Propositional & Predicate Logic

* Propositional logic:

All sentences : either T or F all are declarative statment, or proposition

Ex: $2+3=5$ ✓ T   Ex: $2+5=9$ × F

- Negation:

Representⁿ: $\neg P$ or $\sim P$ or $\overline{P}$

If p = True
 $\neg P$ = False

→ For 2 propositⁿ: p, q, disjunction of p & q is represented as $p \vee q$

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

→ Conjunction of p & q is $p \wedge q$

* Exclusive OR $\oplus$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Answer these q's : Not a statement.

→ $p \rightarrow q$ (conditional statement):
(if p then q)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ $p \leftrightarrow q$ (Biconditional statement): = Negat^n of ⊕

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$(p \leftrightarrow q) = (p \rightarrow q) \wedge (q \rightarrow p)$

Ex: Construct T.T of given composite statement:
$(p \vee \neg q) \rightarrow (p \wedge q)$

Sol^n →

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	Ans
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

* Precedence Order:

(1) ∃ or ∀
(2) ¬
(3) ∧
(4) ∨
(5) →
(6) ↔
(7) ⊕

if true/false - contingency.

Ex: $(p \oplus q) \wedge (p \oplus \neg q)$

Sol^n:

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

} Fallacy or contradic^n

* Propositional Equivalence:

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

} Tautology

Def# Logically Equivalent: The compound proposition p & q are logically equivalent if $p \leftrightarrow q$ is a tautology

Ex: $\neg(P \vee q)$ and $\neg P \wedge \neg q$

Sol^n:
method: 1

P	q	$\neg p$	$\neg q$	$P \vee q$	$\neg(P \vee q)$		$\neg P \wedge \neg q$
T	F	F	T	T	F	=	F
F	T	T	F	T	F	=	F
T	T	F	F	T	F	=	F
F	F	T	T	F	T	=	T

same

OR $\neg(P \vee q) \leftrightarrow \neg p \wedge \neg q$

Method: 2

$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
F	F	T
F	F	T
F	F	T
T	T	T

} Tautology

or

$\equiv$ logically equivalent

Laws/Rules:

→ De-Morgan's law:

(1) $\neg(p_1 \vee p_2 \vee p_3 \cdots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \wedge \neg p_3 \cdots \wedge \neg p_n$

(2) $\neg(p_1 \wedge p_2 \wedge p_3 \cdots \wedge p_n) \equiv \neg p_1 \vee \neg p_2 \vee \neg p_3 \cdots \vee \neg p_n$

→ Identity law:

(1) $p \wedge T \equiv p$

→ Domination law:

(1) $p \vee F \equiv p$

→ Distributive law:

(1) $(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$

→ Associative law:

(1) $(p \vee q) \vee r = p \vee (q \vee r)$

→ $p \vee p \equiv p$

→ $p \vee (p \wedge q) \equiv p$ (Absorption law)

→ $p \wedge p \equiv p$

→ $\neg(\neg p) \equiv p$

→ $p \vee (\neg p) \equiv T$

→ $p \wedge (\neg p) \equiv F$ (Negation)

$p \to q$
$\equiv \neg p \vee q$

Q Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q \equiv \neg p \vee \neg(\neg p \wedge q)$

Sol^n $\neg(p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ (Demorgan's law)

$\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$ (Demorgan's law)

$\equiv \neg p \wedge (p \vee \neg q)$ ($\because \neg(\neg p) = p$)

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ (Distributive law)

$\equiv F \vee (\neg p \wedge \neg q)$ ($\because \neg p \wedge p \equiv F$)

$\equiv \neg p \wedge \neg q$ ($\because F \vee p' \equiv p' \Rightarrow$ Dominat^n law)

Q $[\neg p \wedge (p \vee q)] \to q$

Sol^n

p	q
T	T
T	F
F	T
F	F

method (1)

OR

method (2)

$[\neg p \wedge (p \vee q)]$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge q)$ (Distributive law)

$\equiv F \vee (\neg p \wedge q)$

$\equiv (\neg p \wedge q)$ (Domination law)

Now, check $(\neg p \wedge q) \to q$

p	q	$\neg p$	$\neg p \vee q$	q	$\neg p \wedge q$	$\neg p \wedge q \to q$
T	T	F	T	T	F	T
T	F	F	F	F	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

Tautology

* Subject-Predicate logic:

Ex: x is $\underbrace{\text{greater than } 3}$

$\downarrow$ Subject — Predicate

$\Downarrow$

$P(x)$: x is greater than 3

$P(1)$ is false
$P(4)$ is True

$\forall x P(x) = F$ for $x = 1,2,3$
$\rightarrow$ universal quantifier
$\forall x P(x) = T$ for $x = 4,5,6,\ldots$

$\exists x P(x) = F$ for $x = 1,2,3$
$\rightarrow$ there exists quantifier.

Ex: $x = y+1$
$P(x,y): x = y+1$
$P(2,1)$ is true while $P(3,1)$ is False

Similiarly, $P(x_1, x_2, \ldots, x_n)$ = entouple

Ex: M3 is functioning properly.
Solⁿ: $P(M_3)$ is true if M_3: functioning properly.

Statement	When True?	When False?
$\forall x P(x)$	$P(x)$ is T for every x	$P(x) = F$ for some x
$\exists x P(x)$	$P(x)$ is T for some x	$P(x) = F$ for every x

Ex: ~~Rulester~~ $Q(x)$ be statetement "$x < 2$". What is truth value of the quantification $\forall x Q(x)$ where domain consists of all natural no.

Solⁿ: $Q(x)$ is not true for every real no. x because for instant $P(3)$ = false, that is $x = 3$ is a counter example for this statement $\forall x Q(x)$
Thus, $\forall x Q(x)$ is False.

$\neg(x \rightarrow y) \equiv x \wedge \neg y$

\# $\exists x P(x)$ is same as $P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$

Let

$P(x): x^2 > 10$; Domain: $x = 1, 2, 3, 4$

$P(1) \vee P(2) \vee P(3) \vee P(4)$

$= F \vee F \vee F \vee T$

$= T$

$\therefore$, $\exists x P(x)$ is T

$\rightarrow$ $\exists$ and $\forall$ has higher precedence than $\neg$

$\forall x P(x) \vee Q(x) \neq \forall x (P(x) \vee Q(x))$

$= (\forall x P(x)) \vee Q(x)$

$\rightarrow$ $\neg \forall x P(x) \equiv \exists x \neg P(x)$

$\rightarrow$ $\neg \exists x P(x) \equiv \forall x \neg P(x)$

Ex: $\forall x(x^2 > 2) \wedge \exists x(x^2 = 2)$

Soln: $\neg[\forall x(x^2 > 2) \wedge \exists x(x^2 = 2)]$

$\exists x(x^2 \leq 2)$ $\cancel{\wedge}$ $\vee$ $\forall x(x^2 \neq 2)$

Ex: Show that:

$\neg \forall x (P(x) \rightarrow Q(x))$ and

$\exists x (P(x) \wedge \neg Q(x))$ are logically equivalent.

Soln: $\neg \forall x (P(x) \rightarrow Q(x)) \equiv \exists x \neg (P(x) \rightarrow Q(x))$

$\equiv \exists x (P(x) \wedge \neg Q(x))$

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$\forall x \exists y (x+y=0)$
$\equiv \forall x (\exists y (x+y \not\equiv 0))$

Ex The sum of all +ve integers is always +ve.

Sol^n: $\forall x \forall y ((x>0) \wedge (y>0) \longrightarrow (x+y>0))$
OR

x is positive = $P(x)$
y is positive = $P(y)$

$\therefore \forall x \forall y ((P(x) \wedge P(y)) \longrightarrow P(x,y)) \Rightarrow$ Predicate representation

Ex: Translate $\forall x\, C(x) \vee \exists y (C(y) \wedge F(x,y))$ into English.
$C(x)$ is "x has computer"
$F(x,y)$ is "x & y are friends"
x, y are students in your school.

Sol^n: For every student x in your school, x has computer or there is a student y such that y has a computer and x & y are friends.

Boolean Algebra

$B = \{0, 1\}$ $\quad B^n = \{x_1, x_2, \ldots, x_n\}$

$F : B^n \to B$ $\quad ; F = $ Boolean f^n

Ex: $F(x, y, z)$ s.t. $x, y, z \in \{0, 1\}$

$1 + 1 = 1$	$1 \times 1 = 1$	$\neg 0 = 1$
$1 + 0 = 1$	$1 \times 0 = 0$	$\neg 1 = 0$
$0 + 1 = 1$	$0 \times 1 = 0$	
$0 + 0 = 0$	$0 \times 0 = 0$	
(OR)	(AND)	

Ex: $1 \cdot 0 + \overline{(0+1)} = 0 + \overline{T} = 0 + 0 = 0$ $\quad$ | $1 = T$, $0 = F$

$= T \land F \lor \neg(F \lor T)$

$= F$

Ex: $F(x, y, z) = xy + \overline{z}$

x	y	z	$\overline{z}$	$x \cdot y$	$x \cdot y + \overline{z}$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	1	0	0	0
0	1	0	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	1	1	0	1	1
1	0	0	1	0	1

Pg: 59

* Rules:

1. $x + x = x$
2. $x \cdot x = x$
3. $x + 0 = x$
4. $x \cdot 1 = x$
5. $x + y = y + x$ } Commutative
6. $x \cdot y = y \cdot x$
7. $\overline{x \cdot y} = \bar{x} + \bar{y}$ } Demorgan's
8. $\overline{(x+y)} = \bar{x} \cdot \bar{y} = \bar{x}\bar{y}$
9. $x + (y+z) = (x+y) + z$ } ~~Distrib~~ Associative
10. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$
11. $x + xy = x$ } Absorption law
12. $x(x+y) = x$
13. $x + (y.1)$ has dwell $= x \cdot (y+0)$

* Sum of products:

Ex: $F(x,y,z) = (x+y)\bar{z}$

$= x\bar{z} + y\bar{z}$

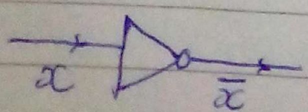
$= x\bar{z} \cdot 1 + y\bar{z} \cdot 1$

$= x\bar{z}(y+\bar{y}) + y\bar{z}(x+\bar{x})$

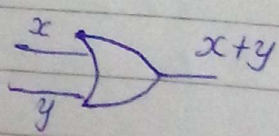
$= x\bar{z}y + x\bar{z}\bar{y} + y\bar{z}x + y\bar{z}\bar{x}$

$= x\bar{z}y + x\bar{z}\bar{y} + y\bar{z}\bar{x}$

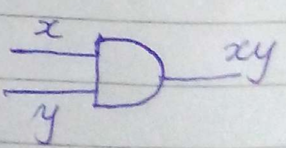
* Logic gate:



Inverter (NOT Gate)



OR Gate



AND gate

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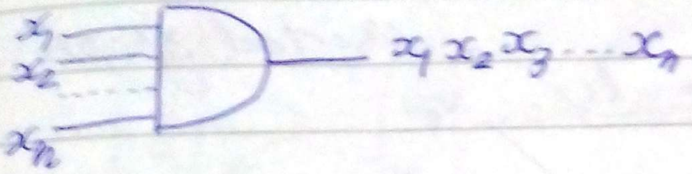
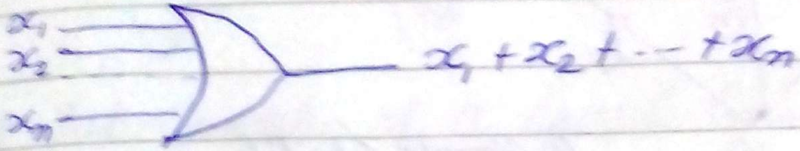
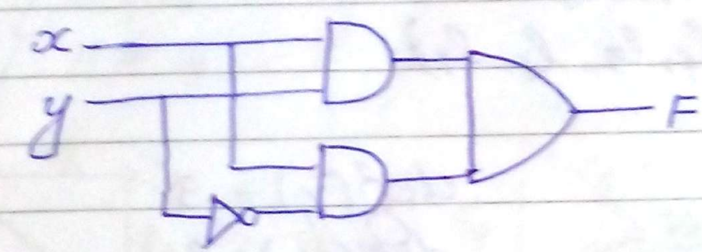
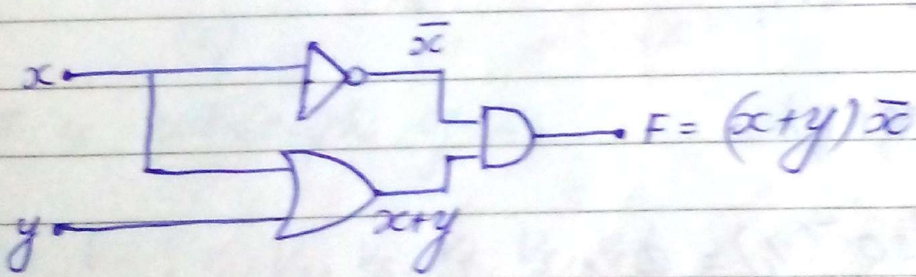




Ex: $F = xy + x\bar{y}$

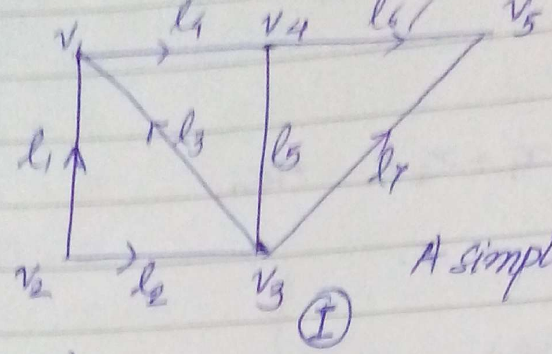


Ex: $F(x, y, z) = (x+y)\bar{x}$



Graph Theory

directed graph

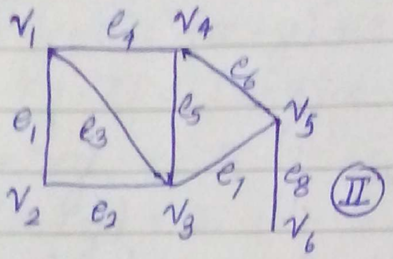


A simple graph

$G = (V, E)$

$V = \{v_1, v_2, v_3, v_4, v_5\}$

$E = \{l_1, l_2, l_3, l_4, l_5, l_6, l_7\}$



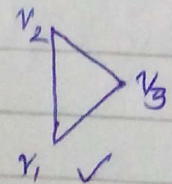
$deg(v_1) = 3$
$deg(v_2) = 2$
$deg(v_3) = 4$

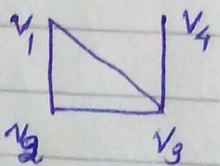
$$2e = \sum_{n=1}^{n} deg(v_n)$$

For ①, $deg^-(v_1) = 2$
$deg^+(v_1) = 1$

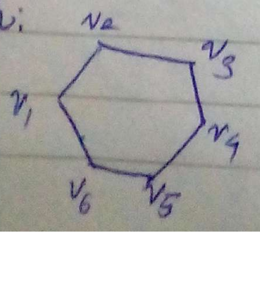
→ The complete graph on n-vertices is denoted by K_n; exactly 1 edge between each pair of distinct vertices.

Ex:

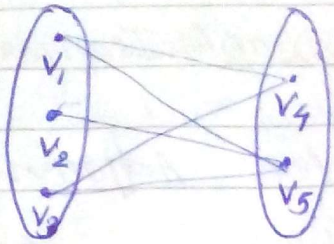




→ Cyclic graph:



$deg(v_2) = 2$

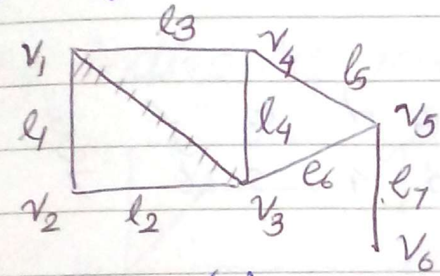


— Bi-parted graph.

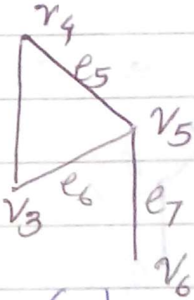
A sub-graph of a graph $G=(V,E)$ is also a graph $H=(W,F)$; where $W \subseteq V$ and $F \subseteq E$

A sub-graph H of G is a proper graph (subgraph) of G if $H \neq G$

Ex



(I)



(II)

II is subgraph of I.

I: $V=\{v_1, v_2, v_3, v_4, v_5, v_6\}$
$E=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

II: $V=\{v_3, v_4, v_5\}$
$E=\{e_4, e_5, e_6, e_7\}$

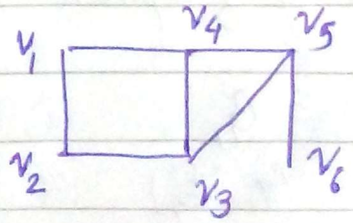
→ $G = G_1 \cup G_2$

$G_1 = (V_1, E_1)$
$G_2 = (V_2, E_2)$

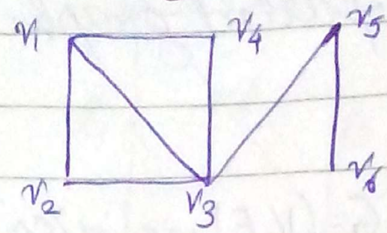
$G = \{(V_1 \cup V_2), (E_1 \cup E_2)\}$

→ Adjacent Vertex:

of $v_1 = v_2, v_4$



* Representing a graph and graph Isomorphism:

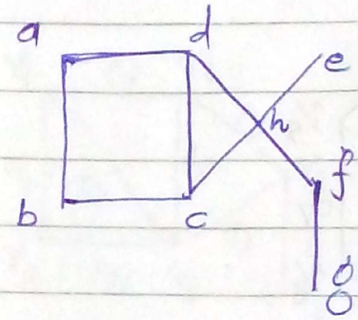


→

Vertex	Adjacency vertices
v_1	v_4, v_2, v_3
v_2	v_1, v_3
v_3	v_1, v_2, v_4, v_5
v_4	v_1, v_3
v_5	v_6, v_3
v_6	v_5

→ Adjacency matrix:

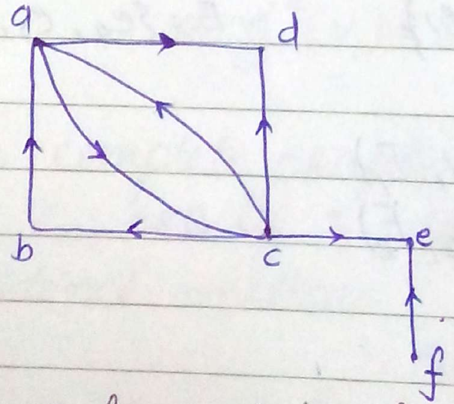
$$A = [a_{ij}] = \begin{cases} 1 & \text{if there's an edge bet}^{n} \text{ vertex } i \,\&\, j \\ 0 & \text{otherwise.} \end{cases}$$



a	b	c	d	e	f	g	h
1	2	3	4	5	6	7	8

$$\begin{array}{c} \begin{array}{cc} a_{11} & a_{12} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & & \\ \vdots & & & & & & & \end{bmatrix} \end{array}$$

→ Adj Terminal Vertices:



Vertex	Terminal Vertex
a	d, c

→ Isomorphism of graph:

G_1
(V_1, E_1)

G_2
(V_2, E_2)

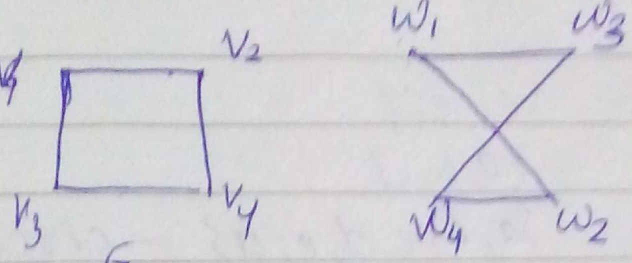
$f : V_1 \rightarrow V_2$ – bijective

$v_1, v_2 \in V_1$

$f(v_1), f(v_2) \in V_2$

If $\exists$ edge betⁿ v_1 & v_2 & a edge also exists betⁿ $f(v_1), f(v_2) \Rightarrow G_1$ & G_2 are isomorphic.

Ex: $(G) = (V, E)$
$H = (W, F)$

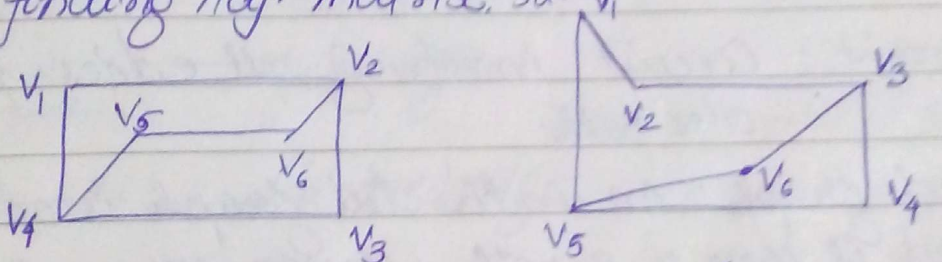


G

w_1	w_2, w_3
w_2	w_1, w_4
w_3	w_1, w_4
w_4	w_2, w_3

✓

(1) By finding Adj. matrix: same



(G) (H)

G:

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	1	0	1	0	0
v_2	1	0	1	0	0	1
v_3	0	1	0	1	0	0
v_4	1	0	1	0	1	0
v_5	0	0	0	1	0	1
v_6	0	1	0	0	1	0

H

	v_6	v_3	v_4	v_5	v_1	v_2
v_6	0	1	0	1	0	0
v_3	1	0	1	0	0	1
v_4	0	1	0	1	0	0
v_5	1	0	1	0	1	0
v_1	0	0	0	1	0	1
v_2	0	1	0	0	1	0

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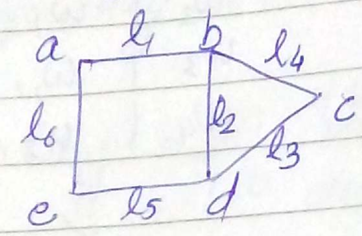
* Connectivity:

Path: It is the sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along the edges of a graph.

Ex:



{e_1, e_4}
{a, b, c}
{a, b, c, d, e, a} – circuit
{e_1, e_4, e_3, e_5, e_6} – x oiler circuit
(∵ e_2 isn't present)

→ Oiler circuit: Circuit involving all edges. path is oiler path.

→ connected graph: An undirected graph is called connected if there is a path between every pair of distinct vertices of this graph.

→ Cut-set:
In connected graph G, a cut-set is a set of edge whose removal from G leaves G disconnected provided removal of no proper subset of these edge disconnects G.

→ Hamiltan path & circuit:
A simple path in a graph G that passes through every vertex exactly once is called Hamiltan path.

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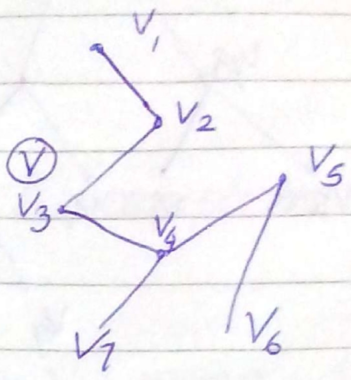
* Tree:

If there exists a unique path betⁿ any 2 distinct vertices.



Tree of n-vertices has $(n-1)$ edges.

$n=1$ —— 0 edge

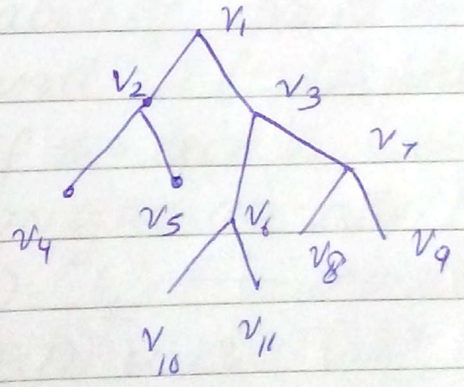
$n=k$ vertices —— $(k-1)$ edges

$n=k+1$ —— $((k-1)+1)$ edges.

* m-ary tree:

→ each-vertex has m-children

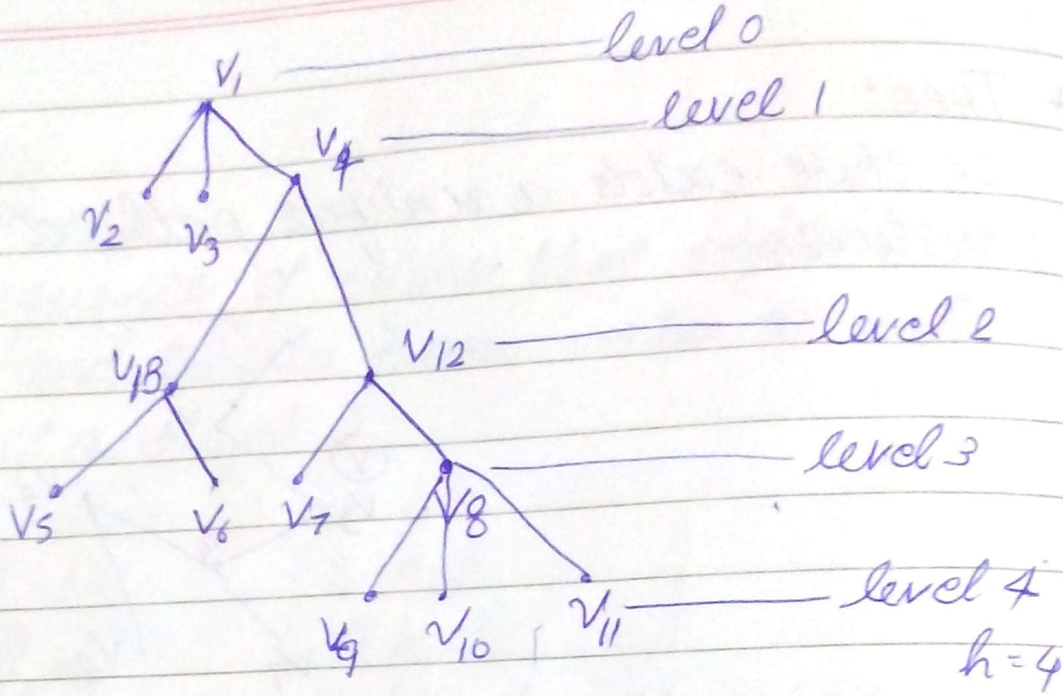
→ A full m-ary tree with i internal vertices contain $n = mi+1$ vertices.



⇒ A full m-ary tree n vertexes.

$i = \frac{n-1}{m}$ internal vertices $\Rightarrow l = \frac{(m-1)n+1}{m}$

l ↓ leaf nodes.



A rooted m-ary tree height h is balanced if all leaves are at level h ~~or h-1~~.

→ Spanning Tree:
Tree formed from a graph G which consists of all vertices of graph is called spanning tree.

Probability Theory

→ E be the event
S be finite sample space
$P(E) = \frac{|E|}{|S|}$

→ Probability of event $\bar{E}$ (complement of E) is:
$P(\bar{E}) = 1 - P(E)$

→ $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

→ Assigning probability:

S be a sample space of an Experiment with a finite or countable no. of outcomes. We assign a probability p(s) to each outcome s.

(i) $0 \leq p(s) \leq 1$ for each $s \in S$
(ii) $\sum_{s \in S} p(s) = 1$

Ex: What probability should be assign to the outcomes H(heads) and T (tails) when a fair coin is flipped.
(i) Case (i): If the coin is biased so that head comes up twice as often as tails?

Soln: $P(H) = P(T) = \frac{1}{2}$ —— unbiased
$P(H) = 2(P(T))$ and $P(H) + P(T) = 1$
$3P(T) = 1 \Rightarrow P(T) = \frac{1}{3}$

$\therefore$, $P(H) = 1 - P(T) \Rightarrow \left(1 - \frac{1}{3}\right) = \frac{2}{3}$

→ Conditional probability:

E is given (condition)

$$P(F|E) = \frac{P(F \cap E)}{P(E)}$$

Ex: Flipping coin 3 times: $|S| = 8$
E = 1st flip given Tail = 4 times ∴ $|E| = 4$
Find probability of event E that odd no. of tails occurs:
$$P = \frac{2}{4} = \frac{1}{2}$$

Ex: A bit string length 4 is generated at random so that each of 16-bit strings of length 4 is equally likely. What is probability that it contains at least 2 consecutive 0's given that 1st bit is 0.

Sol: $E \cap F = \{0000, 0001, 0010, 0011, 0100\}$
$P(E \cap F) = 5/16$
$P(F) = 8/16$
$$\frac{P(E \cap F)}{P(F)} = \frac{5}{8} = P(E|F)$$

→ Independence:
2 events E & F are independent events if & only if
$P(E \cap F) = P(E) \cdot P(F)$

Ex: Let E is the event that a randomly generated bit string length 4 begins with a 1 & F is the event that the bit string contain an even no. of 1.

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$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2 | E_1) \cdot P(E_3 | E_1 \cap E_2)$

Sol^n: $P(E) = \frac{8}{16} = \frac{1}{2}$; $P(F) = \frac{7}{16}$

$P(E \cap F) = \frac{4}{16} = \frac{1}{4}$

$P(E) \cdot P(F) = \frac{7}{16} \times \frac{1}{2} = \frac{7}{32} \neq \frac{1}{4}$

$\therefore$ Dependent events

Ex: A fair die is tossed. Find probability that an even no. will appear if it is given that the tossed no. resulted is a no less than 5.

Sol^n: $E = \{2, 4, 6\}$ $\quad S = \{1, 2, 3, 4, 5, 6\}$

$F = \{1, 2, 3, 4\}$ $\quad E \cap F = \{2, 4\}$

$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2}{4} = \frac{1}{2}$

⇒ Baye's Theorem:

$E_1, E_2, E_3, \ldots, E_n$; $1 \le i \le n$

$\cancel{P(E_i \cap A)}$ $\boxed{P(E_i | A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}}$

Ex: There are 3 bags. 1st bag – 2W & 3R balls,
2nd bag – 1W & 4R balls.
3rd bag – 4W & 6R balls
A bag is chosen at random, then find the probability that a ball is white if a ball is chosen at random from 1st bag.

Sol^n: $P(1^{st} bag | white) = \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{3}\left(\frac{2}{5} + \frac{1}{5} + \frac{42}{105}\right)} = \frac{\frac{2}{5}}{\left(\frac{5}{5}\right)1} = \frac{2}{5}$

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⇒ Discrete probability distribution:

Two coins are tossed simultaneously.
If x denotes the no. of appearance of head, then the sample space
the value of x for each sample point is defined as:

Sample point	TT	HT	TH	HH
x	0	1	1	2

x is a random variable; it takes value 0, 1 & 2.
X is " " if " " countably finite infinite no. of values.

The probability distribut^n corresponding to random variable can be obtained as:

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\therefore$, $P(x=0)=\frac{1}{4}$, $P(x=1)=\frac{2}{4}=\frac{1}{2}$

$P(x=2)=\frac{1}{4}$

$X = x_1, x_2, x_3, \ldots, x_n$

$E(X) = x_1 P(X=x_1) + x_2 P(X=x_2) + \ldots + x_n P(X=x_n)$
(~~expect~~^n)

$$= \sum_{i=1}^{n} x_i P(X=x_i)$$

Ex: If a fair coin is tossed 3 times, find the expected no. of heads.

Sol^n {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$$

→ Prim's Algorithm:

Prim (G) = weighted connected undirected graph with n-vertices.

T = min. weighted edge.

for i : 1 to n-2

begin

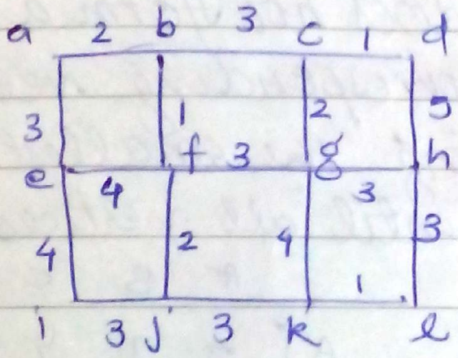
e = an edge of min. weight incident to a vertex in T not forming a simple circuit in T if added to T.

T = T with e added (i.e. T = T ∪ {e})

end

{T is min. spanning tree of G}

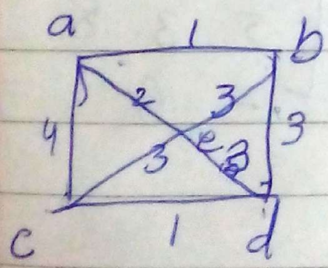
Ex.



choice	edge	weight
1	{b,f}	1
2	{a,b}	2
3	{f,j}	2
4	{a,e}	3
5	{i,j}	3
6	{j,g}	3
7	{c,g}	2
8	{c,d}	1
9	{g,h}	3
10	{h,l}	3
11	{k,l}	1

~~min~~ Total Weight = 24

Ex:



choice	edge	Weight
1	{a,b}	1
2	{a,e}	2
3	{e,d}	3
4	{c,d}	1

Total Weight = 7

→ Prim's Algorithm:

Let G be a graph with n-vertices and e-edges and w_{ij} be the weight of edge (v_i, v_j)

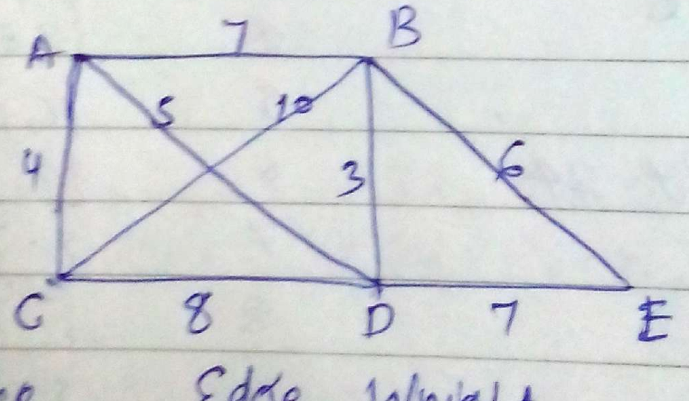
1. Write a matrix $A = [a_{ij}]_{n \times n}$; where $a_{ij} = w_{ij}$

2. Start from first row and choose the smallest entry and the corresponding vertex. Let us assume that the smallest entry is & the vertex is v_k. Draw the edge (v_i, v_k)

3. Choose the smallest entry (for which the corresponding vertex is not already chosen & the corresponding edge does not form a circuit) in row 1 and k and corresponding vertex.

4. Repeat the process of choosing the smallest entry in successive rows until all vertices are covered.

Ex:



$$
\begin{array}{c|ccccc}
 & A & B & C & D & E \\
\hline
A & - & 7 & 4 & 5 & - \\
B & 7 & - & 10 & 3 & 6 \\
C & 4 & 10 & - & 8 & - \\
D & 5 & 3 & 8 & - & 7 \\
E & - & 6 & - & 7 & -
\end{array}
$$

Sol^n

choice	Edge	Weight
1	(B,D)	3
2	(D,A)	5
3	(A,C)	4
4	(B,E)	6

Total Weight = 18

→ smallest entry in 1st row is 4 & corresponding vertex is C ∴ 1st branch is (A, C)

→ The smallest entry in 1st & 3rd row is 5 & corresp. vertex is D. Add 2nd branch (A, D)

→ smallest entry in 1st, 3rd, 4th is 3, corresp. vertex is B, add (B, D)

→ smallest entry in 1st, 3rd, 4th & 2nd is 6. Corres. vertex is E.

1. (A, C) – 4
2. (A, D) – 5
3. (D, B) – 3
4. (B, E) – 6.

⇒ Kruskal's Algorithm:

1. List all edges of the graph in increasing order of their weight.
2. Choose the first edge (mini weight). This is the 1st branch of the spanning tree.
3. Choose the next edge if it does not form a circuit with previously selected edges.
4. Repeat 3, until all vertices are covered.

(B, D)	(B, D) – 3	
(A, C)	(A, C) – 4	total Weight = 18
(A, D)	(A, D) – 5	
(B, E)	(B, E) – 6	
(D, E)		
(A, B)		
(C, D)		
(B, C)		