* Assignment *

1. Ques:- Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces, Construct a table giving the non-zero values of the probability mass function and draw the probability chart. Also find the distribution function on X.

Solⁿ:- The sample space for rolling two dice is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
The possible values of X are the elements in sample spaces.
Probability mass function (pmf) for X can be calculated as follow:

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X	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

* Probability Chart →

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

* Distribution function of X:

X	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
$P(X \le x)$	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	36/36 = 1

Q. Ques:- The following is the distribution function of a discrete random variable X:

X	-3	-1	0	1	2	3	5	8
F(X)	0.10	0.30	0.45	0.50	0.75	0.90	0.95	1.00

(i) Find the probability distribution of X.

(ii) Find P(X is even) and $P(1 \le X \le 8)$

(iii) Find $P(X=-3 \mid X<0)$ and $P(X \ge 3 \mid X>0)$

Soln:- (i) find the Probability distribution of X.

X	-3	-1	0	1	2	3	5	8
P(x)	0.10	0.20	0.15	0.05	0.25	0.15	0.05	0.05

(ii) find P(X is even) and $P(1 \le X \le 8)$

$$P(X \text{ is even}) = P(X=2) + P(X=8)$$
$$= 0.25 + 0.05$$
$$\boxed{P(X \text{ is even}) = 0.30}$$

$$P(1 \le X \le 8) = 0.05 + 0.25 + 0.15 + 0.05 + 0.05$$
$$\boxed{P(1 \le X \le 8) = 0.55}$$

(iii) find $P(X=-3 \mid X<0)$ and $P(X \geq 3 \mid X>0)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(X=-3 \mid X<0) = \frac{P(P(x)=-3 \cap P(x<0))}{P(x<0)}$$

$$= \frac{0.1}{0.3}$$

$$\boxed{P(X=-3 \mid X<0) = \frac{1}{3}}$$

Now $P(X \ge 3 \mid X>0) = \dfrac{P(P(x \geq 3) \cap P(x>0))}{P(X>0)}$

$$P(x \geq 3 \mid x > 0) = \frac{0.15 + 0.05 + 0.05}{0.05 + 0.25 + 0.15 + 0.05 + 0.05}$$

$$= \frac{0.25}{0.55}$$

$$\boxed{P(X \geq 3 \mid X > 0) = \frac{5}{11}}$$

3. Ques:- Suppose that the life in hours of a certain part of radio tube is continuous random variable X with p.d.f given by.

$$f(x) = \begin{cases} \frac{100}{x^2} & , \text{ when } x \geq 100 \\ 0 & , \text{ else where} \end{cases}$$

(i) What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?

(ii) What is the probability that none of three of the original tubes will have to be replaced during that first 150 hours of operation?

(iii) What is the probability that a tube will last less than 200 hours if it is known that the tube is still functionning after 150 hours of service?

Sol^n:- Probability that tube last for first 150 hours is given by

$$P(X \leq 150) = P(0 < X < 100) + P(100 \leq X \leq 150).$$

$$= 0 + \int_{100}^{150} f(x)\, dx$$

$$= \int_{100}^{150} \frac{100}{x^2}\, dx$$

$$= 100 \int_{100}^{150} x^{-2}\, dx$$

$$= 100 \left[-\frac{1}{x} \right]_{100}^{150}$$

$$= \frac{1}{3}$$

$\therefore$ Probability that a tube will not last for the first hours is

$$1 - \frac{1}{3} = \frac{2}{3}$$

(i) The Probability that all three of the original tubes will have to replaced during first 150 hours is $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

(ii) The Probability that none of three tubes will have to replaced during the first 150 hours is $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

(iii) $P(X < 200 \mid x > 150)$

$$\Rightarrow \frac{P(150 < X < 200)}{P(X > 150)}$$

$$\Rightarrow \frac{\int_{150}^{200} \frac{100}{x^2}\, dx}{\int_{150}^{\infty} \frac{100}{x^2}\, dx}$$

$$\Rightarrow \frac{\left[-\frac{100}{x}\right]_{150}^{200}}{\left[-\frac{100}{x}\right]_{150}^{\infty}}$$

$$\Rightarrow \frac{\left[-\frac{100}{200} + \frac{100}{150}\right]}{\left[-\frac{100}{\infty} + \frac{100}{150}\right]}$$

$$\Rightarrow \frac{1/6}{2/3}$$

$$\Rightarrow 1/4$$ Ans

4: **Ques:-** A petrol pump is supplied with petrol once a day. If its daily volume of sales (X) in thousands of liters is distributed by $f(x) = 5(1-x)^4, 0 \le x \le 1$, what must be the capacity of its tank the probability that its supply will be exhausted in a given day shall be 0.01 ?

Sol^n:- Let the capacity of the tank (in 000 of liters) be 'a' such that

$$P(X \geq a) = 0.01$$

$$\Rightarrow \int_a^1 f(x)\,dx = 0.01$$

$$\Rightarrow \int_a^1 5(1-x)^4\,dx = 0.01$$

$$\Rightarrow 5\left[\frac{(1-x)^5}{-5}\right]_a^1 = 0.01$$

$$\Rightarrow (1-a)^5 = 0.01$$

$$\Rightarrow (1-a) = \left(\frac{1}{100}\right)^{1/5}$$

$$\Rightarrow 1-a = 0.3981$$

$$\Rightarrow a = 1 - 0.3981 = 0.6019$$

Hence, the capacity of the tank $= 0.6019 \times 1000$

$= 601.9$ liters Ans

5: Ques:- An experiment consist of three independent tosses of fair coin. Let X = the number of heads, Y = the number of head runs, Z = the length of head runs, a head run being defined as consecutive occurrence of at least two heads, its length then being the number of heads occurring together in three tosses of the coin.
find the probability function of
(i) X (ii) Y (iii) Z (iv) $X+Y$ (v) XY

Sol^n:-

Event	X	Y	Z	X+Y	XY
HHH	3	1	3	4	3
HHT	2	1	2	3	2
HTH	2	0	0	2	0
HTT	1	0	0	1	0
THH	2	1	2	3	2
THT	1	0	0	1	0
TTH	1	0	0	1	0
TTT	0	0	0	0	0

(Random variables: X, Y, Z, X+Y, XY)

Obviously X is a random variable which can take the value 0, 1, 2, 3.

$P(0) = \frac{1}{8}$

$P(1) = 3/8$

$P(2) = P(HHT \cup HTH \cup THH)$
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(3) = P(HHH) = \frac{1}{8}$

(i) p.m.f of X:

X	0	1	2	3
P(X)	$1/8$	$3/8$	$3/8$	$1/8$

(ii) p.m.f of Y:

X	0	1
P(X)	$5/8$	$3/8$

(iii) p.m.f of Z:

X	0	1	2	3
P(X)	$\frac{5}{8}$	0	$\frac{2}{8}$	$\frac{1}{8}$

(iv) p.m.f of x+Z:

X	0	1	2	3	4
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

(v) p.m.f of xy:

X	0	1	2	3
P(X)	$\frac{5}{8}$	0	$\frac{2}{8}$	$\frac{1}{8}$

6. Ques:- The kms x in thousands of kms which car owners get with a certain kind of tyre is a random variable having probability density function

$$f(x) = \begin{cases} \frac{1}{20} e^{-\frac{x}{20}}, & \text{for } x > 0 \\ 0, & \text{for } x \le 0 \end{cases}$$

Find the probabilities that one of these tyres will last

(i) at most 10,000 kms,

(ii) anywhere from 16,000 to 24,000 kms,

(iii) at least 30,000 kms.

Solⁿ:- Let random variable x denote the kms in (000 kms) with a certain kind of tyre. Then required probability is given by -

(i) $P(x \le 10) = \int_0^{10} f(x)\,dx$

$$\Rightarrow \int_0^{10} \frac{1}{20} e^{-\frac{x}{20}}\,dx$$

$$\Rightarrow \frac{1}{20}\left[\frac{e^{-x/20}}{-1/20}\right]_0^{10}$$

$$\Rightarrow 1 - e^{-1/2}$$

$$\Rightarrow 1 - 0.6065$$

$$\Rightarrow 0.3935 \text{ Ans}$$

(ii) $P(16 \le x \le 24) = \frac{1}{20}\int_{16}^{24} e^{-x/20}\,dx$

$$\Rightarrow \frac{1}{20}\int_{16}^{24} e^{-\frac{x}{20}}\,dx$$

$$\Rightarrow \frac{1}{20}\left[\frac{e^{-x/20}}{-1/20}\right]_{16}^{24}$$

$$\Rightarrow e^{-16/20} - e^{-24/20}$$

$$\approx e^{-4/5} - e^{-6/5}$$

$$\approx 0.4493 - 0.3012 = 0.1481 \text{ Ans}$$

(iii) $P(X \geq 30) = \int_{30}^{\infty} f(x)\,dx$

$\Rightarrow \frac{1}{20} \int_{30}^{\infty} e^{-x/20}\,dx$

$\Rightarrow \frac{1}{20} \left[\frac{e^{-x/20}}{-1/20} \right]_{30}^{\infty}$

$\Rightarrow e^{-1.5}$

$\Rightarrow 0.2231$ Ans

7. Ques:- Suppose that the time in minutes that a person has to wait at a certain bus stop is found to be a random phenomenon, with a probability function specified by the distribution function:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{8}, & 0 \leq x < 2 \\ \frac{x^2}{16}, & 2 \leq x \leq 4 \\ 1, & x \geq 4 \end{cases}$$

(i) Is the distribution function continuous? If so, give the formula for its probability density function.

(ii) What is the probability that a person will have to wait (a) more than 2 minutes (b) less than 2 minutes (c) between 1 and 2 minutes

(iii) What is the conditional probability that the person will have to wait for a bus for (a) more than 2 minutes, given that it is more than 1 minute (b) less than 2 minutes given that it is more than 1 minute?

Soln:- (i) $f(x) = \frac{dF(x)}{dx}$

p.m.f $\rightarrow$ $f(x) = \frac{d}{dx} 0, \quad x < 0$

$= 0$

p.m.f $\rightarrow$ $f(x) = \frac{d}{dx} \frac{x}{8}, \quad 0 \leq x < 2$

$= 1/8$

p.m.f → $f(x) = \frac{d}{dx} \frac{x^2}{16}$, $2 \le x < 4$

$= \frac{x}{8}$

p.m.f → $f(x) = \frac{d}{dx}(1)$, $x \geq 4$

$= 0$

(ii) (a) $P(X > 2) = 1 - P(X \le 2)$

$= 1 - F(2)$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

(b) $P(X < 2) = \int_0^2 f(x)\, dx$

$= \int_0^2 \frac{x}{8}\, dx$

$= \frac{1}{8}\left[\frac{x^2}{2}\right]_0^2$

$= \frac{1}{4}$

(c) $P(1 \le x \le 2) = F(2) - F(1)$

$= \frac{x^2}{16} - \frac{x}{8}$

$= \frac{(2)^2}{16} - \frac{1}{8}$

$= \frac{1}{8}$

(ii) (a) $P(x>2 \mid x>1) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{P(x \geq 2)}{P(x > 1)}$$

$$= \frac{3/4}{7/8}$$

$$= \frac{6}{7}$$

(b) $P(x<2 \mid x>1) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{P(1<x<2)}{P(x>1)}$$

$$= \frac{\int_1^2 f(x)\,dx}{7/8}$$

$$= \frac{\frac{1}{8}\left[\frac{x^2}{2}\right]_1^2}{7/8}$$

$$= \frac{1/8\left[2-\frac{1}{2}\right]}{7/8}$$

$$= \frac{3}{2 \times 7}$$

$$= \frac{3}{14} \quad \text{Ans}$$

8. Ques:- A random variabe X is distributed at random between the values 0 and 1 so that its probability density function is :

$f(X) = Kx^2(1-x^3)$, where K is a constant. find the value of K. using the value of k, find its mean and variance.

Sol^n:- (i)

$$\int_0^1 f(x)\,dx = 1$$

$$\int_0^1 Kx^2(1-x^3) = 1$$

$$\int_0^1 Kx^2 - Kx^5 = 1$$

$$\left[\frac{Kx^3}{3} - K\frac{x^6}{6}\right]_0^1 = 1$$

$$K\left[\frac{1}{3} - \frac{1}{6}\right] = 1$$

$$\frac{K}{6} = 1$$

$$\boxed{K = 6}$$

(ii) mean = $\bar{x}$ = $\int_a^b x \cdot Kx^2(1-x^3)$

$$= 6\int_0^1 (x^3 - x^6)\,dx$$

$$= 6\left[\frac{x^4}{4} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{9}{14}$$

(iii) Variance: $\mu_2 = \mu_2' - (\mu_1')^2$

$$\Rightarrow \int_0^1 x^2 f(x)\,dx - \left(\frac{9}{14}\right)^2$$

$$\Rightarrow \int_0^1 x^2 k x^2 (1-x^3)\,dx - \frac{81}{196}$$

$$\Rightarrow 6 \int_0^1 x^4 - x^7 \,dx - \frac{81}{196}$$

$$\Rightarrow 6 \left[\frac{x^5}{5} - \frac{x^8}{8} \right]_0^1 - \frac{81}{196}$$

$$\Rightarrow 6 \left[\frac{1}{5} - \frac{1}{8} \right] - \frac{81}{196}$$

$$\Rightarrow \frac{6 \times 3}{40} - \frac{81}{196}$$

$$\Rightarrow \frac{9}{20} - \frac{81}{196}$$

$$\Rightarrow \frac{9}{245}$$ Ans

9: **Ques:-** In a continuous distribution whose relative frequency density is given by : $f(x) = y_0 x(2-x)$, $0 \le x \le 2$ find the mean, variance median, mode of the distribution and show that for the distribution $\mu_{2n+1} = 0$.

Soln:- Since total probability is unity we have,

$$\int_0^2 f(x)\,dx = 1$$

$$y_0 \int_0^2 x(2-x)\,dx = 1$$

$$\Rightarrow y_0 \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow y_0 = \frac{3}{4}$$

$$\boxed{f(x) = \frac{3}{4} x(2-x)}$$

$$\therefore \quad \mu_r' = \int_0^2 x^r f(x)\,dx$$

$$= \frac{3}{4} \int_0^2 x^{r+1}(2-x)\,dx$$

$$= \frac{3}{4} \left[\frac{2x^{r+2}}{r+2} - \frac{x^{r+3}}{r+3}\right]_0^2$$

$$= \frac{3 \cdot 2^{r+1}}{(r+2)(r+3)}$$

In particular

$$\text{mean} = \mu_1' = \frac{3\ 2^{1+1}}{3 \cdot 4} = 1$$

$$\mu_2' = \frac{3 \cdot 2^3}{4 \cdot 5} = \frac{6}{5}$$

$$\text{Variance} \Rightarrow \mu_2 = \mu_2' - (\mu_1')^2$$

$$= \frac{6}{5} - 1 = 1/5$$

$\rightarrow \mu_{2n+1} = \int_0^2 (x - \text{mean})^{2n+1} f(x)\,dx$

$$\Rightarrow \frac{3}{4}\int_0^2 (x-1)^{2n+1}\, x(2-x)\,dx$$

$$\Rightarrow \frac{3}{4}\int_{-1}^{1} t^{2n+1}(t+1)(1-t)\,dt \qquad \{x-1=t$$

$$\Rightarrow \frac{3}{4}\int_{-1}^{1} t^{2n+1}(1-t^2)\,dt$$

Since, t^{2n+1} is an odd function of t and $(1-t^2)$ is an even function of t, the integral $t^{2n+1}(1-t^2)$ is an odd function of t.

Hence, $\mu_{2n+1} = 0$

$\rightarrow \quad f'(x) = \frac{3}{4}(2-2x) = 0 \Rightarrow x = 1$

and $f''(x) = \frac{3}{4}(-2) = \frac{-3}{2} < 0$

Hence mode $= 1$

$\rightarrow$ if m is the median, then

$$\int_0^m f(x)\,dx = \frac{1}{2}$$

$$\frac{3}{4}\int_0^m x(2-x)\,dx = \frac{1}{2}$$

$$\left[x^2 - \frac{x^3}{3}\right]_0^m = \frac{2}{3}$$

$$3m^2 - m^3 = 2$$

$$(m-1)(m^2 - 2m - 2) = 0$$

The only value of m lying $[0,2]$ is $m=1$

Hence median is 1.

10. Ques:- The amount of bread (in hundreds of pounds) X that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the p.d.f. f(X), given by

$$f(x) = \begin{cases} k \cdot x, & \text{for } 0 \le x < 5 \\ k \cdot (10-x), & \text{for } 5 \le x < 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) find the value of k such that f(X) is a p.d.f.

(b) what is the probability that the number of pounds of bread that will be sold tomorrow is :
(i) more than 500 pounds, (ii) less than 500 pounds, and (iii) between 250 and 750 pound ?

(C) Denoting by A, B and C the events that the pounds of bread sold are as in b(i), b(ii), and b(iii) respectively, find P(A|B) and P(A|C) are (i) A and B independent events ? (ii) A and C independent events ?

Solⁿ:- (a) In order that f(x) should be a probability density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{5} kx\,dx + \int_{5}^{10} k(10-x)\,dx = 1$$

$$\Rightarrow \boxed{k = \frac{1}{25}}$$

(b)
(i) The probability that more than 500 pounds :

$$P(5 \le x \le 10) = \int_{5}^{10} \frac{1}{25}(10-x)\,dx$$

$$= \frac{1}{25}\left[10x - \frac{x^2}{2}\right]_5^{10}$$

$$P(5 \le X \le 10) = \frac{1}{25}\left[\frac{25}{2}\right]$$
$$= \frac{1}{2}$$
$$\boxed{P(5 \le x \le 10) = 0.5}$$

(ii) less than 500 pounds :

$$P(0 \le x \le 5) = \int_0^5 \frac{1}{25}\, x\, dx$$
$$= \frac{1}{25}\left[\frac{x^2}{2}\right]_0^5$$
$$= \frac{1}{2}$$
$$\boxed{P(0 \le x \le 5) = 0.5}$$

(iii) Between 250 and 750 pounds :

$$P(2.5 \le x \le 7.5) = \int_{2.5}^{5} \frac{1}{25}\, x\, dx + \int_5^{7.5} \frac{1}{25}(10-x)\, dx$$
$$= \frac{1}{25}\left[\frac{x^2}{2}\right]_{2.5}^{5} + \frac{1}{25}\left[10x - \frac{x^2}{2}\right]_5^{7.5}$$
$$\boxed{P(2.5 \le x \le 7.5) = \frac{3}{4}}$$

(c) The events A, B and C are given by
$A: 5 < x \le 10$, $B: 0 \le x \le 5$; $C: 2.5 < x < 7.5$
Then from parts b(i), b(ii) and b(iii) we have,
$P(A) = 0.5$, $P(B) = 0.5$, $P(C) = \frac{3}{4}$
The events $A \cap B$ and $A \cap C$ are given by
$A \cap B = \phi$ and $A \cap C$: $5 < x < 7.5$
$\therefore P(A \cap B) = P(\phi) = 0$
and $P(A \cap C) = \int_5^{7.5} f(x)\, dx$

$$= \frac{1}{25} \int_{5}^{7.5} (10-x)\, dx$$

$$= \frac{1}{25} \times \frac{75}{8}$$

$$P(A \cap C) = \frac{3}{8}$$

$$P(A) \cdot P(C) = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8} = P(A \cap C)$$

A and C are indibendent

again $P(A) \cdot P(B) = \frac{1}{4} \neq P(A \cap B)$

A and B are not indibendent

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{3/8}{3/4} = \frac{1}{2}$$

11. Ques:- The diameter, say X of an element of electric cable, is assumed to be a continuous random variable with p.d.f.

$$f(x) = 6x(1-x), \quad 0 \leq x \leq 1$$

(i) Check that $f(x)$ is a p.d.f

(ii) Obtain an expression for the distribution function of X.

(iii) Compute the number K such that $P(X<K) = P(X>K)$.

Sol^n:- (i) for $0 \leq x \leq 1$, $f(x) \geq 0$

$$\int_0^1 f(x)\, dx = 6 \int_0^1 x(1-x)\, dx$$

$$\Rightarrow 6 \int_0^1 (x - x^2)\, dx$$

$$\Rightarrow 6 \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$$

$$\Rightarrow 1$$

hence $f(x)$ is the p.d.f of random variable X.

(ii) $F_x(X) = \int_0^1 f(x)dx$

$$= \int_0^1 6x(1-x)dx$$

$$\Rightarrow \int_0^1 (6x - 6x^2)dx$$

$$\Rightarrow 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$\Rightarrow 6\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$\Rightarrow 6 \times \frac{1}{6} = 1$$

(iii) $P(X < K) = P(X > K)$

$$\Rightarrow \int_0^K f(x)dx = \int_K^1 f(x)dx$$

$$\Rightarrow 6\int_0^K x(1-x)dx = 6\int_K^1 x(1-x)dx$$

$$\Rightarrow \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^K = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_K^1$$

$$\Rightarrow \frac{K^2}{2} - \frac{K^3}{3} = \left[\left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{K^2}{2} - \frac{K^3}{3}\right)\right]$$

$$3K^2 - 2K^3 = [1 - 3K^2 + 2K^3]$$

$$4K^3 - 6K^2 + 1 = 0$$

$$(2K-1)(2K^2 - 2K - 1) = 0$$

$\boxed{K = 1/2}$ is the only real value.

12. Ques:- Verify that the following function is a distribution function of the random variable x.

$$F(x) = \begin{cases} 0, & x < -a \\ \frac{1}{2}\left(\frac{x}{a}+1\right), & -a \le x \le a \\ 1, & x > a \end{cases}$$

Soln:- Now

$$\frac{d}{dx} F(x) = \begin{cases} \frac{1}{2a} & -a \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

$$= f(x), \text{ say}$$

In order that F(x) is a distribution function, f(x) must be a p.d.f.
Thus we have to show that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Now
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-a}^{a} f(x)\,dx$$

$$= \frac{1}{2a}\int_{-a}^{a} 1 \cdot dx$$

$$= 1$$

Hence, F(x) is a distribution function. Varifid