Assignment-3
Course: B.Tech 1st Year
Subject: Discrete Mathematical Structures (DMS)

1. The passcode of an employee in a software organization consists of two letters of the English alphabet, followed by two digits. Calculate the number of different possible passcodes.

2. Find the number of different arrangements of the letters in the words:
   - OMEGA, such that the vowels are always kept together.
   - ASSASSINATION, such that the letters A, N, T are always kept together.
   - BOOLEAN
   - MISSISSIPPIAN, such that the consonants are always kept together.

3. A fair six sided die is tossed four times and the numbers shown are recorded in a sequence. Calculate how many different sequences are possible.

4. Calculate the number of bit strings of length 10 which contain:
   - Exactly four 1's.
   - Almost four 1's.
   - At least four 1's.
   - An equal number of 0's and 1's.

5. Calculate the number of subsets with more than two elements of a set, having 100 elements.

6. Calculate the number of subsets with an odd number of elements for a set, having 15 elements.

7. Calculate the number of diagonals of an n-gon (a closed polygon, with n sides).

8. Compute the sum of the distinct numbers, obtained by all possible permutations of 27583, taken all at a time.

9. In how many ways can eight men and five women stand in a line, so that no two women stand next to each other?

10. In how many ways can seven people be seated in a circular table?

11. Consider that there are three routes from city A to city B, four routes from city B to city C, and two routes from city C to city A. Compute the number of all possible ways:
    - To travel from A to C,
    - To travel from B to A,
    - To make a round trip between B and C, so that the trip does not go through A.

12. Suppose that a bucket contains 20 balls, of which 12 are red and 8 are black. In how many ways can 10 balls be chosen, so that:
    - All of them are red.
    - 4 are red and 6 are black.
    - The number of red balls is always less than or equal to the number of black balls.

13. A committee of six people with one person designated as the chairman of the committee is to be chosen. How many different committees of this type can be made from a group of 11 people?

14. Nine friends have a total of 100 rupees. Show that one of them has at least 12 rupees.

15. Show that if five distinct points are selected in a square whose sides have length 1, then at least two points must be no more than $\sqrt{2}$ inches apart.

16. Show that among 12 different 2-digit numbers, there exists at least one pair of numbers, whose difference is a two-digit number with identical first and second digit.

17. Let A be an 8*8 Boolean matrix, so that the sum of the entries in A is 51. Prove that there is a row and a column in A, whose entries all together add up to more than 13.

18. If 31 players wish to play for five different cricket teams, show that at least seven players must play on the same team.

19. [***Hand-shaking problem***]: Show that if there are n ($n>1$) people who can shake hands with one another then there is always a pair of people who will shake hands with the same number of people.

20. Let (x_i, y_i), $i=1,2,3,4,5$ be a set of five distinct points with integer co-ordinates in the XY plane. Show that the midpoint of the line joining at least one pair of these points has integer co-ordinates.

21. Let, A={0,1}. Give a recurrence relation for s_n, the number of strings in A*,which :
    a) Do not contain adjacent 0's
    b) Do not contain 01
    c) Do not contain three 1's in succession

22. Use backtracking to solve the following recurrence relations:
    i. $a_n = (5/2)\, a_{n-1}$, for all $n \geq 2$,$a_1=4$
    ii. $a_n = 5a_{n-1} + 3$, for all $n \geq 2$,$a_1=3$
    iii. $a_n = a_{n-1} + n$, for all $n \geq 2$,$a_1=4$
    iv. $a_n = a_{n-1} - 2$, for all $n \geq 2$,$a_1=0$
    v. $a_n = n\, a_{n-1}$, for all $n \geq 1$,$a_1=6$
    vi. $a_n = 1 - a_n\, a_{n-1}$, for all $n \geq 2$,$a_1=1$

23. Solve the following homogeneous recurrence relations:
    i. $a_n = 4a_{n-1} + 5a_{n-2}$ for all $n \geq 3$,$a_1=2$, $a_2=6$
    ii. $a_n = 4(a_{n-1} - a_{n-2})$, for all $n \geq 3$,$a_1=1$, $a_2=7$
    iii. $a_n = 2\, a_{n-2}$, for all $n \geq 3$,$a_1=\sqrt{2}$, $a_2=6$
    iv. $a_n = 2(a_{n-1} - a_{n-2})$, for all $n \geq 3$,$a_1=1$, $a_2=4$

24. For the Fibonacci sequence ,prove that:

    $f^2_{n+1} - f^2_n = f_{n-1} f_{n+2}$, for all $n \geq 2$

25. In the following exercise, the n^{th} term a_n of a recurrence relation is provided explicitly. Write down the corresponding recurrence relation in terms of a_n , a_{n-1} and a_{n-2}
    i. $a_n = 2^n + 2.3^n$
    ii. $a_n = 6 - 2^{n+1}$
    iii. $a_n = 2^{n+2} + 3^{n+1}$
    iv. $a_n = 5 - 3^{n+2}$
    v. $a_n = (1+\sqrt{2})^n + (1-\sqrt{2})^n$