National Institute of Technology, Agartala

Mathematical Foundation

Assignment-2.

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1. Find out rank of following matrices:

(a) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_1 \Leftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - 3R_1, \; R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 5 & 3 & 7 \end{bmatrix}$$

$R_4 \to R_4 - R_2$
$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is a row reduced echelon form with rank = 3.

(b) $$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

This is a row reduced echelon form with rank = 3.

(c) $$\begin{bmatrix} 3 & 1 & 1 \\ 4 & 2 & -1 \\ 7 & 3 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 3 & 1 & 1 \\ 4 & 2 & -1 \\ 3 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 3 & 1 & 1 \\ 4 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_2 - R_1$

$R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -2 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

This is a row reduced echelon form with rank $= 3$.

(d) $$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_1$, $R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

This is a row reduced echelon form with rank $= 2$.

2. Apply rank test to examine if system of equation is consistent & if consistent, then find complete solution.

(i) $2x - 3y + z = 1$; $x + 2y - 3z = 4$; $4x - y - 2z = 8$

Matrix is

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -3 \\ 4 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 1 & 2 & -3 & 4 \\ 4 & -1 & -2 & 8 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 2 & -3 & 1 & 1 \\ 4 & -1 & -2 & 8 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$, $R_4 \rightarrow R_4 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 7 & -7 \\ 0 & -9 & 10 & -8 \end{array}\right]$$

$R_3 \rightarrow R_3 - \frac{9}{7} R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 7 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\rho(A) = \rho(A:B) = 3$ = number of unknows

system is consistent & has unique solution.

$$x + 2y - 3z = 4$$

$$-7y + 7z = -7$$

$$\boxed{z = 1}$$

$$-7y = -14$$

$$\boxed{y = 2}$$

$$x + 4 - 3 = 4$$

$$\boxed{x = 3}$$

(ii) $x + y + z = 8$; $x - y + 2z = 6$; $3x + 5y - 7z = 14$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 5 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 14 \end{bmatrix}$$

Augmented matrix :

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 1 & -1 & 2 & 6 \\ 3 & 5 & -7 & 14 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & -2 & 1 & -2 \\ 0 & 2 & -10 & -10 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & -9 & -12 \end{array}\right]$$

This is a row reduced form

$\rho(A) = \rho(A:B) = 3 =$ number of unknowns

$$x + y + z = 8$$
$$-2y + z = -2$$
$$-9z = -12$$

$$\boxed{z = 4/3}$$

$$-2y + 4/3 = -2$$

$$\boxed{y = 5/3}$$

$$x + \frac{5}{3} + \frac{4}{3} = 8$$

$$\boxed{x = 5}$$

(iii) $2y + 4z + 5 = 0$; $8x - y + 4z = 12$; $16x - y + 10x = 1$

$$\left[\begin{array}{ccc|c} 0 & 2 & 4 & -5 \\ 8 & -1 & 4 & 12 \\ 16 & -1 & 10 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 8 & -1 & 4 & 12 \\ 0 & 2 & 4 & -5 \\ 16 & -1 & 10 & 1 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 8 & -1 & 4 & 12 \\ 0 & 2 & 4 & -5 \\ 0 & 1 & 2 & 23 \end{array}\right]$$

$R_3 \to R_3 - \frac{R_2}{2}$

$$\left[\begin{array}{ccc|c} 8 & -1 & 4 & 12 \\ 0 & 2 & 4 & -5 \\ 0 & 0 & 0 & 41/2 \end{array}\right]$$

$\rho(A) = 2$, $\rho(A:B) = 3$

System is inconsistent & has no solution.

3. Find eigen values & corresponding eigen vector of following matrix :

(i) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$-\lambda(\lambda^2 - 1) - (-\lambda - 1) + 1(1 + \lambda) = 0$

$-\lambda^3 + 3\lambda + 2 = 0$

$(\lambda - 2)(\lambda^2 + 2\lambda + 1) = 0$

$\boxed{\lambda = 2, -1, -1}$

For $\lambda = 2$, we have

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank$=2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Sol^n $= 3 - 2 = 1$ independent solution

$$\begin{bmatrix} x_1 - 2x_2 + x_3 \\ -3x_2 + 3x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 = x_3 = k$$

$$x_1 - 2k + k = 0$$

$$x_1 = k$$

Solution is $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

For $\lambda = -1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank=1

$(3-1) = 2$ independent solⁿ

$$\begin{bmatrix} x_1 + x_2 + x_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Two independent solution: $X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ & $X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

Characterstic equation $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 0 & 2-\lambda & 1 \\ -1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[4 + \lambda^2 - 4\lambda - 2] - 1[2 - 4 + 2\lambda] = 0$$

$$\lambda^3 - 5\lambda^2 - 8\lambda + 4 = 0$$

$$(\lambda - 1)(\lambda^2 - 4\lambda + 4) = 0$$

$$\boxed{\lambda = 1, 2, 2}$$

For $\lambda = 1$, $\begin{bmatrix} 0 & 2 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$R_2 \to R_2 - \frac{1}{2}R_1$, $R_3 \leftrightarrow R_1$

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Rank = 2

Solution $\Rightarrow$ $(3-2) = 1$ independent solution

$$\begin{bmatrix} -x_1 + 2x_2 + x_3 \\ 2x_2 + 2x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution is $\Rightarrow \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ [eigen vector of $\lambda = 1$]

For $\lambda = 2$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rank = 2

$(3-2) = 1$ independent solution

$$\begin{bmatrix} -x_1 + 2x_2 + 2x_3 \\ x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = 0$

$x_1 = 2x_2$

Solution is $\Rightarrow \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ [eigen vector of $\lambda = 2$]

(iii) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

Characterstic equation

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & -3 & 3-\lambda \end{bmatrix}$$

$$-\lambda(-3\lambda + \lambda^2 + 3) + 1(1) = 0$$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$(\lambda - 1)^3 = 0$$

$$\boxed{\lambda = 1, 1, 1}$$

For $\lambda = 1$,

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rank $= 2$

$\Rightarrow (3-2) = 1$ independent solution

$$\begin{bmatrix} -x_1+x_2 \\ -x_2+x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 = x_3 = k$$
$$x_1 = k$$

Solution $= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ [eigen vector for $\lambda=1$]

4. Solve the following difference equation :

(i) $u_{n+1} + 4u_{n+1} + 4u_n = 0$

(i) If $u_n = k^n$, $u_{n+1} = k^{n+1}$, $u_{n+2} = k^{n+2}$

Equation is satisfied if

$$k^{n+2} - 4k^{n+1} + 4k^n = 0$$
$$k^n[k^2 - 4k + 4] = 0$$
$$k^2 - 4k + 4 = 0 \quad \text{as} \quad k^n \neq 0$$
$$(k-2)^2 = 0$$
$$k = 2, 2$$

Thus $u_n = A \cdot 2^n$ is a solution but it contains only one arbitary constant. Then

$$u_n = 2^n v_n \quad \text{then}$$

$$2^{n+2} v_{n+2} - 4 \cdot 2^{n+1} v_{n+1} - 4 \cdot 2^n v_n = 0$$
$$v_{n+2} - 2v_{n+1} + v_n = 0$$

or

$$\Delta^2 v_n = 0$$

v_n = constant & v_n is a polynomial of first degree in n

$$v_n = A + Bn$$
$$u_n = (A + Bn)2^n$$

This is the general solution

(ii) $u_{x+2} - 3u_{x+1} - 4u_x = 3^x$

(ii) $U_x = k^n$

$k^{n+2} - 3k^{n+1} - 4k^n = 3^x$

Auxiliary Equation

$k^n(k^2-3k-4)=0$

$k^2-3k-4=0$

$(k-1)(k-4)=0$

$k=-1, 4$

General solution is : $A(-1)^n + B(4)^n$

$PI = \dfrac{3^x}{k^2-3k-4}$

We can take $U_n = a \cdot 3^n$ as a trial solution

$a3^{x+2} - 3^{x+1}a3 - 4 \cdot a3^x = 3^x$

$a(3^{x+2} - 3^{x+1} \cdot 3 - 4 \cdot 3^x) = 3^x$

$a(9-9-4)=1$

$a = -\dfrac{1}{4}$

$U_n = -\dfrac{3^n}{4}$

$U_n = A(-1)^n + B(4)^n - \dfrac{3^n}{4}$

(iii) $U_{x+3} + 5U_{x+2} + 8u_{x+1} + 4u_x = 0$

(iii) $u_0 = 0, \; u_1 = -1, \; u_2 = 2$

$U_x = k^n$

$k^3 + 5k^2 + 8k + 4 = 0$

$(k+1)(k^2+4k+4)=0$

$k = -1, -2, -2$

$u_x = A(-1)^x + (B + Cx)(-2)^x$

$x = 0$

$u_0 \Rightarrow A + B = 0$

$\boxed{A = -B}$ — (1)

$x=1$

$u_1 \Rightarrow A(-1)^1 + (B-C)(-2) = -1$

$\Rightarrow 2C - B = -1$ — ②

$x=2$

$u_2 \Rightarrow A(-1)^2 + (B+2C)(-2)^2 = 2$

$\Rightarrow A + 4B + 8C = 2$ — ③

Solving ①, ② & ③

$$\boxed{A = -\frac{6}{7},\ B = \frac{6}{7},\ C = -\frac{1}{14}}$$

$$u_x = -\frac{6}{7}(-1)^x + \left(\frac{6}{7} - \frac{x}{14}\right)(-2)^x$$

(iii) $u_{x+2} + u_{x+1} - 12u_x = 5^x$

(iii) $u_x = k^x$

$k^{x+2} + k^{x+1} - 12k^x = 5^x$

Auxiliary Equation

$k^{x+2} + k^{x+1} - 12k^x = 0$

$k^x(k^2 + k - 12) = 0$

$k^2 + k - 12 = 0$

$k = 3, -4$

General solution $\Rightarrow A(3)^x + B(-4)^x$

Particular solution

tak $u_x = a \cdot 5^x$ as a trial solution

$a \cdot 5^{x+2} + a \cdot 5^{x+1} - 12 \cdot a \cdot 5^x = 5^x$

$a\,5^x(25 + 5 - 12) = 5^x$

$$\boxed{a = \frac{1}{18}}$$

Solution is

$$A(3)^x + B(-4)^x + \frac{1}{18}5^x$$

5. Solve the differential equation:

(i) $(D^2-2D+10)y = 16e^x \cos 3x + 24e^x \sin 3x$

(i) Auxiliary Equation:

$$m^2-2m+10=0$$

$$m = 1 \pm 3i$$

$$C.F \Rightarrow e^x[c_1 \cos 3x + c_2 \sin 3x]$$

$$P.I \Rightarrow y = \frac{1}{D^2-2D+10}(16e^x \cos 3x + 24e^x \sin 3x)$$

$$\Rightarrow 16e^x \frac{1}{(D+1)^2-2(D+1)+10} \cdot \cos 3x + 24e^x \cdot \frac{1}{(D+1)^2-2(D+1)+10} \sin 3x$$

$$\Rightarrow 16e^x \cdot \frac{1}{-3^2-9} \cos 3x + 24e^x \cdot \frac{1}{-3^2-9} \sin 3x$$

$$\Rightarrow -\frac{8e^x \cos 3x}{9} - \frac{4}{3} e^x \sin 3x$$

Complete Solution:

$$\boxed{y = e^x(c_1 \cos 3x + c_2 \sin 3x) - \frac{8}{9}e^x \cos 3x - \frac{4}{3}e^x \sin 3x}$$

(ii) $(D^2+1)y = e^{2x} + \cosh 2x + x^3$

(ii) Auxiliary Equation:

$$m^2+1=0$$

$$m = \pm 1$$

$$C.F \Rightarrow c_1 \cos x + c_2 \sin x$$

$$P.I \Rightarrow \frac{1}{D^2+1}(e^{2x} + \cosh 2x + x^3)$$

$$\Rightarrow \frac{e^{2x}}{D^2+1} + \frac{1}{D^2+1}\left[\frac{e^{2x}+e^{-2x}}{2}\right] + \frac{x^3}{D^2+1}$$

$$\Rightarrow \frac{e^{2x}}{5} + \frac{1}{2}\left[\frac{e^{2x}}{5} + \frac{e^{-2x}}{5}\right] + (1+D^2)^{-1}x^3$$

$$\Rightarrow \frac{e^{2x}}{5} + \frac{1}{5}\cosh 2x + x^3 - 6x$$

Solution is :

$$\boxed{y = c_1\cos x + c_2\sin x + \frac{e^{2x}}{5} + \frac{1}{5}\cosh 2x + x^3 - 6x}$$

(iii) $(x^2D^2 - 4xD + 6)y = -x^4\sin x$

Putting $x = e^z$

$$[D(D-1) - 4D + 6]y = -e^{4z}\sin e^z$$

$$(D^2 - 5D + 6)y = -e^{4z}\sin e^z$$

Auxiliary Equation :

$$m^2 - 5m + 6 = 0$$

$$\boxed{m = 2, 3}$$

$$C.F \Rightarrow c_1e^{2z} + c_2e^{3z} \Rightarrow c_1x^2 + c_2x^3$$

$$P.I \Rightarrow \frac{1}{D^2 - 5D + 6}(-e^{4z}\sin e^z)$$

$$\Rightarrow -e^{4z}.\frac{1}{D^2 + 3D + 2}\sin e^z$$

$$\Rightarrow -e^{4z}.\frac{3D-1}{9D^2-1}.\sin e^z$$

$$\Rightarrow -e^{4z}\frac{(3D-1)}{-10}.\sin e^z$$

$$\Rightarrow \frac{e^{4z}}{10}[e^z\cos e^z - \sin e^z]$$

Solution is :

$$\boxed{y = c_1x^2 + c_2x^3 + \frac{x^4}{10}[x\cos x - \sin x]}$$