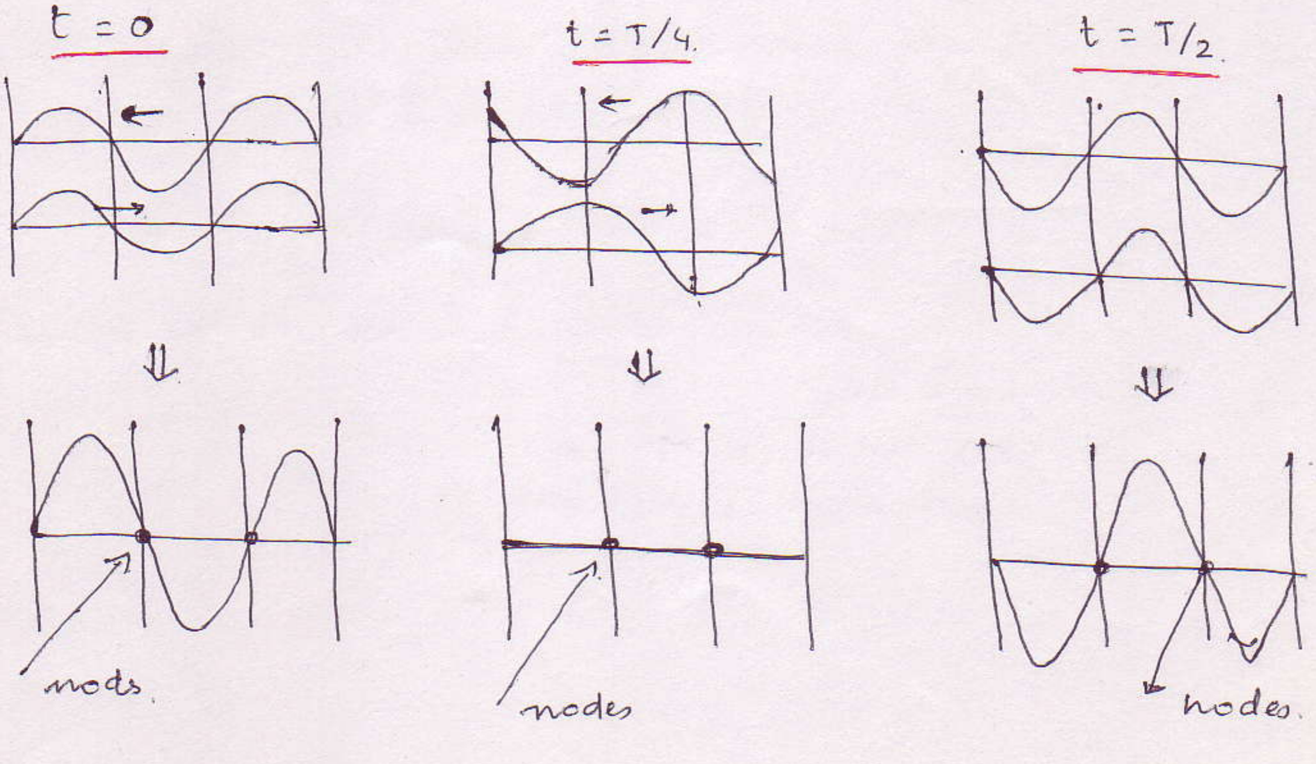
Superposition
- Standing waves
  - Same amplitude
  - Same frequency
- Beats.

Standing Waves.

: When two similar waves of same amplitude and frequency travelling in opposite direction superimpose each other it gives rise to standing wave.

- Nodes : certain points which are permanently at rest, [Though they are moving pts in ~~the~~ each of the individual waves]
- Energy : Energy of one region is always confined.
- Amplitude : Different particle move with diff. amplitude.



Important: The nodes or the nodal points are moving in the both waves but when they are superimposed they are not moving — they are permanently at rest in the superposed or the resultant wave that is formed. This is an important property of a wave — This is not possible in case of particles. This phenomena may help us to understand the double slit phenomena. In double slit interference light is coming from both slit then why should there be a region which is totally dark. This is exactly what happens in the nodal point. In both the waves the nodal points have a final displacement but when they are added they give rise to 0. Similarly the interference dark fringe is dark though light is reaching from both the slit. The dark region of the interference fringes cannot be explained by Corpuscular or particle theory of light. It can only be explained by wave phenomena.

Mathematical Representation.

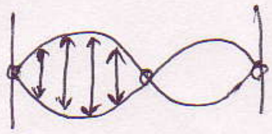
$$y_1 = A \sin(kx - \omega t) \longrightarrow$$

$$y_2 = A \sin(kx + \omega t) \longleftarrow$$

Question: There may be a question about phase of the reflected wave. y_2. In that case one can add $+\delta$ with the phase & find out the value of δ. [δ comes out to be 0]

$$y = y_1 + y_2 = [2A \sin kx] \cos \omega t$$

$$y = A(x) \cos \omega t.$$



Two Important Points.

- The equation does not contain $(x - vt)$ or any derived form of this type. So this is not an equation of a progressive wave. Here x and t are separate.
- The amplitude $A(x)$ is not fixed, it is a function of x. The maximum displacement is $2A$.

Position of Nodes.

$$kx = n\pi \quad \text{for } n = 0, 1, 2.$$

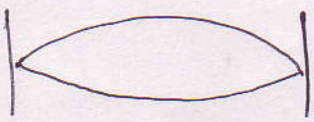
$$x = \frac{n\pi}{k} = n\frac{\lambda}{2} \quad \text{(nodes)} \left[\text{as } k = \frac{2\pi}{\lambda}\right]$$

Node Separation $= \lambda/2$.
(Antinode Separation) $= \lambda/2$

For Antinodes.
$$kx = (n + 1/2)\pi$$
$$x = (n + 1/2)\lambda/2$$

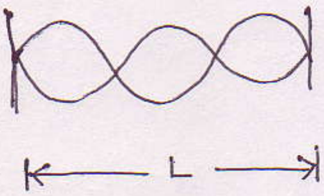
Standing Waves in
A String fixed at both the ends.
(Normal modes of Vibration)

If a string is fixed at both the ends like a guitar or a violin, different mods of Vibration can be produced by plucking the string at different positions. The waves generated get reflected from the end and interferes with all the waves traveling on the string giving rise to standing waves. The different mods are as follows.



$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$$2 \times \frac{\lambda}{2} = L \Rightarrow \lambda = L$$



$$3 \times \frac{\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{3}.$$

So the general condition for setting up stationary wave in a string which is fixed at both ends is

V. Imp. → $\boxed{L = n\,\lambda/2}$ $\quad n = 1, 2, 3.$

or $\boxed{\lambda_n = \frac{2L}{n}}$ $\quad (n = 1, 2, 3)$

Here n is the number of half wavelength or loops. It is only when L is equal to the integral mutiple of half wavelength or those wavelength which.

Satisfy the above equation are allowed and Standing waves are produced.
The other wavelengths die out because of destructive interference.

Allowed frequencies.

$$V = \nu \lambda$$

$$\nu_n = \frac{V}{\lambda} = n\frac{V}{2L} \qquad (n = 1, 2, 3 \ldots)$$

$$\boxed{\nu_n = n\frac{V}{2L}} \quad \text{or} \quad \boxed{\nu_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}} \text{ (string)}$$

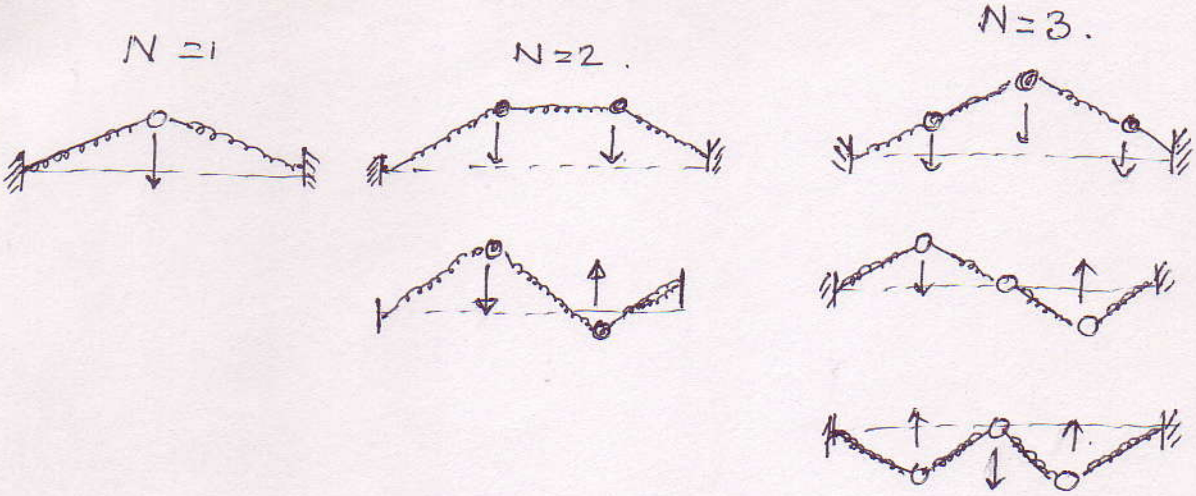
These are the allowed frequencies of the Standing waves on a string.

Here $n=1$ is known as fundamental or first harmonic.

$n=2$ is second harmonic & so on.
(or 1st overtone)

So ν_0 (fundamental), $2\nu_0$, $3\nu_0$ are the allowed frequencies. They are known as natural frequencies of the system. They are also known by the name normal mods.

Question: Why one simple oscillator like a mass spring system has only one allowed frequency where as a string has infinite number of frequency.



In a block-spring system, the inertia is concentrated in a single element of the system but in the string the inertia is distributed throughout the system. In general, a lumped system of N elements has N different pattern of oscillation. The limit as N tends to infinity leads us to the completely distributed system of the stretched string with the infinite number of vibrational frequencies.

Resonance in stretched String.

Suppose a string is fixed at one end & the other ~~hand~~ end is shaken by a student trying to create Standing waves in the string.

At Natural frequency:

When the frequency of oscillation of the free end is almost equal to the natural frequency of the String perfect standing waves will be produced & the string will start oscillating with maximum amplitude which is much larger than the driver's amplitude. Some energy will be lost because of air friction, internal energy of the string etc. Eventually a steady state is reached in which the energy Supplied exactly balances the energy lost by the student. This is the case of Resonance & correspondingly resonant frequency.

At frequency other than Natural frequency.

Here the string vibrates with the same aplitude as the driver & much more energy is lost. The string does work on to the hand holdig the string.

- In practise the resonant frequency is almost but not exactly a natural frequency of the system because some energy is lost due to damping.

If there were no damping what will happen?

The resonant frequency will be exactly a natural frequency and the amplitude would increase without limit as energy continued to be Supplied to the String by the student's hand. Eventually the elastic limit would be exceeded and the string would break.

- If it were possible to shake the string with an assortment of frequencies, the motion of the String would select those frequencies That were equal to its natural frequencies. Motion at those frequencies would be reinforced and would occur at ~~the~~ large amplitude where as motion at other frequencies would be damped or suppressed. — Musical Instrument.

Musical Instruments

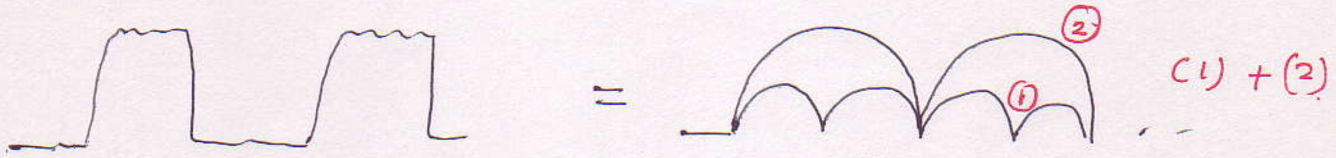
Guitar, violin. (wired Instruments)
flute (air Instrument)

(1) Now. $\nu = \frac{nV}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

How can one change the frequency in these instruments?
By changing L in flute, T in strings.

(2) In all these instruments we don't get the fundamental as 440 Hz. but a combination of the fundamental with harmonics. – Combination of frequencies.
Why?

Like.



How can many frequencies exist together?

Ans: Remember the first figure we studied in the wave superposition.

Why the Combination of frequency in musical instruments?

When we blow air or pluck a wire, a single frequency is never produced but it has a width as our blow will obviously vary and a single frequency which is very narrow or sharp cannot be produced. Like If I have a Thick nib pencil and I am asked to draw a narrow a few micron width it is not possible. The line I will draw will have a width of a mm.

So λ variation is

$$\lambda_i \longrightarrow \lambda_f.$$

All the wavelength in between λ_i and λ_f will be generated. Out of them only those frequency which will satisfy the Condition

$$n\frac{\lambda_n}{2} = L \qquad \frac{n\lambda_n}{2} = L.$$

will remain because of Constructive interference, rest will die out. So those frequencis will exist. They will Superimpose & give the final output.

 Suppose these 3 will satisfy.