

**The LNM Institute of Information Technology**  
**Jaipur, Rajasthan**  
**MATH-II**  
Assignment 8

---

1. Solve the integral equations:

(a)  $y(t) + \int_0^t y(\tau) d\tau = u(t-a) + u(t-b)$

(b)  $e^{-t} = y(t) + 2 \int_0^t \cos(t-\tau)y(\tau) d\tau$

(c)  $3 \sin 2t = y(t) + \int_0^t (t-\tau)y(\tau) d\tau.$

2. Sketch the following functions and find their Laplace transforms:

(a)  $f(t) = \begin{cases} u(t) - 2u(t-1), & 0 \leq t < 2, \\ f(t-2), & t > 2. \end{cases}$       (b)  $f(t) = \begin{cases} t[u(t) - u(t-1)], & 0 \leq t < 2, \\ f(t-2), & t > 2. \end{cases}$

(c)  $f(t) = \begin{cases} tu(t) - 2(t-1)u(t-1), & 0 \leq t < 2, \\ f(t-2), & t > 2. \end{cases}$

3. Find the Fourier series of  $f$  (assuming  $f$  to be periodic with period  $2\pi$ ):

(a)  $f(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & \pi < x < 2\pi \end{cases}$       (b)  $f(x) = \begin{cases} x & -\pi/2 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases}$

(c)  $f(x) = x^2/4, -\pi < x < \pi$       (d)  $f(x) = x, 0 < x < 2\pi.$

In each case, find the sum of the Fourier series at  $x = \frac{101\pi}{2}$ .

4. Show that

(i)  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$  [Hint: use 4(a)]

(ii)  $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12}$  [Hint: use 4(c)]

5. Expand  $f(x)$  in a Fourier series on the interval  $[-2, 2]$  if  $f(x) = 0$  for  $-2 \leq x < 0$  and  $f(x) = 1$  for  $0 \leq x \leq 2$ . (Assume  $f$  to be periodic with period  $p = 2L = 4$ ).

6. Find the (i) Fourier cosine series of  $f(x) = 1 + \sin \pi x, 0 \leq x < 1, f(x+2) = f(x),$

(ii) Fourier sine series for  $f(x) = e^x, 0 \leq x < \pi, f(x+2\pi) = f(x).$

7. Using the Fourier integral representation, show that

(a)  $\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1 + \omega^2} d\omega = \begin{cases} 0 & x < 0 \\ \frac{\pi}{2} & x = 0 \\ e^{-x} & x > 0 \end{cases}$       (b)  $\int_0^\infty \frac{\cos x\omega}{1 + \omega^2} d\omega = \frac{\pi}{2} e^{-x}, x > 0.$

8. Find  $A(\omega)$  such that  $\frac{1}{\pi} \int_0^\infty A(\omega) \cos x\omega d\omega = \frac{1}{1+x^2}.$

**Supplementary problems** from “Advanced Engg. Maths. by E. Kreyszig (8<sup>th</sup> Edn.)”

(i) Section 4.8, Q. 1,7

(ii) Chapter 4, Review Problems, Q. 32,33

(iii) Section 10.1, Q. 7,10,11,17

(iv) Section 10.2, Q. 1,2,9,10

(v) Section 10.3, Q. 8,11

(vi) Section 10.4, Q. 11, 13,21,25

(vii) Section 10.8, Q. 2,3,9,15