DMS ASSIGNMENT #3 SOLUTION

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Sol : 1) Number of n-bit words corresponding to $n=1,2,3,4$ are $a_1 = 2$, $a_2 = 3$, $a_3 = 5$ and $a_4 = 8$. these many words don't have 2 consecutive 1's.

Let's consider an arbitrary n-bit word

Case 1 : the n^{th} bit is 0. then $(n-1)^{th}$ bit can be 0 or 1
There's no restriction on the $(n-1)^{th}$ bit
So, a_{n-1} n bit words end in 0 and contain no consecutive 1's.

Case 2 : The n^{th} bit is 1. Then the $(n-1)^{th}$ bit must be 0.
Now again there's no restriction on the remaining $(n-2)$ bits.
So, a_{n-2}, n bit words end in 1 and don't contain any 2 consecutive 1's.

Above 2 cases are mutually exclusive. So, by Additive principle $a_1 = 2$, $a_2 = 3$ (initial data)

$$a_n = a_{n-1} + a_{n-2} \quad , \quad n \geq 3$$

↳ required recurrence relation

Sol 2) : $\underline{\underline{L_1 = 1}}$, $\underline{\underline{L_2 = 3}}$ (given initial condition)

the next 4 Lucas numbers are

$L_3 = 3+1 = \underline{\underline{4}}$, $L_4 = 4+3 = \underline{\underline{7}}$, $L_5 = 7+4 = \underline{\underline{11}}$, $L_6 = 11+7 = \underline{\underline{18}}$

Sol 37 Objective - find $\gcd\{28, 18\}$

Let $x = 28$, $y = 18$. Using given definition in Q.

$\gcd\{x, y\} = \gcd\{y, x \bmod y\}$ if $y \le x$ and $y > 0$

$= \gcd\{18, 28 \% 18\}$

$= \gcd\{18, 10\}$

$= \gcd\{10, 18 \% 10\}$

$= \gcd\{10, 8\}$

$= \gcd\{8, 10 \% 8\}$

$= \gcd\{8, 2\}$

$\therefore \gcd\{28, 18\} = \underline{\underline{2}}$. (Ans).

Sol 47 (a) With $a_1 = 1$

$\boxed{a_n = a_{n-1} + 3}$ is the recurrence relation which defines the sequence $1, 4, 7, 10, 13, \ldots$

(b) with $a_1 = 1$ and by observation clearly

$\boxed{a_n = (a_{n-1})^2 + 1}$ recursively defines the sequence $1, 2, 5, 26, 677, \ldots$

Sol 57 Given that $f(x) = \begin{cases} x - 10 & \text{if } x > 100 \\ f(f(x+1)) & \text{if } 0 \le x \le 100 \end{cases}$

(a) $f(99) = f(f(99+1))$ $[\because 99 < 100]$

$= f(f(110))$ $[\because 110 > 100]$

$= f(110 - 10)$

$= f(100)$

$$= f(f(100+11))$$

$$= f(f(111))$$

$$= f(\cancel{\#}\,101)$$

$$\boxed{f(99) = \cancel{f}\ 91} \quad //$$

(b) $f(f(99)) = f(91)$ (from above part)

$$= f(f(91+11))$$

$$= f(f(102))$$

$$= f(92)$$

$$= f(f(103))$$

$$= f(93)$$

$$\dots$$

$$\boxed{f(f(99)) = f(99) = 91} \quad //$$

Sol 6) (a) for $i=1$ to n do

for $j=1$ to i do

$x \leftarrow x+1$

No. of times $x \leftarrow x+1$ will execute

$$= 1 + (1+2) + (1+2+3) + \dots + (1+2+3+\dots+n)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{i} j$$

$$= \sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{1}{2} \sum_{i=1}^{n} (i^2+i)$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right]$$

$$= \frac{n(n+1)}{4}\left[\frac{2n+1}{3} + 1 \right]$$

$$= \frac{n(n+1)}{\cancel{4}_2} \cdot \frac{(\cancel{2}n+\cancel{4}^2)}{3} = \boxed{\frac{n(n+1)(n+2)}{6}\ \#\text{ans}}$$

let say $a_1 = 1$, then

$$a_2 = a_1 + 2$$

$$a_3 = a_2 + 3$$

$$a_4 = a_3 + 4$$

$$\vdots$$

$$a_{n-1} = a_{n-2} + (n-1)$$

$$\boxed{a_n = a_{n-1} + n}$$

(b) for $i = 1$ to n do

for $j = 1$ to i do

for $k = 1$ to i do

$x \leftarrow x + 1$

Number of times the statement $x \leftarrow x+1$ will execute is given by $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{i}$

$$= 1(1) + 2(1+2) + 3(1+2+3) + \cdots + n(1+2+3+\cdots+n)$$

Let us say that $a_1 = 1$

By observation $a_2 = 2\left(\frac{a_1}{1} + 2\right)$

$$a_3 = 3\left(\frac{a_2}{2} + 3\right)$$

$$a_4 = 4\left(\frac{a_3}{3} + 4\right)$$

$$\cdots$$

$$a_{n-1} = (n-1)\left(\frac{a_{n-2}}{n-2} + n-1\right)$$

$$\boxed{a_n = n\left(\frac{a_{n-1}}{n-1} + n\right)}$$

(c) for $i = 1$ to n do

for $j = 1$ to $\lfloor i/2 \rfloor i$ do

$x \leftarrow x + 1$

Total no. of times the statement $x \leftarrow x+1$ is executed

$= 0 + 2 + 3 + 8 + 10 + 18 + \cdots$

Sol 7) Given $S(n, r) = \begin{cases} 1 & \text{if } r = 1 \text{ or } r = n \\ S(n-1, r-1) + r S(n-1, r) & \text{if } 1 < r < n \\ 0 & \text{if } r > n \end{cases}$

$\therefore S(2,2) = 1$ (clearly as $n = r = 2$)

Sol 8) $A(m, n) = \begin{cases} n+1 & \text{if } m = 0 \\ A(m-1, 1) & \text{if } n = 0 \\ A(m-1, A(m, n-1)) & \text{otherwise} \end{cases}$

(a) $A(0,7) = 7 + 1 = 8$ $(\because m = 0)$

(b) $A(4,0) = A(3,0)$ $(\because n = 0)$

$= A(2,0)$

$= A(1,0)$

$= A(0,0)$

$= A(-1, A(0,-1))$

$= A(-1, 0)$

$= A(-2$

(b) $A(4,0) = A(3,1)$ $\quad (\because n=0)$

$= A(2, A(3,0))$

$= A(2, A(2,1))$

$= A(2, A(1, A(2,0)))$

$= A(2, A(1, A(1,1)))$

$= A(2, A(1, A(0, A(1,0))))$

$= A(2, A(1, A(0, A(0,1))))$

$= A(2, A(1, A(0, 2)))$

$= A(2, A(1,3))$

$= A(2, A(0, A(1,2)))$

$= A(2, A(0, A(0, A(1,1))))$

$= A(2, A(0, A(0, A(0, A(1,0)))))$

$= A(2, A(0, A(0, A(0, A(0,1)))))$

$= A(2,5)$

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sol: 97 (b) $S_1 = 1$

$S_n = S_{n-1} + n^3, \quad n \geq 2$

$S_{n-1} = S_{n-2} + (n-1)^3$

$S_n = S_{n-2} + n^3 + (n-1)^3$

$S_n = S_1 + n^3 + (n-1)^3 + \dots 2^3$

$= n^3 + (n-1)^3 + \dots 2^3 + 1^3$

$\boxed{S_n = \left[\frac{n(n+1)}{2}\right]^2}$ $\quad \left[\sum_{x=1}^{n} x^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$

(c) $a_1 = 1$; $a_n = 2a_{n-1} + (2^n - 1)$, $n \geq 2$

$a_{n-1} = 2a_{n-2} + (2^{n-1} - 1)$

$a_n = 2(2a_{n-2} + (2^{n-1} - 1)) + (2^n - 1)$

$= 2^2 a_{n-2} + 2 \cdot 2^n - 2 - 1$

$a_{n-2} = 2(a_{n-3}) + (2^{n-2} - 1)$

$a_n = 2^2(2a_{n-3} + 2^{n-2} - 1) + 2 \cdot 2^n - 2 - 1$

$= 2^3 a_{n-3} + 3 \cdot 2^n - 2^0 - 2^1 - 2^2$

similarly $a_n = 2^{n-1} a_1 + (n-1)2^n - (2^0 + 2^1 + 2^2 + \dots + 2^{n-2})$

$= 2 \cdot 2^{n-1} + n \cdot 2^n - (1 + 2 + 2^2 + \dots + 2^n)$

$= (n+1) \cdot 2^n - \left(\frac{2^n - 1}{2 - 1} \right)$

$\boxed{a_n = (n+1) \cdot 2^n - 2^n + 1}$ // (Ans).

(a) $a_0 = 0$, $a_n = a_{n-1} + 4n$, $n \geq 1$

$a_1 = 4$, $a_{n-1} = a_{n-2} + 4(n-1)$

$a_n = a_{n-2} + 4(n + n - 1)$

$a_n = a_0 + 4(n + n-1 + \dots + 1)$

$a_n = 0 + \frac{\cancel{4}^{2} (n)(n+1)}{\cancel{2}}$

$\therefore$ $\boxed{a_n = 2n(n+1)}$ //

x ——— x