Date → 16/01/202

Q) Can there exist any graph without any vertices.

Ans → No,

* 1. vertex ⟶ Initial & Terminal vertex
2. Edge
3. Self loop
4. Parallel Edge (Starting and ending vertex are same).
5. Incidence Property denotes if some v & e are connected or not. or some common vertex and e betn them is there or not.
6. AdJacency Property
7. Pendant vertex → vertex connected with only 1 edge.
8. Degree of vertex is the no. of edges connected with it.

* Hand Shaking Lemma ⟶ $\boxed{\sum_{i=1}^{n} \partial(v_i) = 2e}$

→ The no. of vertices with odd degree in a graph is always even.

$$\sum \partial(v_i) = 2e$$

$$\boxed{\sum_{\text{even}} \partial(v_i) + \sum_{\text{odd}} \partial(v_i) = \underset{\text{even no.}}{2e}}$$

*Finite & Infinite graph

→ with finite no. of edges & infinite no. of v.
(Isolated vertices)

or
→ with finite no. of vertices & infinite no. of e.

Q.) Show that an infinite graph with a finite no. of edges must have an infinite no. of isolated vertices.

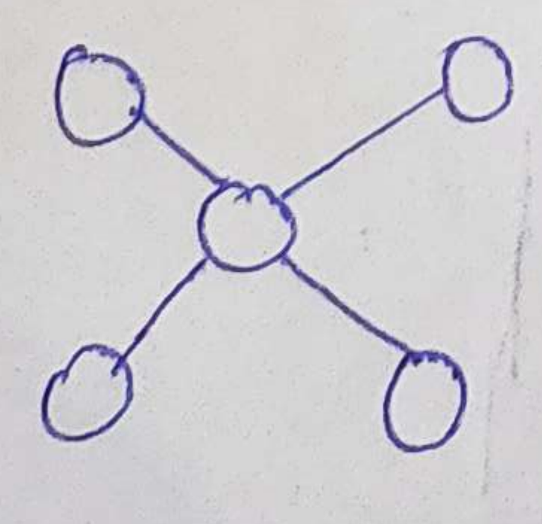
Q) Show that an infinite graph with finite no. of vertices ~~must~~ have one pair of vertices must be join by infinite no. of parallel vertices.

*Simple graph, Multigraph, Complete graph,
(don't have self loop or p.e.) (only p. edge & no self loop) (e b/w every pair of v)
↓
degree of ~~this~~ (n-1)

Regular graph,
(Degree of every v is same)

Q) Explain that every complete graph are regular but not all regular graph is complete.

* Wheel graph, N-cube graph
(center v connected by all v.) (Every v. has 1 bit diff.)



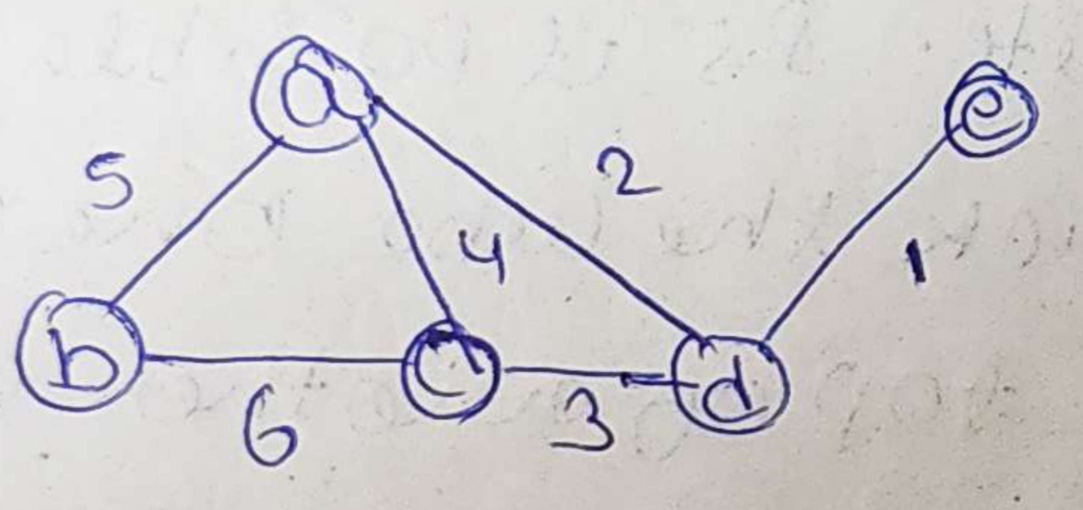
Application

* Konigsberg Bridge Problem (1736) (L. Euler)

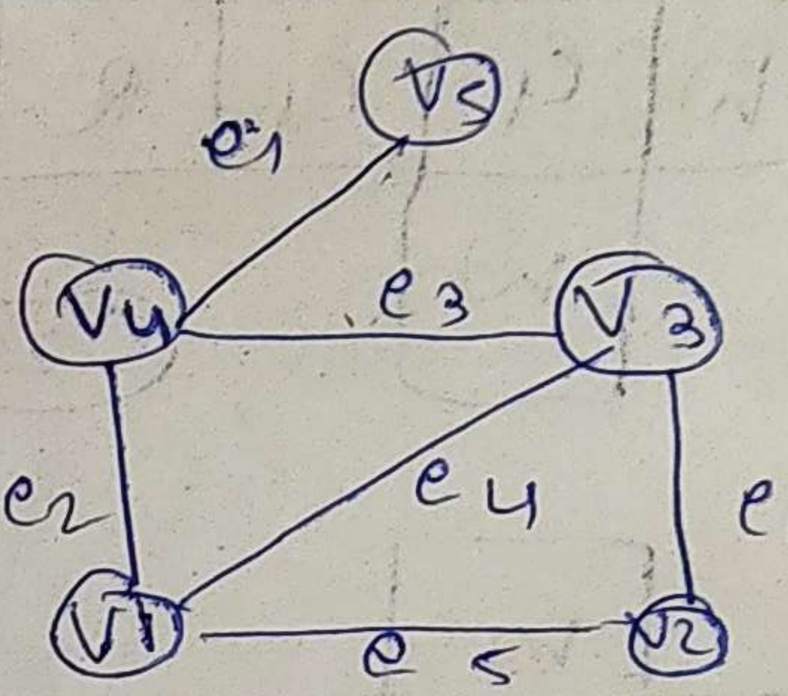
Q) Show that the maximum no. of edges with n vertices is $\frac{n(n-1)}{2}$.

Date → 18/01/23

* Isomorphism



G_1



G_2

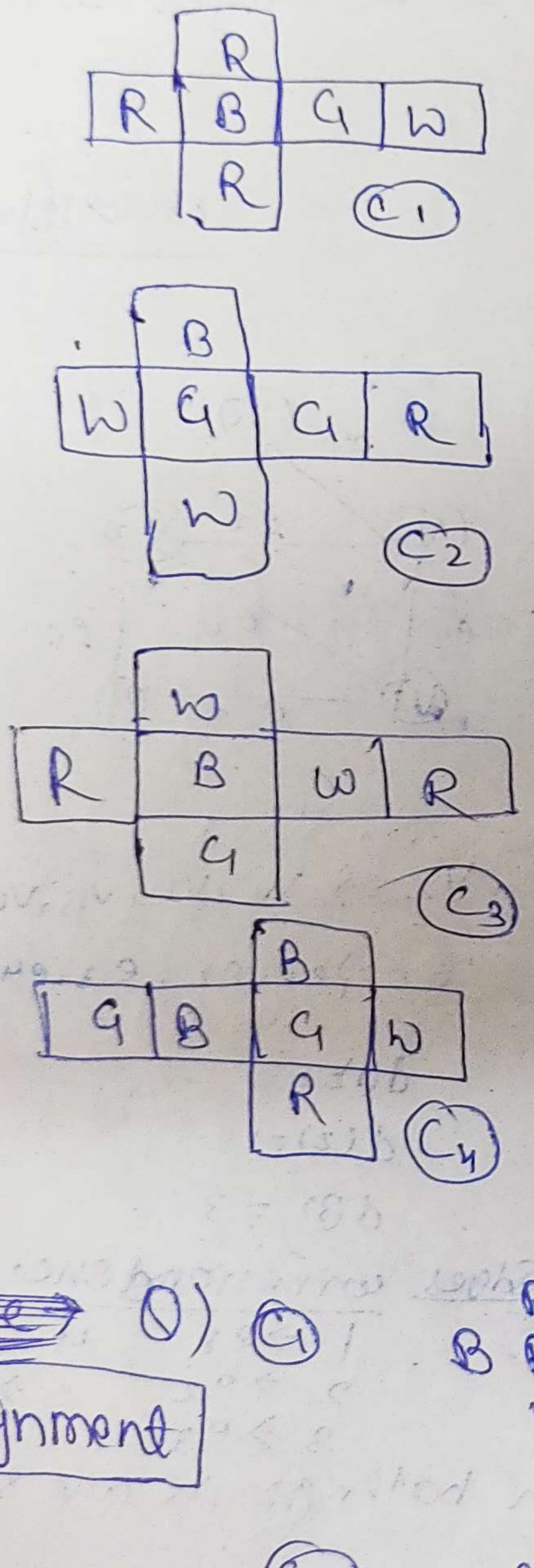
1) $V_1 = \{a, b, c, d, e\}$ $V_2 = \{v_1, v_2, v_3, v_4, v_5\}$
2) $E_1 = \{1, 2, 3, 4, 5, 6\}$ $E_2 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$
3) $d(1) = 1$ $d(1) = 1$
   $d(2) = 2$ $d(2) = 1$
   $d(3) = 3$ $d(3) = 3$
4) vertex correspondence
   $e \to v_5$ $a \to v_1$
   $d \to v_4$ $b \to v_2$
   $c \to v_3$

Edges correspondence
$1 \to e_1$ $4 \to e_4$
$2 \to e_2$ $5 \to e_5$
$3 \to e_3$

Properties :- 1) No. of vertices in both graph are same.
2) No. of edges are same.
3) Both graphs have same no. of vertices with a given degree.
4) There must be some 1 to 1 correspondence betn v & e

* Subgraph → edge disjoint
→ vertex disjoint

* Multicolor cube problem



Q) Given 4 cubes with six faces of every cube are variously colored with Blue, Green, Red & White. Is it possible to stack the cube one on the top of another to form a color such that no color appears twice on any of the four side of this column

~~Solve~~ Q) C1

R
B R G
W
G

C2

W
B G R
G
B

C3

G
B B R
R
W

C4

B
B W G
B
R

Assignment

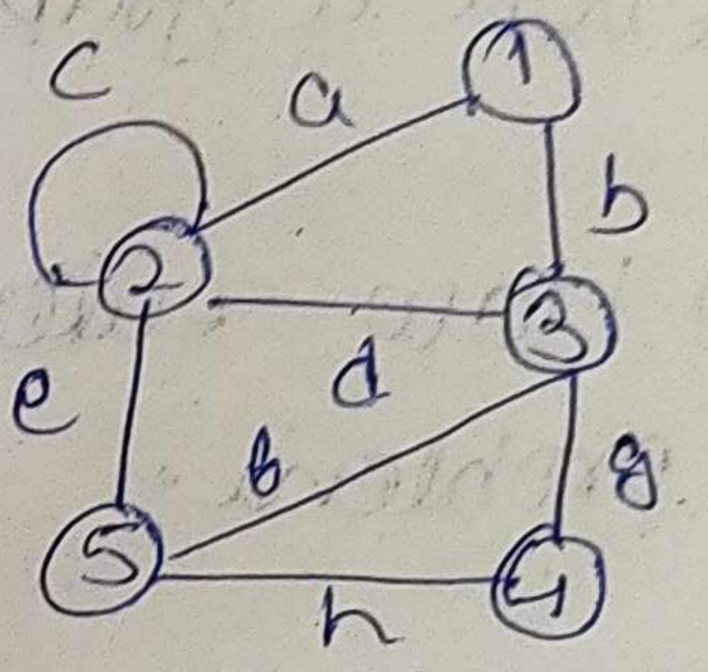
* Walk, Path & Circuit :-

* Walk → Finite, alternating seq^n of edges & v where vertex & edge can be repeated.

Eg → W_1 : → 1 - b - 3 - d - 2 - e - 5 - h - 4

* Trail → It's a walk where v can be repeated but edges can't be repeated.

Eg → T_1 → 1 - b - 3 - d - 2 - e - 5 - h - 4



* Path → It's a walk where v & edges cannot be repeated.

Eg → P_1 → 1 - b - 3 - d - 2 - e - 5 - h - 4

* Circuit → It's a closed walk where v & e can't be repeated except the starting v.

Eg → C_1 → 1 - b - 3 - d - 2 - a - 1

Assign

(Q). Show with proper Example that every self loop is a circuit but not every circuit is self loop.

* Connected Graph :- There exists atleast one path between every pair of vertices otherwise Disconnected.

Date → 25/01/2023

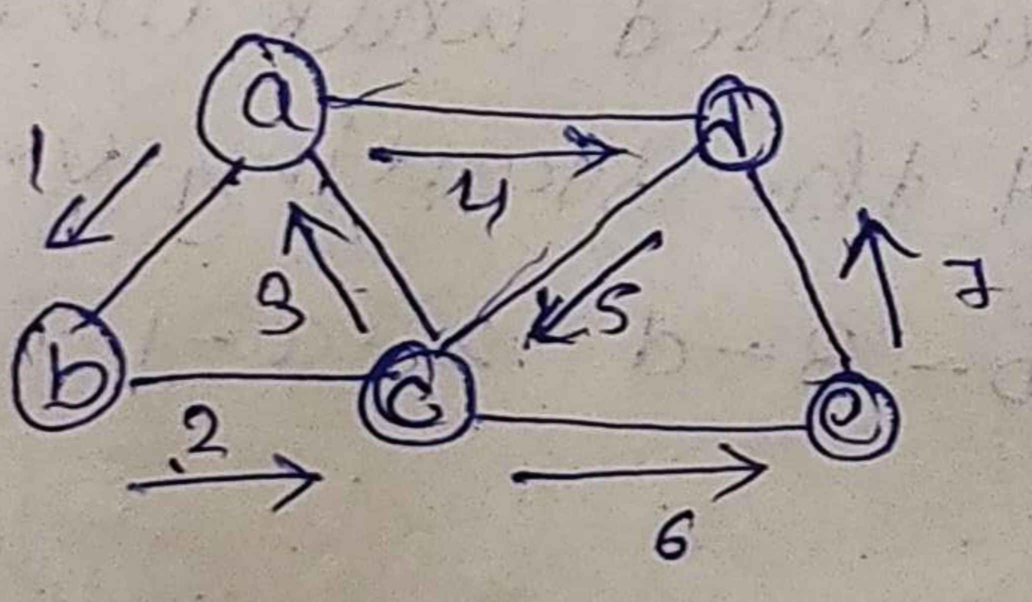
* Component → It is subgraph which is connected from ~~~~ dis connected.

It is denoted by K. connected graph K = 1.

* Euler graph → If degree of every vertex is even a graph is euler graph.

Q) How euler solve the Konigsberg bridge problem.

* Open Euler graph or unicursal graph →

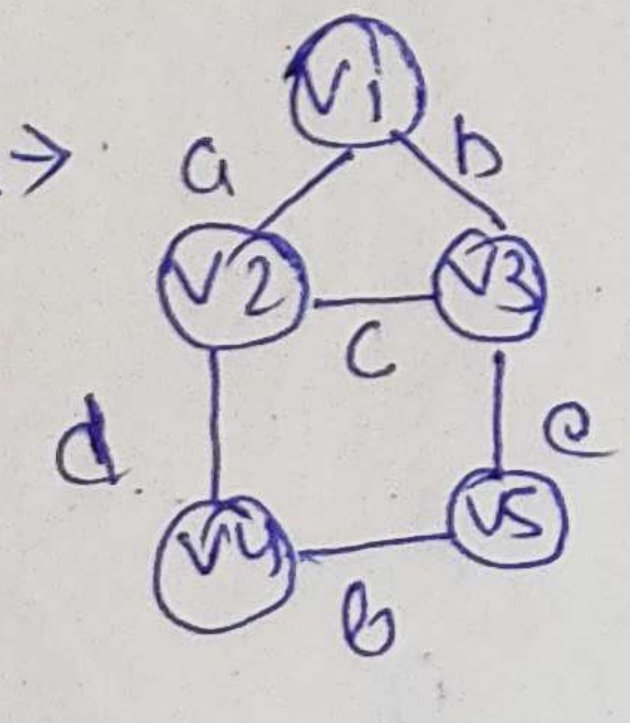


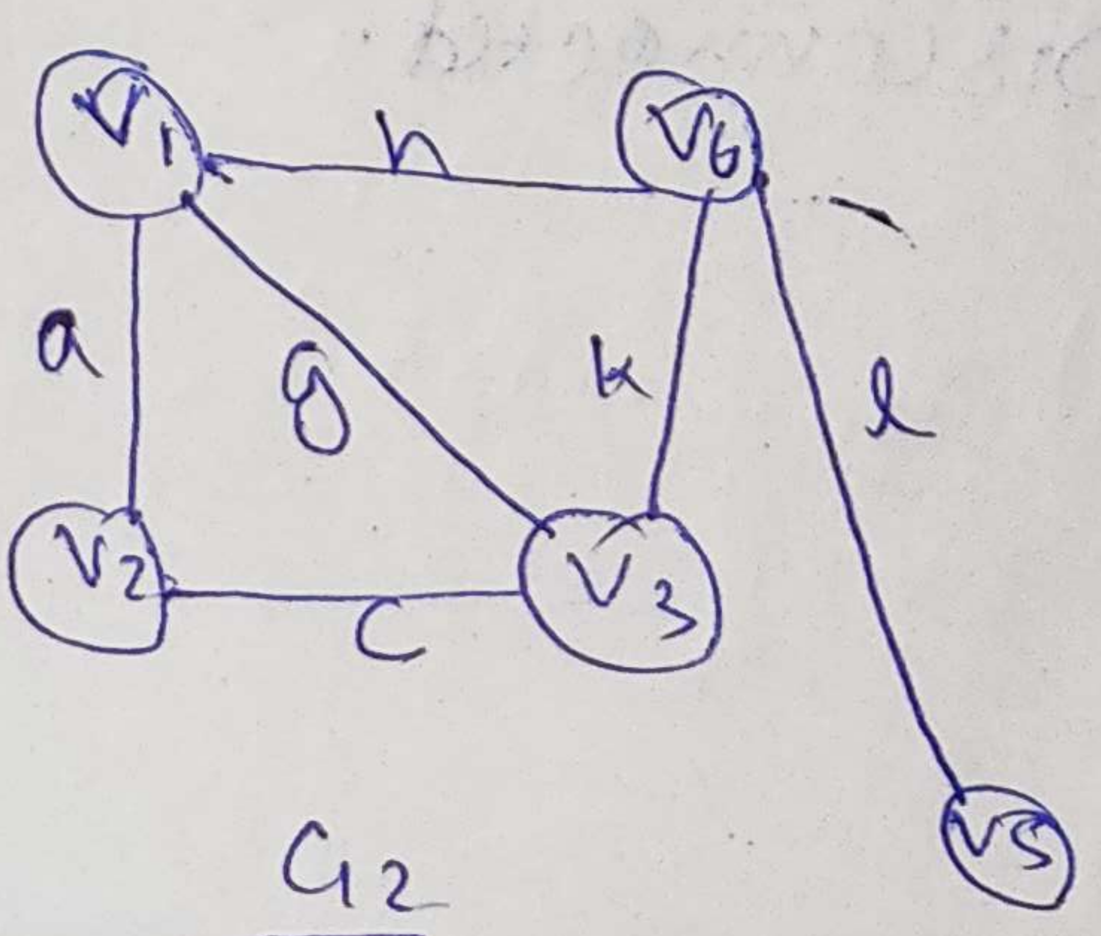
→ Start from a v traverse every e exactly once & reach to another v.

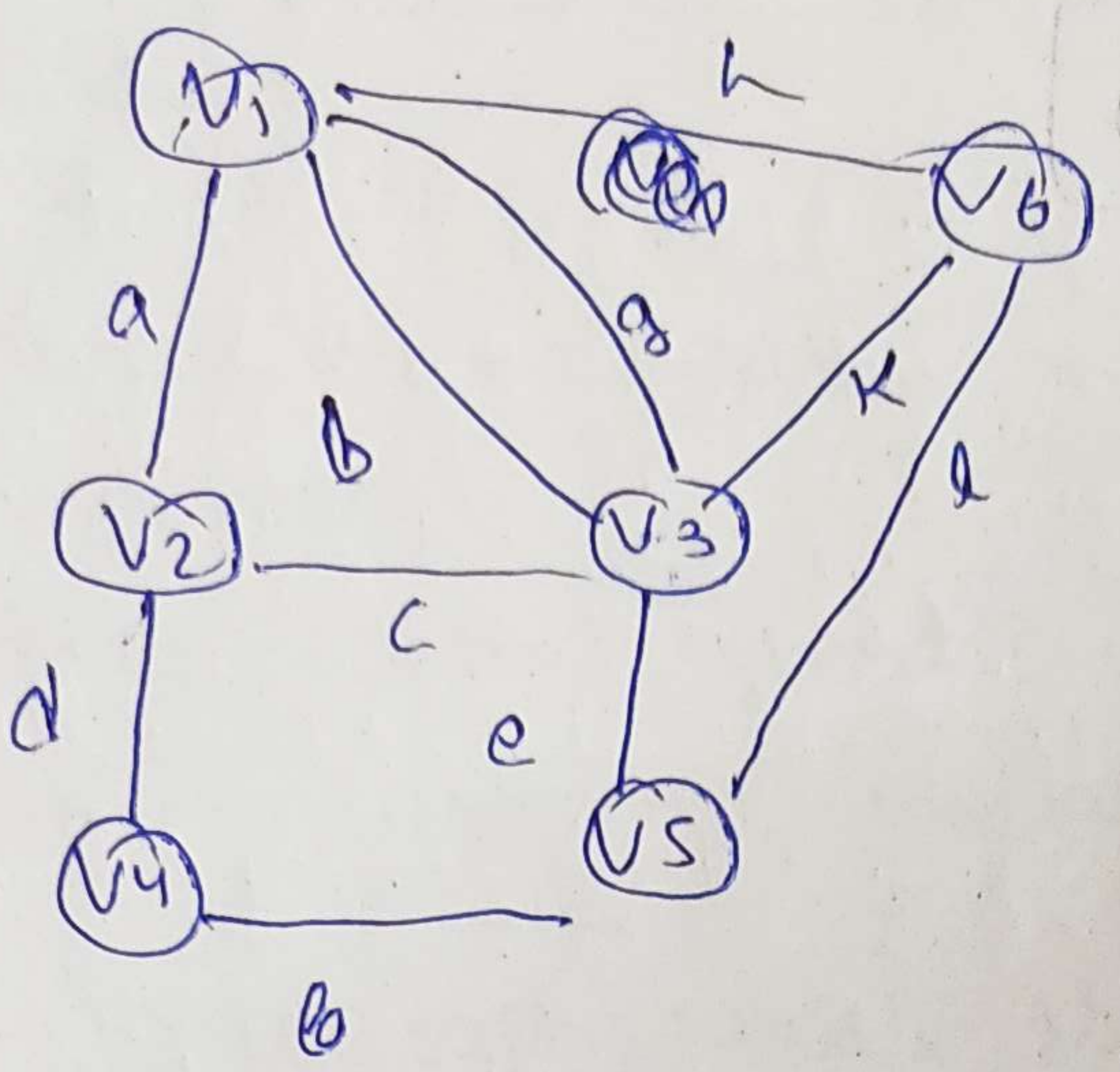
* Operation on graph :-

1. Union 2. Intersection 3. Ring sum 4. Deletion
5. Fusion.

1. Union →





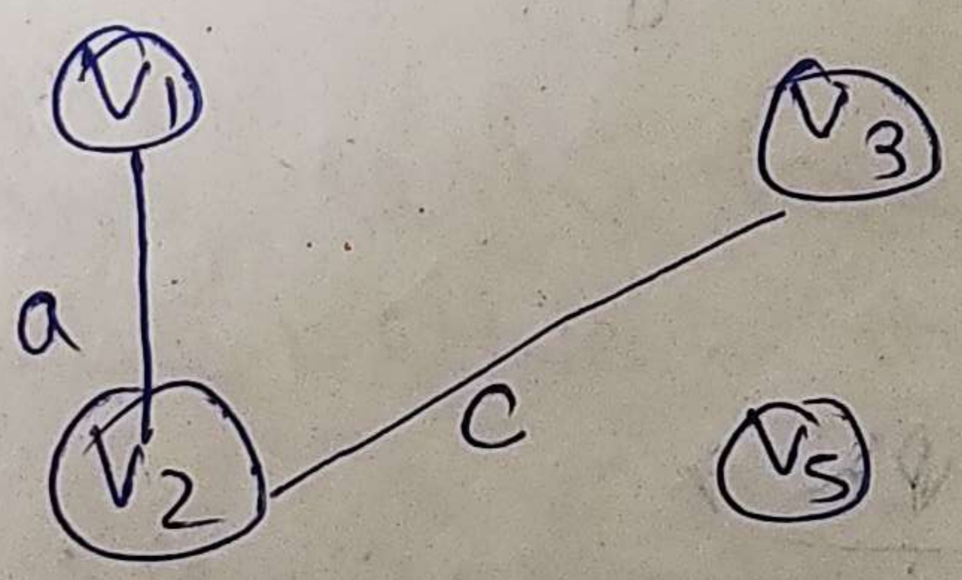


(All edge)

$\boxed{G_1 \cup G_2}$

2. Intersection →

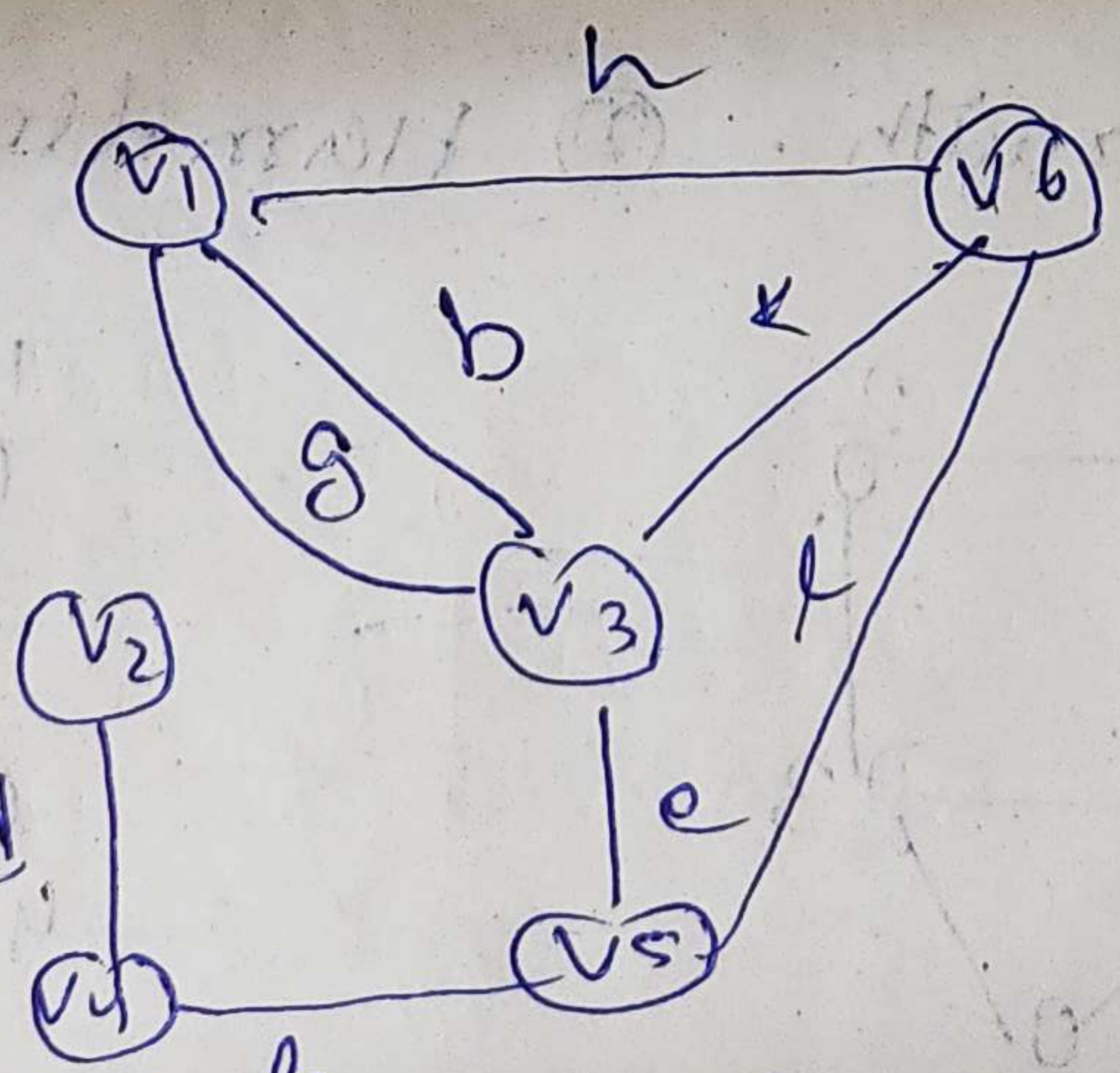
(only common edges)



$\boxed{G_1 \cap G_2}$

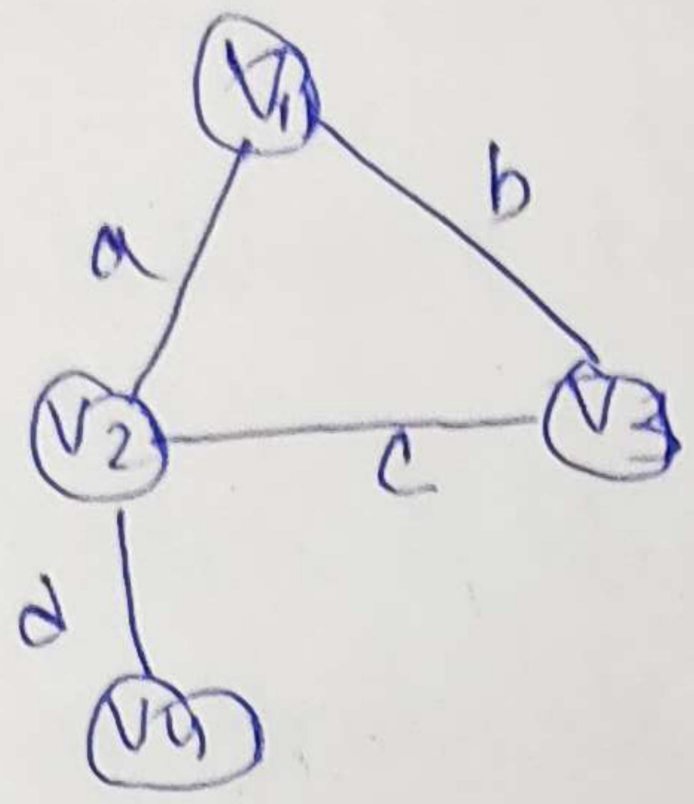
3. Ring sum →

only uncommon (edges)

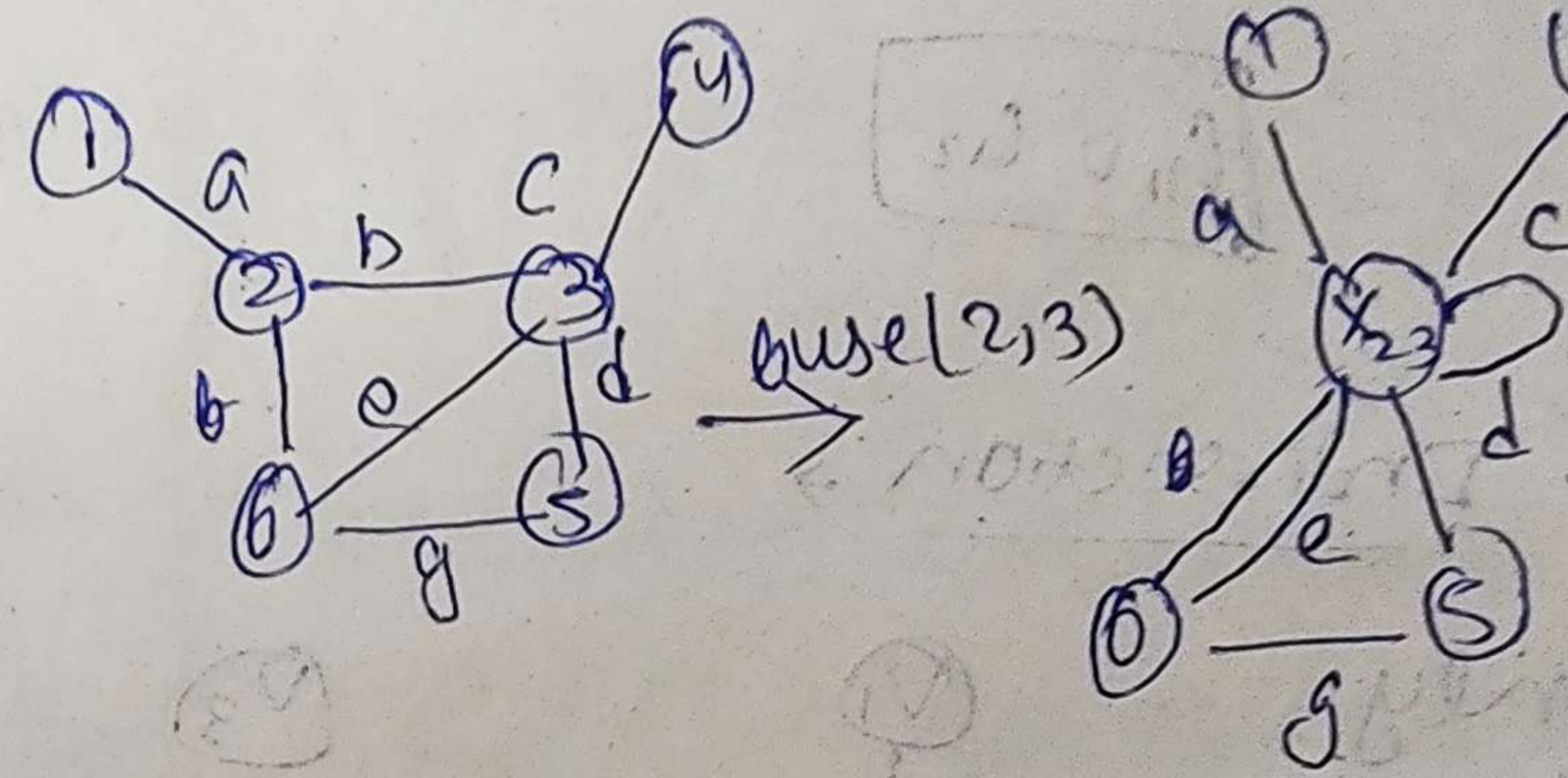


$\boxed{G_1 \oplus G_2}$

4. Deletion → $\boxed{G_1 - V_3}$



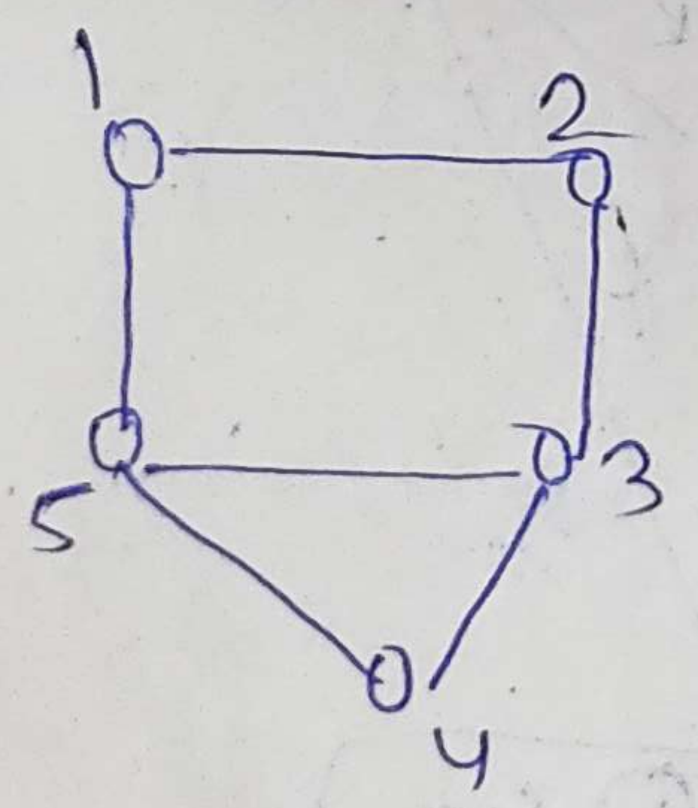
5. Fusion ⇒

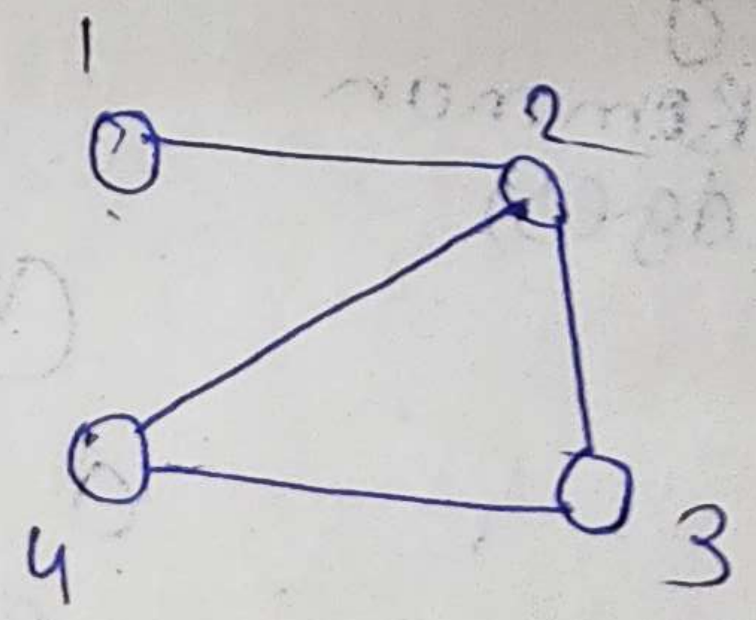


* Hamiltonian graph →

Q.) Differentiate between Hamiltonian graph and euler graph

→ (1) Hamiltonian circuit. (2) Hamiltonian path





✓ H. Ckt
✓ H. path] → H. G.

× H. Ckt
✓ H. Path → $\xrightarrow{\times}$ H.G

Chapter → 3 (TREE)

Properties

→ Tree is a graph without any circuit.

→ There is one path between every pair of v in a tree.

→ A tree with n vertices has $n-1$ Edges.

→ A tree must be minimally connected.

- Tree is a graph which is circuitless, has $(n-1)$ no. of edges for n vertices and has exactly one path betn every pair of vertices.

* **Distance** → Length of path = no. of edges present in that path.

↳ Distance is the length of the shortest path available between the pair of vertices.

path 1 : A - 2 - B - 3 - C → 2

path 2 : A - 1 - D - 4 - C → 2

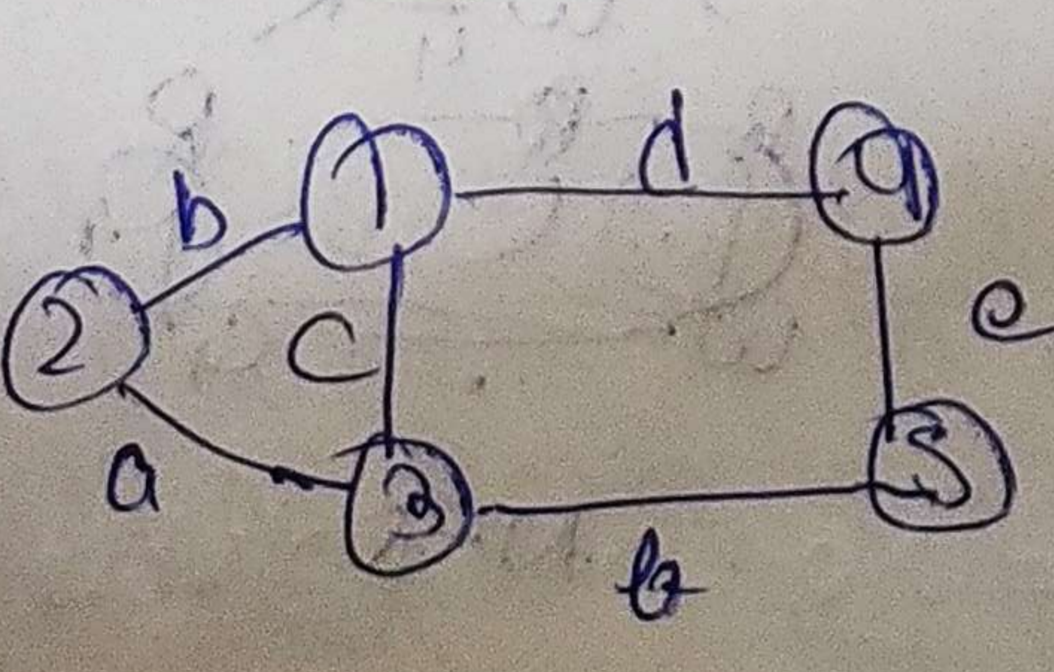
Q) Show that Distance betn vertices of a connected graph is a matric.

Properties of matric

→ Non negativity → $d(x,y) \geq 0$

→ Symmetry → $d(x,y) = d(y,x)$

→ Triangle inequality → $d(x,y) \leq d(x,z) + d(z,y)$



→ Example.

* Eccentricity → $E(v)$ → It is the distance from v to the farthest vertex in the graph.

* The vertex having lowest eccentricity is $\boxed{\text{Centre of tree}}$.

Q) Show that a tree can be either mono centric or by centric.

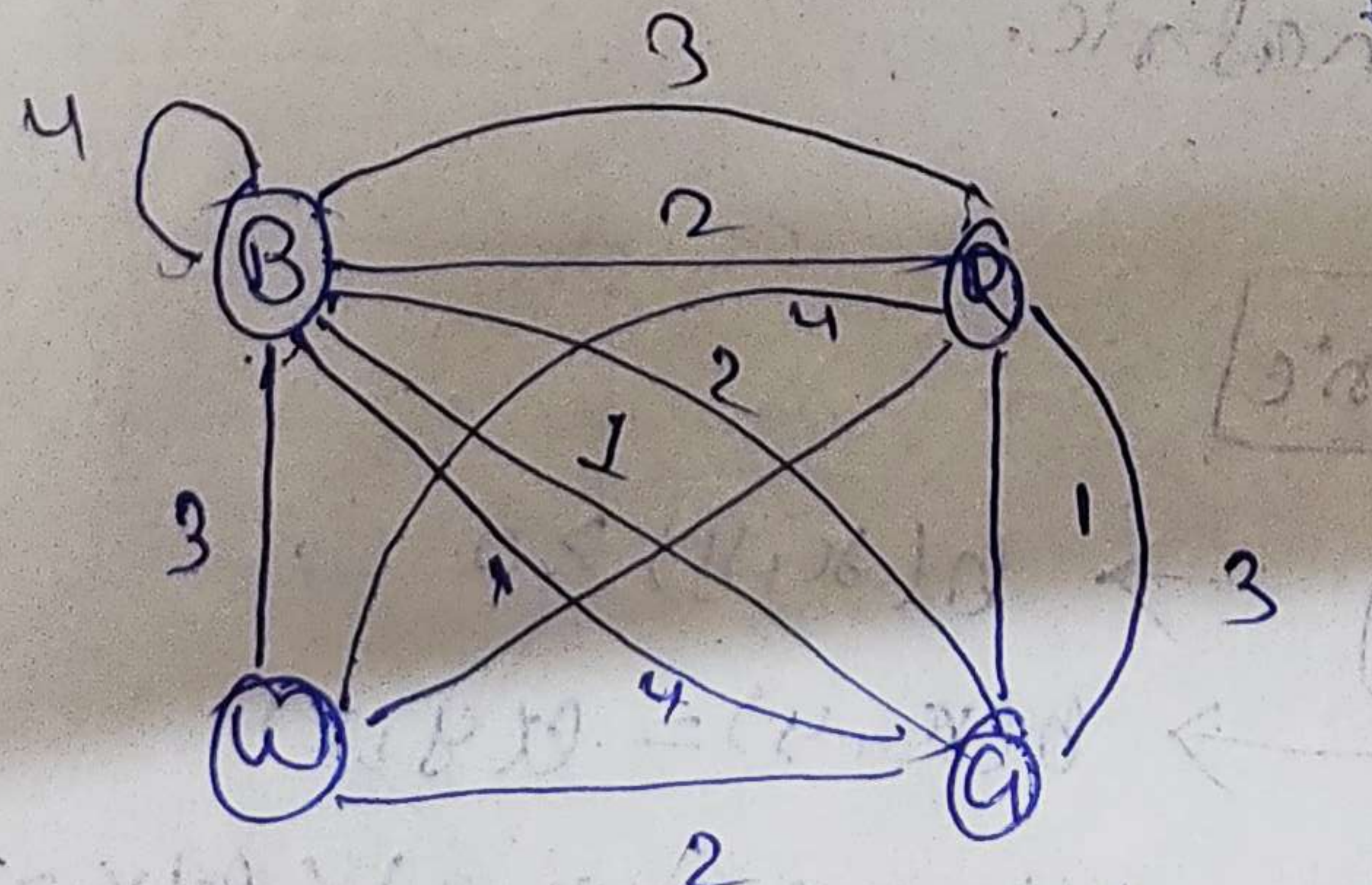
→ To find centre delete pendant of outmost step by step.

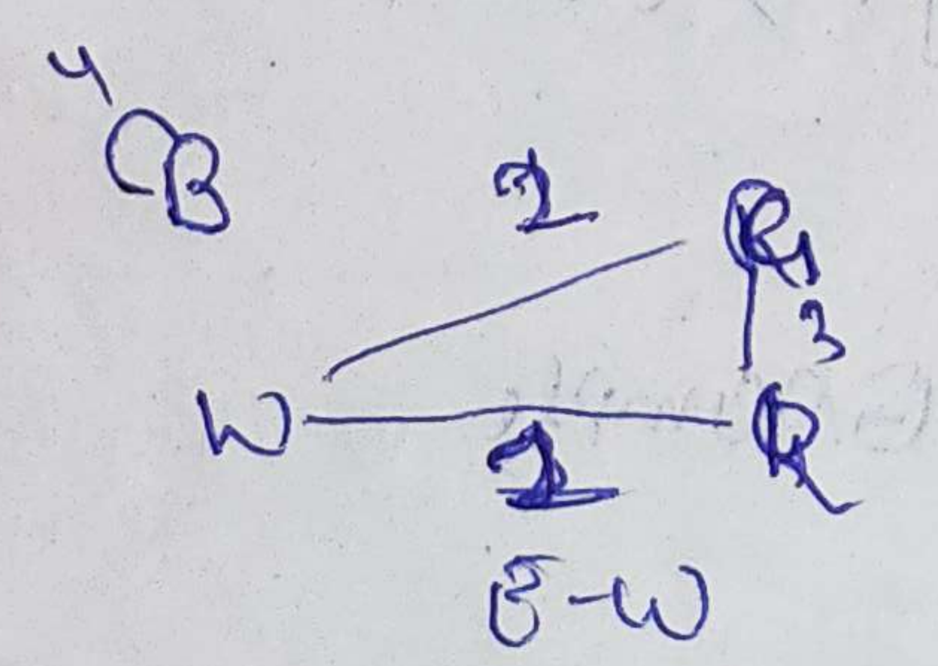
* $\boxed{\text{Radius \& Diameter}}$

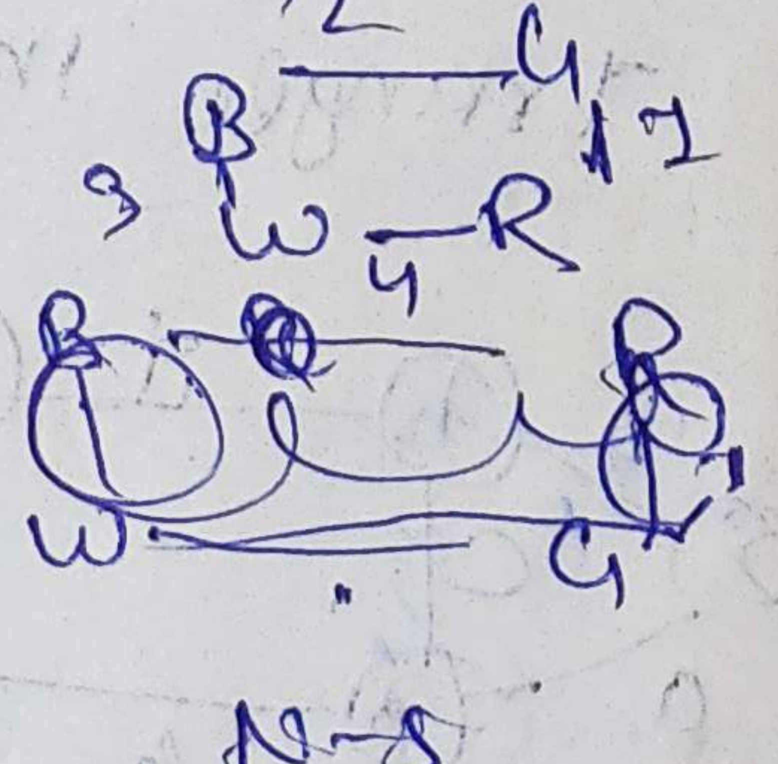
→ Radius is the eccentricity value of the centre.
→ Diameter is the largest path available in a tree.

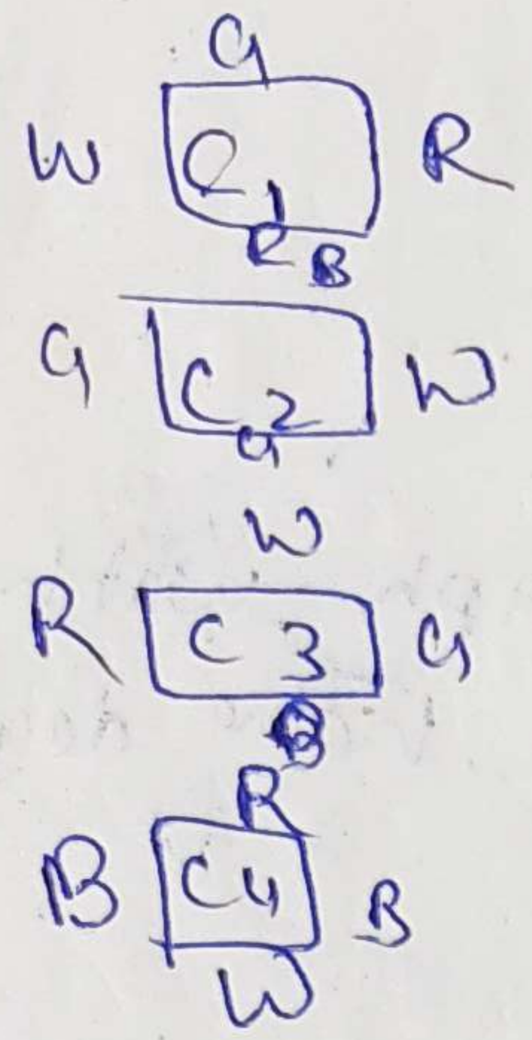
Q) Show that in a tree it's Diameter may not be always equal to twice it's radius.

Date → 30/01/2023

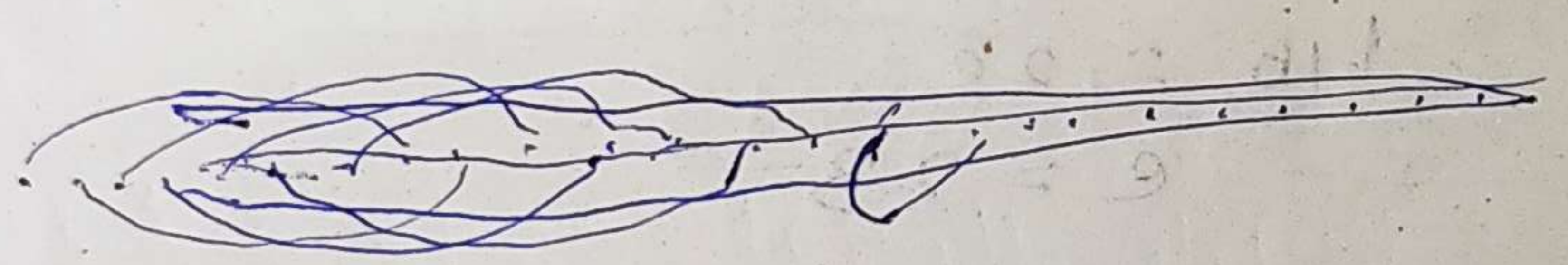








Q.) There are 25 telephones in some office. is it possible to connect them with wires so that each telephone is connected with exactly 7 others.



$$2e = \sum \partial(v)$$

$$2e = 25 \times 7$$

$$e = \frac{25 \times 7}{2}$$

$\therefore$ It is not possible.

* no. of cables connected to all computer

$$n(n-1) = 2e$$

$$\Rightarrow \boxed{e = \frac{n(n-1)}{2}}$$

* Maximum degree is equal to no. of ports.

Q). Determine the number of edges in a graph with six vertices two of which are of degree 4. and 4 of which are of degree 2.

$$(2 \times 4) + (4 \times 2) = 2e$$

$$e = 8$$

Q) How many vertices is needed to construct a graph with six edges in which every vertex is of degree 2.

solve $\Rightarrow$ $2 \times 6 = \cancel{2} \times v$

$v = 6$

Q) Is it possible to construct a graph with 12 vertices such that 2 of the vertices have degree 3 and Remaining have degree 4.

solve $\Rightarrow$ $2 \times 3 + 10 \times 4 = 2e$

$6 + 40 = 2e$

$46 = 2e$

$e = 23$

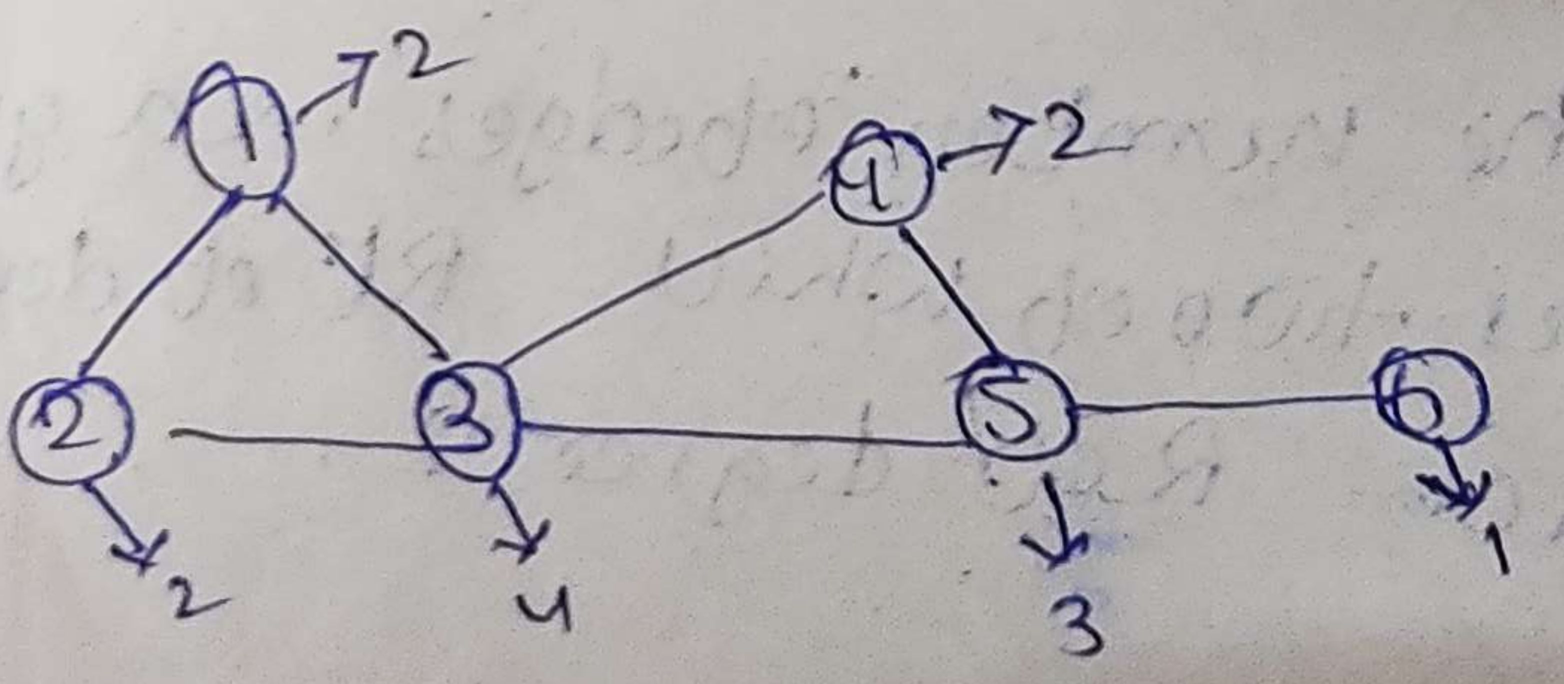
Q) Is it possible to draw a simple graph with 4 vertices and 7 edges.

solve $\Rightarrow$ Max. no. of edges in a simple graph $= \frac{n(n-1)}{2}$

$= \frac{4(4-1)}{2}$

$= 2 \times 3 = 6$

* degree sequence



$\{1, 2, 2, 2, 3, 4\}$

Q) Can there exists a simple graph with their degree sequence $\{2, 2, 3, 4\}$

$\boxed{2+2+3+4 \neq 2e}$ not possible

Q. $\{1,2,2,3,4,5\}$ = NO

Q. $\{2,2,4,6\}$

$\frac{13}{}\ 2$

2, 3, 12, 11

Q) A Tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. Determine no. of edges and vertices in the tree.

In a tree

Solve → $\boxed{\begin{array}{l} n - \text{vertices} \\ n-1 \to \text{edges} \end{array}}$

$$\begin{array}{r} 2n \\ 3n \\ n \\ \hline v = 6n \end{array} \qquad \begin{array}{r} 2n \\ 6n \\ 3n \\ \hline d = 11n \end{array}$$

$11n = 2 \times (6n-1)$

$11n = 12n - 2$

$2 = n$

$v = 12$

$e = 11$

Date → 01/02/23

* Rooted tree

→ There is exactly one vertex with degree 2 and rest of the vertices will be degree of 1 or 3 is called Binary rooted tree (BRT). The particular vertex $d(2)$ is known as the root.

Properties

(i) The no. of vertices in a BRT is odd.

(ii) The no. of odd degree vertices in a tree is always even.

1 vertex is of degree (2)

$(n-1)$ " " $d(1)$ or $d(3)$

3) The no of vertices with the degree (3) is $\left(\frac{n-3}{2}\right)$

Total $\rightarrow n$

Let p no. of vertices $\rightarrow$ d(1) $\rightarrow \frac{n+1}{2}$

1 no. of " $\rightarrow$ d(2)

$(n-1-p)$ " " " $\rightarrow$ d(3) $\rightarrow \frac{n-3}{2}$

$\sum d(v) = 2e$

$(p \times 1) + (1 \times 2) + (n-p-1) \times 3 = (n-1) \times 2$

$\boxed{p = \frac{n+1}{2}}$

→ Height min $= \{\log_2(n+1) - 1\}$

Height max $= \frac{n-1}{2}$

* Counting Tree

$\boxed{\text{1856, Arthur cayley}}$

Q.) How the idea of counting tree was originated.

Q). If n no. of vertices are given, How many distinct trees/ graph can be drawn?

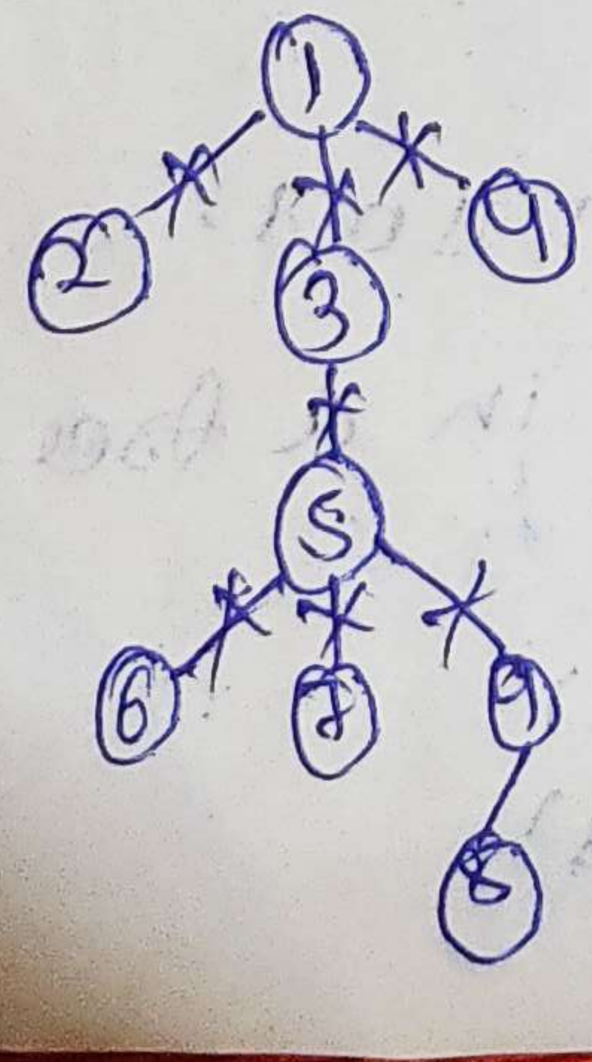
Q P.g.53 Ans P.g. 240

$\left(n^{n-2} \right.$ seperate trees can be drawn).

→ Pruber Encoding Sequence (PES) = {1, 1, 3, 5, 5, 5, 9}

$(n-2)$

until one edge is left



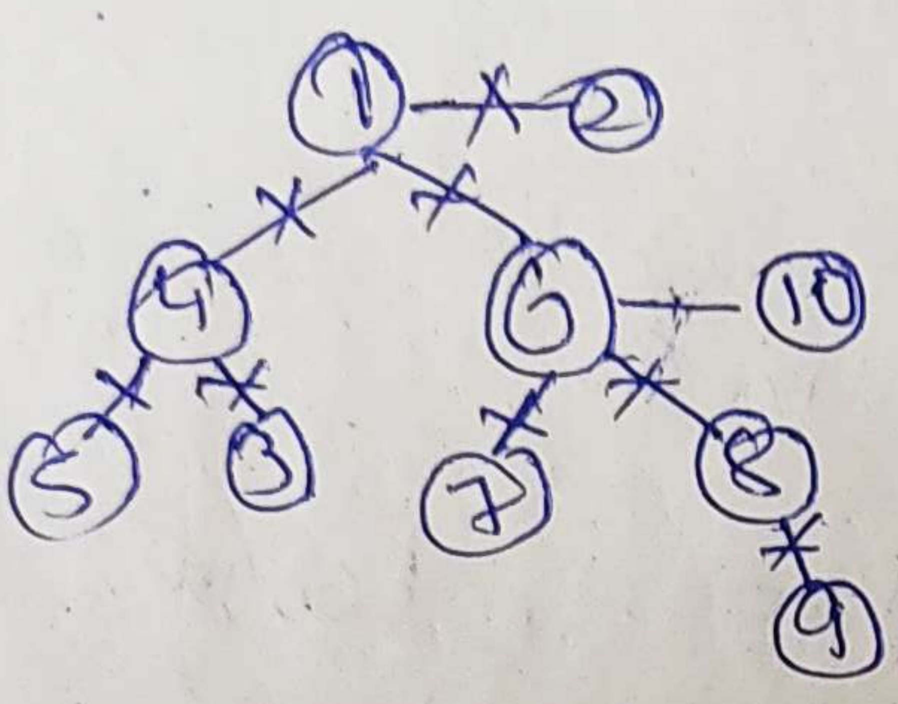
PDS = {2, 4, 1, 3, 6, 7, 5, 8}

$(n-1)$

→ Pruber Decoding sequence (PDS)

3

Q). from the given tree find out PES and PDS & reconstruct the original tree.



PES = {1, 4, 4, 1, 6, 6, 8, 6}

PDS = {2, 3, 5, 4, 1, 7, 9, 8, 10}

Q. PES = {3, 3, 4, 1, 5, 5, 1, 6, 6, 6, 2, 1, 3, 9, 9}

17

1-17 PDS = {7, 8, 10, 4, 11, 12, 5, 13, 14, 10, 6, 2, 16, 17}

* Spanning tree

Q) Show that a hamiltonian path is a spanning tree.

→ branch (n-1) 2

→ Chord (e-n+1)

Q) What is a spanning forest.

Q) Show that the distance between two spanning trees of a graph is a matric.

Ans → $d(T_1, T_2)$ = max. no. of changes b/w them.

Date > 6/02/23

* Matrix tree theorem

Q.) Laplacian matrix

1. If $i \neq j$

If value of i & j are adjacent in G, then $q_{ij} = -1$

Otherwise, $q_{ij} = 0$

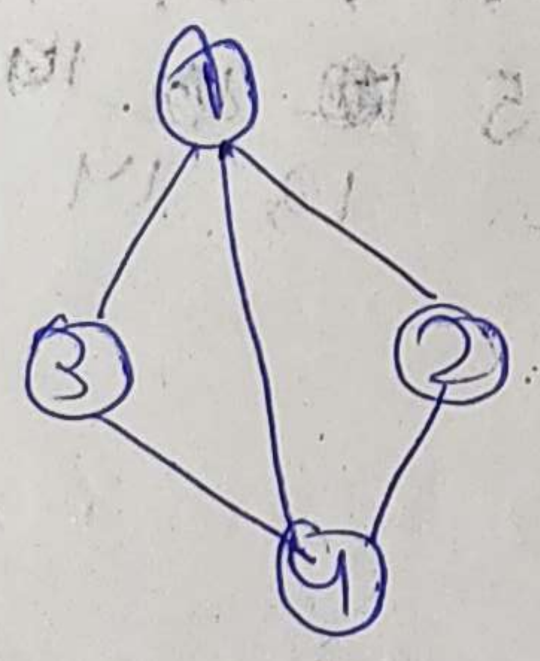
2. If $i = j$

q_{ij} = degree of vertex i in G.

$Q' \rightarrow$ reduced Q with 1 row, 1 coln.

det $(Q') \rightarrow$ No. of ST in G
(determinant)

Eg > 1)



$$Q = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 3 & -1 & -1 & -1 \\ 2 & -1 & 2 & 0 & -1 \\ 3 & -1 & 0 & 2 & -1 \\ 4 & -1 & -1 & -1 & 3 \end{array}$$

delete any row and any column.

$$Q' = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

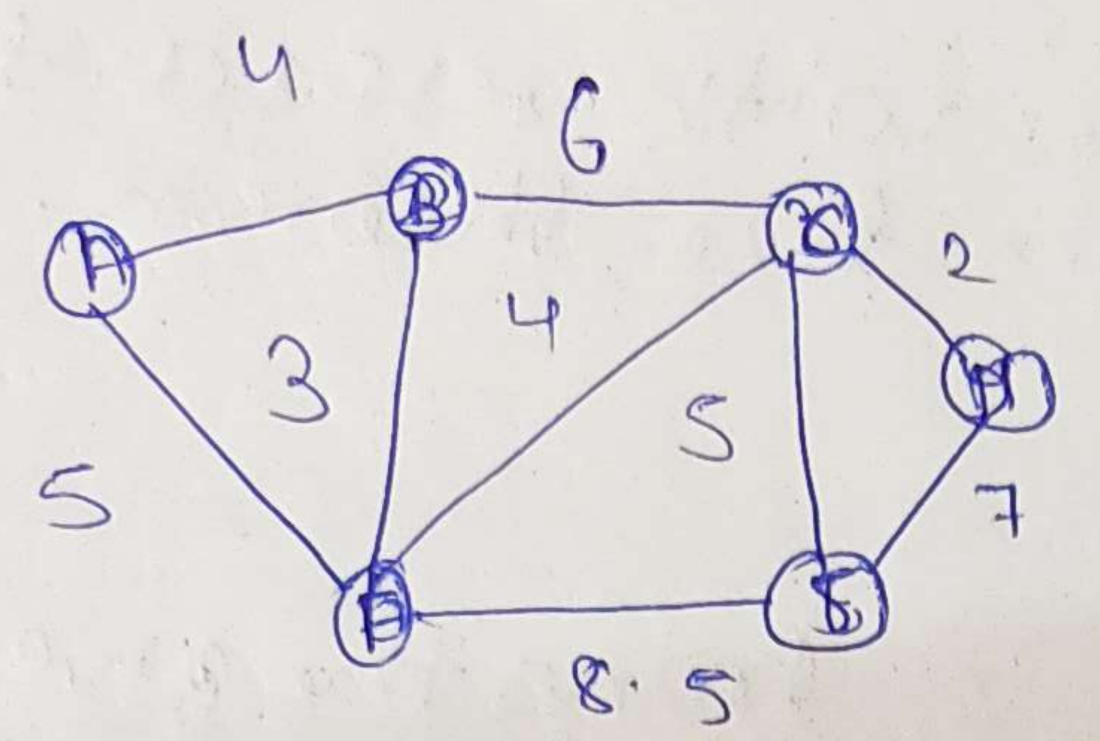
$$\det(Q') = 2(6-1) + 0(1-0) - 1(0+2)$$

$$= 2(5) - 2$$

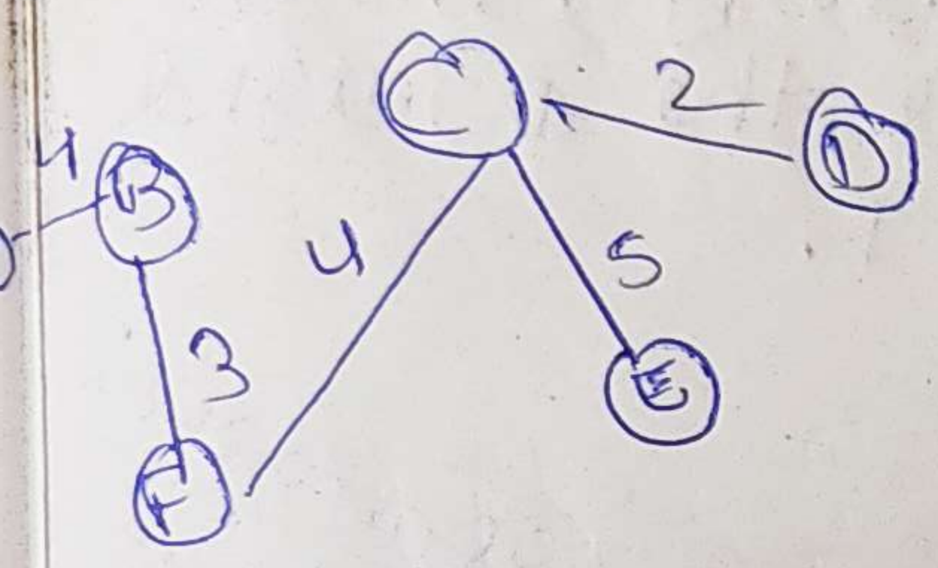
$$= 10 - 2 = 8$$

Maximum no. of spanning tree is 8.

* Minimum Spanning Tree :-



Krushkal's Algorithm →



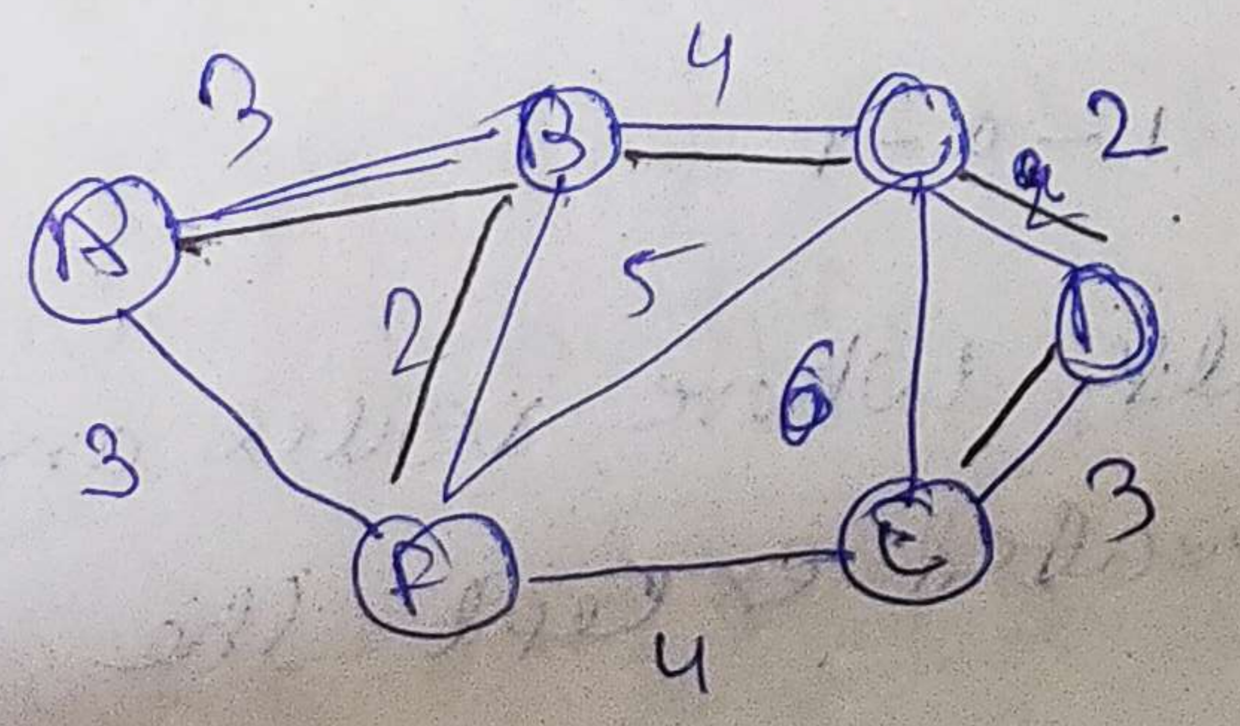
Vertex Name	Distance
✓ C, D	2 ✓
✓ B, F	3 ✓
✓ A + B	4 ✓
✓ C, F	4 ✓
A, F	5
✓ C, E	5 ✓
B, C	6
D, E	7
E, F	8.5

Minimum Spanning Tree → 18
(MST)

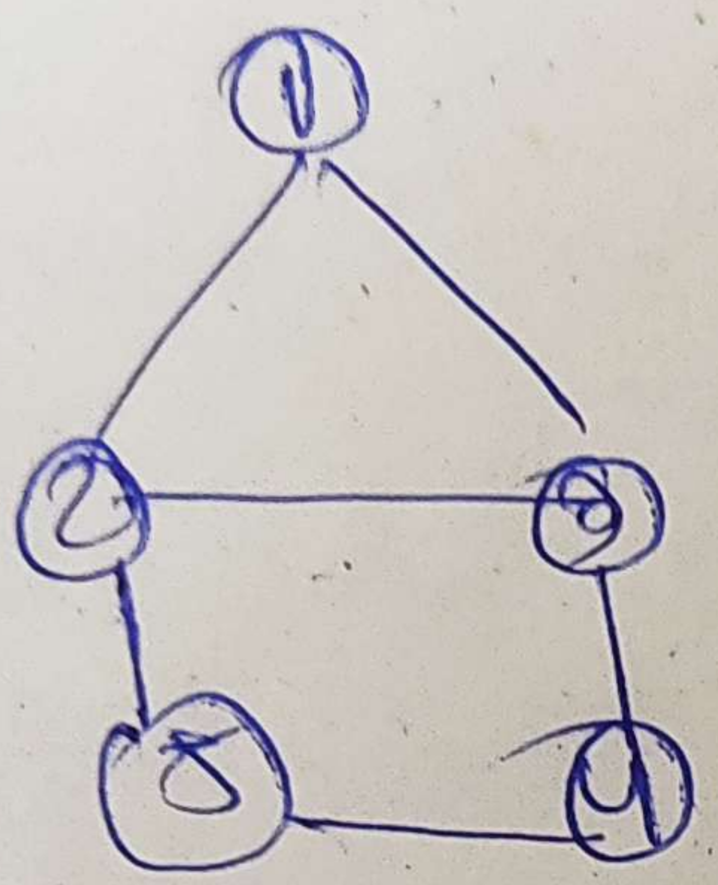
Prim's Algorithm →

$$
X = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}
\begin{array}{c}
\begin{array}{cccccc} A & B & C & D & E & F \end{array} \\
\begin{bmatrix}
- & 3 & \infty & \infty & \infty & 3 \\
3 & - & 4 & \infty & \infty & 2 \\
\infty & 4 & - & 2 & 6 & 5 \\
\infty & \infty & 2 & - & 3 & \infty \\
\infty & \infty & 6 & 3 & - & 4 \\
3 & 2 & 5 & \infty & 4 & -
\end{bmatrix}
\end{array}
$$

* Self loop value not be taken
* Parallel edge of min^m value is taken.

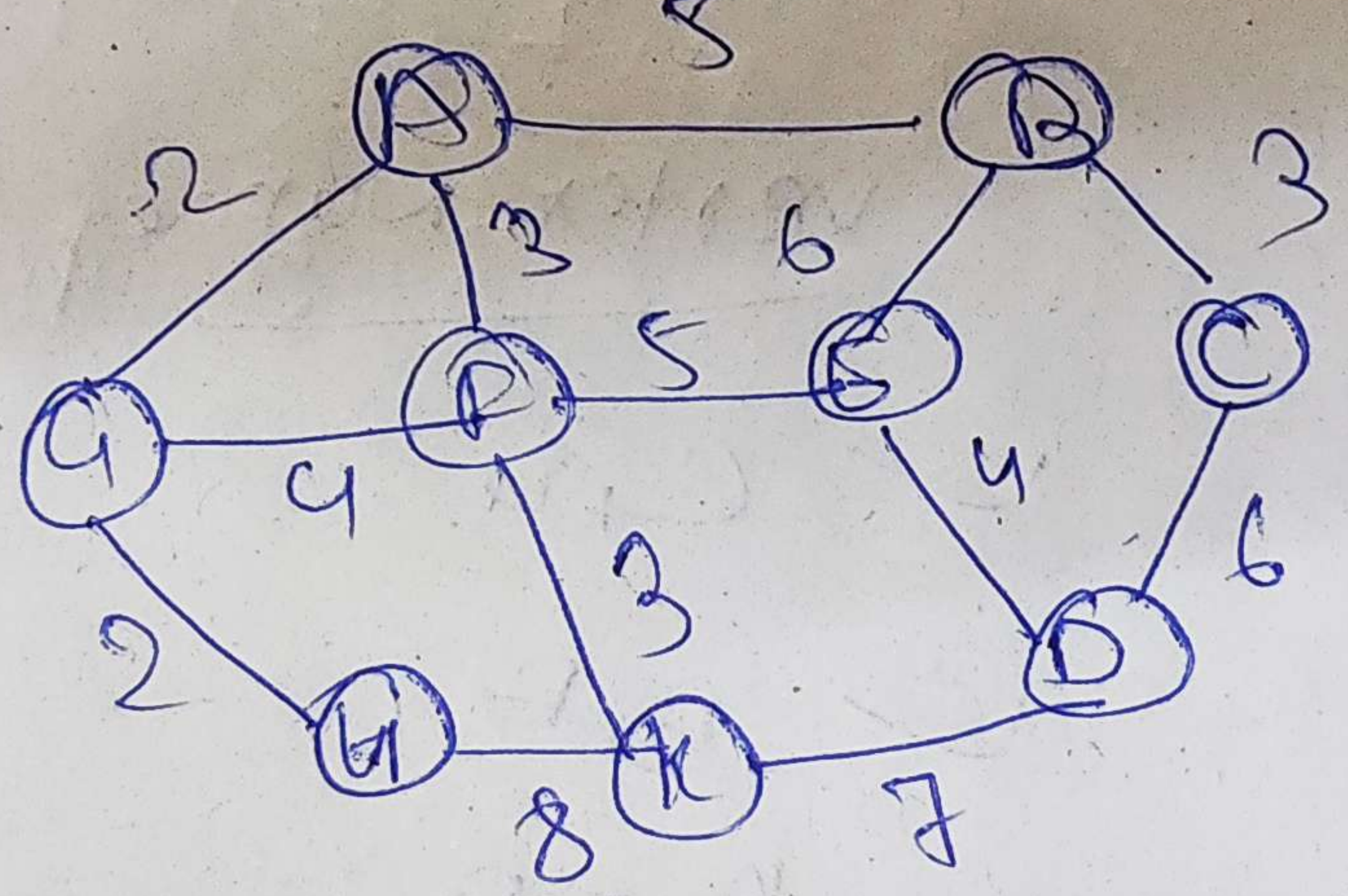


MST = 3+2+4+2+3
= 14

Q) 

From the given graph find out the no. of spanning tree possible with respect to matrix tree theorem.

5.

Q) 

From the given graph find out MST using both Q. Alogorithms seperately. ~~find~~ What is the weight of MST.

1)

M Q.

Group A → $2 \times 4 = 8$

Group B → $4 \times 2 = 8$

Group C → $4 \times 1 = 4$

4 marks / Isomorphism / Dice / Doubre enco dry / matrix tree / ... algorithms