Assignment-2
Course: B.Tech 1st Year
Subject: Discrete Mathematical Structures (DMS)

1. Let, $A=\{0,1\}$. Show that the following expressions are regular over A:

   a) $0^*(0\vee 1)^* 01^* 0$, b) $00^*(0\vee 1)^* 1^* 1$, c) $(01)^*\left(01\vee 1^*\right)^*$.

2. Let, $A=\{a,b,c,d\}$. Show that the following expressions are regular over A:

   a) $abc\vee b(ab)^*(abc\vee a)$, b) $ab^*\left(a^*b\vee c\right)^* d$, c) $\left(a^*b\vee c^*d\right)^*\vee abcd$.

3. Let, $S=\{0,1\}$. Give the regular expressions, corresponding to the regular sets:

   a) $\{00, 010, 0110, 011110, \ldots\}$

   b) $\{0, 001, 000, 00001, 00000, 0000001, \ldots\}$

4. Let, U be an universal set with cardinality 12 and A, B, C be three nonempty subsets of U, so that the bit string representations of $A\cup B$, $A\cap B$, and $A-B$ are as follows: 111010111111, 000010100001, and 011000001010, respectively. Write down the bit string representations for A, B, $B-A$, $A\oplus B$, and A^c-B.

5. Find the product AB for the following matrices:

   i) $A=\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix}$

   ii) $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & -2 & 7 \\ -4 & 3 & -1 \end{bmatrix}$, $B=\begin{bmatrix} -3 & 2 & 1 \\ 8 & 5 & -10 \\ 1 & 1 & -1 \end{bmatrix}$

6. Consider the matrix: $A=\begin{bmatrix} 2 & 3 & -7 \\ 4 & 5 & 8 \\ -1 & 0 & 3 \end{bmatrix}$. Compute the values of A^2 and A^3.

7. For each of the three matrices A in Questions 5 i), ii), and 6, compute the corresponding matrix A^{-1}.

8. Find a matrix A that satisfies the following matrix identity:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} A = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}.$$

9. Show that the following matrix is nilpotent and find its index of nilpotency:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

10. Show that the following matrix is idempotent:

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}.$$

11. Compute $A \vee B$, $A \wedge B$, and $A \odot B$ for the following two Boolean matrices:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

12. Show that the set of all 5×5 Boolean matrices is a mathematical structure that is closed with respect to the operations: *meet*, *join*, and *Boolean product*. Write down the identity elements for each of these three operations.

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