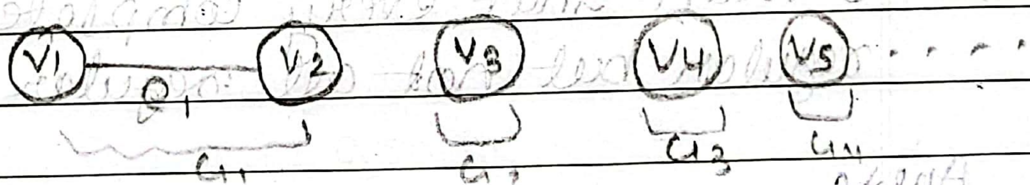
Assignment.

Q1. Show that an infinite graph with a finite no. of edges must have an infinite no. of isolated vertices.

Ans → If a graph contains infinite no. of isolated vertices but a finite no. of edges then the graph contains atleast one edges. If a graph has n vertices and m edges then it has to satisfy $m \le \binom{n}{2}$. So, if no. of edges be constant. So there must be infinite vertices. And the vertices are isolated because if they are connected then edges also must be infinite. So, from this if we take infinite isolated vertices then only finite no. of edges be possible.



Q2. Show that an infinite graph with finite no. of vertices must have one pair of vertices must be Join by infinite no. of parallel vertices.

Ans→ If a graph has finite no. of vertices and the graph is infinite graph then there must be a infinite no. of edges. But if the vertices are finite and only one edges is connect with two vertices then edges also be finite. So, to make edges infinite we must have to draw parallel edges. So that the graph must be infinite graph.

Q) 3. Explain that every complete graph are regular but not all regular graph is complete.

Ans→ Complete graph: A graph in which edge is present between every pair of vertices

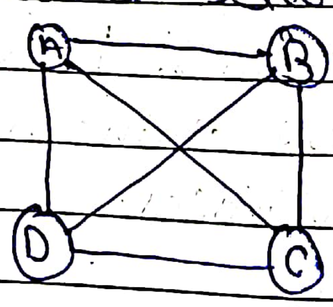
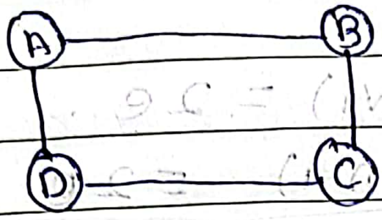


Fig (1)

Q) 4

A

Regular graph: A graph in which every vertex has the same degree.



Fig(2).

In fig (1). every vertex have degree 3 and there is edge present between every vertex but in fig(2) every vertex have degree 2 but there is no edge between A & C and B & D. Therefore, complete graph satisfy Regular graph property as well but not regular graph satisfy complete graph property.

Q)4. Show that the maximum no. of edges with n vertices is $n(n-1)/2$.

Ans→ Let a complete graph with n no. of vertices. Hence, each vertices of complete graph degree is $(n-1)$.

$\therefore$ Total degree $= n(n-1)$.

from Handshaking lemma theorem total degree is equal to twice of edges.

$$\therefore \sum \partial(v_i) = 2e.$$

$$n(n-1) = 2e$$

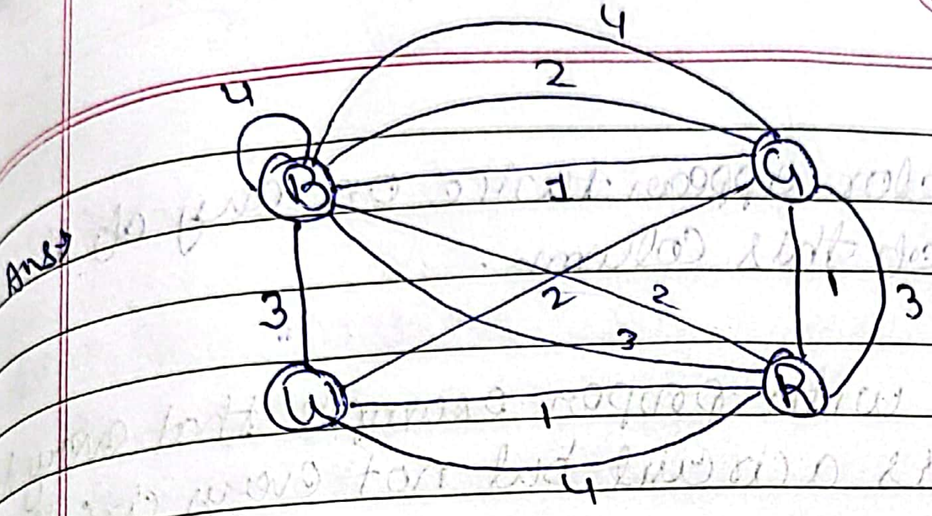
$$\boxed{e = \frac{n(n-1)}{2}}$$

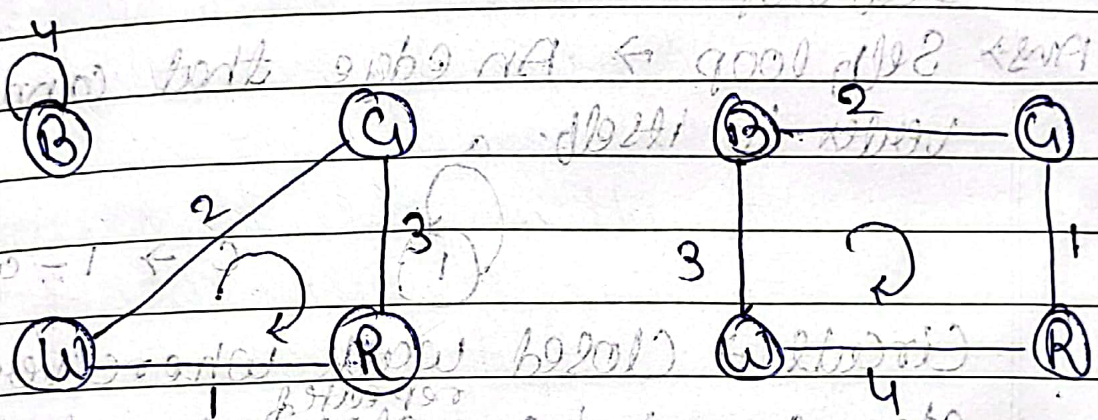
$\therefore$ maximum no. of edges with n vertices is $\frac{n(n-1)}{2}$ is proved.

Q5. Given 4 cubes with six faces of every cube are variously colored with Blue, green, Red & white. Is it possible to stack the cube one on the top of another to form an color such that no. of color appears twice on any of four side of this column.

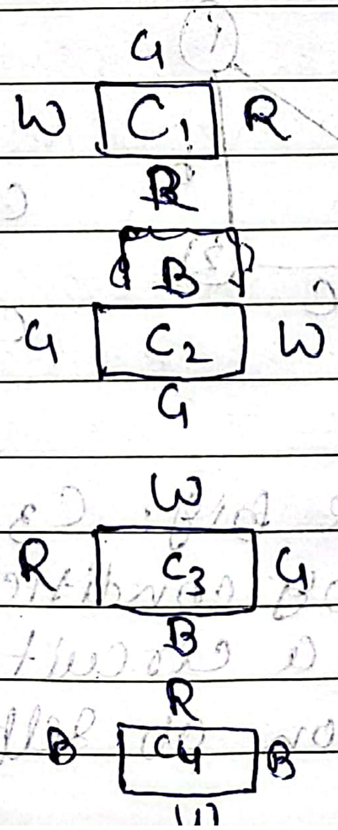
C1			
R	B G R	G	B
B R G	G	B B R	B W G
W	B	R	B
G		W	R
	C2	C3	C4

Ans:


4
4
2
B
G
1
3
2
2
1
3
3
W
R
1
4


4
B
G
2
3
W
R
1
2
B
G
3
1
W
R
4

E-W


G
W
C1
R
R
B
G
C2
W
G
W
R
C3
G
B
R
B
C4
B
W

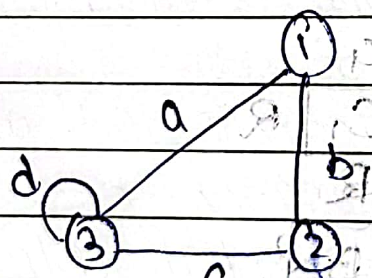
No, Color appear twice on any of the 4 side of this column.

Q)6. Show with proper example that every self loop is a circuit but not every circuit is self loop.

Ans→ Self loop → An edge that connects a vertex to itself.

$C \rightarrow 1 - a - 1$

Circuit → Closed walk where vertex and edge cannot be ~~replaced~~ repeated except the starting vertex.



$C_1 \rightarrow 1-b-2-c-3-a-1$

$C_2 \rightarrow 1-b-2-c-3-d-3-a-1$

$C_3 \rightarrow 3-d-3$

In the above fig. C_3 is a self loop and also satisfy condition for circuit but C_1 is a circuit and doesn't satisfy the condition of self loop.

Q) 7. How euler solve Konigsberg bridge problem?

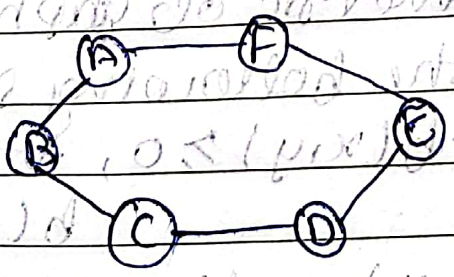
Ans→ The problem was to perform a closed walk. The graph he had drawn did not have all the vertex an even degree. If all the vertices had an even degree the problem would have been solved.

Q) 8. Differentiate between Hamiltonian graph and euler graph.

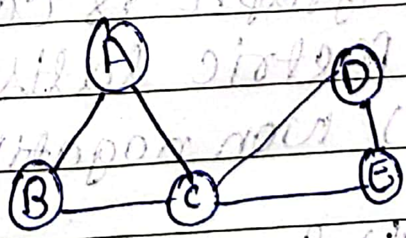
Ans→

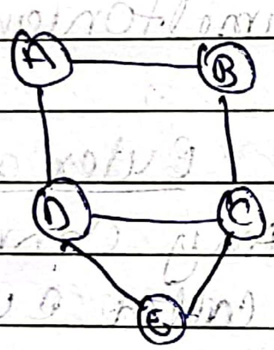
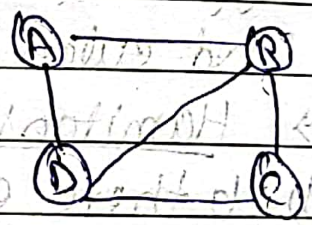
Hamiltonian graph	Euler graph
1) If there exists a closed walk in the connected graph that visits every vertex of graph exactly once.	i) Any connected graph is called euler graph if all its vertices are of even degree.

Example



Example



(ii) If there exists a walk in the connected graph that visits every vertex of the graph exactly once without repeating the edges then such a walk is known as Hamiltonian path.	(ii) if there exists a walk in the connected graph that visits every edge of the graph exactly once with or without repeating the vertex, then such a walk is euler's path.
	
Path: ABCDE	Path: BCD BAD

Q) 9. Show that distance between vertices of a connected graph is a metric.

Ans→ Shortest distance between vertices of a connected graph is called metric of graph.

Metric satisfy the following properties:-

(i) Non negativity → $d(x,y) \geq 0, \forall (x,y)$
$d(x,y) = 0$

(ii) Symmetry: $d(x,y) = d(y,x)$

(iii) Triangle inequality → $d(x,y) \leq d(x,z) + d(z,y)$

Proof → We define $d(v_i, v_j)$ = length of the shortest path from vertex v_i, v_j.

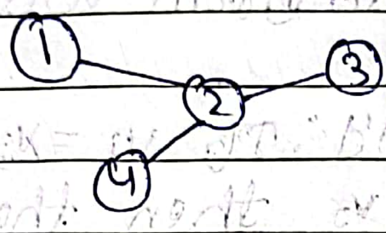
(i) non-negativity: If $v_i = v_j$ then $d(v_i, v_j) = 0$. If $v_i \neq v_j$ then there is atleast one path between them ⇒ $d(v_i, v_j) > 0$. Hence $d(v_i, v_j) \geq 0$.

(ii) Symmetry: $d(v_i, v_j)$ is the length of the shortest path from v_i to v_j. Since the same path can be traced from v_j to v_i to be shortest path from v_j to v_i, hence, it follows that ~~that~~ $d(v_i, v_j) = d(v_j, v_i)$.

(iii) Triangle inequality: Consider the vertex v_k. If v_k lies in the shortest path between v_i and v_j, then $d(v_i, v_j) = d(v_i, v_k) + d(v_k, v_j)$. If v_k doesn't lie in the shortest path, then $d(v_i, v_j) < d(v_i, v_k) + d(v_k, v_j)$. Hence, $d(v_i, v_j) \leq d(v_i, v_k) + d(v_k, v_j)$.

Q) 10. Show that a tree can be either mono-centric or by centric.

Ans> let a tree as a example(1)

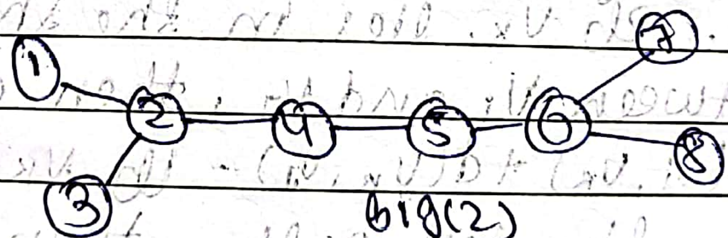


fig(1)

To find centre we have to delete pendant vertices of tree. so the tree becomes.

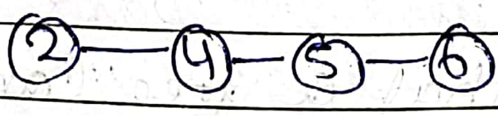
Hence, only one vertices is ~~same~~ remaining so, it is monocentric tree.

Let another example(2)

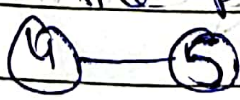


fig(2)

To find centre we have to delete pendant vertices.



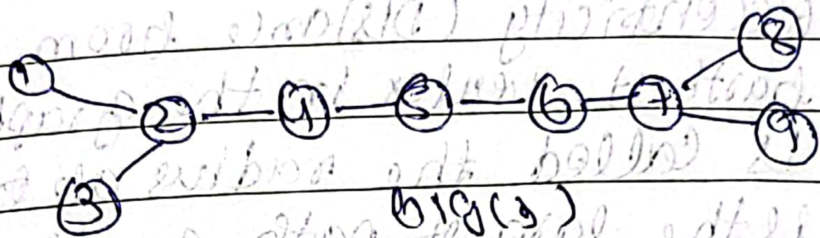
Again repeat the process,



Hence, here two vertices is remaining

So it is bicentric.

Let another example (3)



fig(3)

If we delete pendant vertice we find,

2 — 4 — 5 — 6 — 7

Again delete pendant vertice to find centre.

4 — 5 — 6

Again delete we find

5

Hence, here is one vertice remaining so it is mono centre.

From above example we have clearly seen that, a tree is either having one centre or two centre. If we increase there vertices as well.

classmate
Date
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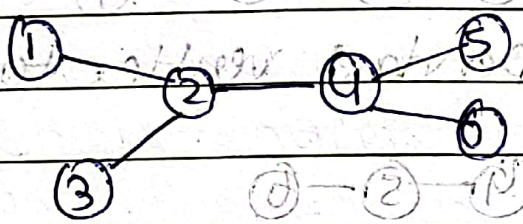
Q) 11. Show that in a tree it's diameter may not be always equal to twice it's radius.

Ans ⇒ Eccentricity (Distance from vertex to the farthest vertex in the graph) of the centre is called the radius of tree. Diameter is the largest path available in a tree.

let an example of tree.



here, Radius of tree is '2' and diameter of tree is '3'

hence, $2 \times 2 \neq 3$
$2 \times R \neq D$

hence from above example we prove that Diameter may not be always equal to twice of it's radius.

Q) 12 How the idea of counting tree was originated.

Ans→ In 1857, Arthur Cayley discovered trees while he was trying to count the number of structural isomers of the saturated hydrocarbons C_kH_{2k+2}. He used a connected graph to represent the C_kH_{2k+2} molecules. Since, the graph is connected and the number of edges is one less than the number of vertices, it is a tree. Thus the problem of counting structural isomers of a given hydrocarbon becomes the problem of counting trees.

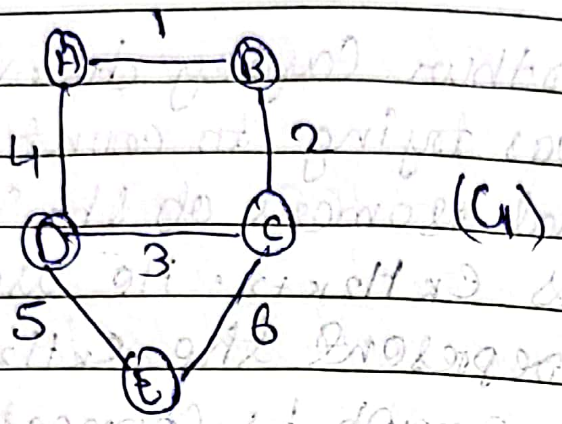
Total no. of vertices $= 3K+2$

Total no. of edges $= 3K+1$.

13. Show that a hamiltonian path is a spanning tree.

Ans→ Spanning tree is a subgraph of an un-directed connected graph. It includes all the vertices in the subgraph and the least number of edges that can connect every vertex without forming a loop or cycle.

Let a graph G.

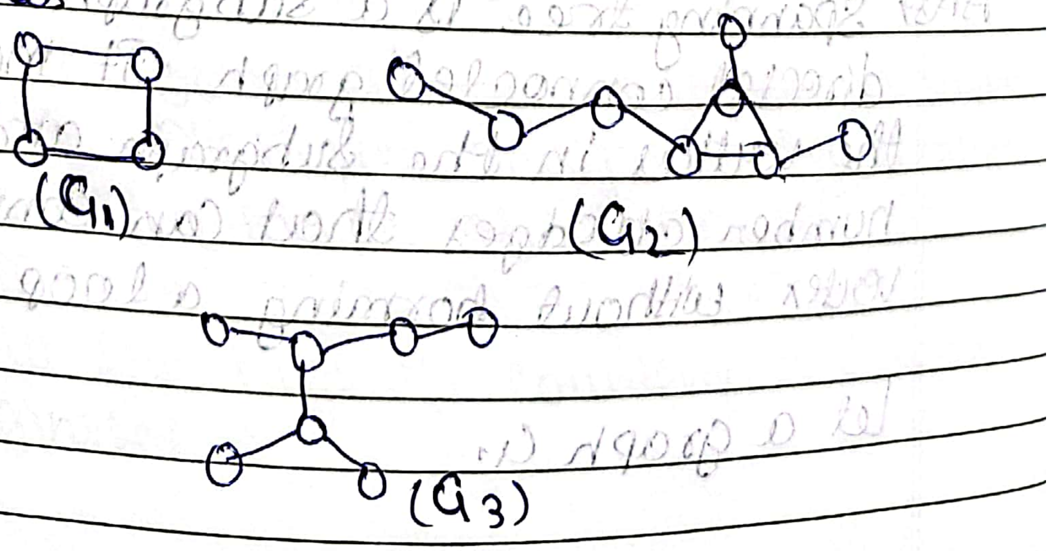


Hamiltonian path of graph G is A-1-B-2-C-3-D-5-E and from the above definition of spanning tree path of spanning tree is A-1-B-2-C-3-D-5-E which is same as Hamiltonian ~~graph~~ path.

∴ Hence, Hamiltonian path is spanning tree.

Q14 What is spanning forest?

Ans→ Spanning forest is collection of spanning Trees



G_1, G_2, G_3 are spanning tree and when they are collectively called then it is known as spanning forest.

Q) 15. Show that the distance between two spanning tree of a graph is a metric.

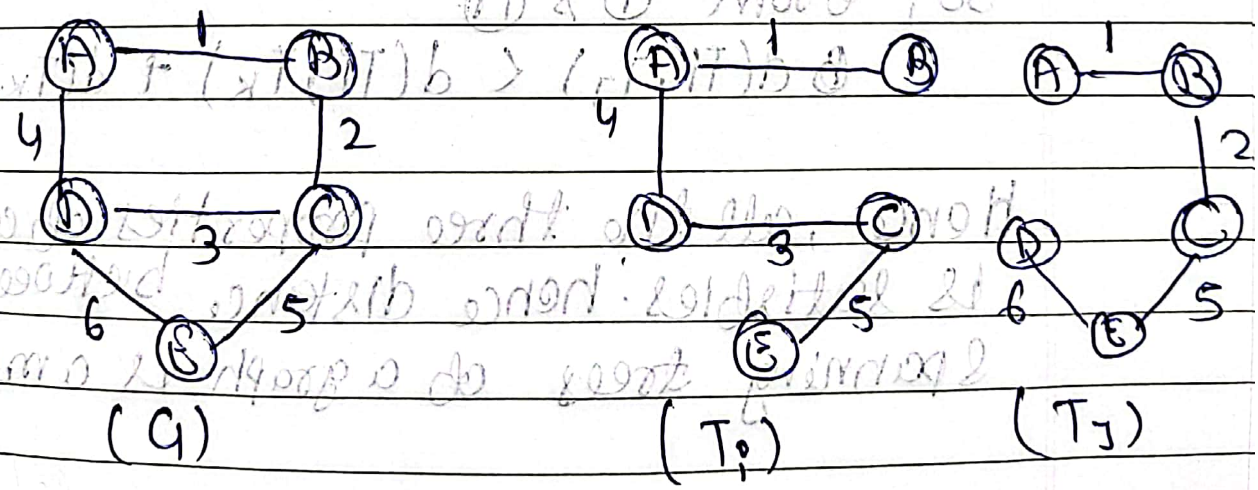
Ans) The distance between the spanning trees in a graph is a metric, if it satisfies

(i) $d(T_i, T_j) \geq 0$ and $d(T_i, T_j) = 0$ if and only if $T_i = T_j$,

(ii) $d(T_i, T_j) = d(T_j, T_i)$

(iii) $d(T_i, T_j) \leq d(T_i, T_k) + d(T_k, T_j)$.

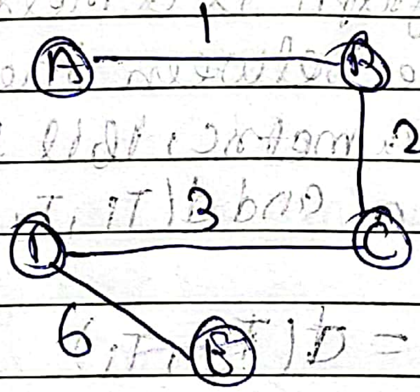
Let G be a graph and T_i and T_j are 2 spanning tree of graph.



(i) $d(T_i, T_j) = 2$ — (I)

(ii) $d(T_i, T_j) = d(T_j, T_i) = 2$

(iii) Let T_K be the third spanning tree of graph



Here, $d(T_i, T_K) = 2$ & $d(T_K, T_j) = 2$

$\therefore$ $d(T_i, T_K) + d(T_K, T_j) = 2+2=4$ — (II)

So, from (I) & (II)

$d(T_i, T_j) < d(T_i, T_K) + d(T_K, T_j)$

Hence, all the three properties of a metric is satisfies. hence distance between two spanning trees of a graph is a metric.