The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment #2

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1. Find general solution of the following differential equations:

$$(i)\ (x+2y+1)-(2x+y-1)y'=0 \qquad (ii)\ y'=(8x-2y+1)^2/(4x-y-1)^2.$$

2. Show that the following equations are exact and hence find their general solution:

$$(i)\ (\cos x\cos y-\cot x)=(\sin x\sin y+1)y' \qquad (ii)\ y'=2x(ye^{-x^2}-y-3x)/(x^2+3y^2+e^{-x^2}).$$

3. Verify that

$$\frac{1}{2}(Mx+Ny)d(\ln(xy))+\frac{1}{2}(Mx-Ny)d(\ln(x/y))=Mdx+Ndy.$$

Hence show that (i) if the differential equation $M(x,y)dx+N(x,y)dy=0$ is homogeneous, then $1/(Mx+Ny)$ is an integrating factor unless $Mx+Ny\equiv 0$, (ii) if the differential equation $M(x,y)dx+N(x,y)dy=0$ is not exact but is of the form $f_1(xy)ydx+f_2(xy)xdy=0$, then $1/(Mx-Ny)$ is an integrating factor unless $Mx-Ny\equiv 0$.

4. Show that if the differential equation $Mdx+Ndy=0$ is of the form

$$x^ay^b(mydx+nxdy)+x^cy^d(pydx+qxdy)=0,$$

where a, b, c, d, m, n, p, $q\,(mq\neq np)$ are constants, then x^hy^k is an integrating factor. Hence find a general solution of $(x^{1/2}y-xy^2)dx+(x^{3/2}+x^2y)dy=0$.

5. Assuming that the differential equation $M(x,y)dx+N(x,y)dy=0$ has an integrating factor which is a function of $x+y^2$, find the relation to be satisfied by M and N. Show that the equation$(3y^2-x)+2y(y^2-3x)y'=0$ admits an integrating factor which is a function of $(x+y^2)$. Hence solve this differential equation.

6. Show that $2sin(y^2)+xycos(y^2)y'=0$ admits an integrating factor which is a function of x only. Hence solve the differential equation.

7. Reduce the following differential equations into linear form and solve:

$$(i)\,y^2y'+y^3/x=\sin x \qquad (ii)\,y'\sin y+x\cos y=x \qquad (iii)\,y'=y(xy^3-1).$$

8. Find the orthogonal trajectories of the following families of curves:

$$(i)\,e^x\sin y=c \qquad (ii)\,y^2=cx^3.$$

9. Find the family of oblique trajectories which intersect the family of straight lines $y=cx$ at an angle of $45°$.
Note: An oblique trajectory is a curve that intersect each member of a given family of curve at a constant angle $\alpha\neq 90°$.

10. Show that the following families of curves are self-orthogonal:

$$(i)\,y^2=4c(x+c) \qquad (ii)\,x^2/c^2+y^2/(c^2-1)=1.$$

Supplementary problems from “Advanced Engg. Maths. by E. Kreyszig (8^{th} Edn.)
(i) Page 32, Q.10,12,17,26,29,35
(ii) Page 39, Q.13,18,20,28,29,33,34
(iii) Page 51, Q.7,9,10,15,17