Binomial distribution.

1. Tens, coins are thrown simulotaneously. Find the probability of getting at least seven heads.

Answer:-

p = Probability of getting head $= 1/2$

q = probability of not getting $= 1/2$

$\therefore$ Probability of getting seven at least seven head out of ten throws

$$P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$$

$$= {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right)$$

$$= \left(\frac{1}{2}\right)^{10}(120 + 45 + 10 + 1)$$

$$= \frac{176}{1024} = 0.1718$$

2. An irregular six-faced die thrown and the expectation that in the 10 throws it will give five even number is twice the expectation that it will givero four even numbers. How many items in 10,000 sets of 10 throws each would you expect it to given not an even numbers?

Let p be the probability of getting an even number.

probability of x even number in a throws

$$P(X=x) = {}^{10}C_x \cdot p^x (1-\cancel{q}p^x)^{10-x}$$

~~we are~~ given that

$$P(X=5) = 2P(X=4)$$

$${}^{10}C_5 \cdot p^{\cancel{5}}(1-p)^{\cancel{5}} = 2\,{}^{10}C_4 \cdot p^{\cancel{4}}(1-p)^{\cancel{6}}$$

$$\frac{\cancel{10!}}{5!\,5!} \cdot P = \frac{2\times \cancel{10!}}{6!\,4!}(1-P)$$

$$\frac{1}{5}P = \frac{\cancel{2}}{\cancel{6}}\frac{1}{3}(1-P)$$

$$\frac{8}{\cancel{15}}P = \frac{1}{\cancel{3}} \qquad P = \frac{5}{8} \qquad q = \frac{3}{8}$$

$$P(X = x) = {}^{10}C_x \left(\frac{5}{8}\right)^x \left(\frac{3}{8}\right)^{10-x}$$

Hence the required number of times that in 10,000 sets of 10 throw each we get no even number.

$$10,000 \times P(X=0) = 10000 \times \left(\frac{3}{8}\right)^{10} = 1 \text{ (approx)}$$

In a binomial distribution consisting of 5 independent trials probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution.

Answer:- given that

$$p(x=1) = 0.4096 \quad \text{— (i)}$$

$$p(x=2) = 0.2048 \quad \text{— (ii)}$$

divided equation (i) by (ii)

$$\frac{{}^5C_1 \cdot \cancel{p^1} (1-p)^{\cancel{4}}}{{}^5C_2 \cdot p^{\cancel{2}} (1-p)^{\cancel{3}}} = \frac{0.4096}{0.2048}$$

$$\frac{\frac{\cancel{5!}}{4!}(1-p)}{\frac{\cancel{5!}}{3!\,2!}\,p} = 2$$

$$\frac{\cancel{2}}{\cancel{4}} \frac{(1-p)}{p} = 2$$

$$(1-p) = 4p$$

$$p = \frac{1}{5}$$

Answer:- $p = \frac{1}{5}$

4. With the usual notations find 'p' for the binomial variate x, if $n=6$ and $9P(X=4) = P(X=2)$

Answer:

$$9P(X=4) = P(X=2)$$

$$9 \cdot {}^{6}C_{4}(p)^4(1-p)^2 = {}^{6}C_{2}\, p^2(1-p)^4$$

$$9p^2 = (1-p)^2$$

$$9p^2 = 1+p^2-2p$$

$$8p^2+2p-1=0$$

$$8p^2+4p-2p-1=0$$

$$4p(2p+1)-1(2p+1)=0$$

$$(4p-1)(2p+1)=0$$

$$p = 1/4 \text{ or } p = -1/2$$

p is can not be a negative value

so $p = 1/4$

Answer:- $p = 1/4$

poisson distribution.

A car hires firm has two cars, which it hires out day by day the demands for a car on each day is distributed as a possion distribution with mean 1.5 Calculate the proportion of day on which

(i) Neither car is used.

$$P(\text{of } x \text{ demands}) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

by the $\lambda = 1.5$

so

$$P(X=x) = \frac{e^{-1.5}(1.5)^{x}}{x!}$$

Proportion of day on which neigther car is used

$$P(X=0) = \frac{e^{-1.5}\cdot(1.5)^{0}}{0!}$$

$$= e^{-1.5}$$

$$= 0.2231$$

Answer: 0.2231

(ii) some demand is refused.

Proportion of day on which some demands is refused is

$$p(x>2) = 1 - p(x \le 2)$$

$$= 1 - (p(x=0) + p(x=1) + p(x=2))$$

$$= 1 - \left(e^{-1.5} + e^{-1.5}(1.5) + \frac{e^{-1.5}(1.5)^2}{2!} \right)$$

$$= 1 - e^{-1.5}\left(1 + 1.5 + \frac{(1.5)^2}{2!}\right)$$

$$= 1 - 0.2231(3.628)$$

$$= 1 - 0.80873$$

$$= .19126$$

Answer:- 0.19126

6. An insurance company insures 4,000 people against loss of both eye in a car accident. Based on the previous data the rates were computed on the assumption that average 10 person in 100,000 will have car accident each year that results in this type of injury, what is probability that more than 3 of the insured will collect on their policy in a given year.

Answer:-

According to the question

$n = 4000$

$p = \frac{1\cancel{0}}{10000\cancel{0}}$

Since $\lambda = np$

$\lambda = \frac{4\cancel{000} \times 1}{10\cancel{000}} = \frac{2}{5} = 0.4$

So $p(x) = \frac{e^{-0.4} \cdot (0.4)^x}{x!}$

For more than 3 of the insured will collect on their policy in a given year.

$$P(x>3) = 1 - P(X \leq 3)$$

$$= 1 - \left(P(X=0) + P(X=1) + P(X=2) + P(X=3) \right)$$

$$= 1 - \left(e^{-0.4} + \frac{e^{-0.4}(0.4)}{1!} + \frac{e^{-0.4}(0.4)^2}{2!} + \frac{e^{-0.4}(0.4)^3}{3!} \right)$$

$$1 - e^{-0.4}\left(1 + 0.4 + \frac{(0.4)^2}{2!} + \frac{(0.4)^3}{3!}\right)$$

$$1 - e^{-0.4}(1 + 0.4 + 0.0800 + .001066)$$

$$1 - 0.6703 \times (\cancel{1.4866}\ 1.4906)$$

$$1 - 0.9991$$

$$= 0.0009.$$

Answer: 0.0009

7. A manufacturer who produces medicine bottles find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the produce of bottles. Using poisson distribution find how many boxes will contain:

(i) No defectives

Answer:- P = Probability of defective bottles = 0.1%

$$= \frac{1}{1000}$$

λ = mean number of defective bottles in a box of 500

$$500 \times \frac{1}{1000} = 0.5$$

P is small we may use poisson distribution

$$P(X=x) = \frac{e^{-0.5} (0.5)^x}{x!}$$

for $x=0$

$$P(X=0) = \frac{e^{-0.5} (0.5)^0}{0!}$$

$$= e^{-0.5}$$

$$= .6065$$

(ii) At least two defectives

$$P(x \geq 2) = 1 - P(x<2)$$

$$= 1-(P(X=0) + P(X=1))$$

$$= 1-\left(\frac{e^{-0.5} \cdot (0.5)^0}{0!} + \frac{e^{-0.5} \cdot (0.5)^1}{1!}\right)$$

$$= 1-(0.6065 + 0.3032)$$

$$= 1-(0.9097)$$

$$= 0.0903$$

Answer: 0.0903

8 In a book of 520 pages 390 typographical error occur. Assuming poisson law for the number of errors per page find the probability that a random sample of 5 pages will contain no error.

The average number of error per page in the book is given by

$$\lambda = \frac{390}{520} = 0.75$$

Using possion law

$$p(x=x) = \frac{e^{-\lambda} \lambda^{x}}{x!} = \frac{e^{-0.75} \cdot (0.75)^{x}}{x!}$$

for $x = 5$

$$[p(x=0)]^{5} = \left[\frac{e^{-0.75} \cdot (0.75)^{0}}{0!}\right]^{5}$$

$$= [e^{-0.75}]^{5}$$

$$= [0.4723]^{5}$$

$$= 0.0235$$

answer: 0.0235

Normal Distribution

If x is normally distributed and the mean x is 12 SD is 4

(a) Find out the probability of the following.

(i) $x \geq 20$ (ii) $x \leq 20$ (iii) $0 \leq x \leq 12$

we have $\mu = 12$ $\sigma = 4$

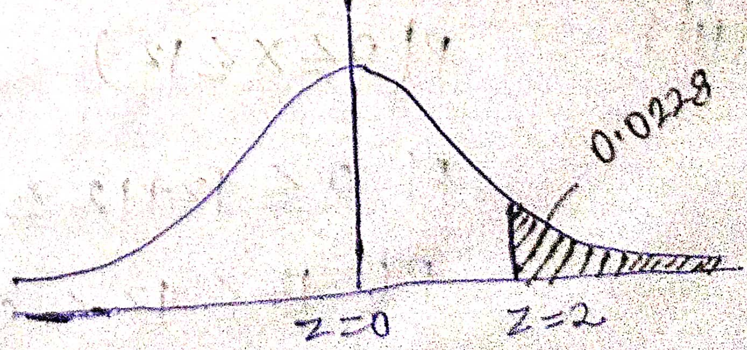
i.e $x \sim N(12, 16)$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{x - 12}{4}$$

$$4z + 12 = x$$

$P(x \geq 20) = ?$



$P(4z + 12 \geq 20) =$

$P(z \geq 2)$

$= 0.5 - P(0 \leq z \leq 2)$

by Normal Probability Tables

$0.5 - 0.4772$

$= 0.0228$ Answer:-

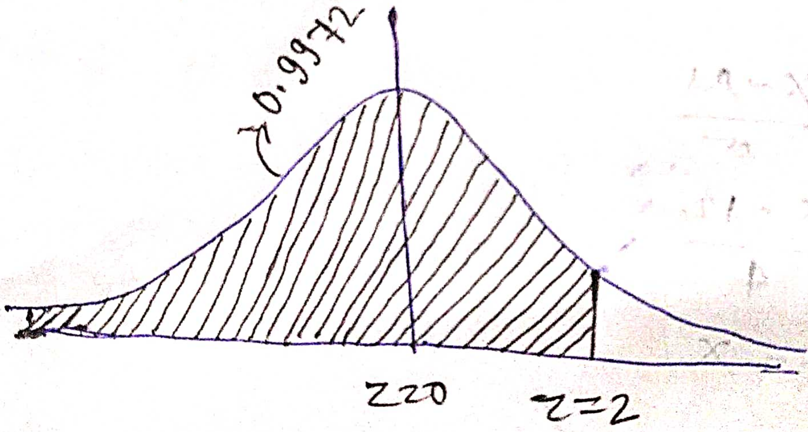
(ii) $X \le 20$

$P(X \le 20) = 1 - P(X \ge 20)$

$= 1 - P(Z \ge 2)$

$= 1 - 0.0228$ by $P(Z \ge 2) = 0.0228$

$= 0.9972$



(iii) $P(0 \le X \le 12)$

$P(0 \le 4Z + 12 \le 12)$

$P(-12 \le 4Z \le 0)$

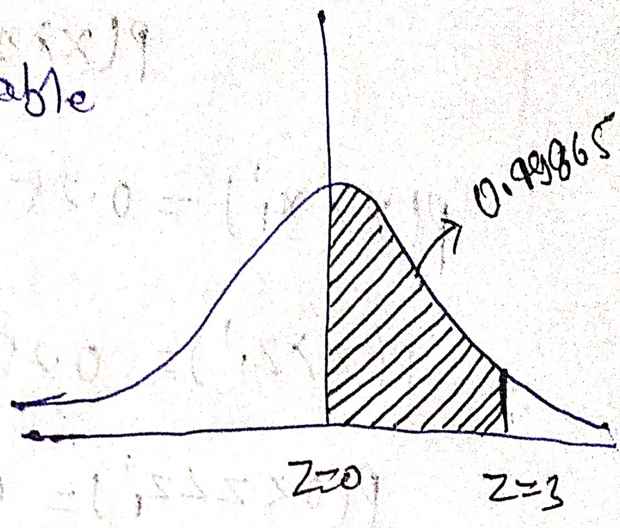
$P(-3 \le Z \le 0)$ by symmetry

$P(0 \le Z \le 3)$

$p(0 \leq z \leq 3)$

by Normal probability table

$= 0.49865$



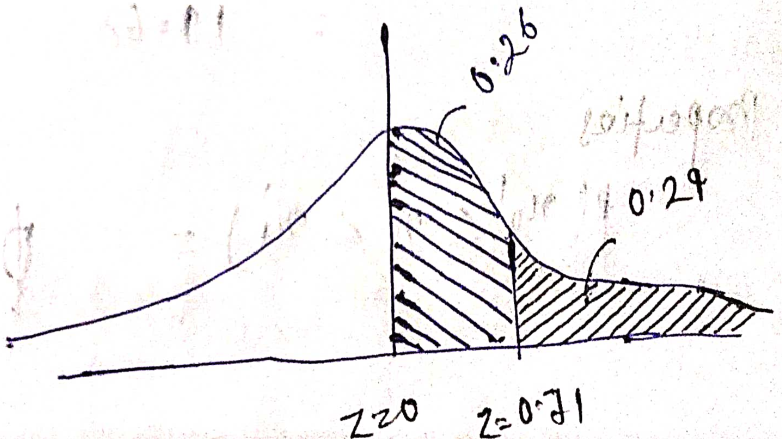
(b) Find x' when $p(x > x') = 0.24$

$$P(x > x') \Rightarrow P(z > z') = 0.24$$

$$\Rightarrow P(0 < z < z') = 0.26$$

From normal tables

$\therefore \; z' = 0.71$



$$\frac{x' - 12}{4} = 0.71$$

$x' = 12 + 0.71 \times 4$

$x' = 14.84$

$\boxed{x' = 14.84}$

c. Find x_0' and x_1' when $P(x_0' < X < x_1') = 0.50$ and

$$P(X > x_1') = 0.25$$

$$P(X > x_1') = 0.25$$

$$P(Z > Z_1') = 0.25$$

$$P(0 < Z < Z_1') = 0.25$$ From normal table

$$Z_1' = 0.67$$

$$\frac{x_1' - 12}{4} = 0.67$$

$$x_1' = 12 + 0.67 \times 4$$

$$= 14.68$$

Properties

$$P(x_0' < x < x_1') = \phi\left(\frac{x_1' - \mu}{\sigma}\right) - \phi\left(\frac{x_0' - \mu}{\sigma}\right)$$

$$0.50 = \phi\left(\frac{x_1' - \mu}{\sigma}\right) - \phi\left(\frac{x_0' - \mu}{\sigma}\right)$$

$$0.50 = 0.25 - \phi\left(\frac{x_0' - \mu}{\sigma}\right)$$

$$\phi\left(\frac{x_0' - \mu}{\sigma}\right) = -0.25$$

from normal table

$$\frac{x_0' - 12}{4} = -.67$$

$$x_0' - 12 = -2.68$$

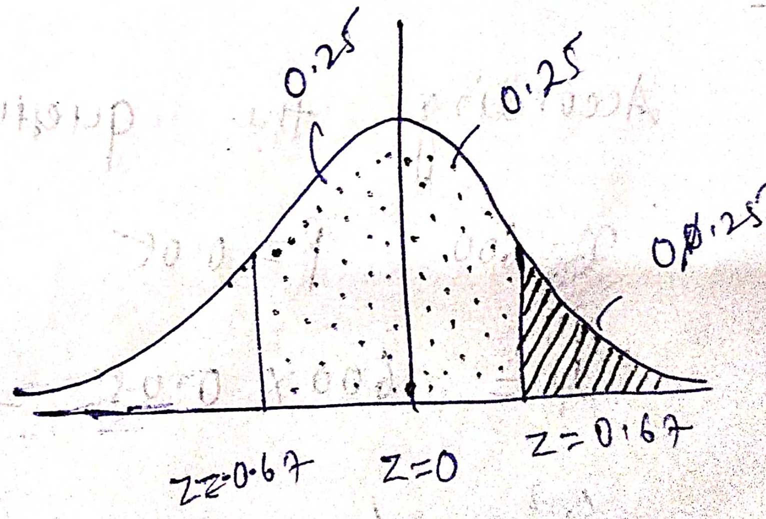
$$x_0' = 12 - 2.68$$

$$= 9.32$$

$$x_0' = 9.32$$

$$x_1' = 14.68$$

$$x_0' = 9.32$$



(10) There are six hundred Economics Students in the post-graduate classes of a university and the probability for any student to need a copy of a particular book from the university library on any day is 0.05. How many copies of the book should be kept in the university library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed!

Answer:-

According the question

$n = 600 \quad p = 0.05$

$\mu = 600 \times 0.05 = 30$

$\sigma^2 = 600 \times 0.05 \times 0.95$

$\sigma^2 = 28.5$

$\sigma = \sqrt{28.5} = 5.3$

we want x_1 such that

$P(X < x_1) > 0.90$

$P(Z < Z_1) > 0.90$

$P(0 < Z < Z_1) > 0.90 - 0.50$

$P(0 < Z < Z_1) > 0.40$

$Z_1 > 1.28$ From Normal tables

$$\frac{x_1 - 30}{5.031} > 1.28$$

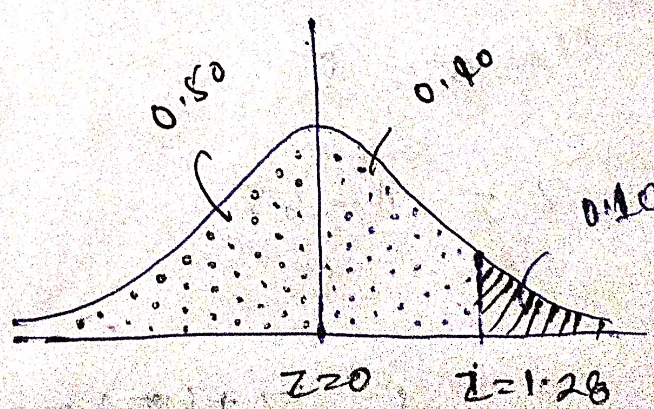
$x_1 > 30 + 5.3 \times 1.28$

$x_1 > 30 + 6.784$

$x_1 > 36.784$

$x_1 \simeq 37$

Hence library should keep at least 37 copies of the book



11. Q. the marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean value 65 and with a standard deviation of 5. If 3 students are taken at random from this set what is the probability that exactly 2 of them will have marks over 70.

Let r.v x denote the marks obtained by student

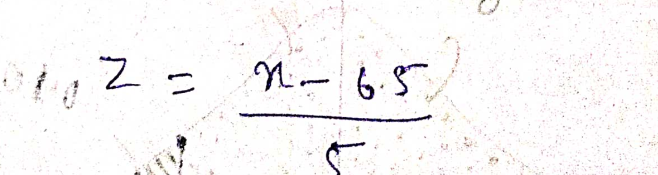
from the question

$P(x>70) = p$ {p is probability that a random select student who has mark over 70}

Z: $\mu = 6.5$

$\sigma = 5$

$Z = \dfrac{x - 6.5}{5}$



$P(5Z + 65 > 70) = p$

$P(5z > 5) = p$

$P(z > 1) = p$

$0.5 - P(0 \leq z \leq 1) = p$

from normal table

$0.5 - 0.3413 = p$

$p = 0.1587$

by binomial probability law

$n = 3$, $x = 2$, $p = 0.1587$

$q = 0.8413$

$^{3}C_{2} \cdot p^{2} q^{1}$

$3 \times (0.1587)^{2} \cdot (0.8413)$

$3 \times (0.0251) \times (0.8413)$

$3 \times (0.0211)$

0.0633

Answer $= 0.0633$