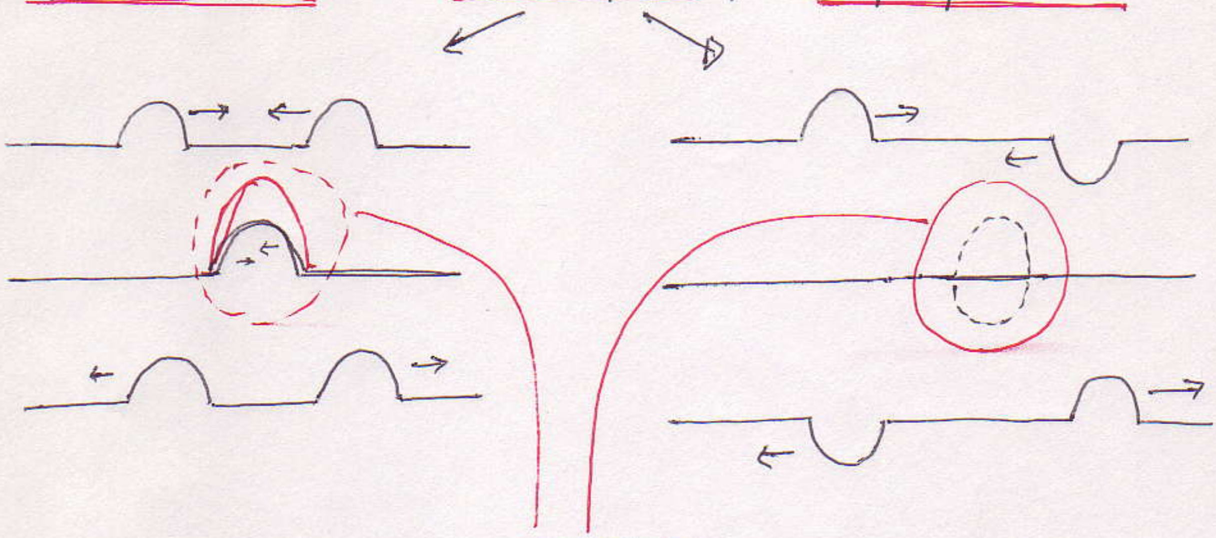
Interference & Principle of Superposition



Superposition

Constructive Interference

Destructive Interference

- This is particular of a Wave & not of a Particle.
- Two or more waves can exist simultaneously.

Principle of Superposition:

When two or more waves simultaneously pass Through a point, the ~~distance~~ displacement at That is given by sum of disturbances each wave would produce in the absence of the other wave.

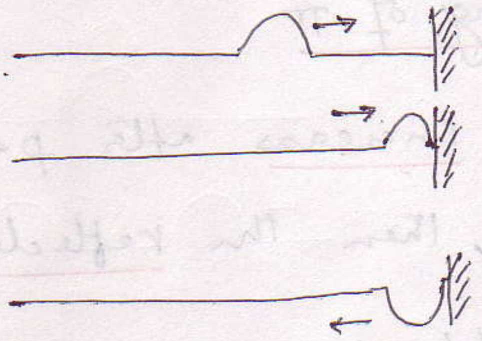
- Valid for small disturbances
- not valid for large disturbances — nonlinear waves.

Here.

$$\boxed{y = y_1 + y_2}$$

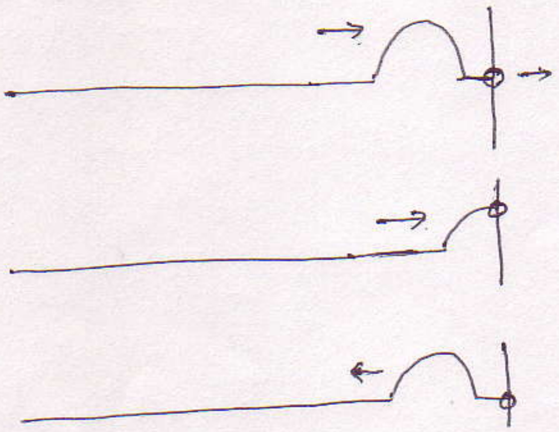
Reflections at a Boundary.

(a) string tightly bound.



Phase change of π

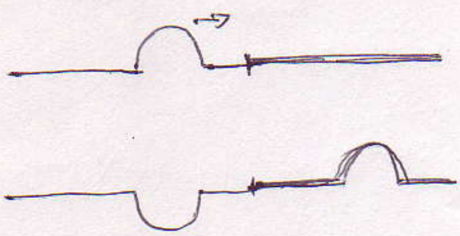
(b) String loosely bound



string attached to a ring.

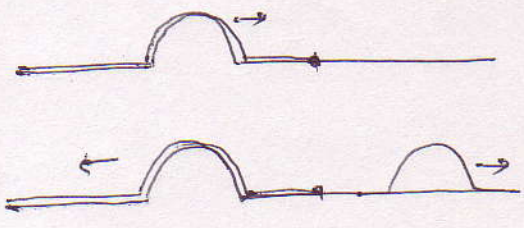
No phase change

(c) One light string attached to a heavier string.



Light string. Heavier string.

- reflected wave - phase change of π
- transmitted - No change.



Heavier. Light string.

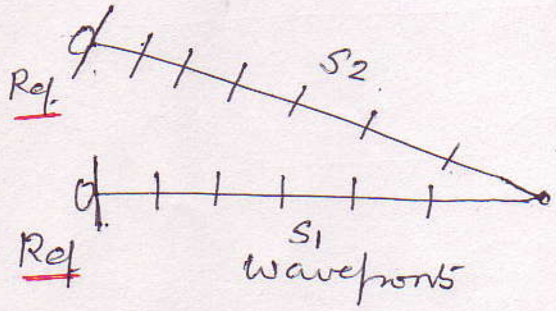
- reflected - No change.
- transmitted - No change.

Superposition of Waves.

- Same frequency – Why? Why not with same amplitude, same λ. etc.

- Mathematical Representation
  - Simple Case. 2 waves.
    - Special Cases.
      - Constructive Interference
      - Destructive Interference.
  - General Method.
    - All other Cases than Special case.
    - n waves. superposition/Interference.

Simple Case.



Imp.

What is the significance of this figure? $S1$ & $S2$ and not x_1 and x_2

$$y_1 = A_1 \sin(kS_1 - \omega t + \phi_1).$$

$$y_2 = A_2 \sin(kS_2 - \omega t + \phi_2)$$

S & ϕ both vary – so couple it together in one variable – A standard outlook.

$$\boxed{\begin{aligned} kS_1 + \phi_1 &= \alpha_1 \\ kS_2 + \phi_2 &= \alpha_2 \end{aligned}}$$

Here t is const

$y_1 = A_1 \sin(\alpha_1 - \omega t)$

$y_2 = A_2 \sin(\alpha_2 - \omega t)$.

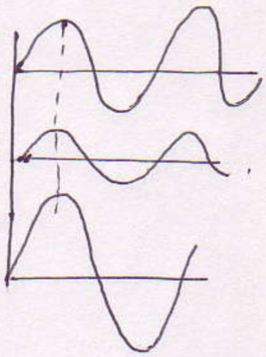
$$y = y_1 + y_2 = A_1 \sin(\alpha_1 - \omega t) + A_2 \sin(\alpha_2 - \omega t)$$

Special Cases.

Phase difference $= \alpha_2 - \alpha_1 = k(S_2 - S_1) + (\phi_2 - \phi_1)$

Constructive Interference

$\alpha_2 - \alpha_1 = 2m\pi$



Peaks are together

This means when the phase differs by 0 or $2m\pi$ the peak of one falls at the same place as of the other & the resultant amplitude is sum of individual amp $= A_1 + A_2$

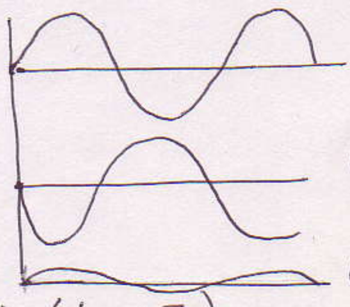
put $\alpha_1 = \alpha_1 \qquad \alpha_2 = \alpha_1 + 2m\pi$

So $\quad y = A_1 \sin(\alpha_1 - \omega t) + A_2 \sin(\alpha_1 + 2m\pi - \omega t)$

$\quad = (A_1 + A_2) \sin(\alpha_1 - \omega t)$

Destructive Interference

$\alpha_2 - \alpha_1 = (2m+1)\pi$



Peaks of one coincide with the trough of other

$y = (A_1 - A_2) \sin(\alpha_1 - \omega t)$

General Superposition

Using Complex form & Phasor method.

This method can be applied for n number of waves.
Let us first try for 2 waves.

$$\tilde{y} = A_1 e^{i(\alpha_1 - \omega t)} + A_2 e^{i(\alpha_2 - \omega t)}$$

[In place of $\tilde{y}$ one can write E, A1 as E_{01}, A2 as E_{02}]

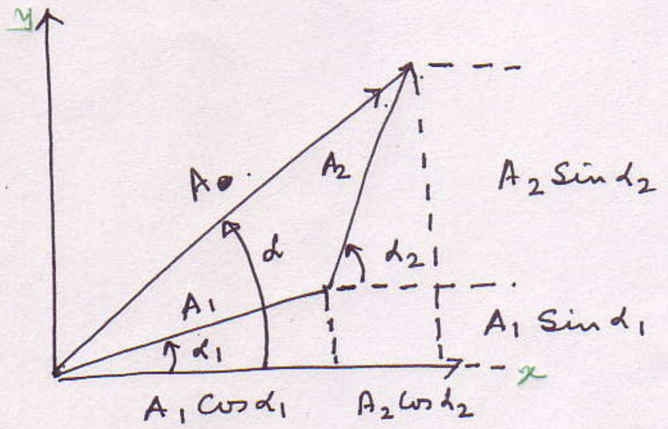
$$= e^{-i\omega t}\left[A_1 e^{i\alpha_1} + A_2 e^{i\alpha_2} \right] = A e^{i(\alpha - \omega t)}$$

$$\Downarrow$$

$$\text{Say} = A_0 e^{i\alpha}$$

Now what is the value of A and α.

Two ~~Methods~~ ways of addition: 1) Apply cosine formula directly.
2) Sum the vertical & horizontal comp.



(1) Cosine formula - Two vectors can be added as

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_2 - \alpha_1)$$

(2) Vert comp = $A_2 \sin\alpha_2 + A_1 \sin\alpha_1$

Horiz. comp = $A_1 \cos\alpha_1 + A_2 \cos\alpha_2$

So $$A^2 = (A_1\cos\alpha_1 + A_2\cos\alpha_2)^2 + (A_1\sin\alpha_1 + A_2\sin\alpha_2)^2$$

$$= A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_2 - \alpha_1)$$

~~Cos~~ Laws of cosine

$$\tan\alpha = \frac{A_2\sin\alpha_2 + A_1\sin\alpha_1}{A_1\cos\alpha_1 + A_2\cos\alpha_2}$$

To generalize for $\underline{n}$ waves.

$$\boxed{A^2 = \left(\sum_{i=1}^{n} A_i \cos\alpha_i\right)^2 + \left(\sum_{i=1}^{n} A_i \sin\alpha_i\right)^2}$$ ✓

Further expanding.

$$\left(\sum_{i=1}^{n} A_i \sin\alpha_i\right)^2 = \sum_{i=1}^{n} A_i^2 \sin^2\alpha_i + 2\sum_{j>i}^{n}\sum_{i=1}^{n} A_i A_j \sin\alpha_i \sin\alpha_j$$

$$\left(\sum_{i=1}^{n} A_i \cos\alpha_i\right)^2 = \sum_{i=1}^{n} A_i^2 \cos^2\alpha_i + 2\sum_{j>i}^{n}\sum_{i=1}^{n} A_i A_j \cos\alpha_i \cos\alpha_j$$

Adding

$$A_0^2 = \sum_{i=1}^{n} A^2 (\sin^2\alpha_i + \cos^2\alpha_i) + 2\sum_{j>i}^{n}\sum_{i=1}^{n} A_i A_j (\cos\alpha_i \cos\alpha_j + \sin\alpha_i \sin\alpha_j)$$

✓ $$\boxed{A_0^2 = \sum_{i=1}^{n} A_i^2 + 2\sum_{j>1}^{n}\sum_{i=1}^{n} A_i A_j \cos(\alpha_i - \alpha_j)}$$ ✓

This is amplitude of adding n harmonic waves.

Phase is given by.

✓ $$\boxed{\tan\alpha = \frac{\sum_{i=1}^{n} A_i \sin\alpha_i}{\sum_{i=1}^{n} A_i \cos\alpha_i}}$$

The above Summation means.

Suppose you have 3 waves — 1, 2, 3. Then $A_1 A_2 \cos(\,) + A_1 A_3 \cos(\,) + A_2 A_3 \cos(\,)$.

Random & Coherent Sources.

Two Important class of Superposition.

(1) n random phased sources of equal amplitude & frequency.

(2) n coherent sources of the same type.

(1). Random Phased sources.

$\cos(\alpha_j - \alpha_i) \Rightarrow$ Sum of these cosine terms is 0 because of its variation bet +1 & -1.

So $\underline{A^2} = \sum_{i=1}^{n} A_i^2 = \underline{n A_1^2}$

as n sources of equal amplitude.

In light irradiance (W/m^2) is proportional to the square of the amplitude. So the resultant irradiance of n identical but randomly phased sources is the sum of individual irradiances.

(2). n Coherent Sources and in phase so that

* all α_i are equal or their diff is 2π.

$$A^2 = \sum_{i=1}^{n} A_i^2 + 2\sum_{j>i}^{n} \sum_{i=1}^{n} A_i A_j$$

all cosine factors = 1. For equal amplituds

$$\underline{A^2} = \left(\sum_{i=1}^{n} A_i\right)^2 = (n A_i)^2 = \underline{n^2 A_i^2}$$

Here is $\underline{n^2}$ times the irradiance of individual source.