Objectives

Black Body Radiation

1. Inadequacy of wave theory in explaining blackbody radiation spectrum
2. Planck hypothesis of on atoms absorbing radiation in quanta of energy.

Black Body Radiation

When heated, a solid object glows and emits thermal radiation. As the temperature increases, the object becomes red, then yellow then white. The thermal radiation emitted by glowing solid objects consists of a continuous distribution of ~~en~~ frequencies ranging from ultraviolet to Infrared.

Understanding the continuous character of the radiation emitted by a glowing solid object constituted on the major unsolved problem during the second half of the nineteenth century. All attempts to explain this phenomenon by means of the available theories of classical physics (statistical thermodynamics and classical electromagnetic theory) ended up in miserable failure. This problem consisted in essence of specifying the proper theory of thermodynamics that describes how energy gets exchanged between radiation and matter.

When radiation falls on object, some of it might be absorbed and some reflected. An idealized "**black body**" is a material object that absorbs all the radiation falling on it, and hence appears as black under reflection when illuminated from outside.

An object in thermal equilibrium with its surrounding radiates as much energy as it absorbs. It thus follows that a blackbody is perfect emitter of radiation.

B②

A practical blackbody can be constructed by taking a hallow cavity whose internal walls ~~pef~~ perfectly reflect electromagnetic radiation (e.g. metallic walls) and which has a very small hole on its surface. Radiation that enters through the hole will be trapped inside the cavity and gets completely absorbed after successive reflections on the inner surface of the cavity. The hole thus absorbs radiation like a black body. On the other hand, when this cavity ~~two~~ is heated to T (due to thermal agitation or vibrations of the electrons in the metallic walls). the radiation that leaves the hole is blackbody radiation.

In 1859 Kirchoff unveiled the design that has since been accepted as a good design for a black body.

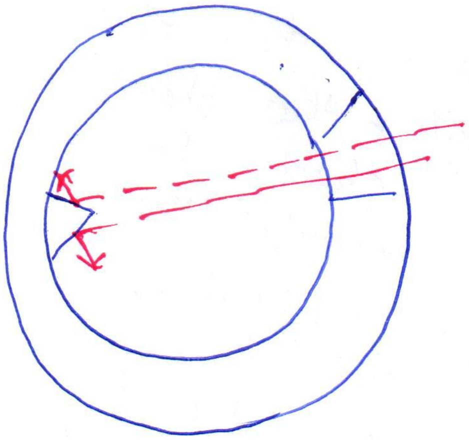


figure I

Schematic of → kirchoff designed black body

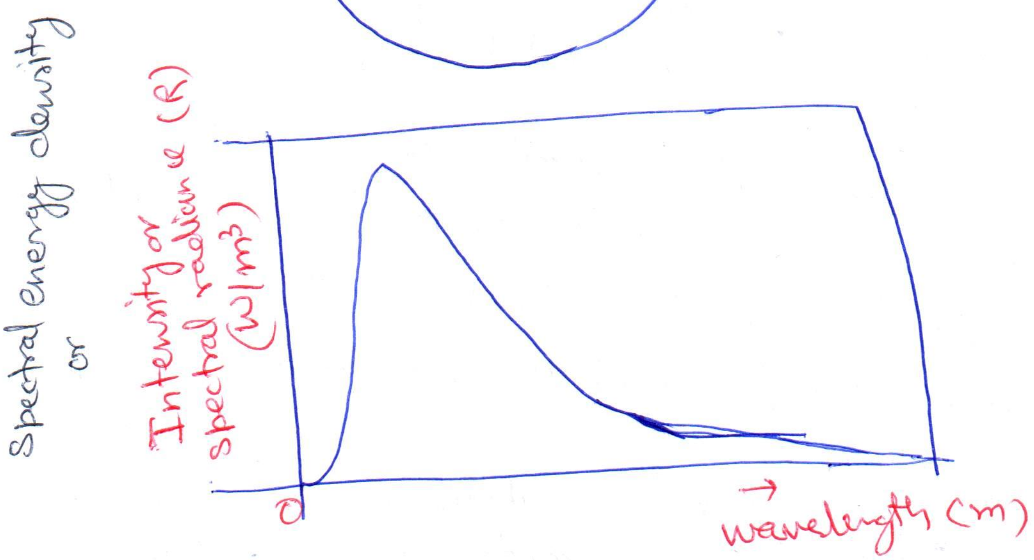


figure II

In figure 2

x-axis is wavelength

Y axis is Spectral radiance or Intensity

Spectral radiance can be understood as follows

Energy is measured in Joules (J)

Energy per unit area is measured in J/m^2

Power is measured in J/s = Watt (W)

Power per unit area is measured in (W/m^2)

Spectral radiance is power per unit area per unit wavelength and therefore written by W/m^3.

Intensity, which is power per unit area is therefore the area under the curve in figure 2.

$$I = \int_0^{\infty} R(\lambda)\, d\lambda$$

Treatment Under Classical Physics

There are two observations that can be made about BBR.

1. As the temperature T of the body increases, intensity of the radiation from the body increases.

2. Higher the temperature, lower is the wavelength of the most intense part of the spectrum.

B4

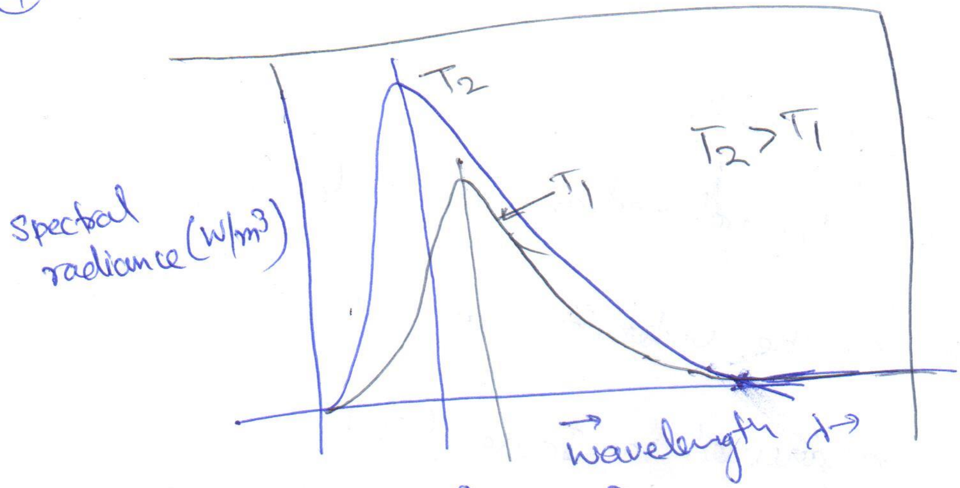


figure 3

A number of attempts aimed at explaining the origin of the continuous character of this radiation curve. ~~The m~~

① Stefan Boltzmann Law

In 1879 J. Stefan found experimentally that the total intensity radiated by glowing object of temperature is T is given by $\boxed{I = a\sigma T^4}$ → known as stefan Boltzmann law

where $\sigma = 5.67 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$ is the stefan Boltzmann constant

$a = 1$ for black body

which is the based on I^{st} property of ~~radia~~ BBD.

② Weins displacement law

$$\lambda_{max} T = \text{constant} = 2898\ \mu m\,K$$

This is based on second property of BBD " higher temperat lower ~~temperature~~ wavelength of the most intense part of the spectra.

The intensity predicted by Stefan Boltzmann law should math the expression for the curve indicated ~~or~~ earlier, therefore;

$$I = \int_0^{\infty} R(\lambda)\, d\lambda$$

~~The s~~

The scientific challenge that remainded was to determine the exact form of the spectral radiance, or power per unit area at a particular wavelength $R(\lambda)$. Obtaining the equation for $R(\lambda)$ was expected to result in a fundamental understanding of how matter interacted with matter.

The first attempt made by Rayleigh-Jeans law.

In classical Physics, radiation is considered as waves and calculation of radiant energy emitted by a black body is carried out in the following steps.

① We consider the black body to be in the shape of cubical metal cavity of side "L" with a small hole on it. The radiation which emerges from the hole has the characteristic of the radiation that is trapped inside the cavity.

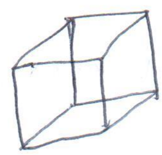
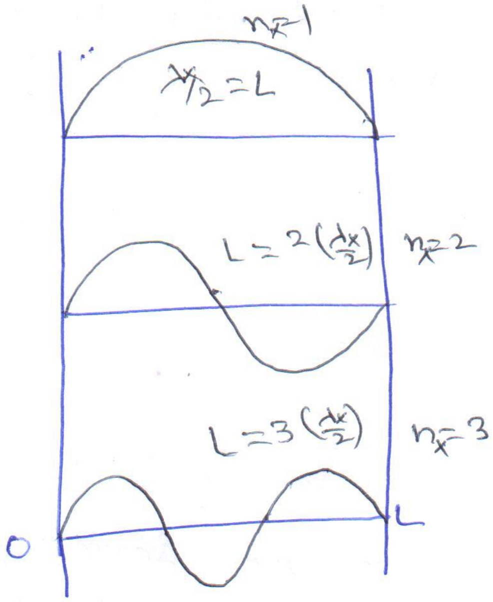


Figure 3

② The waves inside the cavity form standing wave pattern with nodes at walls of the cavity since the electric field must vanish inside a metal.

Let us consider a standing wave in on dimension, the electric field having nodes at $x=0$ & $x=L$.



the electric field

$$E = E_0 \sin\left(\frac{n_x\pi}{L}x\right)\sin\omega t.$$

$$= E_0 \sin(K_x x)\sin\omega t$$

$$1\cdot\left(\frac{\lambda_x}{2}\right)=L$$

$$2\cdot\left(\frac{\lambda_x}{2}\right)=L$$

$$3\cdot\left(\frac{\lambda_x}{2}\right)=L$$

$$\vdots$$

$$n_x\left(\frac{\lambda_x}{2}\right)=L \quad \text{where } n_x=1,2,3,\ldots$$

and $K_x=\dfrac{n_x\pi}{L}$

~~E~~ electric field in three dim.

$$E = E_0 \sin(k_x x)\sin(k_y y)\sin(k_z z)\sin\omega t \quad -(1)$$

where $k_x=\dfrac{n_x\pi}{L}$, $k_y=\dfrac{n_y\pi}{L}$, $k_z=\dfrac{n_z\pi}{L}$

where $\{n_x, n_y, n_z\}$ is a set of positive integers.

we know electromagnetic wave equation.

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \text{where } \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

put value of E

$$\left(\frac{n_x^2\pi^2}{L^2}+\frac{n_y^2\pi^2}{L^2}+\frac{n_z^2\pi^2}{L^2}\right)E=\frac{\omega^2}{c^2}E=k^2E$$

Unit vector $\vec{K} = K_x\hat{i} + K_y\hat{j} + K_z\hat{k}$

$$K^2 = K_x^2 + K_y^2 + K_z^2$$

$$K^2 = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2}{L^2}$$

where, $\vec{K} = \frac{2\pi}{\lambda}$

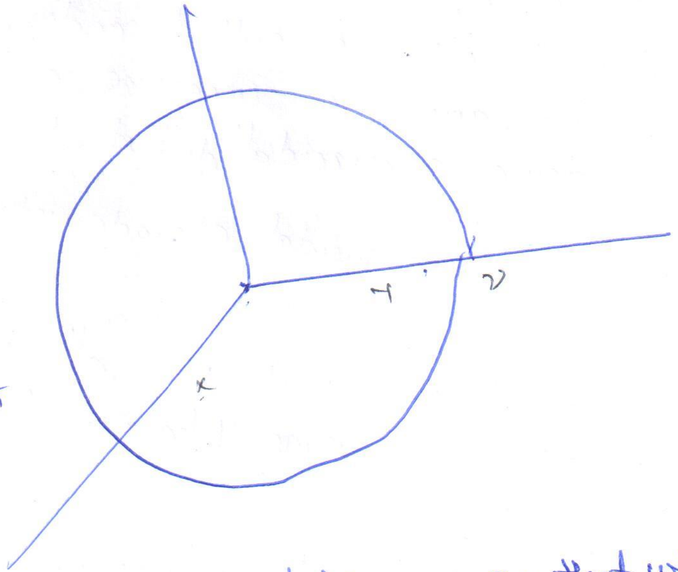
$$\left(\frac{2\pi}{\lambda}\right)^2 = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2}{L^2}$$

or $$\boxed{n_x^2 + n_y^2 + n_z^2 = \frac{4L^2}{\lambda^2}}$$

If we compare with equation of sphere where R is radius $(x^2 + y^2 + z^2 = R^2)$

$$\boxed{R = \frac{2L}{\lambda} = \frac{2L\nu}{c}}$$

For a given frequency, the equation above represents a sphere of radius $R = \frac{2L\nu}{c}$ in three dimensional space of n_x, n_y, n_z and each value of ν represents a distinct point in this space.



Since n_x, n_y, n_z are +ve integers so that will be in I^{st} octant.

Volume of the octant $= \frac{1}{8} \times \frac{4}{3} \times \pi R^3$

$$= \frac{\pi R^3}{6}$$

number of modes for frequency $\leq \nu$.

$$\& N(\nu) = \frac{\pi R^3}{6}$$

put the value of R

$$N(\nu) = \frac{\pi}{6} \times \frac{8L^3\nu^3}{c^3} \quad \{\text{and } L^3 = \text{Volume})$$

$$N(\nu) = 2\times\left(\frac{4}{3}\pi\frac{V\nu^3}{c^3}\right)$$

Where factor 2 takes into account the fact there are two transverse modes"

~~About~~ Mode

The point where there is no motion satisfy the condition

$\sin k_x x = 0, \rightarrow k_x x = 0, \pi, 2\pi, 3\pi$ ——— $n\pi$.

Thes points are called nodes.

Between any two successive nodes every point moves up and down sinusoidally, but the pattern of motion stays fixed in space.

This is femdamental characteristic of what we called the mode.

Normal mode

An oscillation in which all particles moves with the same frequency and same phase.

The number of modes in the frequency interval ν & $\nu + d\nu$.

$$N(\nu)\,d\nu = N(\nu + d\nu) - N(\nu)$$

$$= \frac{8\pi V}{3c^3}\left\{(\nu + d\nu)^3 - \nu^3\right\}$$

as we know $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 - - - .$

$$= \frac{8\pi V}{3c^2}\left\{ \nu^3\left(1+\frac{3}{\nu}d\nu\right) - \nu^3 \right\}$$

$$= \frac{8\pi V}{3c^3}\left\{ 3\nu^2 d\nu \right\}$$

$$\boxed{N(\nu)\,d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu}$$

no of modes per unit valume

$$N(\nu) = \frac{8\pi}{c^3}\nu^2$$

electromagnetic density in the frequency ν & $\nu+d\nu$ is given by

$$U(\nu,T) = N(\nu)\langle E\rangle$$

where $\langle E\rangle$ = average energy of the oscillators presents at wall.

$\langle E\rangle = ?$ According to the equipartition theorem of classical thermodynamics, all the oscillaters in the cavity have the same mean energy, irrespective of their frequencies.

$$\langle E\rangle = \frac{\int_0^\infty E e^{-E/kT} dE}{\int_0^E e^{-E/kT} dE} = kT$$

where $k = 1.3807 \times 10^{-23}$ J/K

k = Baltzmann constant.

$$\boxed{U(\nu,T) = \frac{8\pi}{c^3}\nu^2 \cdot kT}$$

This is know as "Rayleigh - Jeans law"

Rayleigh Jeans law in (λ)

$$\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$$

put $\nu = \frac{c}{\lambda}$

$$\cancel{u(\lambda,T) = \frac{8\pi kT}{c^3}}$$

energy per unit valume

$$u(\nu)\,d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu$$

$$= \frac{8\pi kT \, \cancel{c^2}}{\lambda^2 \cancel{c^3}} \left(-\frac{c}{\lambda^2}\right) d\lambda$$

$$u(\nu)\,d\nu = \frac{-8\pi kT}{\lambda^4} d\lambda$$

$$\boxed{R(\lambda) = \frac{8\pi kT}{\lambda^4}}$$

-ve sign here just reminds us that a 'decrease with wavelength' implies an increase with increasing frequency.

Rayleigh-Jean's law is roughly in agreement with the thermal radiation curves at long wavelengths. However, at short wavelengths, it gives infinite energy density as $u(\lambda \to \infty$ as $\lambda \to 0)$. This is clearly unphysical. The failure of the classical wave theory to explain the observed radiation curve in the ultraviolet end of the electromagnetic spectrum is known as "ultraviolet catastrophe".

$$\boxed{I(\lambda)\,d\lambda = \frac{2\pi c}{\lambda^4} kT \, d\lambda}$$

↳ Rayleigh Jean's Intensity.

The radiant intensity can be obtained from the expression ~~from the ex~~ for the energy density by multiplying the above expression by $c/4$. The curious factor ~~of~~ $\frac{1}{4}$ arises because.

1. At any instant, on an average, half of the waves are directed towards the wall of the cavity and another half is directed away from it. This gives a factor of $\frac{1}{2}$. we need to average over all angles. In computing the radiant power, we get a factor of $\cos^2\theta$, which average to $\frac{1}{2}$.

The radiant intensity $\boxed{I(\lambda)d\lambda = \frac{2\pi c}{\lambda^4} kT\, d\lambda}$

Quantum view

Planck's Theory

Planck modified Rayleigh - Jean's law in such a way that it fitted with experimental curve precisely and then he looked for sound theorectical basis for this formula.

He assumed that the atoms of walls of black body behave like a tiny electromagnetic oscillators each with the characteristic frequency of oscillations

The oscillators emit electromagnetic energy into the cavity and absorb electromagnetic energy from it.

Planck then boldly put forth the following suggestions regarding the atomic oscillators.

(1) An oscillator can have energies given by

$$E_n = nh\nu \quad \text{where } n = 0, 1, 2, 3 \ldots$$

where ν is the oscillator frequency and h is the planck constant.

In other words oscillator energy is quantized.

(2) ~~Oscilla~~ Oscillator can absorb or emit energy only in discrete units called quanta.

that is $\boxed{\Delta E_n = \Delta n h\nu = h\nu}$

An oscillator in a quantized states neither emits nor absorbs energy. Oscillator is said to be in a stationary state.

According to Boltzmann distribution, the probability of a mode having an energy E at a temperature T is given by $\exp(-\beta E)$, where $\beta = \frac{1}{KT}$. Here K is Boltzmann constant and T is the absolute temp.

Thus the average energy of a mode is

$$\bar{\mathcal{E}} = \frac{\sum_{n=0}^{\infty} nh\nu \exp(-nh\beta\nu)}{\sum_{n=0}^{\infty} \exp(-nh\beta\nu)}$$

$$\bar{\mathcal{E}} = \frac{h\nu}{\exp(h\nu\beta) - 1}$$

energy density as derived previously

$$u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(\frac{h\nu}{KT} - 1)}$$

radiant Intensity

$$I(\nu,T) = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT} - 1\right)}$$

or

$$I(\lambda,T) = \frac{2\pi hc}{\lambda^5} \cdot \frac{1}{\left(\exp\frac{hc}{\lambda kT} - 1\right)}$$