The LNM Institute of Information Technology
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Quiz-1, Section-A (Solution)

1. Show that $\sqrt{3}$ is an irrational number. **[5]**

Ans. Suppose $\sqrt{3}$ is rational. Then $\sqrt{3} = \frac{m}{n}$ for some $m, n \in \mathbb{Z}, n \neq 0$

Then $3n^2 = m^2 \implies 3|m^2 \implies 3|m$, Putting $m = 3k$ for some $k \in \mathbb{Z}$. Hence $3n^2 = (3k)^2$.

We get $n^2 = 3k^2 \implies 3|n^2 \implies 3|n$, that is $gcd(n, m) = 3$. This is contradiction

2. Discuss the convergence of the following recursive sequence: **[5]**

$$a_1 = 1 \text{ and } a_{n+1} = 1 + \frac{1}{a_n} \text{ for } n \in \mathbb{N}.$$

Ans. It is clear that $a_n \geq 1$ for all $n \in \mathbb{N}$ and hence

$$a_n a_{n-1} = 1 + \frac{1}{a_{n-1}} a_{n-1} = a_{n-1} + 1 \geq 2 \text{ for all } n \in \mathbb{N} \text{ with } n \geq 2.$$

Since

$$a_{n+1} - a_n = (1 + \frac{1}{a_n}) - (1 + \frac{1}{a_{n-1}}) = \frac{a_{n-1} - a_n}{a_{n-1} a_n}.$$

We have

$$|a_{n+1} - a_n| = \frac{|a_{n-1} - a_n|}{a_{n-1} a_n} \leq \frac{1}{2} |a_n - a_{n-1}| \text{ for all } n \in \mathbb{N} \text{ with } n \geq 2.$$

So $\alpha = \frac{1}{2}$. By contractive condition it is a cauchy sequence hence convergent.
Suppose it converges to l. then $l = \frac{1+\sqrt{5}}{2}$

Remark: This is not monotonic sequence.