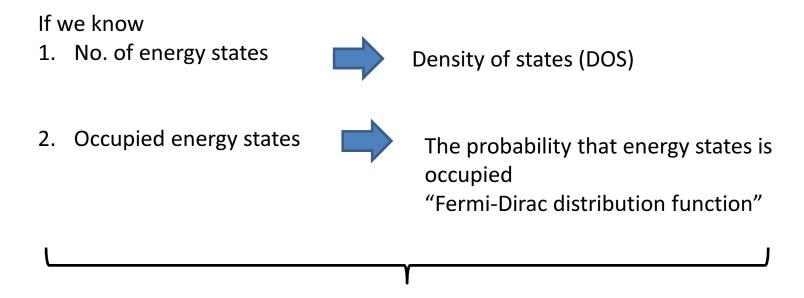
The semiconductor in equilibrium

Equilibrium (Thermal equilibrium)

- No external forces such as voltages, electric fields, magnetic fields, or temperature gradients are acting on the semiconductor
- All properties of the semiconductor will be independent of time at equilibrium
- Equilibrium is our starting point for developing the physics of the semiconductor. We will then be able to determine the characteristics that result when deviations from equilibrium occur

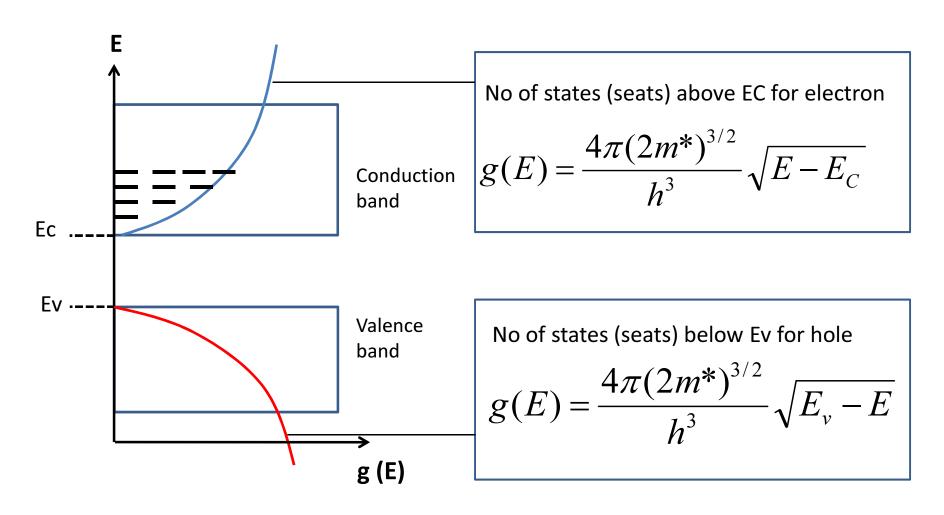
≻How to count number of carriers,n?

Assumption; Pauli exclusion principle



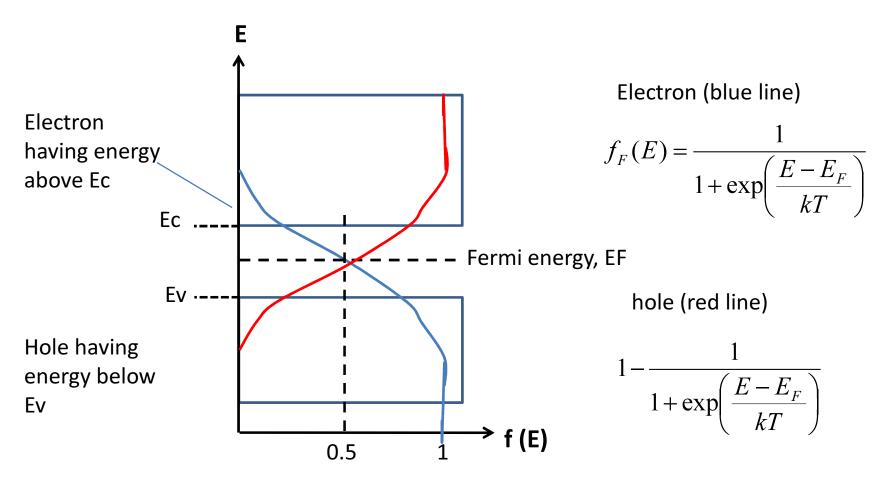
n = DOS x "Fermi-Dirac distribution function"

Density of state



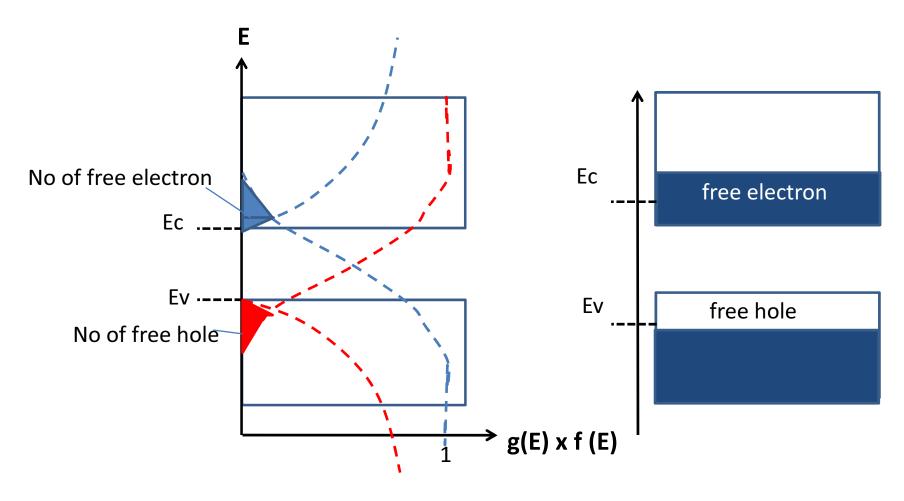
Fermi-Dirac distribution

Probability of electron having certain energy



EF; the energy below which all states are filled with electron and above which all states are empty at 0K

No of carrier



Charge carriers in semiconductors

- Current is the rate at which charge flow
- Two types of carriers can contribute the current flow
 - (a) Electrons in conduction band
- (b) Holes in valence band
- The density of electrons and holes is related to the density of state function and Fermi-Dirac distribution function

$$n(E)dE = g_c(E)f_F(E)dE$$

n(E)dE: density of electrons in CB at energy levels between E and E+dE

$$p(E)dE = g_v(E)[1 - f_F(E)]dE$$

p(E)dE: density of holes in VB at energy levels between E and E+dE

Carriers concentration in intrinsic semiconductor at equilibrium

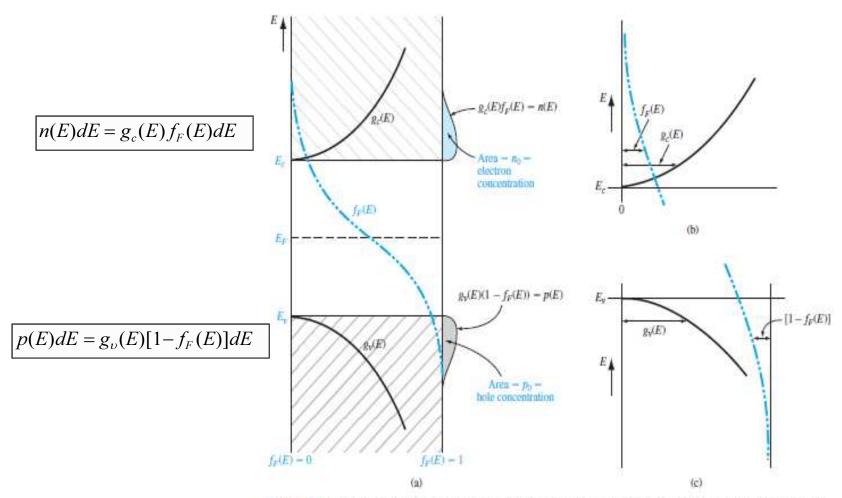


Figure 4.11(a) Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_r is near the midgap energy; (b) expanded view near the conduction-band energy; and (c) expanded view near the valence-band energy.

Thermal equilibrium concentration of electron, n

$$n_o = \int_{E_C}^{\infty} g(E) f(E) dE$$

$$g(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} \sqrt{E - E_C}$$

$$g(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} \sqrt{E - E_C}$$

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \approx \exp\left(\frac{-(E - E_F)}{kT}\right)$$

Boltzmann approximation

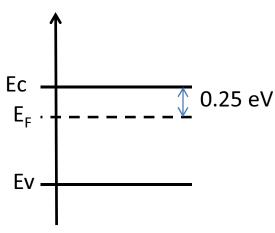
$$n_o = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= N_C \exp \left[\frac{-(E_C - E_F)}{kT} \right]$$

N_C; effective density of states function in conduction band

<u>Ex. 1</u>

Calculate the thermal equilibrium electron concentration in Si at T= 300K. Assume that Fermi energy is 0.25 eV below the conduction band. The value of Nc for Si at T=300 K is 2.8×10^{19} cm⁻³.



Thermal equilibrium concentration of hole, po

$$p_o = \int_{\infty}^{Ev} g(E) [1 - f(E)] dE$$

$$g(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} \sqrt{E_v - E}$$

$$g(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} \sqrt{E_v - E}$$

$$1 - f_F(E) = \frac{1}{1 + \exp\left(\frac{E_F - E}{kT}\right)} \approx \exp\left(\frac{-(E_F - E)}{kT}\right)$$
Boltzmann approximation



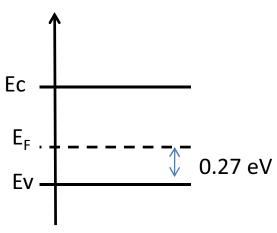
$$p_o = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

$$= N_{v} \exp \left[\frac{-(E_{F} - E_{v})}{kT} \right]$$

N_v; effective density of states function in valence band

<u>Ex.2</u>

Calculate the thermal equilibrium hole concentration in Si at T= 300K. Assume that Fermi energy is 0.27 eV above the valence band. The value of Nv for Si at T=300 K is 1.04×10^{19} cm⁻³.



The n₀ and p₀ equations

$$n_o = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{3/2} \exp \left[\frac{-(E_C - E_F)}{kT} \right] = N_C \exp \left[\frac{-(E_C - E_F)}{kT} \right]$$

$$p_{o} = 2 \left(\frac{2\pi m_{p}^{*} kT}{h^{2}} \right)^{3/2} \exp \left[\frac{-(E_{F} - E_{v})}{kT} \right] = N_{v} \exp \left[\frac{-(E_{F} - E_{v})}{kT} \right]$$

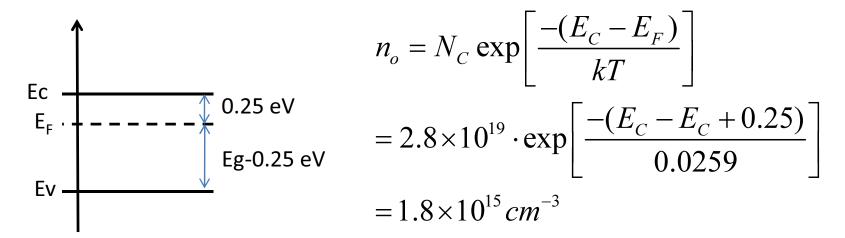
➤ N_C and N_v are constant for a given material (effective mass) and temperature ➤ Position of Fermi energy is important

> If E_F is closer to E_C than to E_V , n>pIf E_F is closer to E_V than to E_C , n< p

Table 4.1 | Effective density of states function and effective mass values

| | N_c (cm ⁻³) | N_v (cm ⁻³) | m_n^*/m_0 | m_p^*/m_0 |
|------------------|---------------------------|---------------------------|-------------|-------------|
| Silicon | 2.8×10^{19} | 1.04×10^{19} | 1.08 | 0.56 |
| Gallium arsenide | 4.7×10^{17} | 7.0×10^{18} | 0.067 | 0.48 |
| Germanium | 1.04×10^{19} | 6.0×10^{18} | 0.55 | 0.37 |

Consider ex. 1



Hole concentration

Eg=1.12 eV
$$p_o = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$
$$= 1.04 \times 10^{19} \cdot \exp\left[\frac{-(1.12 - 0.25)}{0.0259}\right]$$
$$= 2.68 \times 10^4 cm^{-3}$$

Intrinsic semiconductor; A pure semiconductor with no impurity atoms and no lattice defects in crystal

- 1. Carrier concentration(n_i, p_i)
- 2. Position of E_{Fi}
- 1. Intrinsic carrier concentration

Concentration of electron in in conduction band, ni



Concentration of hole in in valence band, pi

$$n_i = p_i = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right] = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$n_i^2 = N_C N_v \exp\left[\frac{-(E_C - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

Independent of Fermi energy

Ex. 3; Calculate the intrinsic carrier concentration in gallium arsenide (GaAs) at room temperature (T=300K). Energy gap, Eg, of GaAs is 1.42 eV. The value of Nc and Nv at 300 K are 4.7×1017 cm-3 and 7.0×1018 cm-3, respectively.

2. Intrinsic Fermi level position, E_{Fi}

If E_F closer to Ec, n>p If E_F closer to Ev, n<p

Intrinsic; n=p



E_F is located near the center of the forbidden bandgap

$$M_p = m_n$$
 $E_{Fi} = E_{midgap}$

$$M_p \neq m_n \implies E_{Fi}$$
 shifts slightly from E_{midgap}

Dopant atoms and energy levels

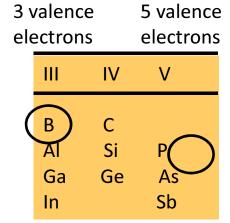
adding small, controlled amounts of specific dopant, or impurity, atoms



Increase no. of carrier (either electron or hole)



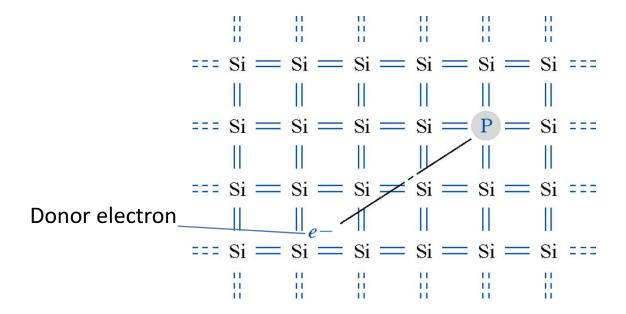
Alter the conductivity of semiconductor



Consider Phosphorus (P) and boron (B) as impurity atoms in Silicon (Si)

N type semiconductor

1. P as substitutional impurity (group V element; 5 valence electron)

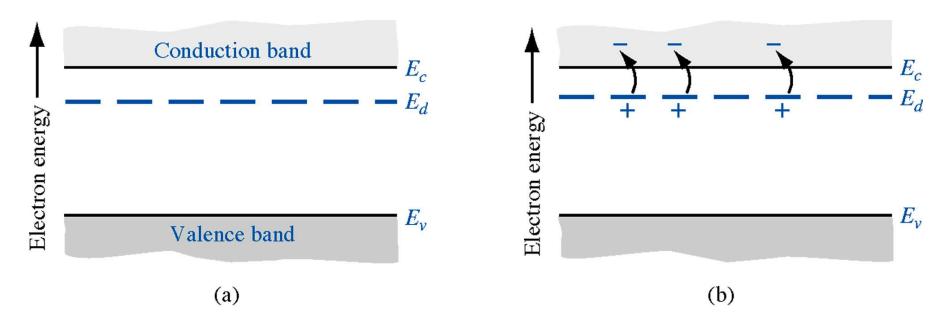


➤ In intrinsic Si, all 4 valence electrons contribute to covalent bonding.

➤ In Si doped with P, 4 valence electron of P contribute to covalent bonding and <u>1</u> electron loosely bound to P atom (Donor electron).



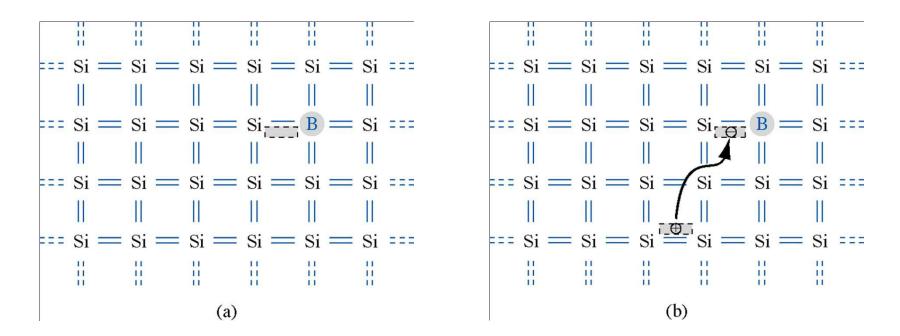
Energy to elevate the donor electron into conduction band is less than that for the electron involved in covalent bonding



- ➤ Ed(; energy state of the donor electron) is located near Ec
- ➤ When small energy is added, donor electron is elevated to conduction band, leaving behind positively charged P ion
- ➤ P atoms donate electron to conduction band → P; donor impurity atom
- \triangleright No. of electron > no. of hole \rightarrow n-type semiconductor (majority carrier is electron)

P type semiconductor

2. B as substitutional impurity (group III element; 3 valence electron)



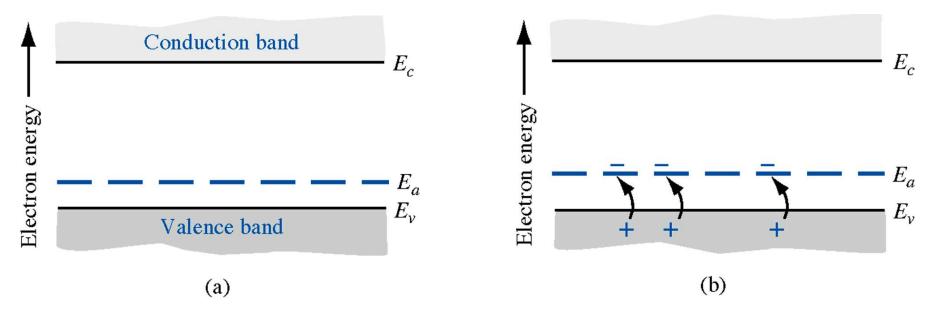
➤ In Si doped with B, all 3 valence electron of B contribute to covalent bonding and one covalent bonding is empty

➤ When small energy is added, electron that involved in covalent bond will occupy the empty position <u>leaving behind empty position</u> that associated with Si atom



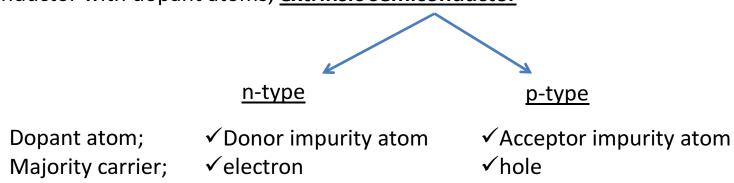
Hole is created

➤ Electron occupying the empty state associated with B atom does not have sufficient energy to be in the conduction band → no free electron is created



- ➤ Ea (;acceptor energy state) is located near Ev
- ➤ When electron from valence band elevate to Ea, hole and negatively charged B are created
- ➤ B accepts electron from valence band → B; acceptor impurity atom
- \rightarrow No. of hole > no. of electron \rightarrow p-type material (majority carrier is hole)

- ➤ Pure single-crystal semiconductor; intrinsic semiconductor
- ➤ Semiconductor with dopant atoms; **extrinsic semiconductor**



Ionization Energy

The energy that required to elevate donor electron into the conduction (in case of donor impurity atom) or to elevate valence electron into acceptor state (in case of acceptor impurity atom).

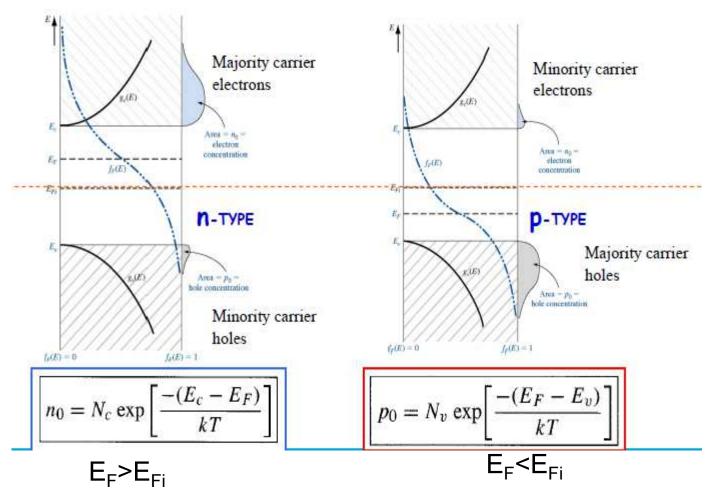
- Ionisation energy of Hydrogen in lowest energy state = -13.6eV
- For Si, it is -25.8meV << band gap.
- Calculations using Bohr model give only the order of magnitude of the ionisation energy. Actual values differ.

Impurity ionization energies in gallium arsenide

| | | | Impurity | Ionization energy (eV) |
|---------------|------------------------|---------------|-----------|------------------------|
| Impurity ioni | zation energies in | n Silicon and | Donors | |
| Germanium | | Selenium | 0.0059 | |
| Impurity | Ionization energy (eV) | | Tellurium | 0.0058 |
| | Si | Ge | Silicon | 0.0058 |
| Donors | | | Germanium | 0.0061 |
| Phosphorus | 0.045 | 0.012 | | |
| Arsenic | 0.05 | 0.0127 | Acceptors | |
| | | | Beryllium | 0.028 |
| Acceptors | | | Zinc | 0.0307 |
| | | | Cadmium | 0.0347 |
| Boron | 0.045 | 0.0104 | Silicon | 0.0345 |
| Aluminum | 0.06 | 0.0102 | Germanium | 0.0404 |
| | | | | |

Carrier concentration of extrinsic semiconductor

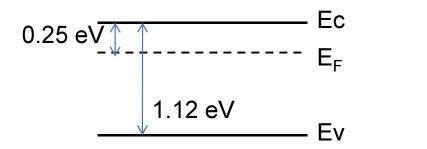
When dopant atoms are added, Fermi energy and distribution of electron and hole will change.



$$n_o = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$
 Thermal equilibrium concentration of electron

$$p_o = N_v \exp \left[\frac{-(E_F - E_v)}{kT} \right]$$
 Thermal equilibrium concentration of hole

<u>Ex. 4</u>



Band diagram of Si. At T= 300 K, Nc=2.8x10¹⁹cm⁻³ and Nv=1.04x10¹⁹cm⁻³. Calculate n_o and p_o.

$$n_o = (2.8 \times 10^{19}) \exp\left(\frac{-0.25}{0.0259}\right) = 1.8 \times 10^{15} cm^{-3}$$

N-type Si

$$p_o = (1.04 \times 10^{19}) \exp\left(\frac{-(1.12 - 0.25)}{0.0259}\right) = 2.7 \times 10^4 cm^{-3}$$

➤ Change of Fermi energy causes change of carrier concentration.

n_o and p_o equation as function of the change of Fermi energy

$$\begin{aligned} &\text{n-type} \quad n_o = N_C \exp \frac{-(E_C - E_f)}{kT} \\ &\text{intrinsic} \quad n_o = n_i = N_C \exp \frac{-(E_C - E_{f_i})}{kT} \\ &\Rightarrow n_o = N_C \exp \left[\frac{-(E_C - E_{f_i}) + (E_F - E_{f_i})}{kT} \right] \\ &n_o = N_C \exp \left[\frac{-(E_C - E_{f_i})}{kT} \right] \exp \left[\frac{(E_F - E_{f_i})}{kT} \right] \\ &n_o = n_i \exp \left[\frac{(E_F - E_{f_i})}{kT} \right] \qquad &\text{(n-type } n_o > n_i, \ E_f > E_{f_i}) \\ &p_o = n_i \exp \left[\frac{-(E_F - E_{f_i})}{kT} \right] \qquad &\text{(p-type } n_o > n_i, \ E_f > E_{f_i}) \end{aligned}$$

The n_op_o product

$$n_{o}p_{o} = N_{C}N_{v} \exp\left[\frac{-(E_{C} - E_{F})}{kT}\right] \exp\left[\frac{-(E_{F} - E_{v})}{kT}\right]$$

$$= N_{C}N_{v} \exp\left[\frac{-E_{g}}{kT}\right]$$

$$= n_{i}^{2}$$

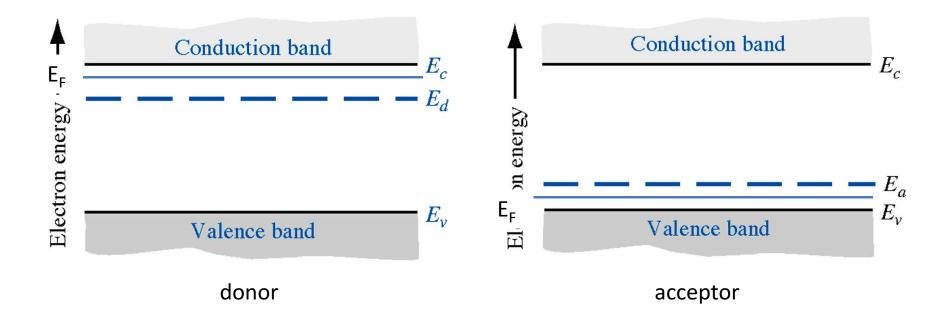
$$n_{o}p_{o} = n_{i}^{2}$$

Product of n_o and p_o is always a constant for a given material at a given temperature.

Degenerate and Non degenerate semiconductors

Nondegenerate semiconductor

If the doping is low, the impurity atoms are spread far enough apart so that there is **no interaction** between donor (acceptor) electrons (holes), the impurities introduced **discrete**, **non interacting energy states**



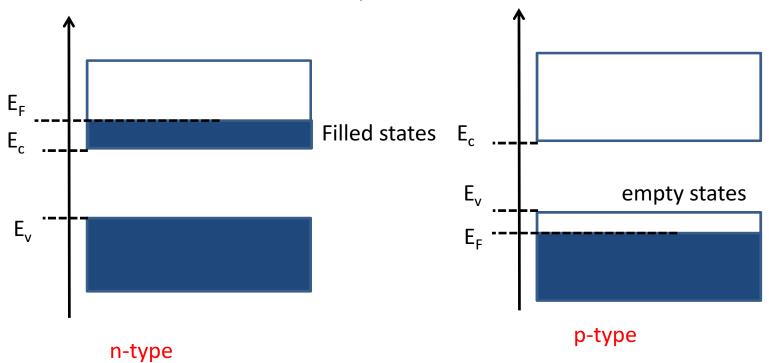
Nondegenerate semiconductor

Degenerate semiconductor

If the doping is high, the impurity atoms are close enough so that donor/acceptor electrons/holes will begin to **interact** with each other. When this occur, the single discrete energy state will split into a **band of energy**. When the concentration of donor (acceptor) electrons (holes) exceeds the effective density of states $(N_c \text{ or } N_v)$, the Fermi level lies with the band. This type of semiconductors are called degenerate semiconductors.

n - type degeneratesemiconductor $n_{\rm o} > N_c$, $E_f > E_c$

p - type degeneratesemiconductor $p_o > N_v$, $E_v > E_f$

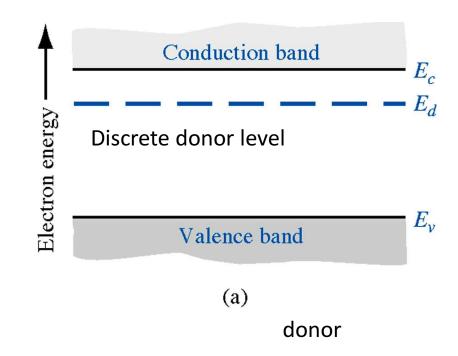


Statistic donors and acceptors

Suppose we have Ni electrons and gi quantum states at ith impurity energy level.

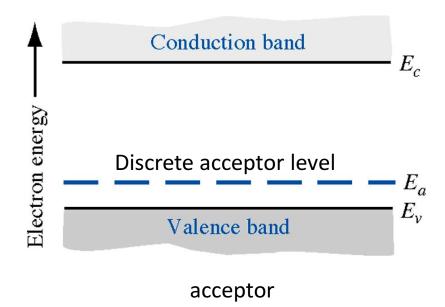
Each donor level at least has two possible spin orientations for donor electron, thus each donor has at least two quantum states.

The distribution function of donor electrons in the donor energy states is slightly different than the Fermi-Dirac function



The probability function of electrons occupying the donor state is $f_F(E) = n_d/N_d$

$$n_d = \frac{N_d}{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)} = N_d - N_d^+$$
 Density of electron occupying the donor
$$\frac{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)}{kT} = N_d - N_d^+$$
 level
$$\frac{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)}{kT} = N_d - N_d^+$$
 Concentration of the donor donors ionized donors



The probability function of holes occupying the acceptor state is $f_F(E)=p_a/N_a$

$$p_a = \frac{N_a}{1 + \frac{1}{g} \exp\left(\frac{E_F - E_a}{kT}\right)} = N_a - N_a^-$$
 Concentration of holes in the acceptor states
$$\frac{\text{Concentration of acceptors}}{\text{Concentration of ionized acceptor}}$$

g; degeneracy factor (Si; 4)

Complete Ionization and Freeze-Out

For non-degenerate semiconductor,

If
$$E_d - E_f >> kT$$

$$n_{d} = N_{d} f_{F}(E) = \frac{N_{d}}{1 + \frac{1}{2} \exp(\frac{E - E_{f}}{kT})} \approx \frac{N_{d}}{\frac{1}{2} \exp(\frac{E_{d} - E_{f}}{kT})} = 2N_{d} \exp\left[\frac{-(E_{d} - E_{f})}{kT}\right]$$

If $(E_d - E_F) >> kT$, then the Boltzmann approximation is also valid for the electrons in the conduction band so that,

$$n_o = N_C \exp \frac{-(E_C - E_f)}{kT}$$

The electron concentration in the conduction band is

$$n_o \approx n_i + (N_d - n_d)$$
 for n - type

The total number of electrons in and near the conduction band is

$$n_o + n_d$$

The ratio of electrons in the donor states to the total number of electrons in and near the conduction band is

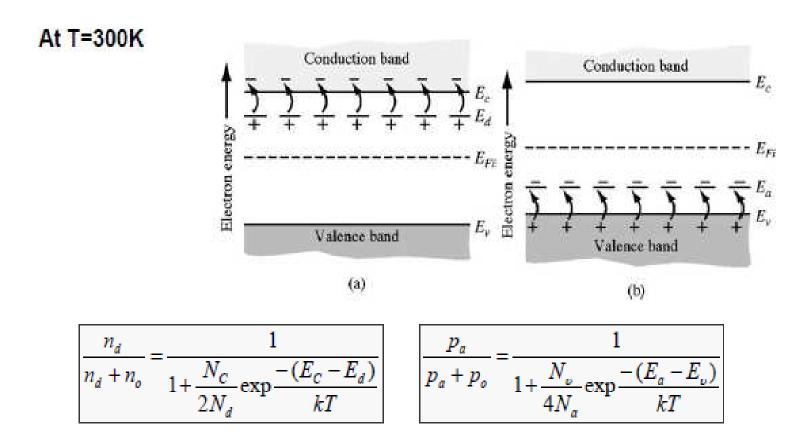
$$\frac{n_d}{n_d + n_o} = \frac{2N_d \exp\left[\frac{-(E_d - E_f)}{kT}\right]}{2N_d \exp\left[\frac{-(E_d - E_f)}{kT}\right] + N_C \exp\left[\frac{-(E_C - E_f)}{kT}\right]}$$

$$\frac{n_d}{n_d + n_o} = \frac{1}{1 + \frac{N_C}{2N_d} \exp{\frac{-(E_C - E_d)}{kT}}}$$
 ionization energy
$$E_c$$

$$E_d$$

$$E_d$$

$$E_d$$

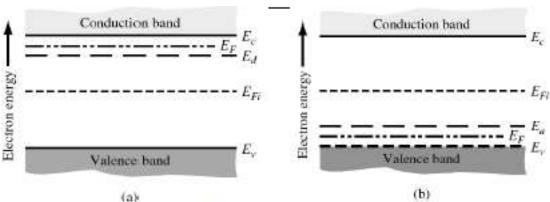


Complete ionization; The condition when all donor atoms are positively charged by giving up their donor electrons and all acceptor atoms are negatively charged by accepting electrons

Freeze-out;

At T=0 K, all electron in their lowest possible energy state

$$N_d^+=0$$
 and $N_a^-=0$



All electrons are in their lowest possible energy states

$$f_F(E) = \frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2} \exp(\frac{E_d - E_f}{kT})} = 1 \\ \Rightarrow \exp(\frac{E_d - E_f}{kT}) = 0 \\ = \exp(-\infty) \\ \Rightarrow E_f > E_d$$

- At T= 0 K, all the energy states below the Fermi level are full, and all the states above the Fermi level are empty. Since the donor states is fully occupied by donor electrons, the Fermi level must be well above the donor energy state.
- At T= 0K, no electrons from the donor state are thermally elevated into conduction band; this effect is called freeze-out.

Charge neutrality

Charge neutrality

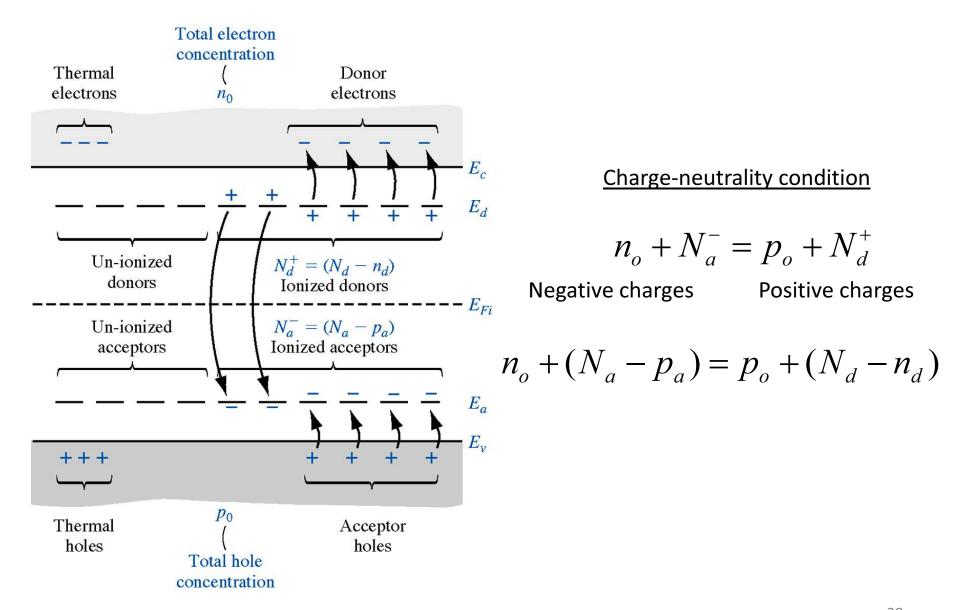
In thermal equilibrium, the semiconductor crystal is electrically neutral. The chargeneutrality condition is used to determine the thermal equilibrium electron and hole concentration as a function of impurity doping concentration.

Compensated semiconductor

A semiconductor contains both **donor and acceptor impurity atoms** in the same region

- □ N-type compensated semiconductor (Nd>Na)
- ☐ P-type compensated semiconductor (Na>Nd)
- ☐ Completely compensated semiconductor (Na=Nd)

Equilibrium electron and hole concentration



electron concentration

$$n_o + (N_a - p_a) = p_o + (N_d - n_d)$$

If we assume complete ionization ($p_a=0$, $n_d=0$)

$$n_o + N_a = p_o + N_d$$

From $n_o p_o = n_i^2$

$$n_o + N_a = \frac{n_i^2}{n_0} + N_d$$

For n-type
$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Electron concentration is given as function of donors and acceptors concentrations

Example;

Consider an n-type silicon semiconductor at T=300 K in which $N_d=10^{16}$ cm⁻³ and $N_a=0$. The intrinsic carrier concentration is assumed to be $n_i=1.5 \times 10^{10}$ cm⁻³. Determine the thermal equilibrium electron and hole concentrations.

Electron,
$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$
$$= \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right) + (1.5 \times 10^{10})^2}$$
$$\approx 10^{16} cm^{-3}$$

hole,
$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 cm^{-3}$$

Electron concentration vs. temperature

■ Freeze-out (0 K)

$$n_d = N_d, \ n_o = 0$$

■ Partial ionization (0- ~100K)

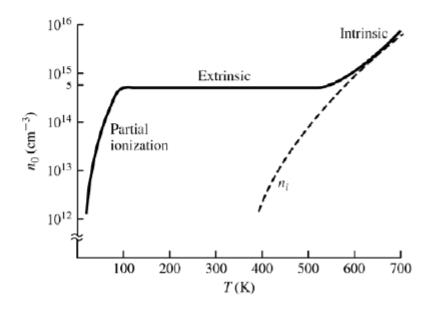
$$\frac{n_d}{n_d + n_o} = \frac{1}{1 + \frac{N_C}{2N_d} \exp{\frac{-(E_C - E_d)}{kT}}}$$

Extrinsic region (~100K-400K)

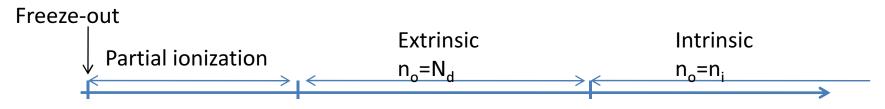
$$n_o \approx n_i + (N_d - n_d)$$
 for n-type

■ Intrinsic region (>~400 K)

$$n_i^2 = N_C \cdot N_v \exp \frac{-E_g}{kT}$$



The above regions are determined by type of semiconductor and doping concentration



Hole concentration

From charge-neutrality condition and $n_o p_o$ product

$$n_o + (N_a - p_a) = p_o + (N_d - n_d)$$
$$n_o p_o = n_i^2$$



$$\frac{n_i^2}{p_o} + N_a = p_o + N_d$$

$$p_{o} = \frac{N_{a} - N_{d}}{2} + \sqrt{\left(\frac{N_{a} - N_{d}}{2}\right)^{2} + n_{i}^{2}}$$

Example;

Consider an p-type silicon semiconductor at T=300 K in which $N_a=10^{16}$ cm⁻³ and $N_d=3 \times 10^{15}$ cm⁻³. The intrinsic carrier concentration is assumed to be $n_i=1.5\times 10^{10}$ cm⁻³. Determine the thermal equilibrium electron and hole concentrations.

Hole,
$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \frac{10^{16} - 3 \times 10^{15}}{2} + \sqrt{\left(\frac{10^{16} - 3 \times 10^{15}}{2}\right) + (1.5 \times 10^{10})^2}$$

$$\approx 7 \times 10^{15} cm^{-3}$$
 electron,
$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{7 \times 10^{15}} = 3.21 \times 10^4 cm^{-3}$$

approximation
$$p_o = N_a - N_d$$

Position of Fermi Energy Level

As a function of doping concentration and temperature

Equations for position of Fermi level (n-type)

$$E_C - E_F = kT \ln \left(\frac{N_C}{n_o} \right)$$

Compensated semiconductor, n_o=N_d-N_a

$$E_C - E_F = kT \ln \left(\frac{N_C}{N_d - N_a} \right)$$

$$E_F - E_{Fi} = kT \ln \left(\frac{n_o}{n_i}\right)$$

Equations for position of Fermi level (p-type)

$$E_F - E_C = kT \ln \left(\frac{N_v}{p_o} \right)$$

Compensated semiconductor, p_o=N_a-N_d

$$E_F - E_v = kT \ln \left(\frac{N_v}{N_a - N_d} \right)$$

$$E_{Fi} - E_F = kT \ln \left(\frac{p_o}{n_i}\right)$$

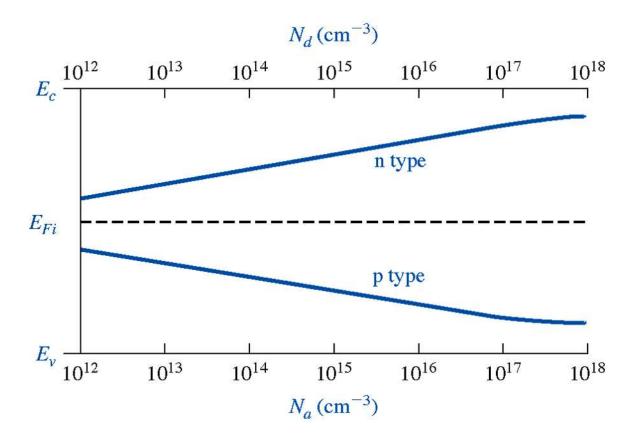
Example;

Silicon at T=300 K contains an acceptor impurity concentration of $N_a=10^{16}$ cm⁻³. Determine the concentration of donor impurity atoms that must be added so that the Silicon is n-type and Fermi energy is 0.20 eV below the conduction band edge.

$$\begin{split} E_C - E_F &= kT \ln \left(\frac{N_C}{N_d - N_a} \right) \\ N_d - N_a &= N_C \exp \left[\frac{-(E_C - E_F)}{kT} \right] \\ &= 2.8 \times 10^{19} \exp \left[\frac{-0.2}{0.0259} \right] = 1.24 \times 10^{16} \, cm^{-3} \end{split}$$

$$N_d = 1.24 \times 10^{16} \, cm^{-3} + N_a = 2.24 \times 10^{16} \, cm^{-3}$$

Position of E_F as function of donor concentration (n-type) and acceptor concentration (p-type)



Position of E_F as function of temperature for various doping concentration

