The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-I ■ Assignment #2

(Sequences Cont.)

Q1. Investigate the convergence/divergence of the following sequences:
(a) $x_n = \frac{n^s}{(1+p)^n}$ for some $s > 0$ and $p > 0$
(b) $x_n = \frac{2^n}{n!}$

Q.2 Is the sequence $a_n = 1 + (-1)^n$ a cauchy sequence ?

Q.3 Is the sequence $a_n = \frac{1}{n}$ a cauchy sequence ?

Q4. Suppose that $0 < \alpha < 1$ and that (x_n) is a sequence which satisfies one of the following conditions:
(a) $|x_{n+1} - x_n| \leq \alpha^n, \quad n = 1, 2, 3, \dots$
(b) $|x_{n+2} - x_{n+1}| \leq \alpha|x_{n+1} - x_n|, \quad n = 1, 2, 3, \dots$
Then prove that (x_n) satisfies the Cauchy criterion. *Whenever you use this result, you have to show that the number* α *that you get, satisfies* $0 < \alpha < 1$. *The condition* $|x_{n+2} - x_{n+1}| \leq |x_{n+1} - x_n|$ *does not guarantee the convergence of* (x_n). *Give examples.*

Q5. Let $x_1 \in \mathbb{R}$ and let $x_{n+1} = \frac{1}{7}(x_n^3 + 2)$ for $n \in \mathbb{N}$. Show that (x_n) converges for $0 < x_1 < 1$. Also conclude that it converges to a root of $x^3 - 7x + 2$ lying between 0 and 1. Does the sequence converge for any starting value of $x_1 > 1$.