The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 10

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1. Define the domain in which the following partial differential equations can be classified as either elliptic or parabolic or hyperbolic:
   (a) $yu_{xx} = xu_{yy}$
   (b) $u_{yy} - xu_{xy} + yu_x + xu_y = 0$
   (c) $y^2u_{xx} + 2xyu_{xy} + x^2u_{yy} = 0$
   (d) $u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} = 0$

2. Reduce the following equations to canonical form:
   (a) $u_{xx} - x^2yu_{yy} = 0, \qquad (y > 0)$
   (b) $e^{2x}u_{xx} + 2e^{x+y}u_{xy} + e^{2y}u_{yy} = 0$
   (c) $x^2u_{xx} + y^2u_{yy} = 0, \qquad (x > 0, y > 0)$
   (d) $u_{xx} + 2u_{xy} + 5u_{yy} = xu_x$.

3. Reduce the following equations to canonical forms and hence find solutions:
   (a) $u_{xx} + 2\sqrt{3}u_{xy} + u_{yy} = 0$
   (b) $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$
   (c) $u_{xx} - 2\sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$
   (d) $u_{xx} + 4u_{xy} + 4u_{yy} = 0$

4. Using D'Alembert's formula, solve the Cauchy problem
$$u_{tt} - c^2u_{xx} = 0, \qquad x > 0,\ t > 0,$$
with initial conditions
$$u(x, 0) = g(x), \quad u_t(x, 0) = h(x), \quad x > 0,$$
and boundary conditions
$$u(0, t) = 0, \qquad t \geq 0.$$

5. Solve the Cauchy problem
$$u_{tt} = 16u_{xx}, \qquad -\infty < x < \infty,\ t > 0,$$
$$u(x, 0) = 6\sin^2 x, \quad u_t(x, 0) = \cos 6x, \quad -\infty < x < \infty.$$

6. Solve the wave equation (Hint: Duhamel's principle):
$$u_{tt} = u_{xx} + x^2 - t, \qquad -\infty < x < \infty,\ t > 0,$$
$$u(x, 0) = u_t(x, 0) = 0, \quad -\infty < x < \infty.$$