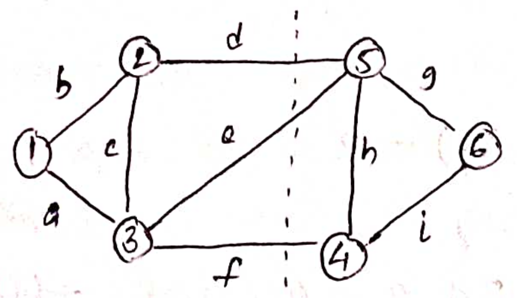
Q Define a cut-set with suitable diagram and mention its properties.

Ans- Set of edges whose removal from graph will make the graph G disconnected, provided no subset of those edges can do so, i.e. minimal number of edges to be removed.



for example, edges {d, e, f} make the graph disconnected.

Properties :-

i) cut set is the minimal set of edges i.e. no more edges can be added to it.

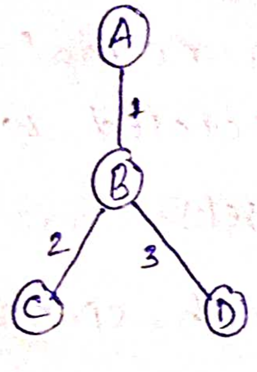
ii) a cut set always cuts the graph into two parts.

iii) the cut set is the set of edges, whose removal destroys all path between the two parts of the graph.

iv) Every edge of a tree is cut-set.

Q. Explain with suitable diagram, that every edge of a tree is cut-set.

Ans. Let us consider a tree with 4 vertices,

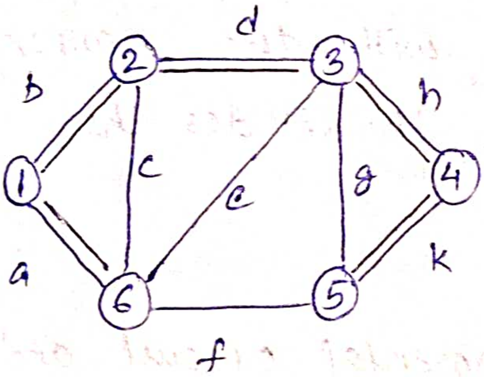


G

Suppose, in the graph G if we remove any of the ~~a~~ edges, the graph turns disconnected. Hence, this is ~~the~~ true for any graph which is a tree.

. what is a fundamental cut-set? And how is it different from a normal cut-set? Show with example.

.



Graph: G

Spanning Tree: T

Consider a spanning tree T of a graph G.

Branches = {a, b, c, d, h, k}

Chords = {c, e, f, g}

Every branch of any spanning tree has fundamental cut-set associated with it. Let S be fundamental cut-set associated with a (branch), consisting of other chords in addition to the branch. i.e.

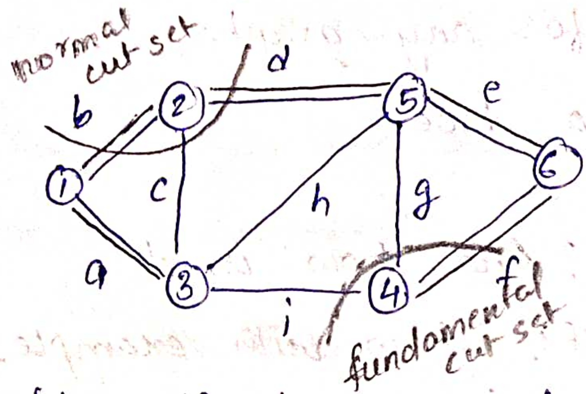
Cut set, S = {a, c, e, f}.

Similarily, other cut-sets of given graphs are

{f, g, k}, {f, g, h}, {d, e, f}, {b, c, e, f}

Fundamental cut-set is different from normal cut-set. As exactly one branch is associated with spanning tree in a fundamental cut-set.

For-example: Consider the graph G, with spanning tree T.

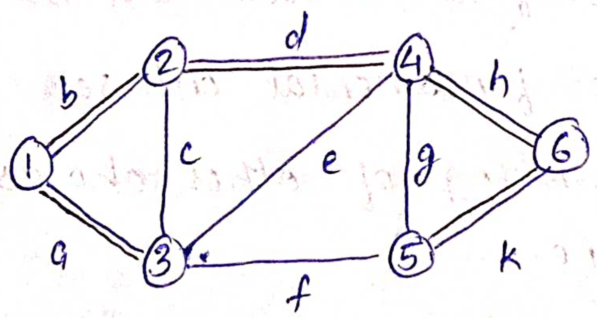


{f, g, i} is a fundamental cut set as only branch f is associated with spanning tree

{b, c, d} is not a fundamental cut-set as, two branches b and d are associated with the spanning tree. But it is normal cut-set as it devides the graph into two parts.

Q. Find the relation between, fundamental circuit and fundamental cut-set with proper example.

Ans.



Graph : G

Spanning Tree : T

Consider a graph G with spanning tree T.

Fundamental cut-set is the set of edges which has exactly one branch associated with the spanning tree consisting other ~~branch~~ chords.

Fundamental cut-sets = {a, c, e, f}, {b, c, e, f}, {d, e, f}

{h, g, f}. {f, g, k}

whereas fundamental circuit is the set of edges which makes the circuit consisting exactly one chord and other branches.

Fundamental circuits = { c, a, b }, { e, d, b, a }, { f, a, b, d, h, k }
{ g, h, k }.

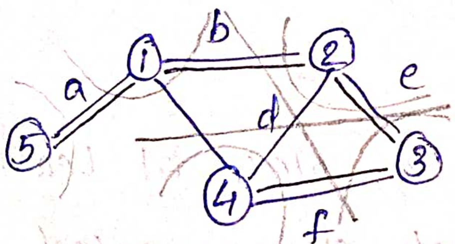
what do you mean by connectivity and seperability ?

Connectivity : denotes wheather the graph is connected or not.

There are two types of connectivity.

1) Edge connectivity : Minimum number of edges whose removal makes the graph disconnected.

Each cut-set of connected graph G consists of certain number of edges. The no. of edges in the smallest cut-set is defined as edge connectivity of G.



G

for ex. smallest cut-set here can ~~be~~ be { a }
hence, edge connectivity = 1.

2) Vertex connectivity : Minimum number of vertex whose removal will make the graph disconnected.

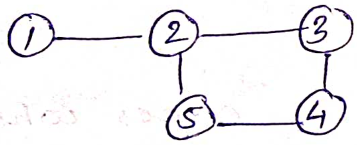
Here, vertex connectivity = 1, since by removing 1 we make graph disconnected.

Separability

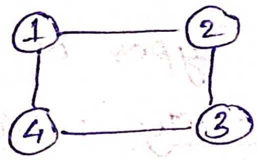
A connected graph is said to be seperable if its vertex connectivity is one. All other connected graphs are called non-seperable.

An equivalent definition is that a connected graph G is said to be seperable if there exists a subgraph g in G such that $\bar{g}$ (the complement of g in G) and g have only one vertex in common.

for ex: consider graph G_1 & G_2



G_1



G_2

Vertex connectivity of graph G_1 is 1, since removal of ② makes graph disconnected hence, it is separable.

Vertex connectivity of graph G_2 is 2, since at least two vertices are needed to make graph disconnected. Hence, it is non-separable.

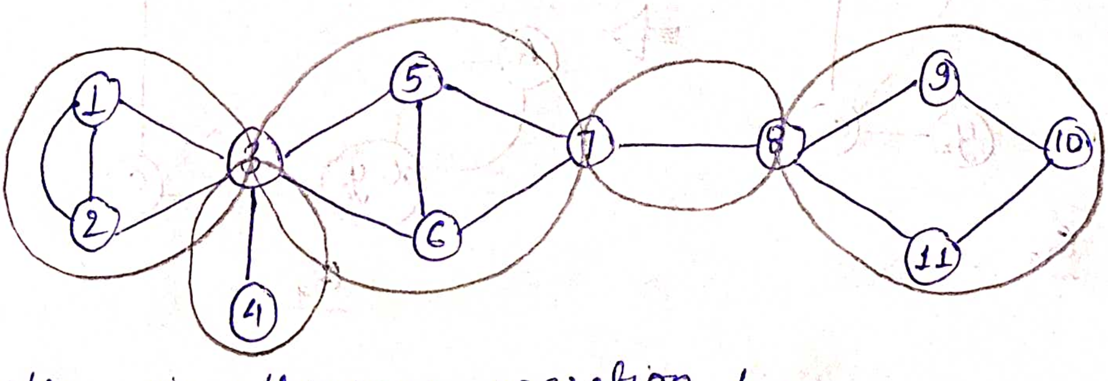
Q. what is an articulation point or cut-vertex?

Ans Consider graph G. which is separable. Hence, having vertex connectivity = 1. The vertex which is deleted to make the graph disconnected is articulation point or cut vertex.

Q. Explain 1-isomorphism and 2-isomorphism with suitable diagram.

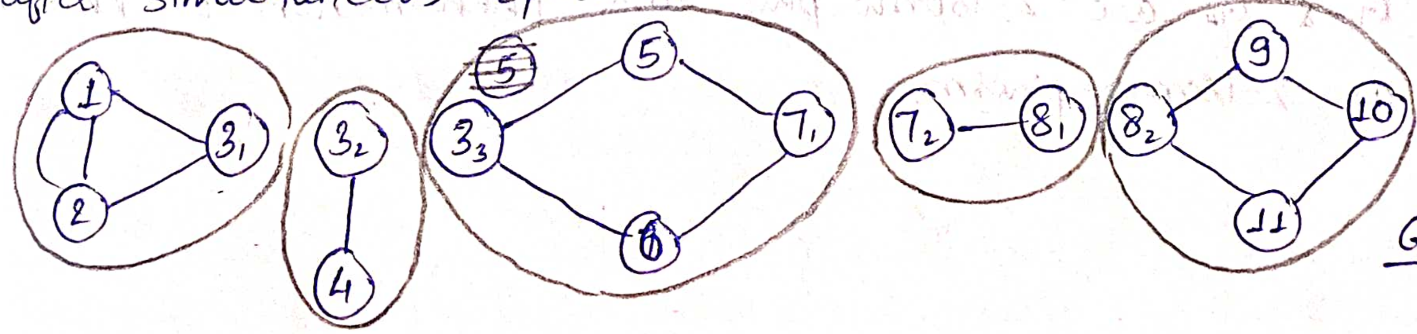
Ans 1-isomorphism: (for separable graph) Two graphs G_1 and G_2 are said to be 1-isomorphic if they become isomorphic to each other under repeated application of following operation.

operation 1: "Split" a cut-vertex into two vertices to produce two disjoint subgraphs.



G

after simultaneous operation 1.



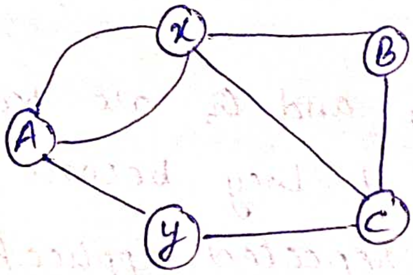
G'

Graph G & G' are 1-isomorphism.

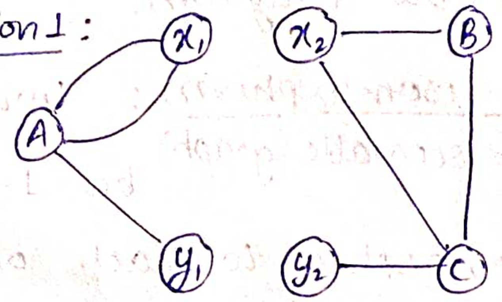
2- isomorphism: (for non-separable graph) Two graphs are said to be ~~non-separable~~ 2-isomorphism if they become isomorphic after undergoing operation 1 or operation 2, or both operations any number of times.

Operation 2: Join the cut-vertices in such a way, that x_1 is connected with y_2 and x_2 is connected with y_1. Also known as twist operation.

G_1

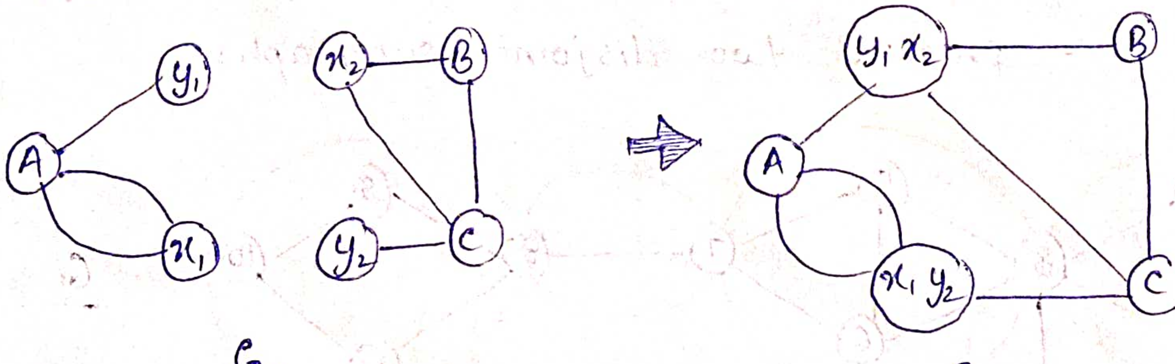


operation 1:



G_2

operation 2:



G_3

G_4

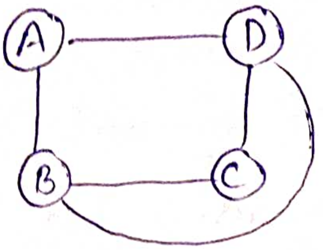
G_1 & G_4 are 2-isomorphic and phenomenon is known as 2-isomorphism.

Q. Define a planar graph with suitable example.

Ans- Consider a graph G, which does not have any edge crossover in single plane, i.e. when a graph can be made to draw in a plane.

for ex.

G

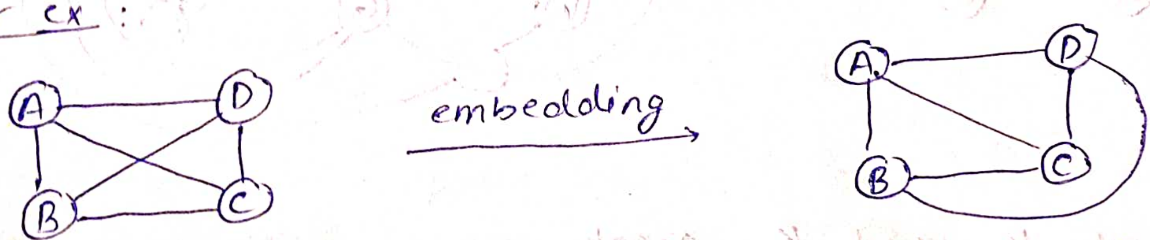


Graph G is planar graph, since it has no edge crossovers.

Q. what is embedding?

Ans- A drawing of geometric representation of a graph on any surface such that no edges intersect is called embedding

for ex :

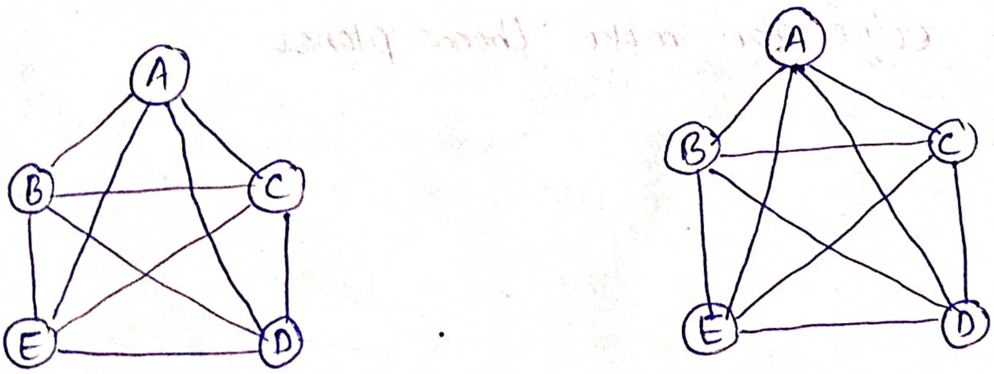


edges intersect here

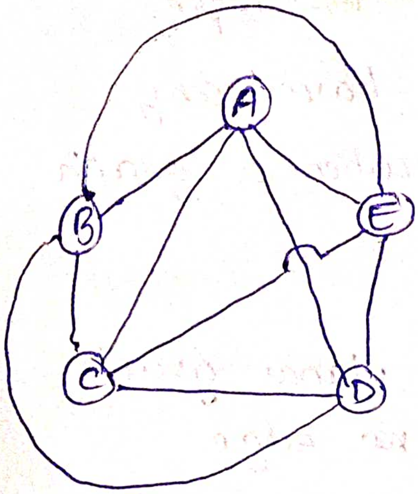
edges do-not intersect here.

Q. What are the properties of K_5 and $K_{3,3}$?

Ans. K_5 → complete graph with 5 vertices.

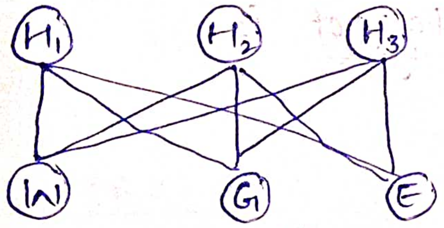


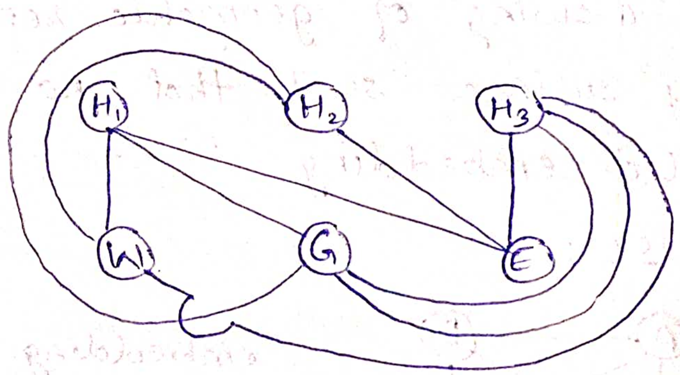
K_5 K_5



K_5

K_{33} → six vertex and nine edge graph.
or bipartite graph.





$K_{3,3}$

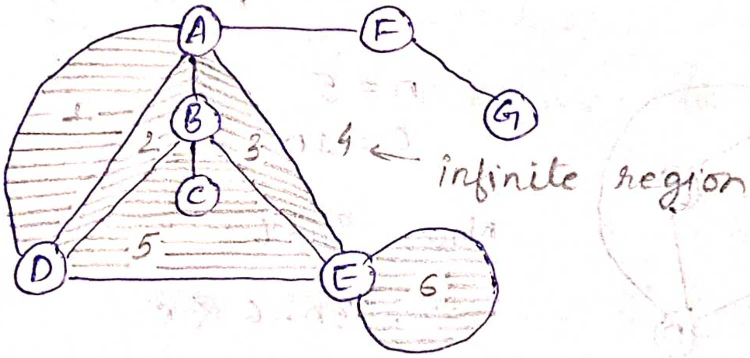
Properties of K_5 and K_{33} graph.

1) Both graph are non-planar.
2) Both graph are regular.
3) Removal of one vertex can make them planer.
4) Removal of one edge can make them planar.

Q What is a region and infinite region. show with proper diagram.

Ans **Region**: A plane representation of a graph divides the plane into regions (also called windows, faces or meshes), A region is characterized by set of edges (or the set of vertices) forming its boundary.

Region is not defined in a non-planar graph or even in a planar graph not embeded in plane.



1, 2, 3, 4, 5, 6 all are regions.

Infinite Region: The portion of the plane lying outside a graph embedded in plane, such as region-4 is infinite in its extent. Such a region is called the infinite, unbounded, outer or exterior region for that particular representation. Like other regions, infinite region is also characterised by set of edges (or vertices). By changing the embedding of the given planar graph, we can change the infinite region.

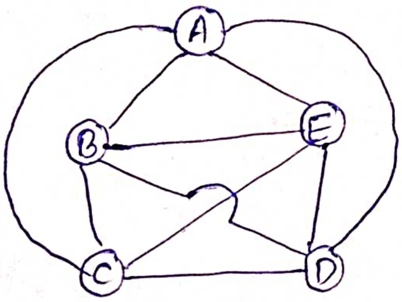
Q. Using Euler's inequality check the planarity of K_5 and $K_{3,3}$.

Ans- Euler's inequality - in any simple, connected planar graph with f regions, n vertices and e edges $(e > 2)$ the following inqualily must be hold.

$$e \geq \frac{3}{2} f \quad \text{— (i)} \quad \text{where } f = e - n + 2$$

$$e \leq 3n - 6 \quad \text{—(ii)}$$

In case of K_5, the complete graph of five vertices



$n = 5$

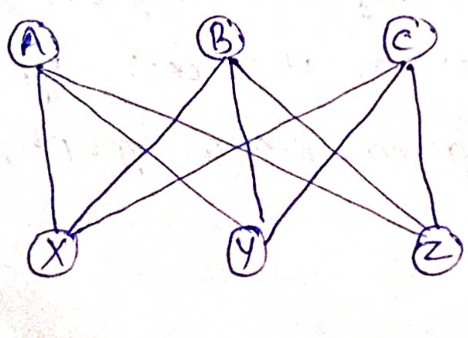
$e = 10$

Now, $3n - 6 = 9$

$\Rightarrow \quad 3n - 6 < e$

which does not satisfy the Euler's inequation. Hence, graph K_5 is non-planar.

In case of $K_{3,3}$, no region in this graph is bounded fewer than four edges. Hence, if this graph wer planar, we would have



$$2e \geq 4f$$

Substituting value of f from Euler's formula

$n = 6$

$e = 9$

$$2e \geq 4(e - n + 2)$$

$$2 \cdot 9 \geq 4(9 - 6 + 2)$$

$18 \geq 20$ is contradiction.

Q. Mention the steps to detect planarity with resultant graph.

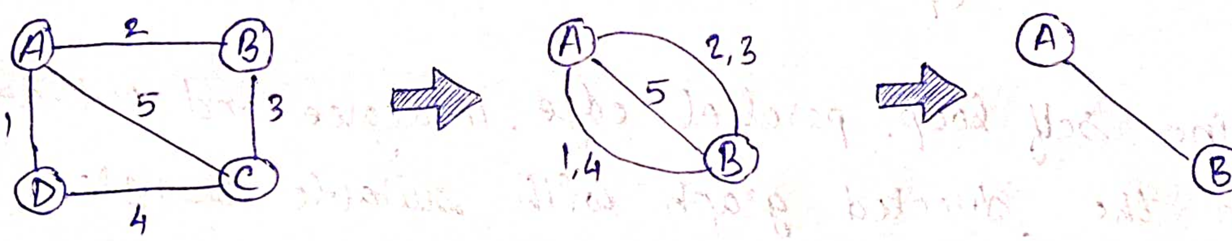
Ans. Steps to detect planarity with resultant graph.

1) If a disconnected graph G is having k-components check the planarity of each of the k components if any of the components is non planar then graph is also non-planar

2) Remove all self loop from given graph.

3) Remove all parallel edges by keeping only one edge.

4) Remove all series edges by merging them.



Resultant graph :-

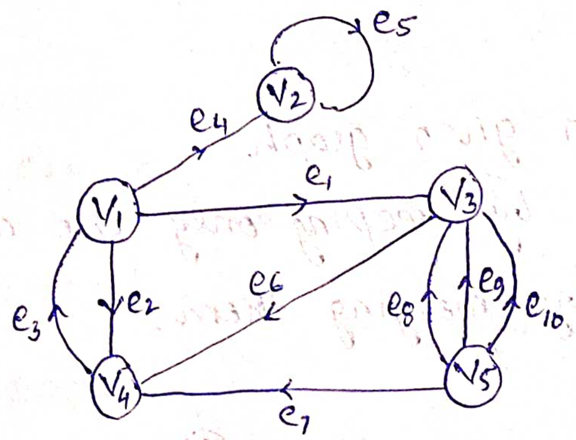
1) If single graph, then planar

2) A complete graph with 4 vertices, then planar

3) A non-separable simple graph with $n \geq 5$ & $e \geq 7$ then use Euler's inequality.

Q. What is directed graph? Explain with proper diagram?

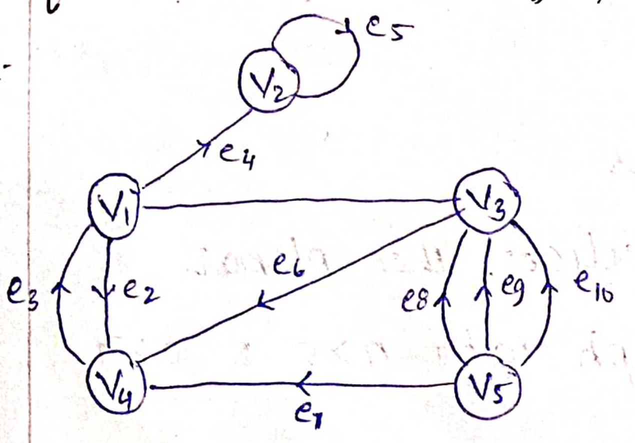
Ans A directed graph (digraph) G consists of a set of vertices $V = \{V_1, V_2, V_3, \ldots\}$, a set of edges $E = \{e_1, e_2, \ldots\}$ and a mapping ψ that maps every edge onto some ordered pair of vertices (V_i, V_j).



Directed graph with 5 vertices and 10 vertices.

Q. Define self loop, parallel edge, in-degree and out-degree from the directed graph with suitable example.

Ans-



G

Self loop: A edge for which the initial and terminal vertices are same forms a self loop. ex. e_5

Parallel edge: Two directed edges are said to be parallel if they are mapped onto same pair of vertices. That in addition to being parallel in the sense of undirected graph edges, parallel directed edges must also agree in

In-degree - The number of edges incident ~~out~~ in of a vertex v_i is called the in-degree (or in-valence or inward demi-degree) of v_i and written as $d^-(v_i)$

for ex: $d^+(v_1) = 3$, $d^+(V_2) = 1$, $d^+(V_5) = 4$

Out-degree. The number of edges incident out of a vertex v_i is called the out-degree (or out-valence or outward demi-degree) of v_i and written as $d^+(v_i)$

for ex: $d^-(v_1) = 1$, $d^-(v_2) = 2$, $d^-(v_5) = 0$

Q what is isomorphic digraph?

Ans. Isomorphic graphs were defined such that they are identical behavior in terms of graph properties. In other words, if their labels are removed, two isomorphic graphs are indistinguishable. For two digraphs to be isomorphic not only must their corresponding undirected graph be isomorphic, but the direction of corresponding edges must also agree.

For example.

Both the graphs are isomorphic.

the direction of their arrows. For ex. $e_8 . e_9 . e_{10}$ are parallel whereas edges e_2 and e_3 are not.

Q. Explain Handshaking dilemma with proper example.

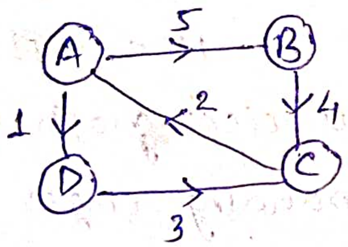
Ans For a digraph G, the sum of all the out-degrees in a graph is equal to the sum of all the in-degrees.

or the sum of sum of all the out-degrees in the graph and sum of all the in-degrees is equal to twice the number of edges.

i.e.

$$\boxed{\Sigma \partial^{+}(v_i) + \Sigma \partial^{-}(v_i) = 2e}$$

for example consider graph G,



In-degree	Out-degree
$\partial^{-}(A) = 1$	$\partial^{+}(A) = 2$
$\partial^{-}(B) = 1$	$\partial^{2}(B) = 1$
$\partial^{-}(C) = 2$	$\partial^{+}(C) = 1$
$\partial^{-}(D) = 1$	$\partial^{+}(D) = 1$

no. of edges $= 5$

Now,

$$\Sigma \partial^{+}(v_i) + \Sigma \partial^{-}(v_i) = (1+1+2+1) + (2+1+1+1)$$

$$= 5 + 5 = 10$$

$$= 2 \times \text{no. of edges}$$

$$= 2e.$$

Hence, Handshaking dilemma is satisfied.

In-degree - The number of edges incident ~~out~~ in of a vertex v_i is called the in-degree (or in-valence or inward demi-degree) of v_i and written as $d^-(v_i)$

for ex: $d^+(v_1) = 3$, $d^+(v_2) = 1$, $d^+(v_5) = 4$

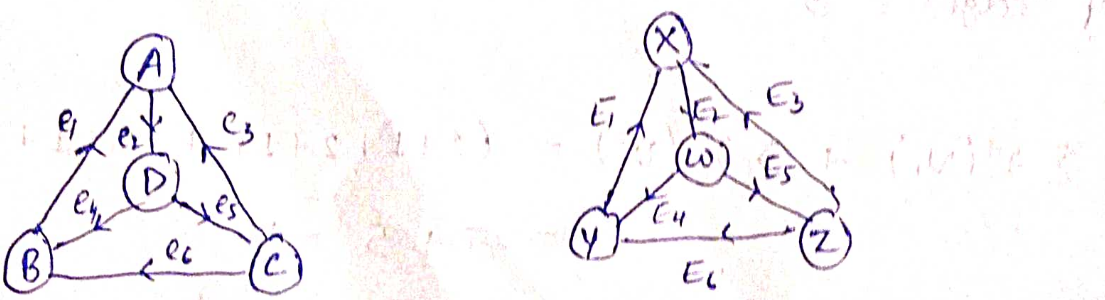
Out-degree - The number of edges incident out of a vertex v_i is called the out degree (or out-valence or outward demi-degree) of v_i and written as $d^+(v_i)$

for ex: $d^-(v_1) = 1$, $d^-(v_2) = 2$, $d^-(v_5) = 0$

Q What is isomorphic digraph?

Ans. Isomorphic graphs were defined such that they are identical behavior in terms of graph properties. In other words, if their labels are removed, two isomorphic graphs are indistinguishable. For two digraphs to be isomorphic not only must their corresponding undirected graph be isomorphic, but the direction of corresponding edges must also agree.

For example.

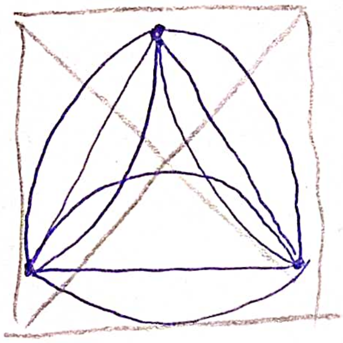


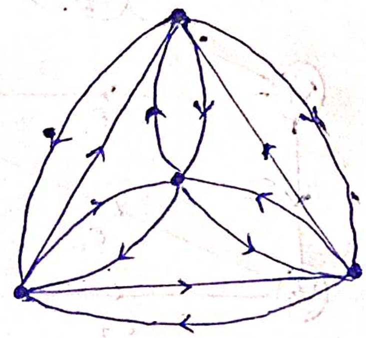
Both the graphs are isomorphic.

Define complete symmetric graph for digraph ?

A complete symmetric digraph is a simple digraph in which there is exactly one edge directed from every vertex to every other vertex.

for example

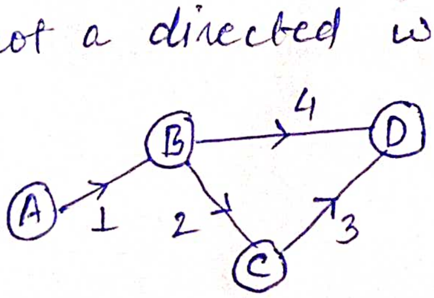




Q Define directed walk, semi-directed walk, directed path, semi-directed path, semi-circuit, directed circuit, strongly connected graph and weakly connected graph.

Ans Directed walk - A directed walk from a vertex v_i to v_j is an alternating sequence of vertices and edges, beginning with v_i and ending with v_j, such that each edge is oriented from the vertex preceding it to the vertex following it. No edge in directed walk appears more than once, but a vertex may appear.

Semi walk - A semi walk in a directed graph is walk in the corresponding undirected graph, but is not a directed walk.



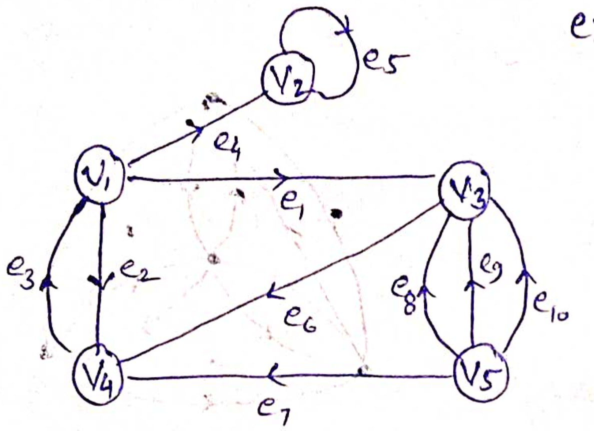
Directed walk : A 1 B 2 C 3 D

Semi walk : A 1 B 4 D 3 C

Directed path - the sequences of vertices and edges is directed path if it has consistant direction from v_i to v_j. ex. $\{v_5 - e_8 \, v_3 \, e_6 \, v_4 \, e_3 \, v_1\}$

Semi path - The path which is the path in corresponding undirected graph but not in directed graph is semipath.

ex. $\{v_5 \, e_7 \, v_4 \, e_6 \, v_3 \, e_1 \, v_1\}$

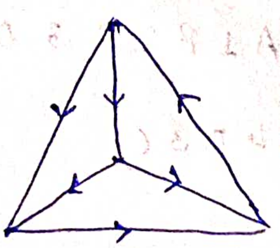


Directed circuit - the sequences of vertices and edges is directed circuit if the path from v_i to v_j is closed path and has consistent direction. for ex. $\{e_1 \, e_6 \, e_3\}$

Semi-circuit - The circuit which is not directed circuit but circuit corresponding to undirected graph. for ex $\{e_1 \, e_6 \, e_2\}$

Strongly connected digraphs - A digraph G is said to be strongly connected if there is atleast one directed path path from every vertex to other vertex.

weakly connected digraphs - A digraph G is said to be weakly connected if its corresponding undirected graph G is connected by but G is not strongly connected.



Strongly connected

weakly connected

Q why self loop are not considered in Incedence matrix?

Ans 1) If self loop is considered the edge is incident on same vertex which contradicts the property of incidence matrix which has exactly two 1's but for self loop the column of that edge will contain only single 1.

2) Number of 1's in each row equals the degree of corresponding vertex but the if self loop is considered we write only one 1 for that edge whereas degree of that vertex is 2.

Q why parallel edges are not considered in adjacency matrix?

Ans. 1) If there is an parallel edge between two verteices then we have to write 1 two times, but only one value can be written