The LNM Institute of Information Technology

Jaipur, Rajasthan

Discrete Mathematical Structures

Assignment # 1

Group Theory

Q1. Let G be a group and let a, b $\in$ G. For any + ve integer n, we have $(a^{-1}ba)^n = a^{-1}b^na$. (Prove this by induction)

Q2. The set of 2X2 matrices of the form $\begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$ with entries from Q and satisfying $a \neq 0$, $d \neq 0$ forms a group under ordinary multiplication. (Over what other number systems, besides Q, is this conclusion still valid?)

Q3. Let G be a group and a,b $\in$ G. Then $(ab)^2 = a^2b^2$ if and only if ab=ba.

Q4. If G be a group in which $a^2 = e$ for all a $\in$ G, then G is abelian.

Q5. Which of the following subsets of $\mathbb{Z}$ are groups under addition? Give brief reasons for your answer.

a. A=Set of even integers.
b. B=Set of odd integers.
c. C=Set of non-negative integers.
d. D= {0}.
e. E=Set of integers which are expressible as 42m + 1023n for integers m, n.

Q6. Which of the following subsets of C (set of complex nos.) are groups under multiplication? Give brief reasons for your answers.

a. A={a+bi : a>0}
b. B={1, π, π^2 , π^3 , …}

Q7. Check whether the following forms a group or not?

G: Set of rational numbers under composition * defined as a*b=ab/2, a, b $\in$ G.

Q8. Let H be a subgroup of G and let N = $\cap$ xHx^{-1} $\forall x \in G$ then show that N is a normal subgroup of G.

Q9. Let G be a group. Suppose a,b $\in$ G such that

a. ab = ba
b. (o(a) , o(b)) = 1

Show that o(ab) = o(a).o(b).

Q10. Let G be a group. Show that

$$O(a^n) = \frac{O(a)}{(O(a),n)} \quad \forall\, a \in G$$

where n is an integer and (o(a) , n) = gcd(o(a) , n).

Q11. Converse of Lagrange's Theorem holds on finite cyclic groups.

Q12. Let G be a finite group whose order is not divisible by 3. Suppose $(ab)^3 = a^3b^3 \;\; \forall\, a,b \in G$ then show that G is abelean.

Q13. If $a \in G$ be of finite order n and also $a^m = e$ then show that $n|m$.