Date → 18/01/23

Probability

* Random (R.V.) variable → 1) Discrete   2) Continuous.

$X: S \to R$

X: no. of heads

$X(HH) = 2$

$X(HT) = 1$

$X(TT) = 0$

→ A random variable is function X with domain (S) and range (R).

→ If a random variable take finite no. of value then it is called random variable.

→ If a random variable take all the possible value of the space and infinite value.

* Probability function :- 1) Prob. mass function (pmf) (discrete)

$P_X(x) = \begin{cases} P(X = x_i) = p_i & \text{if } x = x_i \\ 0, & x \neq x_i \end{cases}$

1) $0 \leq P_X(x) \leq 1$

11) $\sum P_X(x) = 1$

2) Prob. density function (pdf) (continuous R.v.)

$$f_X(x) = \lim_{\delta x \to 0} \frac{P(x \le X \le x+\delta x)}{\delta x}$$

i) $0 \le f_X(x) \le 1$

ii) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$P(a \le X \le b) = \int_a^b f_X(x)\,dx$$

3) Commulative distribution system (cdf)

$$F_X(x) = \begin{cases} P(X \le x_i), & \text{discrete r.v.} \\ \int_{-\infty}^{x_i} f(x)\,dx, & \text{Continous r.v.} \end{cases}$$

Q) A R.v. X has the following probability function. (discrete r.v.)

x: 0, 1, 2, 3, 4, 5, 6, 7

$P(x)$: 0, K, 2K, 2K, 3K, K^2, $2K^2$, $(7K^2+K)$

(i) find K (ii) find $P(X \le 5)$

(iii) $P(0 < X < 5)$

(iv) if $P(X \le a) > \frac{1}{2}$, find min. value of a.

(v) Complete distribution function of X.

(i) $0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$K = -1$ or $K = \frac{1}{10}$

(ii) $\sum_{i=0}^{5} P(x_i) = 0 + \cancel{0}k + 2k + \cancel{3}k + 3k + k^2$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$

$= \frac{81}{100} = 0.81$

(iii) $\sum_{i=1}^{4} (Px_i) =$

(iv) $0 \quad \frac{1}{10} \quad \cancel{} \frac{2}{10} \quad \frac{2}{10} \quad \frac{3}{10} \quad \frac{1}{100} \quad \frac{2}{100} \quad \left(\frac{7}{100} + \frac{1}{10}\right)$

$\frac{17}{100}$

~~$0 \quad \frac{1}{10} \quad \frac{3}{10} \quad \frac{5}{10}$~~

min. value = 4.

(v) $F(x) = P(X \leq x_i)$

x :	$P(x)$	$F(x)$
0	0	0
1	0.1	0.1
2	0.2	0.3
3	0.2	0.5
4	0.3	0.8
5	0.01	0.81
6	0.02	0.83
7	0.17	1

Q) The diameter of a cable is a continuous r.v. with pdf.

$f(x) = Kx(1-x), 0 \le x \le 1$

① find K

② find a no. c such that $(P x \le c) = P(x > c)$

③ Compute $P(x \le \frac{1}{2} \mid \frac{1}{3} \le x \le \frac{2}{3})$

④ find c.d.f of x

①
$$\int_0^1 f(x)\,dx = 1$$

$$\int_0^1 kx - kx^2 = 1$$

$$\left[\frac{kx^2}{2} - \frac{kx^3}{3}\right]_0^1 = 1$$

$$k\left[\frac{1}{2} - \frac{1}{3}\right] = 1$$

$$k = 6$$

②
$$\int_0^c kx - x^2 = \int_c^1 kx - x^2$$

$$k\left[\frac{x^2}{2} - \frac{x^3}{3}\right] = k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_c^1$$

$$\left[\frac{c^2}{2} - \frac{c^3}{3}\right] = \frac{1}{2} - \frac{1}{3} - \frac{c^2}{2} + \frac{c^3}{3}$$

$$\frac{3c^2 - 2c^3}{6} = \frac{3 - 2 - 3c^2 + 2c^3}{6}$$

$6c^2 - 4c^3 = 1$

$4c^3 - 6c^2 - 1 = 0$

$c = \frac{1}{2}$

9 | 18, 81
2, 9

2 | 12, 164
3 | 6, 81
2, 27

3 | 18, 81
3 | 6, 27
2, 9

(3)

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P\left(\frac{1}{3} \le x \le \frac{1}{2}\right)}{\cancel{P(B)}\; P\left(\frac{1}{3} \le x \le \frac{2}{3}\right)}$

$\Rightarrow \int_{1/3}^{1/2} kx - kx^2$

$6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1/3}^{1/2} = \frac{1}{8} - \frac{1}{24} - \frac{1}{18} + \frac{1}{81}$

Numerator

$\frac{3-1}{24} - \frac{9+2}{164}$

$\frac{\cancel{2}}{\cancel{24}\ 12} - \frac{11}{164}$

$\frac{27 - 22}{2 \times 3 \times 2 \times 27}$

$= \frac{5}{2 \times 3 \times 2 \times 27}$

Denominator

$\frac{4}{18} - \frac{4}{81} - \frac{1}{18} + \frac{1}{81}$

$\frac{3}{18} - \frac{3}{81}$

$\cancel{2 \times 3 \times 2 \times 27}$

$\frac{27 - 6}{3 \times 3 \times 2 \times 9} = \frac{19}{3 \times 3 \times 2 \times 9}$

$\frac{\frac{5}{2 \times \cancel{3} \times \cancel{2} \times 2\cancel{7}}}{\frac{19}{\cancel{3 \times 3 \times 2 \times 9}}} = \boxed{\frac{5}{38}}$

(4) $\underline{0 \leq x \leq 1}$ $\quad F(x) = \int_{-\infty}^{x} f(x)\,dx$

$-\infty < x \leq 0 \rightarrow Fx = \int_{-\infty}^{0} f(x)\,dx = 0$

$0 < x < 1 \rightarrow f(x) = \int_{-\infty}^{0} \text{limit} \int_{0}^{x=1} f(x)\,dx$

$1 \leq x < \infty$

$= 0 + \int_{0}^{x} f(x)\,dx$

$\int_{-\infty}^{0} + \int_{0}^{1} + \int_{1}^{x=\infty} f(x)\,dx$

$= 3x^2 - 2x^3$

$= 1$ (Put $x=1$)

$0 + 1 + 0$

$= \cancel{0}\ 1$

(5) $f(x) = ax, \quad 0 \leq x \leq 1$

$a, \quad 1 \leq x \leq 2$

$-ax + 3a, \quad 2 \leq x \leq 3$

$0, \quad$ else.

$f(x) = \int_{0}^{1} ax = a\left[\frac{x^2}{2}\right]_{0}^{1} = \frac{a}{2}$

$f(x) = \int_{0}^{1} a + \int_{1}^{2} a$

$= a + a[x]_{1}^{2}$

$= a + a = 2a$

$f(x) = \int_{0}^{1} -ax + 3a + \int_{1}^{2} -ax + 3a + \int_{2}^{3} -ax + 3a$

$$\Rightarrow \left[-a\frac{x^2}{2}\right]_0^1 + 3a[x]_0^1 + \left[-\frac{ax^2}{2}\right]_1^2 + 3a[x]_1^2$$

$$+ \left[-\frac{ax^2}{2}\right]_2^3 + 3a[x]_2^3$$

$$\Rightarrow -\frac{a}{2} + 3a + \left(-\frac{a \cancel{4}^2}{\cancel{2}} + \frac{a}{2}\right) + 3a + 3a + \left[-\frac{9a}{2} + \frac{\cancel{4}^2 a}{\cancel{2}}\right]$$

$$\Rightarrow a + 3a + 3a - \frac{9a}{2} + 2a$$

$$9a - \frac{9a}{2} = \frac{9a}{2}$$

~~$\frac{a}{2} + 2a + 9a \quad \frac{9a}{2} = \frac{11a}{2}$~~

(i) find cdf

(ii) find $P(X \leq 1.5)$

(iii) $\frac{d}{dx} F(x) = f_x(x)$

$= e^{-2x} e^{t} \Rightarrow e^{-2x} e^{e^x}$ Ans

$\frac{a + 2a - 5a + 6a}{2}$

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(5) $f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \\ 0 & \text{else} \end{cases}$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\underbrace{\int_{-\infty}^{0} f(x)\, dx}_{=0} + \int_{0}^{1} f(x)\, dx + \int_{1}^{2} f(x)\, dx + \int_{2}^{3} f(x)\, dx + \underbrace{\int_{3}^{\infty} f(x)\, dx}_{=0} = 1$$

$$\int_{0}^{1} a(x)\, dx + \int_{1}^{2} a\, dx + \int_{2}^{3} (-ax + 3a)\, dx = 1$$

$$a\left[\frac{x^2}{2}\right]_0^1 + a[x]_1^2 + a\left[-\frac{x^2}{2} + 3x\right]_2^3 = 1$$

$$\frac{a}{2} + a + 3a - \frac{5a}{2} = 1$$

$$2a = 1$$

$$a = \frac{1}{2}$$

(6) C.d.f = Case 1 $x < 0$

$$\int_{-\infty}^{x} f(x)\, dx = 0$$

Case 2 $0 \le x \le 1$

$$\int_{-\infty}^{x} f(x)\, dx = \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{x} f(x)\, dx$$

$0 + \int_0^x \frac{1}{2}x \, dx = \frac{1}{4}x^2$ $\boxed{= \frac{1}{4} \quad (\because x=1)}$

Case 3 :- $1 \le x \le 2$

$\int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$

$= 0 + \int_0^1 \frac{1}{2}x + \int_1^x \frac{1}{2}$

$= 0 + \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$

$= \frac{x}{2} - \frac{1}{4}$ $\boxed{= 1 - \frac{1}{4} = \frac{3}{4} \quad (\because x=2)}$

Case 4 :- $2 \le x \le 3$

$\int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^x f(x)\,dx$

$0 + \frac{1}{4} + \frac{1}{2}[x]_1^2 + \left[\frac{3}{2}x - \frac{1}{4}x^2\right]_2^x$

$0 + \frac{1}{4} + \frac{1}{2} + \left[\frac{3x}{2} - \frac{1}{4}x^2 - 3 + 1\right]$

$\frac{3}{4} - 2 + \frac{3x}{2} - \frac{1}{4}x^2$

$= \frac{3x}{2} - \frac{1}{4}x^2 - \frac{5}{4}$ $\boxed{= 0 - \frac{5}{4} + \frac{9}{2} - \frac{9}{4} \quad [\because x=3] \quad = \frac{-5+18-9}{4} = 1}$

Case 5 $x \geq 3$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx + \int_{3}^{x} f(x)\,dx$$

$$0 + \frac{1}{4} + \frac{1}{2} \left[\frac{3x}{2} - \frac{1}{4}x^2 \right]_2^3 + 0$$

$$\frac{3}{4} + \left[\frac{9}{2} - \frac{9}{4} - \frac{6}{2} + 1 \right]$$

$$\frac{3}{4} + \frac{9}{4} - \frac{6}{2} + 1$$

$$= 1$$

Q) The time (in min.) that a person has to wait at a certain bus stop for a bus is found to be a random phenomena. with a function specified by,

$$F(x) = \begin{cases} 0 & , x < 0 \\ \frac{x}{8} & , 0 \leq x < 2 \\ \frac{x^2}{16} & , 2 \leq x < 4 \\ 1 & , x \geq 4 \end{cases}$$

① find Pdf of x.

② find the prob. that a person has to wait

a) more than 2 min.

b) betn 1 & 2 min.

③ find the prob. that a person has to wait more than 2 min. given it in more than 1 min.

① $\frac{d}{dx} F(x) = f(x)$

$$f(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{8} & , 0 \le x < 2 \\ \frac{x}{8} & , 2 \le x \le 4 \\ 0 & , x \ge 4 \end{cases}$$

② (a) $P(x \ge 2) = 1 - P(x \le 2)$

$1 - P(x \le 2)$ or $= 1 - \int_0^2 f(x)\,dx$

$1 - F(2)$ or

$1 - \frac{1}{4}$ or $= 1 - \left[\cancel{\int_0^2 f(x)dx + \int_1^2 f(x)dx} \right.$

$\rightarrow \frac{3}{4}$ $= 1 - \int_0^2 \frac{x}{8}$

$= 1 - \frac{1}{8} \left[\frac{x^2}{2} \right]_0^2$

$= 1 - \frac{1}{8} \left(\frac{2}{4} \right) = \frac{3}{4}$

b) $P(1 \le x \le 2)$ $\boxed{P(a \le x < b) = F(b) - F(a)}$

$F(2) - F(1)$

$\frac{(2)^2}{16} - \frac{1}{8}$

$\frac{4}{16} \cdot 4 - \frac{1}{8} = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$

(3) $P(x>2 \mid x>1) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{P(x>2)}{P(x>1)}$$

$$= \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$$

* Let $f(x)$ be a pdf defined on $[a,b]$ Then, rth moment about any Point A is

$$\mu_r^A = \int_a^b (x-A)^r f(x)\,dx$$

Put $A=0 \Rightarrow \mu_r' = \int_a^b x^r f(x)\,dx \rightarrow$ rth moment about origin.

$A=\bar{x} \Rightarrow \mu_r = \int_a^b (x-\bar{x})^r f(x)\,dx \rightarrow$ " " " mean

* → Arithmatic mean (AM)
→ Geometric mean (GM)
→ Harmonic mean (HM)

mean

$\mu_1' = \int_a^b x f(x)\,dx$ (Put $r=1$)

$= \text{mean} = \bar{x}$

$\mu_1 = \int_a^b (x-\bar{x}) f(x)\,dx$ (Put $r=1$)

$3ab(a-b)$

$$\mu_1 = \int_a^b x\, f(x)\,dx - \int_a^b \bar{x}\, f(x)\,dx$$

$$= \bar{x} - \bar{x}\int_a^b f(x)\,dx$$

$$\Rightarrow \bar{x} - \bar{x} = 0$$

$\boxed{\text{Put } r = 2}$

$$\mu_2 = \int_a^b (x-\bar{x})^2 f(x)\,dx$$

$$= \int_a^b \cancel{(x-\bar{x})(x+\bar{x})\, f(x)\,dx}$$

$x^2 f(x) + (\bar{x})^2 f(x) - 2x\bar{x}\, f(x$

$$= \int_a^b \cancel{x^2 + x\bar{x} - x\bar{x}}$$

$x^2 + (\bar{x})^2$

$2(\bar{x})^2 + 2(\bar{x})^2$

$\boxed{4(\bar{x})^2}$

variance $\boxed{\mu_2 = \mu_2' - (\mu_1')^2}$

$$\mu_3' = \int_a^b x^3 f(x)\,dx$$

$$= x^3 \int_a^b f(x)\,dx.$$

$$\mu_3 = \int_a^b (x-\bar{x})^3 f(x)\,dx.$$

$$= \int_a^b \left(x^3 - (\bar{x})^3 - 3x^2\bar{x} + 3x(\bar{x})^2\right) f(x)\,dx$$

$= \cancel{\mu_3'}$

$= \cancel{3x^2\bar{x} + 3x(\bar{x})^2}$

$\mu_3 = \mu_3' - 3\mu_1' \mu_2' + 2(\mu_1')^3$

$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

<u>meadium</u> : $\int_a^m f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$

<u>mode</u> ! $f'(x) = 0$ & $f''(x) < 0$

Q) In a continuous distribution with relative density function $f(x) = Kx(2-x)$; $0 \le x \le 2$

(i) find K

(ii) find mean, variance e.t.c.

solve→ (i)

$$\int_0^2 2Kx - Kx^2 = 1$$

$$\frac{2K}{2}\left[x^2\right]_0^2 - K\left[\frac{x^3}{3}\right]_0^2 = 1$$

$$K \times 4 - K \times \frac{8}{3} = 1$$

$$4K - \frac{8K}{3} = 1$$

$$\frac{4K}{3} = 1$$

$$K = \frac{3}{4}$$

(ii) mean $= \bar{x} = \int_a^b Kx(2-x)\,dx$

$$= \frac{3}{4}\int_0^2 2x - x^2\,dx$$

$$= \frac{3}{4}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= \frac{3}{4}\left[4 - \frac{8}{3}\right] = 1$$

(iii) variance $\Rightarrow \mu_2 = \mu_2' - (\mu_1')^2$

$$\int_0^2 x^2 f(x)\,dx - 1$$

$$\frac{3}{4}\int_0^2 x^3(2-x)\,dx - 1$$

$$\frac{3}{4}\left[\cancel{2}\frac{x^4}{\cancel{4}_2} - \frac{x^5}{5}\right]_0^2 - 1$$

$$\frac{3}{4}\left[8 - \frac{32}{5}\right] - 1$$

$$\frac{3}{\cancel{4}} \times \frac{\cancel{8}2}{5} - 1$$

$$\frac{6}{5} - 1 = \frac{1}{5}$$

$$= \cancel{\int_0^2 x\,k\,x^2(2-x)} - 1$$

$$= \cancel{\frac{3}{4}\int_0^2 2x^2 - x^3} - 1$$

$$= \cancel{\frac{3}{4}\left(\left[\frac{2x^3}{3}\right]_0^2 - \left[\frac{x^4}{4}\right]_0^2\right)} - 1$$

$$= \cancel{\frac{3}{4}\left(\frac{16}{3} - 4\right)} - 1$$

$$\cancel{= \frac{4}{3}}$$

(iv) Meadiann $= \int_a^m \frac{3}{4}(2x - x^2)\,dx$

$$= \frac{3}{4}\int_0^m 2x - x^2$$

$$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^m$$

$$\frac{3}{4}\left[m^2 - \frac{m^3}{3}\right] = \frac{1}{2}$$

$$\frac{3}{4}\int_m^2 2x - x^2\,dx$$

$$\frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_m^2$$

$$\frac{3}{4}\left[m^2 - \frac{m^3}{3} - 4 + \frac{8}{3}\right]$$

$$\frac{3}{4}\left[m^2 - \frac{m^3}{3}\right] + \frac{3}{4}\left(\frac{8}{3}\right)$$

$x^3 - x^6$

$$\frac{x^4}{4} - \frac{x^7}{7}$$

$$\frac{1}{4} - \frac{1}{7} \quad \frac{3}{28} \quad \frac{m^3}{4}$$

$$= \frac{1}{2} + \frac{x}{x} = \frac{4}{x}$$

* Geometric mean $\Rightarrow \log G = \int_a^b \log x \, f(x) \, dx.$

* Harmonic mean $\frac{1}{H} = \int_a^b \frac{f(x)}{x} dx$

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Q) $(D^2 + a^2) y = \sec ax$

Auxiliary Eqn $\quad m^2 + a^2 = 0$

$m = \pm ai$

$C.F. = (c_1 \cos ax + c_2 \sin ax)$

$P.I. = \frac{1}{D^2 + a^2} \sec ax$

$= \frac{1}{(D + ai)(D - ai)} \sec ax$

$= \frac{1}{2ia} \left[\frac{1}{D - ia} - \frac{1}{D + ia} \right] \sec ax$

$= \frac{1}{2ia} \left[\frac{1}{D - ia} \sec ax - \frac{1}{D + ia} \sec ax \right]$

$\frac{1}{D - ia} \sec ax = e^{iax} \int \sec ax \, e^{-iax} dx$

$\cos ax = \frac{e^{iax} + e^{-iax}}{2}$

$\sec ax = \frac{2}{e^{iax} + e^{-ia}}$

$e^{iax} \int \frac{2}{e^{iax} + e^{-iax}} \times e^{-iax} dx$

$e^{iax} \int \frac{2 \cdot e^{-2iax}}{1 + e^{-2iax}} dx.$

* Mathematical expectation :- (expected value)

$$E(x^r) = \begin{cases} \sum_x x^r P(x) , & \text{discrete} \\ \int_{-\infty}^{\infty} x^r f(x)\, dx , & \text{continuous} \end{cases}$$

$$\mu'_r = E(x^r)$$

$$\mu_r = E[x - \bar{x}]^r$$

$$\text{mean} = E(x) = \begin{cases} \sum x\, p(x) \\ \int x\, f(x)\, dx \end{cases}$$

$$\text{var.} = E[x - \bar{x}]^2 = \int_{-\infty}^{\infty} (x - \bar{x})^2 f(x)\, dx$$

$$= \mu'_2 - (\mu'_1)^2$$

$$= E(x^2) - (E(x))^2$$

Property :

① $E(X+Y) = E(X) + E(Y)$

② $E(XY) = E(X)\, E(Y)$ if X, Y are independent.

③ $E(KX) = K E(X)$

④ $E(K) = K$

⑤ $\text{var}(ax+b) = a^2 V(x)$

* Covariance

$$\text{cov}(x, y) = E(x - \bar{x})(y - \bar{y})$$

$$= E(xy) - 2E(x)E(y) + E(x)E(y)$$

$$= E(xy) - E(x)E(y)$$

$$E(xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y})$$

$$E(xy) + E(\bar{x}\bar{y}) - [E(x\bar{y}) + E(\bar{x}y)]$$

$$E(x)E(y) + E(\bar{x})E(\bar{y}) - [E(x)E(\bar{y}) + E(\bar{x})E(y)]$$

$$E(x)(E(y) - E(\bar{y})) + E(\bar{x})(E\bar{y} - E(y))$$

$$(E(x) - E(\bar{x}))(E(y) - E(\bar{y})) \times$$

(H/W) 1. $cov(ax, by) = ab\,cov(x,y)$

2. $cov(x+a, y+b) = cov(x,y)$

Q) Two unbiased dice are thrown find the expected value of the sum of nos. of points on the face.

$\Rightarrow$

$x \rightarrow$	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	5	4	3	2	1

7
6 - 6 5 - 8
5 - 4 4 - 9
4 - 3 3 - 10
3 - 2 2 - 11
2 - 1 1 - 12

112
40
152
36
188
30
218
240

$E(x) = \sum x P(x)$

$E(x) = 2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12$

$= \frac{252}{36} = 7$ Ans

x_i : no. of points on ith dice

$E(x_1 + x_2) = E(x_1) + E(x_2)$

$= \frac{7}{2} + \frac{7}{2}$

$= 7$

$\boxed{\text{For n dice} = \frac{7n}{2}}$

(H/W) In four tosses of coin find the expected values of no. of heads.

Q) A coin is tossed until the head is appear. find the expected of toss required.

1 2 3

$\frac{1}{2} + \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$

$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

$X = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \ldots$

$P(x) = \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{32} \quad \ldots$

$E(x) = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{16} + \ldots$

$\frac{1}{2}\left(\frac{1}{1} + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \ldots\right)$

$\frac{\frac{1}{2}}{1-2} \cdot \frac{1\times\frac{1}{2}}{(1-r)^2}$

$\frac{1}{1-\frac{1}{2}} + \frac{\frac{1}{2} \times 2}{(1-\frac{1}{2})^2} = 2$

$2 - 1 = 1$

$S_n = \frac{1}{2} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{8} - \ldots$

$\frac{1}{2} S_n = \frac{1}{4} + \ldots$

$S_n - \frac{1}{2} S_n =$

* Chebychev's inequality

$$P(|x-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

or $P(|x-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

let $k\sigma = c$

$$P(|x-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$$

$$\Rightarrow P(|x-\mu| \geq c) \leq \frac{Var(x)}{c^2}$$

Q. A symmetric dice is thrown 600 times, find the lower bound of probability of getting 80 to 120 sixes.

$$\boxed{\begin{aligned} \mu &= n \times p \\ \sigma^2 &= npq \quad ; q = 1-p \end{aligned}}$$

Solve: $n \times p = 100$ $\qquad$ $K = 80 - 120$

$= \frac{500}{6}$

$\Rightarrow P(|x - 100| < K \cdot \sqrt{\frac{500}{6}}) \geq 1 - \frac{1}{K^2}$

$\Rightarrow P(100 - K\sqrt{\frac{500}{6}} < x < 100 + K\sqrt{\frac{500}{6}}) \geq 1 - \frac{1}{K^2}$

$\Rightarrow P(80 < x < 120) \geq \frac{19}{24}$

Q.) Two unbiased dice. If x is the sum of the no. showing up then find the upper bound of the probability of $P(|x - 7| \geq 3)$.