The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment #4

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1. Verify that $y = x^2 \sin x$ and $y = 0$ both are solutions of the initial value problem:
$$x^2 y'' - 4xy' + (x^2 + 6)y = 0, \quad y(0) = y'(0) = 0.$$
Does it contradict the uniqueness?

2. Find general solution of the following differential equations given a known solution y_1:

   (i) $x(1-x)y'' + 2(1-2x)y' - 2y = 0 \qquad y_1 = \frac{1}{x}$

   (ii) $(1-x^2)y'' - 2xy' + 2y = 0 \qquad y_1 = x.$

3. Reduce the following second order differential equations to system of first order differential equations and hence solve.
$$(i)\ xy'' + y' = y'^2 \qquad (ii)\ yy'' - y'^2 = 0 \qquad (iii)\ yy'' + y'^2 + 1 = 0 \qquad (iv)\ y'' - 2y' \coth x = 0.$$

4. Find the values of m such that $y = x^m$ $(x > 0)$ is a solution of
$$(i)\ x^2y'' - 4xy' + 4y = 0 \qquad (ii)\ x^2y'' - 3xy' - 5y = 0.$$

5. If $p(x)$, $q(x)$ are continuous functions on the interval I, then Show that $y = x$ and $y = \sin x$ are not solutions of the linear homogeneous equation
$$y'' + p(x)y' + q(x)y = 0, \qquad x \in I. \tag{1}$$

6. (a) Let $y_1(x)$, $y_2(x)$ be two linearly independent C^2 functions on the interval I, such that the Wronskian $W(y_1, y_2)$ is not zero at any point on I. Show that there exists unique $p(x)$, $q(x)$ on I such that (2) has y_1, y_2 as fundamental solutions.

   (b) Construct equations of the form (2), from the pairs of linearly independent solutions:
$$(i)\ e^{-x},\ xe^{-x} \qquad (ii)\ e^{-x}\sin 2x,\ e^{-x}\cos 2x$$

7. Show that a solution to (1) with x-axis as tangent at any point in I must be identically zero on I.

8. Let $y_1(x)$, $y_2(x)$ are two solutions of (1) with a common zero at any point in I. Show that y_1, y_2 are linearly dependent on I.

9. Solve the following initial value problems:

   (i) $y'' + 4y' + 4y = 0 \qquad y(0) = 1, y'(0) = -1$

   (ii) $y'' - 2y' - 3y = 0 \qquad y(0) = 1, y'(0) = 3.$

10. The equation
$$x^2\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0,$$
where a, b are constants, is called the Euler-Cauchy equation. Show that under the transformation $x = e^t$ for the independent variable, the above reduces to
$$\frac{d^2y}{dt^2} + (a-1)\frac{dy}{dt} + by = 0,$$
which is an equation with constant coefficients. Hence solve:
$$(i)\ x^2y'' + 2xy' - 12y = 0 \quad (ii)\ x^2y'' + xy' + y = 0 \quad (iii)\ x^2y'' - xy' + y = 0.$$

11. Find a particular solution of each of the following equations by the method of undetermined coefficients and hence find its general solution:

(i) $y'' + 4y = 2\cos^2 x + 10e^x$ (ii) $y'' + y = \sin x + (1 + x^2)e^x$

(iii) $y'' - y = e^{-x}(\sin x + \cos x)$ (iv) $y''' - 3y'' - y' + 3y = x^2 e^x$.

12. By using the method of variation of parameters, find the general solution of:

(i) $y'' + 4y = 2\cos^2 x + 10e^x$ (ii) $y'' + y = x\sin x$

(iii) $y'' + y = 1 + \sin x$ (iv) $xy'' - y' = x^2(3 + x)e^x$.