Assignment 3 (Quantum Mechanics)

1. Draw the wave function for a particle in a box at the n = 4 energy level.
2. Draw the probability distribution for a particle in a box at the n = 3 energy level.
3. What is the probability of locating a particle of mass m between x = L/4 and x = L/2 in a 1-D box of length L? Assume the particle is in the n=1 energy state.

4. State, giving your reasons, which of the following functions would make satisfactory wavefunctions for all values of the variable x: :: (i) Ne^{ax^2}; (ii) Ne^{-ax^2}; (iii) $Ne^{-ax^2}/(3-x)$; and (iv) Ne^{-ax}, where N and a are constants.

5. Consider a particle whose normalized wave function is

$$\psi(x) = 2a\sqrt{a}xe^{-ax} \qquad x > 0$$
$$= 0 \qquad x < 0$$

(a) For what value of x does $P(x) = |\psi(x)|^2$ peak?

(b) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$.

(c) What is the probability that the particle is found between $x = 0$ and $x = 1/a$?

6. The wavefunction of a particle moving in the x-dimension is

$$\psi(x) = \begin{cases} Nx(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

(a) Normalize the wavefunction.
(b) Calculate $\langle x \rangle$, $\langle x^2 \rangle$, Δx.
(c) Calculate $\langle p_x \rangle$, $\langle p_x^2 \rangle$, Δp_x.

7. If the normalized wave function of a particle in a box is given by

$$\psi(x) = \begin{cases} \sqrt{\frac{30}{L^5}}x(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

what is the probability of obtaining the energy of the ground state, E_1, if a measurement of the energy is carried out?

8. Determine <E> for a particle in a box with wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{30}{L^5}}x(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$