DMS ASSIGNMENT #2 SOLUTIONS

NAME- AASHISH RAJ GUPTA
ROLL NO- 16UCS004

Sol: 2> Suppose S has the property that every pair of distinct $a, b \in S$. Also assume that a doesn't divide b and vice-versa. Since we have $(n+1)$ elements, we partition $S = \{1, 2, \cdots, 2n\}$ into n sets $T_1, T_2, \cdots, T_n$. The Pigeonhole principle says that there should be at least two element for some i. this should be useful to conclude that there should be $a, b \in S$ such that $a | b$.

Sol 3> Applying Pigeonhole principle :

let no. of Pigeons(n) = 37 (~~alphanumeric~~ characters)

and pigeonholes no.(m) = 26 + 9 (total alphanumeric characters)
= 35

$\therefore \lfloor (n-1)/m \rfloor + 1$

$= \lfloor (37-1)/35 \rfloor + 1 = 1+1 = 2$ // (proved).

Sol: 4> Applying Pigeonhole principle :

let no. of pigeons(n) = ~~#~~ 12,305 (total cost)

" pigeonholes (m) = 13 (no. of refrigerators)

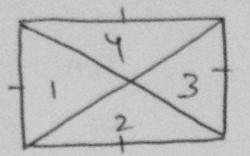
$\therefore \lfloor (n-1)/m \rfloor + 1$

$= \lfloor (12305-1)/13 \rfloor + 1$

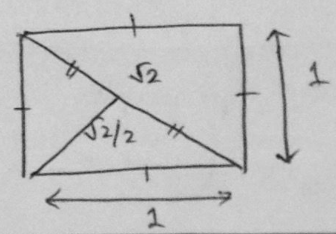
$= \lfloor 946.4 \rfloor + 1$

$= 947$ // (proved)

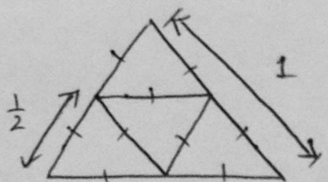
Sol 5) Divide the square into 4 triangles as shown:



If 5 points are chosen then we can assign each of them to a triangle that contains it. If the point belongs to several triangles, arbitrarily assign it to one of them. Then the 5 points are assigned to 4 triangular regions; so, by the Pigeonhole principle at least two points must belong to the same region. these two points cannot be more than $\frac{\sqrt{2}}{2}$ units aparts. This is clear from the diagram below:-



Sol 6)



Divide the Δ in 4 smaller Δ's. If 5 pts. are chosen, then we can assign each of them to a triangle that contains it. If the points belong to several triangles, arbitrarily assign it to one of them. then the 5 points are assigned to 4 triangular regions. So, by the Pigeonhole principle at least two points must belong to the same region. these two points can't be more than $\frac{1}{2}$ units apart (This is clear from above diagram).

---

sol:7) $f: ASCII \rightarrow W$

$f(c) =$ ordinal number of c

the function is clearly one-one because every c belonging to ASCII code will have a unique ordinal decimal number.

The function is not-onto as the co-domain is the set W (whole no's) whereas the range has limited members, even though they are whole no's.

Hence function is not bijective $\Rightarrow$ not invertible //

---

sol 8)

$$LHS = \sum_{i=m}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{m-1} i$$

$$= \frac{n(n+1)}{2} - \frac{(m-1)m}{2} = \frac{n^2 - m^2 + n + m}{2}$$

$$RHS = \sum_{i=m}^{n} (n+m-i)$$

$$= \sum_{i=m}^{n} (n+m) - \sum_{i=m}^{n} i$$

$$= (n-m+1)(n+m) - \frac{n^2 \cancel{+} - m^2 + n - m}{2}$$

$$= \frac{\cancel{n^2} - \cancel{nm} + \cancel{n} + \cancel{mn} - \cancel{m^2} + m - \cancel{n^2} + \cancel{m^2} - \cancel{n} + m}{2}$$

$$= \frac{2m}{2} = m$$

$\therefore$ RHS $\neq$ LHS

Hence given statement is False.

P.T.O

Sol : 9> ⓐ $S = \sum_{i=m+1}^{n} (a_i - a_{i-1})$

$$= (\cancel{a_{m+1}} - a_m) + (\cancel{a_{m+2}} - \cancel{a_{m+1}}) + (\cancel{a_{m+3}} - \cancel{a_{m+2}}) + \vdots + (\cancel{a_{n-1}} - \cancel{a_{n-2}}) + (a_n - \cancel{a_{n-1}})$$

which clearly is equal to $a_n - a_m$ (proved).//

ⓑ $\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$

$\therefore \sum_{i=1}^{n} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \frac{1}{i} - \frac{1}{i+1}$

$= -\left(\sum_{i=1}^{n} a_i - a_{i-1} \right)$

(where $a_i = \frac{1}{i+1}$, $a_{i-1} = \frac{1}{i}$)

$= -(a_n - a_1) = a_1 - a_n = \boxed{\frac{1}{2} - \frac{1}{n+1}}$ .//

Sol : 10> $\sum_{1 \le i \le j < 3} (a_i + a_j)$

$= \sum_{i=1}^{2} \sum_{j=i}^{2} (a_i + a_j)$

$= (a_1 + a_1) + (a_1 + a_2) + (a_2 + a_2)$

$= \boxed{3(a_1 + a_2)}$ .//

Sol : 11) (a) $A+B$ is defined when $m=p$ & $n=q$

size $(A+B) = m\times n$ or $p\times q$

(b) $B-C$ is defined when $m=r$ & $n=s$

size $(B-C) = m\times n$ or ~~$p\times q$~~ $r\times s$

(c) BC is defined when $q=r$

size $(BC) = p\times s$

(d) A^2 is always defined

size $(A^2) = m\times m$

Sol : 12) (a) No. of grams of each type of insulin is given by

$$\begin{bmatrix} 25 & 40 & 35 & 0 \end{bmatrix}_{1\times4} \times \begin{bmatrix} 7 \\ 14 \\ 21 \\ 28 \end{bmatrix}_{4\times1}$$

$= 175 + 560 + 735 + 0$

$= \boxed{1470\ g}$ # (Semi-Lente) //

$$\begin{bmatrix} 20 & 0 & 15 & 15 \end{bmatrix}_{1\times4} \times \begin{bmatrix} 7 \\ 14 \\ 21 \\ 28 \end{bmatrix}_{4\times1}$$

$= 140 + 0 + 315 + 420$

$= \boxed{875\ g}$. (Lente) //

$$\begin{bmatrix} 20 & 0 & 30 & 40 \end{bmatrix}_{1\times4} \times \begin{bmatrix} 7 \\ 14 \\ 21 \\ 28 \end{bmatrix}_{4\times1}$$

$= 140 + 0 + 630 + 1120$

$= \boxed{1290\ g}$ (Ultra).

(b) Total cost of insulin

$= ₹ [10 \times 1470 + 11 \times 875 + 12 \times 1290]$

$= ₹ [14700 + 9625 + 15480]$

$= \boxed{₹\ 39805}$

(c) Let's calculate the insulin requirements for additional 3, 5, 8, 13 days separately.

<u>Semi-Lente</u> → $\begin{bmatrix} 25 & 40 & 35 & 0 \end{bmatrix}_{1\times4} \times \begin{bmatrix} 3 \\ 5 \\ 8 \\ 13 \end{bmatrix}_{4\times1}$

$= 75 + 200 + 280 + 0$

$= 555$

Total $= 555 + \cancel{2025}\ 1470$

$= \boxed{2025\ g}$

<u>Lente</u> → $\begin{bmatrix} 20 & 0 & 15 & 15 \end{bmatrix}_{1\times4} \times \begin{bmatrix} 3 \\ 5 \\ 8 \\ 13 \end{bmatrix}_{4\times1}$

$= 60 + 0 + 120 + 195$

$= 375$

Total $= 375 + 875$

$= \boxed{1250g}$

~~Ut~~ <u>Ultra</u> → $\begin{bmatrix} 20 & 0 & 30 & 40 \end{bmatrix}_{1\times4} \times \begin{bmatrix} 3 \\ 5 \\ 8 \\ 13 \end{bmatrix}_{4\times1}$

$= 60 + 0 + 240 + 520$

$= 820$

Total $= 820 + 1290$

$= \boxed{2110\ g}$

ⓓ semi-lente → $3 \times 1470g$ = $\boxed{4410\,g}$

lente → $3 \times 875g$ = $\boxed{2625\,g}$

~~U~~ Ultra → $3 \times 1290g$ = $\boxed{3870\,g}$

Sol : 13) Objective – find gcd {2076, 1024}

Let $a = 2076$, $b = $ ~~120~~ 1024 ∴, Using Euclidean algorithm we have:

divide 2076 by 1024 : $2076 = 1024 \times 2 + 28$

" 1024 " 28 : $1024 = 28 \times 36 + 16$

" 28 by 16 : $28 = 16 \times 1 + 12$

" 16 by 12 : $16 = 12 \times 1 + 4$

" 12 by 4 : $12 = 4 \times 3 + 0$

∴ $\boxed{GCD = 4}$ (last of the non-zero divisors)

Sol : 14) ⓐ $1 + 2 + 3 + \cdots + 12$

$= \frac{\cancel{12}^{6}(13)}{\cancel{2}} = \boxed{78}$ //

$\left[\text{as } \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \right]$

ⓑ $\sum_{i=1}^{12} \sum_{j=1}^{i} j$

$= \sum_{i=1}^{12} \frac{i(i+1)}{2}$

$= \frac{1}{2} \sum_{i=1}^{12} (i^2 + i)$

$= \frac{1}{\cancel{2}} \left[\frac{\cancel{12}^{2}(13)(25)}{\cancel{6}} + \frac{\cancel{12}^{\cancel{6}\,3}(13)}{\cancel{2}} \right]$

$= 13\,(28) = \boxed{364}$ //

Sol : 157 (a) Basis step →

for $n=2$

$P(2) : \sum_{i=1}^{2} (2i-1) = (2\times1-1)+(2\times2-1) = 1+3=4=2^2$

therefore $P(2)$ is true.

Let $P(k)$ be also true then we need to prove that $P(k+1)$ is also true.

$\therefore \sum_{i=1}^{k} (2i-1) = k^2$ ($P(k)$ is true)

$P(k+1) : \sum_{i=1}^{k+1} (2i-1)$

$= 2\sum_{i=1}^{k+1} i - \sum_{i=1}^{k+1} 1$

$= \cancel{2}\dfrac{(k+1)(k+2)}{\cancel{2}} - (k+1)$

$= (k+1)(k+2-1) = (k+1)^2$.

Hence $P(k+1)$ is also true.

So, by PMI $P(n)$ is true for all $n>1$

(b) Basis step:

for $n=2$, $P(2) = 2^4 + 2\cdot2^3 + 2^2$

$= 2^2(2^2+2^2+2^0)$ which is divisible by 4

so $P(2)$ is true.

Let $P(k)$ be also true

$\therefore k^4 + 2k^3 + k^2 = 4m$ ($m \in N$)

Now, we need to prove that $P(k+1)$ is also true

$P(k+1):\quad (k+1)^4 + 2(k+1)^3 + (k+1)^2$

$= (k+1)^2 \left[k^2+2k+1 + 2k+2+1 \right]$

$= (k+1)^2 (k^2+4k+4)$

$= (k^2+2k+1)(k^2+4k+4)$

$= \underline{k^4+2k^3+k^2} + 4k^3 + 8k^2 + 4k + 4k^2 + 8k + 4$

$= 4m + 4(l) \qquad (l \in N)$

$= 4(m+l)$ which is divisible by 4.

$\therefore$ $P(k+1)$ is also true.

Hence, by PMI, $P(n)$ is true for all $n \geq 1$.

Sol 16) (a)

$x \leftarrow 0$
for $i=1$ to n
do $x \leftarrow x + (2i-1)$

$x = 1+3+5+\cdots+2n-1$

$= \left(\frac{2n-1+1}{2}\right)^2 = \boxed{n^2}$ //

(b)

$x \leftarrow 0$
for $i=1$ to n
do for $j=1$ to i
$x \leftarrow x+1$;

$1 + (1+2) + (1+2+3) + \cdots + (1+2+3+\cdots+n)$

$= \sum_{i=1}^{n} \sum_{j=1}^{i} j$

$= \sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{1}{2} \sum_{i=1}^{n} i^2 + i$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{4}\left[\frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)(\overset{1}{\cancel{2}}n+\overset{2}{\cancel{4}})}{\cancel{12}\,6} = \boxed{\frac{n(n+1)(n+2)}{6}} \quad //$$
