Propositional & Predicate Logic

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* Propositional logic:

All sentences: either T or F all are declarative statment, or proposition.

Ex: $2+3=5$ ✓ T   Ex: $2+5=9$ × F

- Negation:

Representⁿ: $\neg P$ or $\sim P$ or $\overline{P}$

If p = True

$\neg P$ = False

→ For 2 propositⁿ: p, q, disjunction of p & q is represented as $p \vee q$.

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

→ Conjunction of p & q is $p \wedge q$

* Exclusive OR $\oplus$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Answer these q's : Not a statement.   Pg no. 16, 17, 18, 19

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→ $p \rightarrow q$ (conditional statement):
(if p then q)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ $p \leftrightarrow q$ (Biconditional statement): = Negat^n of $\oplus$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$$(p \leftrightarrow q) = (p \rightarrow q) \wedge (q \rightarrow p)$$

Ex: Construct T.T of given composite statement:
$(p \vee \neg q) \rightarrow (p \wedge q)$

Sol^n:

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	Ans
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

* Precedence Order:

(2) $\neg$ (1) $\exists$ or $\forall$
(3) $\wedge$
(4) $\vee$
(5) $\rightarrow$
(6) $\leftrightarrow$
(7) $\oplus$

if true/false – contingency.

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Ex: $(p \oplus q) \wedge (p \oplus \neg q)$

Sol^n:

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

} Fallacy or contradic^n

* Propositional Equivalence:

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

} Tautology

Def# Logically Equivalent: The compound proposition p & q are logically equivalent if $p \leftrightarrow q$ is a tautology

Ex: $\neg(p \vee q)$ and $\neg p \wedge \neg q$

Sol^n:

Method: 1

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$		$\neg p \wedge \neg q$
T	F	F	T	T	F	=	F
F	T	T	F	T	F	=	F
T	T	F	F	T	F	=	F
F	F	T	T	F	T	=	T

same

OR $\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$

Method: 2

$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
F	F	T
F	F	T
F	F	T
T	T	T

} Tautology

or

$\equiv$ logically equivalent

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Laws/Rules:

→ De-Morgan's law:

(1) $\neg(p_1 \vee p_2 \vee p_3 \cdots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \wedge \neg p_3 \cdots \wedge \neg p_n$

(2) $\neg(p_1 \wedge p_2 \wedge p_3 \cdots \wedge p_n) \equiv \neg p_1 \vee \neg p_2 \vee \neg p_3 \cdots \vee \neg p_n$

→ Identity law:

(1) $p \wedge T \equiv p$

→ Domination law:

(1) $p \vee F \equiv p$

→ Distributive law:

(1) $(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$

→ Associative law:

(1) $(p \vee q) \vee r = p \vee (q \vee r)$

→ $p \vee p \equiv p$

→ $p \vee (p \wedge q) \equiv p$ (Absorption law)

→ $p \wedge p \equiv p$

→ $\neg(\neg p) \equiv p$

→ $p \vee (\neg p) \equiv T$

→ $p \wedge (\neg p) \equiv F$ (Negation)

$p \to q$
$\equiv \neg p \vee q$

Q Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q \equiv \neg p \vee \neg(\neg p \wedge q)$

Sol^n: $\neg(p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ (Demorgan's law)

$\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$ (Demorgan's law)

$\equiv \neg p \wedge (p \vee \neg q)$ ($\because \neg(\neg p) = p$)

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ (Distributive law)

$\equiv F \vee (\neg p \wedge \neg q)$ ($\because \neg p \wedge p \equiv F$)

$\equiv \neg p \wedge \neg q$ ($\because F \vee p' \equiv p' \Rightarrow$ Dominat^n law)

Q $[\neg p \wedge (p \vee q)] \to q$

Sol^n:

p	q
T	T
T	F
F	T
F	F

method (1)

OR

method :(2)

$[\neg p \wedge (p \vee q)]$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge q)$ (Distributive law)

$\equiv F \vee (\neg p \wedge q)$

$\equiv (\neg p \wedge q)$ (Domination law)

Now, check $(\neg p \wedge q) \to q$

p	q	$\neg p$	$\neg p \vee q$	q	$\neg p \wedge q$	$\neg p \wedge q \to q$
T	T	F	T	T	F	T
T	F	F	F	F	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

Tautology

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* Subject-Predicate logic:

Ex: x is greater than 3

x → Subject

is greater than 3 → Predicate

P(x): x is greater than 3

P(1) is false
P(4) is True.

$\forall x P(x)$ = F for $x = 1,2,3$
→ universal quantifier
$\forall x P(x)$ = T for $x = 4,5,6, \ldots$

Ex: $x = y+1$
P(x,y): $x = y+1$
P(2,1) is true while P(3,1) is False.

$\exists x P(x)$ = F for $x = 1,2,3$
→ There exists quantifier.

Similiarly, $P(x_1, x_2, \ldots, x_n)$ = entouple

Ex: M3 is functioning properly.
Sol^n: $P(M_3)$ is true if M_3: functioning properly.

Statement	When True?	When False?
$\forall x P(x)$	P(x) is T for every x	P(x)=F for some $\exists$ x
$\exists x P(x)$	P(x) is T for some x	P(x)=F for every x.

Ex: ~~Rule~~ Let Q(x) be statetement "$x < 2$". What is truth value of the quantification $\forall x Q(x)$ where domain consists of all natural no.

Sol^n: Q(x) is not true for every real no. ~~x~~ x because for instance P(3) = false, that is $x = 3$ is a counter example for this statement $\forall x Q(x)$.
Thus, $\forall x Q(x)$ is False.

$\neg(x \rightarrow y) \equiv x \wedge \neg y$

\# $\exists x P(x)$ is same as $P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$

Let

$P(x): x^2 > 10$; Domain : $x = 1, 2, 3, 4$

$P(1) \vee P(2) \vee P(3) \vee P(4)$

$= F \vee F \vee F \vee T$

$= T$

$\therefore$, $\exists x P(x)$ is T

$\rightarrow$ $\exists$ and $\forall$ has higher precedence than $\neg$

$\forall x P(x) \vee Q(x) \neq \forall x (P(x) \vee Q(x))$

$= (\forall x P(x)) \vee Q(x)$

$\rightarrow \neg \forall x P(x) \equiv \exists x \neg P(x)$

$\rightarrow \neg \exists x P(x) \equiv \forall x \neg P(x)$

Ex: $\forall x (x^2 > 2) \wedge \exists x (x^2 = 2)$

Soln: $\neg [\forall x (x^2 > 2) \wedge \exists x (x^2 = 2)]$

$\exists x (x^2 \leq 2) \ \cancel{\wedge} \vee \ \forall x (x^2 \neq 2)$

Ex: Show that:

$\neg \forall x (P(x) \rightarrow Q(x))$ and

$\exists x (P(x) \wedge \neg Q(x))$ are logically equivalent.

Soln: $\neg \forall x (P(x) \rightarrow Q(x)) \equiv \exists x \neg (P(x) \rightarrow Q(x))$

$\equiv \exists x (P(x) \wedge \neg Q(x))$

predicate logic: p: 54, 43
propositⁿ Logic: p: 28, 16
p: 273, 289

$\forall x \exists y (x+y=0)$
$\equiv \forall x(\exists y(x+y \neq 0))$

Ex: The sum of all +ve integers is always +ve.

Solⁿ: $\forall x \forall y((x>0) \wedge (y>0) \longrightarrow (x+y>0))$
OR
x is positive = $P(x)$
y is positive = $P(y)$

$\therefore \forall x \forall y((P(x) \wedge P(y)) \longrightarrow P(x,y)) \Rightarrow$ Predicate representation

Ex: Translate $\forall x\, C(x) \vee \exists y(C(y) \wedge F(x,y))$ into English.
$C(x)$ is "x has computer"
$F(x,y)$ is "x & y are friends"
x, y are students in your school.

Solⁿ: For every student x in your school, x has computer or there is a student y such that y has a computer and x & y are friends.

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Boolean Algebra

$B = \{0, 1\}$ $\quad B^n = \{x_1, x_2, \dots, x_n\}$

$F : B^n \to B$ $\quad$; F = Boolean f^n

Ex: $F(x, y, z)$ s.t. $x, y, z \in \{0, 1\}$

$1 + 1 = 1$	$1 \times 1 = 1$	$\neg 0 = 1$
$1 + 0 = 1$	$1 \times 0 = 0$	$\neg 1 = 0$
$0 + 1 = 1$	$0 \times 1 = 0$	
$0 + 0 = 0$	$0 \times 0 = 0$	
(OR)	(AND)	

Ex: $1 \cdot 0 + \overline{(0+1)} = 0 + \overline{T} = 0 + 0 = 0$ $\quad$ | $1 = T$, $0 = F$

$= T \wedge F \vee \neg(F \vee T)$

$= F$

ex: $F(x, y, z) = xy + \bar{z}$

x	y	z	$\bar{z}$	$x \cdot y$	$x \cdot y + \bar{z}$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	1	0	0	0
0	1	0	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	1	1	0	1	1
1	0	0	1	0	1

* Rules:

1. $x + x = x$
2. $x \cdot x = x$
3. $x + 0 = x$
4. $x \cdot 1 = x$
5. $x + y = y + x$
6. $x \cdot y = y \cdot x$
7. $\overline{x \cdot y} = \bar{x} + \bar{y}$
8. $\overline{(x+y)} = \bar{x} \cdot \bar{y} = \bar{x}\bar{y}$
9. $x + (y+z) = (x+y) + z$
10. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$
11. $x + xy = x$
12. $x(x+y) = x$
13. $x + (y.1)$ has dual $= x.(y+0)$

5–6: } Commutative

7–8: } Demorgan's

9–10: } ~~Distrib~~ Associative

11–12: } Absorption law

* Sum of products:

Ex: $F(x, y, z) = (x+y)\bar{z}$

$= x\bar{z} + y\bar{z}$

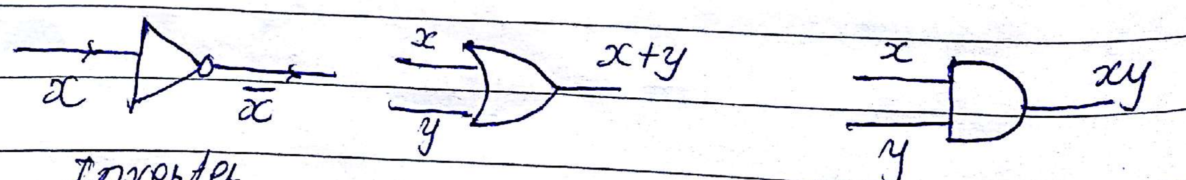
$= x\bar{z} \cdot 1 + y\bar{z} \cdot 1$

$= x\bar{z}(y+\bar{y}) + y\bar{z}(x+\bar{x})$

$= x\bar{z}y + x\bar{z}\bar{y} + y\bar{z}x + y\bar{z}\bar{x}$

$= x\bar{z}y + x\bar{z}\bar{y} + y\bar{z}\bar{x}$

* Logic gate:



Inverter (NOT gate) OR Gate AND gate

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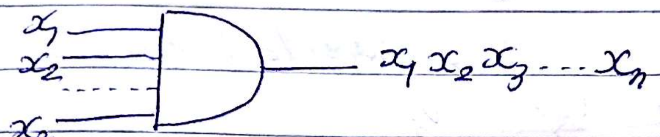
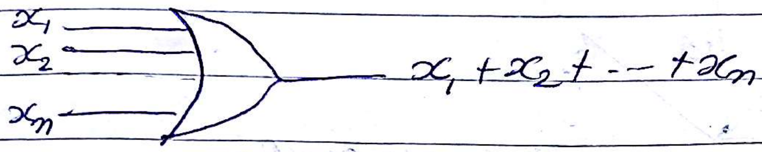
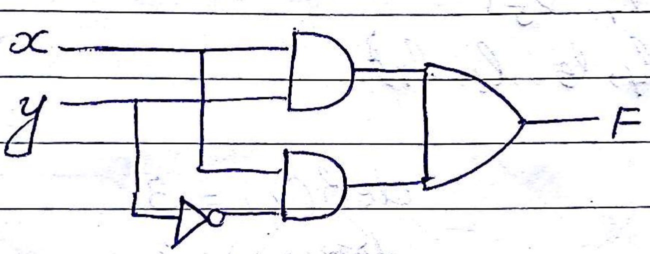
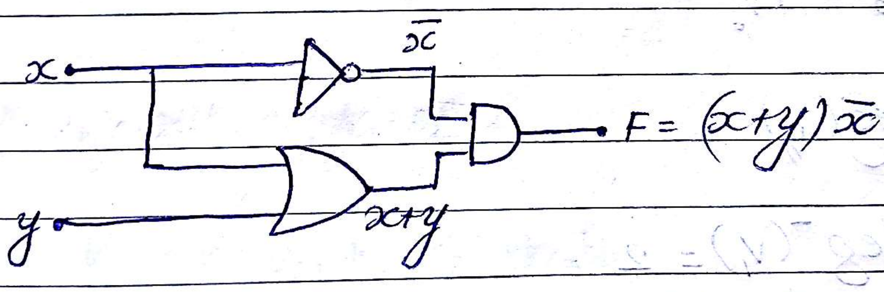




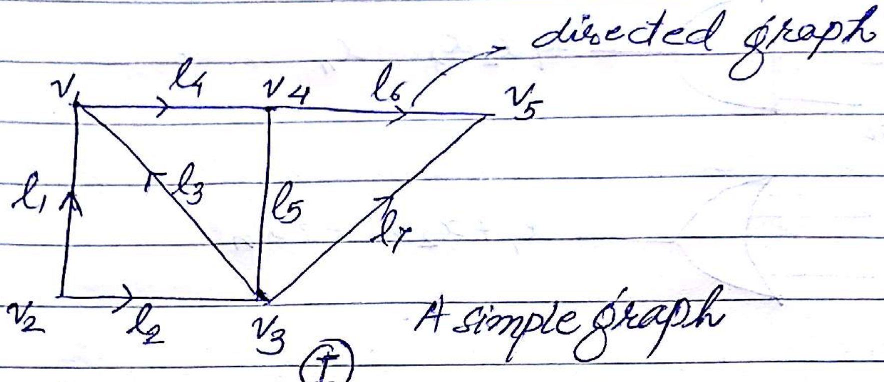
Ex: $F = xy + x\bar{y}$



Ex: $F(x, y, z) = (x+y)\bar{x}$



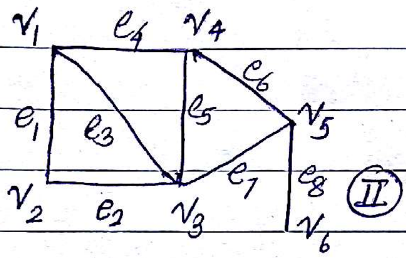
Graph Theory



$G = (V, E)$

$V = \{v_1, v_2, v_3, v_4, v_5\}$

$E = \{l_1, l_2, l_3, l_4, l_5, l_6, l_7\}$



$deg(v_1) = 3$

$deg(v_2) = 2$

$deg(v_3) = 4$

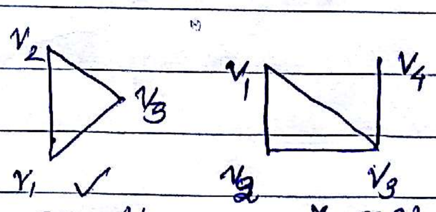
$$2e = \sum_{n=1}^{n} deg(v_n)$$

For (I), $deg^-(v_1) = 2$

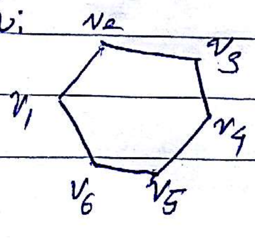
$deg^+(v_1) = 1$

→ The complete graph on n-vertices is denoted by K_n; exactly 1 edge between each pair of distinct vertices.

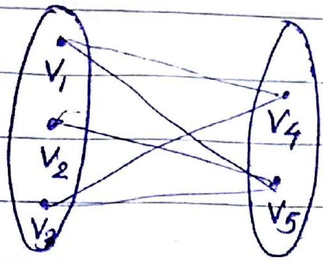
Ex:



→ Cyclic graph:



$deg(v_2) = 2$

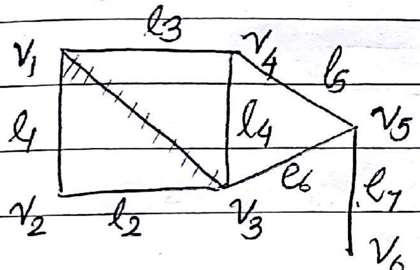


— Bi-parted graph.

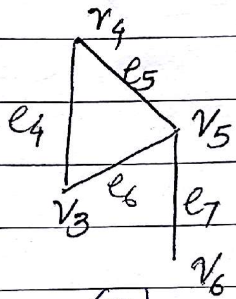
A Sub-Graph of a graph $G=(V,E)$ is also a graph $H=(W,F)$; where $W \subseteq V$ and $F \subseteq E$

A sub-graph H of G is a proper graph (subgraph) of G if $H \neq G$

Ex:



(I)



(II)

II is subgraph of I.

I: $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

II: $V = \{v_3, v_4, v_5\}$
$E = \{e_4, e_5, e_6, e_7\}$

→ $G = G_1 \cup G_2$ $\qquad G_1 = (V_1, E_1)$
$G_2 = (V_2, E_2)$

$G = \{(V_1 \cup V_2), (E_1 \cup E_2)\}$

→ Adjacent Vertex:

of:
$v_1 = v_2, v_4$