The LNM Institute of Information Technology
Department of Mathematics
Mathematics-II & Course code
End Term

Time: 180 minutes Date: 28/04/2017 Maximum Marks: 100

Instruction: You should attempt all questions. Your writing should be legible and neat. Marks awarded next to the question. Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

Question No.				
Page No.				

1. (a) Let $W = \{(a, b, c) : a - 2b + c = 0\}$. Show that W is a subspace of R^3. Find the basis and dimension of W. [6]

Ans Let $\alpha = (a_1, b_1, c_1) \in W$ and $\beta = (a_2, b_2, c_2) \in W$

As $\alpha, \beta \in W$, $a_1 - 2b_1 + c_1 = 0$ and $a_2 - 2b_2 + c_2 = 0$

Now for $a, b \in R$, $a\alpha + b\beta = (aa_1 + ba_2, ab_1 + bb_2, ac_1 + bc_2)$

$$\begin{aligned} aa_1 + ba_2 - 2(ab_1 + bb_2) + (ac_1 + bc_2) &= aa_1 + -2ab_1 + ac_1 + ba_2 - 2bb_2 + bc_2 \\ &= a(a_1 - 2b_1 + c_1) + b(a_2 - 2b_2 + c_2) \\ &= a0 + b0 = 0 \end{aligned}$$

It shows that $a\alpha + b\beta \in W$ for arbitrary $\alpha, \beta \in W and a, b \in R$. So W is a subspace of R^3.
$(a, b, c) = (2b - c, b, c) = b(2, 1, 0) + c(-1, 0, 1)$ using $a - 2b + c = 0$
Thus, the vector $\alpha = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \beta = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ will generate the W. Additionally, $\{\alpha, \beta\}$ are L.I.
So the basis of $W = \{\alpha, \beta\} = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$
The dimension of W = number of element in basis = 2.

(b) Consider a second order linear differential equation $xy'' - (1 + x)y' + y = x^2e^{2x}, x > 0$. If e^x is a solution of corresponding homogeneous equation then find the general solution of the given differential equation. [8]

Sol: By divind x throughout the equation we write the standard form as

$$y'' - \frac{(1 + x)}{x}y' + \frac{1}{x}y = xe^{2x}.$$

$y_1 = e^x$ is one of the solution of corresponding homogeneous part $y'' - \frac{(1+x)}{x}y' + \frac{1}{x}y = 0$.
The second LI solution of corresponding homogeneous equation is $y_2 = y_1(x)v(x)$, where

$$v(x) = \int \frac{1}{y_1^2} e^{-\int P(x)dx} dx = \int \frac{1}{y_1^2} e^{\int \frac{1+x}{x} dx} dx = -e^{-x}(1 + x)$$

and $y_2 = y_1v = -(1 + x)$
Now the particular solution of the given non-homogeneous equation is given by

$$y_p(x) = y_1 \int \frac{-y_2xe^{2x}}{W(y_1, y_2)} + y_2 \int \frac{y_1xe^{2x}}{W(y_1, y_2)}$$

Note that $W(y_1, y_2) = W(e^x, -(1+x)) = xe^x$
$\displaystyle\int \frac{-y_2 xe^{2x}}{W(y_1, y_2)} = xe^x$ and $\displaystyle\int \frac{y_1 xe^{2x}}{W(y_1, y_2)} = \frac{e^{2x}}{2}$.
So $y_p(x) = xe^{2x} - (1+x)e^{2x}/2 = \frac{e^{2x}}{2}(x-1)$.
So the general solution of the given ODE is $y(x) = c_1 y_1 + c_2 y_2 + y_p(x) = c_1 e^x + c_2(1+x) + \frac{e^{2x}}{2}(x-1)$

2. (a) Find the value of k in the matrix $A = \begin{pmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & k & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ such that the eigenspace for the eigenvalue $\lambda = 5$ is two-dimensional. Justify answer. [6]

Ans The eigenspace corresponding to eigenvalue $\lambda = 5$ is the Null space of the matrix $A - 5I$. Compute the null space of $A-5I$ using row echelon form of $A-5I = \begin{pmatrix} 0 & -2 & 6 & -1 \\ 0 & -2 & k & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 & 6 & -1 \\ 0 & 0 & k-6 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 & 6 & 0 \\ 0 & 0 & k-6 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Now $rank(A) + dim(Nul(A)) =$ no of column (4). It will also true for $A - 5I$ and we get

$$rank(A-5I) + dim(Nul(A-5I)) = 4$$

It is clear that if $k \neq 6$, we have 3 pivot columns, which gives $dim(Nul(A-5I)) = 4 - rank(A-5I) = 4 - 3 = 1 < 2$.

If $k = 6$, then $rank(A-5I) = 2$, which gives $dim(Nul(A-5I)) = 4 - 2 = 2$.

We conclude that k must equal 6 in order for the eigenspace corresponding to eigenvalue $\lambda = 5$ to be two-dimensional.

(b) Solve $y'' + 2xy = 0$ using power series method. [6]

Sol: Note that here $P(x) = 0, Q(x) = 2x$ which are analytic everywhere and hence every point is an ordinary point. The given DE admits a power series solution of the form

$$y = \sum_{n=0}^{\infty} a_n x^n$$

By differentiating term by term twice we get

$$y''(x) = \sum_{n=0}^{\infty} (n+1)(n+2)a_{n+2}x^n, \quad 2xy(x) = 2\sum_{n=0}^{\infty} a_n x^{n+1}.$$

Substituting into the DE, we get

$$\sum_{n=0}^{\infty} (n+1)(n+2)a_{n+2}x^n = -2\sum_{n=0}^{\infty} a_n x^{n+1} = -2\sum_{n=1}^{\infty} a_{n-1}x^n.$$

By rewriting

$$2.1a_2 + \sum_{n=1}^{\infty} (n+1)(n+2)a_{n+2}x^n = -2\sum_{n=1}^{\infty} a_{n-1}x^n.$$

For this equation to be satisfied for all x in some interval, the coefficients of like powers of x must equal; hence $a_2 = 0$, and we obtain the following recurrence relation

$$(n+2)(n+1)a_{n+2} = -2a_{n-1} \forall n = 1, 2, 3 \cdots$$

Since $a_2 = 0$, $a_5 = a_8 = a_{11} = \cdots = 0$.

For the sequence $a_0, a_3, a_6, a_9, \ldots$

$$a_3 = -2\frac{a_0}{2\cdot 3}, \qquad a_6 = -2\frac{a_3}{5\cdot 6} = (-1)^2 2^2 \frac{a_0}{2\cdot 3\cdot 5\cdot 6}, \qquad a_9 = -2\frac{a_6}{8\cdot 9} = (-1)^3 2^3 \frac{a_0}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9}, \ldots.$$

This suggests the general formula

$$a_{3n} = (-1)^{[n]} 2^{[n]} \frac{a_0}{2\cdot 3\cdot 5\cdot 6\cdots(3n-1)(3n)}, n \geq 4.$$

For the sequence $a_4, a_7, a_{10}, \ldots$

$$a_4 = -2\frac{a_1}{3\cdot 4}, \qquad a_7 = -2\frac{a_4}{6\cdot 7} = (-1)^2 2^2 \frac{a_1}{3\cdot 4\cdot 6\cdot 7}, \qquad a_{10} = -2\frac{a_7}{9\cdot 10} = (-1)^3 2^3 \frac{a_1}{3\cdot 4\cdot 6\cdot 7\cdot 9\cdot 10}, \ldots.$$

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This suggests the general formula

$$a_{3n+1} = (-1)^{[n]} 2^{[n]} \frac{a_1}{3\cdot 4\cdot 6\cdot 7\cdots(3n)(3n+1)}, n \geq 4.$$

Thus the general solution for the given equation is

$$\begin{aligned} y &= a_0\left[1 - \frac{2x^3}{2\cdot 3} + \frac{2^2x^6}{2\cdot 3\cdot 5\cdot 6} - \cdots + \frac{(-1)^{[n]}2^{[n]}x^{3n}}{2\cdot 3\cdots(3n-1)(3n)} + \cdots\right] \\ &\quad + a_1\left[x - \frac{2x^4}{3\cdot 4} + \frac{2^2x^7}{3\cdot 4\cdot 6\cdot 7} - \cdots + \frac{(-1)^{[n]}2^{[n]}x^{3n+1}}{3\cdot 4\cdots(3n)(3n+1)} + \cdots\right] \\ &= a_0y_1 + a_1y_2. \end{aligned}$$

By ratio test y_1 & y_2 converge for all x and since the ratio of y_1 & y_2 is a non-constant function, they are LI and hence forms a basis for the given problem.

3. (a) Solve $(xy\sin xy + \cos xy)y\ dx + (xy\sin xy - \cos xy)x\ dy = 0$ [8]

Ans $M = y(xy\sin xy + \cos xy)$ and $N = x(xy\sin xy - \cos xy)$
Then, $\frac{\partial M}{\partial y} = 2xy\sin xy + x^2y^2\cos xy + \cos xy - xy\sin xy$
$\frac{\partial N}{\partial x} = 2xy\sin xy + x^2y^2\cos xy - \cos xy + xy\sin xy$
Clearly, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$
However, $Mx - Ny = 2xy\cos xy \neq 0$. Hence Integrating factor $(IF) = \frac{1}{Mx-Ny} = \frac{1}{2xy\cos xy}$

Multiplying the DE with IF, we get

$\frac{1}{2}(y\tan xy + \frac{1}{x})\ dx + \frac{1}{2}(x\tan xy - \frac{1}{y})\ dy = 0$, which gives
$M = \frac{1}{2}(y\tan xy + \frac{1}{x})$ and $N = \frac{1}{2}(x\tan xy - \frac{1}{y})$

Now clearly it is exact, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{1}{2}(\tan xy + xy\sec^2 xy)$

So sol,
$\int(y\tan xy + \frac{1}{x})\ dx + \int(-\frac{1}{y})\ dy = logc$

$\Rightarrow \log\sec xy + \log\frac{x}{y} = logc$ or $\frac{x}{y}\sec xy = c$.

(b) If y_1 and y_2 are two linearly independent solutions of $y'' + p(x)y' + q(x)y = 0, x \in I$, then prove that between two consecutive zeros of y_1 there exists unique zero of y_2 in I. (Here I is an open interval) [6]

Sol. On the contrary, suppose there is no zeros of y_2 between two zeros of y_1 at $x = a, b$. Without loss of generality, suppose $y_2(x) > 0$ for $x \in [a, b]$ and ${y'}_1(a) > 0, {y'}_1(b) < 0$.
Then $W(y_1, y_2) < 0$ at $x = a$ and $x = b$. This contradicts that y_1, y_2 are basis solutions.
For uniquness, let there exist two zeros between $x = a$ and $x = b$. Using the same argument (reversing the role of y_1, y_2), we conclude that y_1 has a zero between zeros of y_2 and hence in (a, b), which is a contardiction.

4. Find all the nontrivial solutions of the boundary value problem $y'' + \lambda y = 0$ for $x \in (0, \frac{\pi}{2})$ with $y(0) = 0, y'\left(\frac{\pi}{2}\right) = 0$. Compute the coefficients c_n for the eigenfunction expansion of the function $f(x) = x$ on $[0, \frac{\pi}{2}]$. [9]

Sol. Case I: $\lambda = 0$ gives $y(x) = Ax + B$. Boundary Condition implies $y \equiv 0$.
Case II: Let $\lambda < 0$ and $\lambda = -n^2$. The solution is $y(x) = Ae^{nx} + Be^{-nx}$. Boundary condition implies $y \equiv 0$.
Case III: Let $\lambda > 0$ and $\lambda = n^2$. The solution is $y(x) = A\cos nx + B\sin nx$.
$y(0) = 0 \Rightarrow A = 0$. $y'(\pi/2) \Rightarrow Bn\cos\frac{n\pi}{2} = 0$. $B = 0$ implies $y \equiv 0$, so assume $B \neq 0$.
Hence, $cos\frac{n\pi}{2} = 0$ and $\frac{n\pi}{2} = \frac{2m-1}{2}\pi, m = 0, \pm1, \pm2, \cdots$.

The eigen values are $\lambda_m = n^2 = (2m-1)^2, m = 1, 2, \cdots$.
By taking $B = 1$, the corresponding eigen functions are

$$y_m(x) = \sin(2m-1)x, m = 1, 2, 3, \cdots$$

For $f(x) = x$ defined on $[0, \frac{\pi}{2}]$, we can represent $f(x) = x$ as $f(x) = x = \sum_{m=1}^{\infty} c_m y_m(x)$.
Multiply y_k both sides and integrate from 0 to $\pi/2$ w.r.t. x. Since eigenfunctions are orthogonal we get

$$c_m = \frac{\int_0^{\pi/2} x\sin(2m-1)x dx}{\int_0^{\pi/2}\sin^2(2m-1)x dx} = \frac{4}{\pi}\int_0^{\pi/2} x\sin(2m-1)x dx.$$

By solving we get $c_m = \begin{cases} \frac{4}{\pi(2m-1)^2} & m = 1, 3, 5\cdots \\ -\frac{4}{\pi(2m-1)^2} & m = 2, 4, 6\cdots \end{cases}$

5. (a) Let $P_n(x)$ be the Legendre polynomial of degree n. Then prove that [6]

$$\|P_n(x)\|^2 = \int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$$

(Hint: Use Rodrigues' formula $P_n(x) = \dfrac{1}{2^n n!}\dfrac{d^n}{dx^n}[(x^2-1)^n]$)
Sol: Let $f(x)$ be any function with at least n continuous derivatives in $[-1, 1]$. Consider the integral

$$I = \int_{-1}^{1} f(x)P_n(x)dx = \frac{1}{2^n n!}\int_{-1}^{1} f(x)\frac{d^n}{dx^n}(x^2-1)^n dx.$$

Repetition of integration by parts gives

$$I = \frac{(-1)^n}{2^n n!}\int_{-1}^{1} f^n(x)(x^2-1)^n dx.$$

If $f(x) = P_n(x)$, then

$$f^n(x) = \frac{1}{2^n n!}\frac{d^{2n}}{dx^{2n}}(x^2-1)^n = \frac{(2n)!}{2^n n!}.$$

Thus

$$I = \frac{(2n)!}{2^{2n}(n!)^2}\int_{-1}^{1}(1-x^2)^n dx = \frac{2(2n)!}{2^{2n}(n!)^2}\int_0^1 (1-x^2)^n dx.$$

Substituting $x = \sin\theta$, we get

$$I = \frac{2(2n)!}{2^{2n}(n!)^2}\int_0^{\pi/2}\cos^{2n+1}\theta d\theta = \frac{2(2n)!}{2^{2n}(n!)^2}I_n.$$

Since

$$\int\cos^{2n+1}\theta d\theta = \frac{1}{2n+1}\cos^{2n}\theta\sin\theta + \frac{2n}{2n+1}\int\cos^{2n-1}\theta d\theta,$$

we find

$$I_n = \frac{2n}{2n+1} I_{n-1} = \frac{2n}{2n+1} \frac{2n-2}{2n-1} \cdots \frac{2}{3} I_0.$$

Now

$$I_0 = \int_0^{\pi/2} \cos\theta d\theta = 1.$$

Hence,

$$I_n = \frac{2^n n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)(2n+1)} = \frac{2^{2n}(n!)^2}{(2n)!(2n+1)}.$$

Thus, we get the desired result.

(b) Let A be a 6×9 matrix ($A = [a_{ij}]_{6\times 9}$). Could A have a two dimensional null space? What is the largest possible dimension of the column space of A? Justify each answer. [6]

Ans The dimension of null space can not be two as Maximum rank is 6 and number of columns are 9, then by rank-nullity theorem,

$$rank + nullity = 9 \ (number of column)$$

Nullity represents the dimension of null space. So, minimum dimension of null space means Maximum rank. This shows that the Nullity cannot be less than 3 (9 - 6).

The largest possible dimension of the column space of A can be achieved by minimizing the dimension of null space. As minimum dimension of null space is 3, largest possible dimension of the column space of A is 6 (9 - 3).

6. (a) Using the **Laplace transformation**, solve the differential equation $xy'' + (1-x)y' + ny = 0$ where n is an non-negative integer. [8]

Sol: We determine a solution of it with $n = 0, 1, 2, \cdots$. The subsidary equation is

$$-\frac{d}{ds}(s^2Y(s) - sy(0) - y'(0)) + (sY(s) - y(0)) + \frac{d}{ds}(sY(s) - y(0)) + nY(s) = 0$$

$$\text{or, } (-2sY(s) - s^2\frac{dY(s)}{ds} + y(0)) + (sY(s) - y(0)) + (Y(s) + s\frac{dY(s)}{ds}) = 0$$

$$\text{or, } (s - s^2)\frac{dY(s)}{ds} + (n - 1 + s)Y(s) = 0 \text{ or, } \frac{dY}{Y} = (\frac{n}{s-1} - \frac{n+1}{s})ds$$

Integrating, we get $Y = \dfrac{(s-1)^n}{s^{n+1}}$. We write $l_n = \mathcal{L}^{-1}(Y)$ and prove Rodrigues's formula

$$l_0 = 1, l_n(t) = \frac{e^t}{n!}\frac{d^n(t^n e^{-t})}{dt^n}, \ n = 1, 2, 3, \cdots$$

These are polynomials because the exponential terms canceal if we perform the indicated differentiations.

They are called Laguerre polynomials. We know by shipting theorem $\mathcal{L}(t^n e^{-t}) = \dfrac{n!}{(s+1)^{n+1}}$.

Hence by the Theorem of Laplace Transform of the derivative f^n we can write

$\mathcal{L}(\dfrac{d^n(t^n e^{-t})}{dt^n}) = \dfrac{n! s^n}{(s+1)^{n+1}}$ because the derivatives to the order $n-1$ are zero at 0.

Then make another sift and dividing by $n!$ to get $\mathcal{L}(l_n) = \dfrac{(s-1)^n}{s^{n+1}} = Y$

(b) Show that the following second order linear non-homogeneous IVP has unique solution. [5]

$$y'' + 2x^2y'e^x + y\cos x = \sin x$$
$$y(0) = 2, \qquad y'(0) = 1.$$

Sol: Let y_1 and y_2 are two solutions of the given IVP. Take $z = y_2 - y_1$. Then z satisfies the following IVP:

$$z'' + 2x^2 z' e^x + z \cos x = 0$$
$$z(0) = 0, z'(0) = 0.$$

This is a homogeneous equation with initial conditions. Since the coefficient functions $x^2 e^x$ and $\cos x$ are continuous, by existence and uniqueness theorem this IVP has unique solution z.

$z = 0$ is a trivial solution of this ODE and also satisfy the initial conditions and hence $z \equiv 0$ is the only solution. Therefore, $y_1 = y_2$.

7. (a) Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0. [6]

Ans Suppose that λ is an eigenvalue of A. Then there is an eigenvector v such that $Av = \lambda v$. Multiplying both sides by A, we get that

$$0 = A^2 v = A\lambda v = \lambda A v = \lambda(\lambda v) = \lambda^2 v$$

Thus $\lambda^2 v = 0$. As per the definition of eigenvector, v must be non-zero. So, we conclude that $\lambda^2 = 0$, and hence $\lambda = 0$.

(b) Find Fourier sine and cosine series of $f(x) = x,\ 0 < x < \pi$ and use this to find the sum of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$. [8]

Sol: For sin series, $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$. Here $b_n = \frac{2}{\pi}\int_0^\pi x \sin nx dx = -\frac{2x\cos nx}{n\pi}|_0^\pi + \frac{2}{n\pi}\int_0^\pi \cos nx dx = -\frac{2\cos n\pi}{n} + \frac{2\sin nx}{n^2\pi}|_0^\pi = \frac{-2(-1)^n}{n}$. Hence $x = 2(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \cdots)$

For cosine series, $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$. Here $a_0 = \frac{2}{\pi}\int_0^\pi x dx = \frac{2}{\pi}\frac{\pi^2}{2} = \pi$. For $n = 1, 2, 3, \cdots$; $a_n = \frac{2}{\pi}\int_0^\pi x \cos nx dx = \frac{2x \sin nx}{n\pi}|_0^\pi - \frac{2}{n\pi}\int_0^\pi \sin nx dx = 0 + \frac{2\cos nx}{n^2\pi}|_0^\pi = \frac{2(\cos n\pi - 1)}{n^2\pi}$. So $a_{2n} = 0$ $a_{2n-1} = -\frac{4}{(2n-1)^2\pi}$, $n = 1, 2, 3\cdots$. Hence $x = \frac{\pi}{2} - \frac{4}{\pi}(\cos x + \frac{\cos 3x}{9} + \frac{\cos 5x}{25} + \frac{\cos 7x}{49} + \cdots)$

If we put $x = 0$ in the cosine series, then $0 = \frac{\pi}{2} - \frac{4}{\pi}(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots)$. Hence $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

8. (a) Sketch the following functions and then find it's Laplace transforms: [6]

$$f(t) = \begin{cases} t[u(t) - u(t-1)], & 0 \le t < 2, \\ f(t-2), & t > 2. \end{cases}$$

Sol:

$$f(t) = \begin{cases} t, & 0 < t < 1, \\ 0, & 1 < t < 2, \\ f(t-2), & t > 2. \end{cases}$$

Here f is periodic of period 2. The sketch of the function is given in Figure 1.

Then $\mathcal{L}(f(t)) = \dfrac{1}{1-e^{-2s}}\displaystyle\int_0^2 e^{-st} f(t) dt = \dfrac{1}{1-e^{-2s}}\int_0^1 te^{-st} dt$

$= \dfrac{1}{1-e^{-2s}}(\dfrac{te^{-st}}{-s}|_0^1 + \dfrac{1}{s}\displaystyle\int_0^1 e^{-st} dt) = \dfrac{1}{1-e^{-2s}}(-\dfrac{e^{-s}}{s} - \dfrac{e^{-st}}{s^2}|_0^1) = \dfrac{1}{1-e^{-2s}}(-\dfrac{e^{-s}}{s} - \dfrac{e^{-s}-1}{s^2}) = \dfrac{1 - e^{-s} - se^{-s}}{s^2(1-e^{-2s})}$

(b) Find sine integral of $f(x) = e^{-x}, x > 0$ and then evaluate the integral $\displaystyle\int_0^\infty \frac{\omega \sin x\omega}{1+\omega^2} d\omega$. [6]

Sol: We have $B(\omega) = \frac{2}{\pi}\int_0^\infty e^{-v}\sin\omega v dv$. By integration by parts, $\int e^{-v}\sin\omega v dv = -\frac{\omega e^{-v}}{1+\omega^2}(\frac{1}{\omega}\sin\omega v + \cos\omega v)$.

This equal to $-\frac{\omega}{1+\omega^2}$ if $v = 0$ and approaches 0 as $v \to \infty$. Thus $B(\omega) = \frac{2\omega}{\pi(1+\omega^2)}$.

Therefore $f(x) = e^{-x} = \int_0^\infty B(\omega)\sin\omega x d\omega = \frac{2}{\pi}\int_0^\infty \frac{\omega \sin \omega x}{(1+\omega^2)} d\omega$. From this we see that $\int_0^\infty \frac{\omega\sin\omega x}{(1+\omega^2)} d\omega = \frac{\pi}{2}e^{-x}$

End of paper

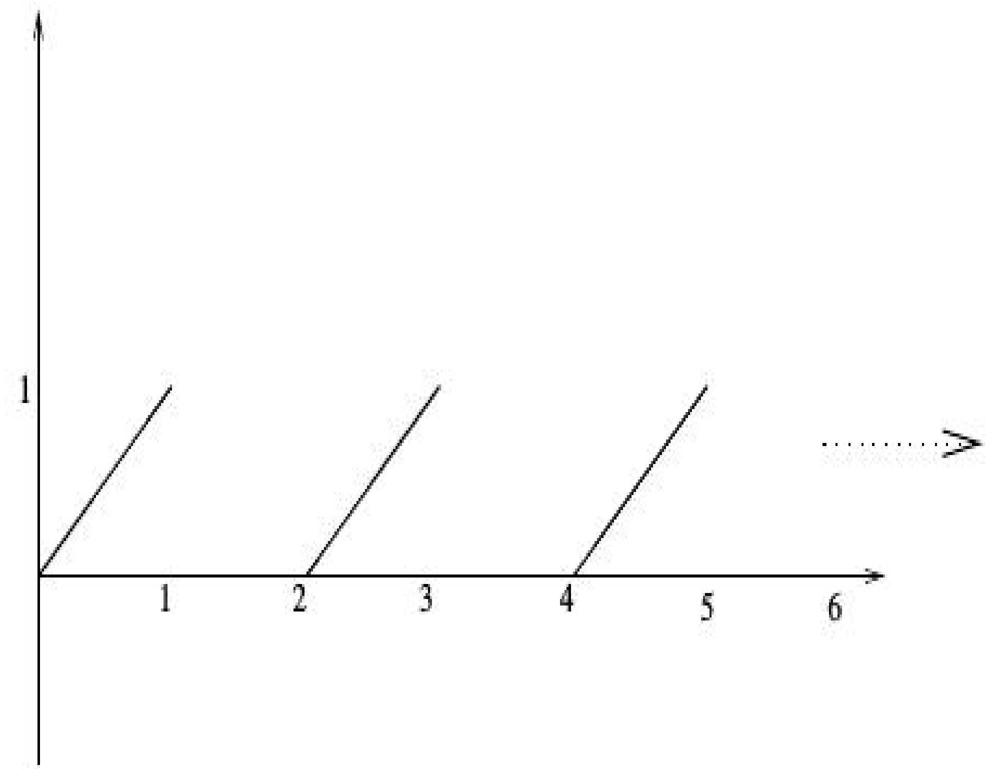

Figure 1: The sketch