equation: $x(2y^3 - x^3)y' = y(y^3 - 2x^3)$.
   Note: Such a solution (implicitly defined) is called an *implicit* solution.

   (b) Verify that $y = ce^{-x} + x^2 - 2x + 4$ is general solution of $y' + y = x^2 + 2$.
   Note: If the one-parameter family of curves $G(x, y, c) = 0$ satisfies a first order ordinary differential equation, then $G(x, y, c)$ is a *general* solution of the given differential equation.

   (c) Verify that $y = cx - c^2$ is a general solution of $y'^2 - xy' + y = 0$. Also show that $y_1 = \frac{x^2}{4}$ is also a solution.
   Note: We can not obtain solution y_1 from the general solution by choosing a suitable c. Such a solution y_1 is called *singular* solution.

4. Verify that $y = -1/(x + c)$ is general solution of $y' = y^2$. Find particular solutions such that (i) $y(0) = 1$, and (ii) $y(0) = -1$. In both the cases, find the largest interval I on which y is defined.

5. Verify that $y = x^2 + a$ and $y = -x^2 + b$ are solutions of $y'^2 = 4x^2$.
   Note: Interestingly, this differential equation has 2 sets of general solutions.

6. Consider the differential equations $y' = \alpha y$, $x > 0$, where α is a constant. Show that (i) if $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant; (ii) if $\alpha < 0$, then every solution tends to zero as $x \to \infty$.

7. For each of the following differential equations, draw several *isoclines* with appropriate lineal elements and hence sketch some solution curves:

$$(i)\ y' = x \qquad (ii)\ y' = x^2 + y^2$$

8. Reduce the differential equation $y' = f\left(\frac{ax+by+m}{cx+dy+n}\right)$, $ad - bc \neq 0$ to a separable form. Also discuss the case of $ad = bc$.

Supplementary problems from "Advanced Engg. Maths. by E. Kreyszig (8^{th} Edn.)

$$(i)\ \text{Page}\ 8-9:\ Q.\ 9, 11, 12 \qquad (ii)\ \text{Page}\ 13:\ Q.\ 7, 16, 18$$
$$(iii)\ \text{Page}\ 18:\ Q.\ 7-11, 17, 22, 25 \qquad (iv)\ \text{Page}\ 23-24:\ Q.\ 1, 2, 6, 9, 11, 12, 16$$