Q ~~#Friend Function~~ class. → can access private and protected members.

A
```
class Base {
    friend Derived;    } // Base class
};                     ← syntax (declaration)
class Derived {
};                     } ← Friend class.
```

# Friend function.	
Class Base {	class Base {
friend void fn(); // declaration	friend void fn();
};	}.
	another class otherClass {
void Base :: public fn() { // definition	void Base :: fn();
}	}.

Polymorphism

The word polymorphism means having many forms. In simple words, we can define polymorphism as the ability of message to be displayed in more than one form.

- compile time
  - fn overloading
  - operator overloading
- run time
  - virtual functions

GRAPH THEORY :-

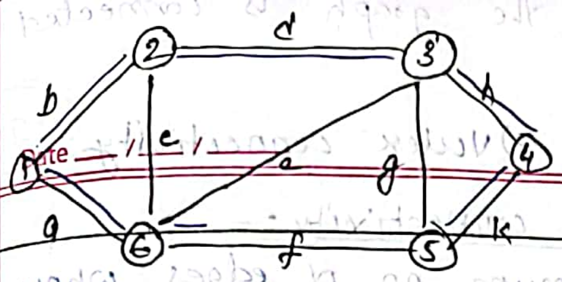
13/03/2023

Cut-Set → set of edges whose removal from ~~edit~~ graph will move G disconnected, provided no subset of those edges can do so.

→ minimal no. of edges

Properties

1) cut set is the minimal set of edges. (no more edge can be added to it
2) a cut set always cuts the graph into 2 parts.
3) The cut set is the set of edges whose removal destroys all path between the 2 parts of a graph.
4) Every edge of a tree is a cut set.



Q what is relation b/w fundamental circuit & fundamental cut set show with proper example.

Fundamental cut set:

branch = {a, b, d, h, k}

chords = {c, e, f, g} $(e-n+1)$

If add one chord → fundmental circuit

If add one branch → fundmental cutset

ex {a, b+c} → abc → fundmental cutset

{a, b} → cut set

→ not fundmental cut set.

In a fundmental cut set there will be exactly one one branch of a spanning tree.

No. of fundmental cut set = $(n-1)$ = branches.

Fund. cut set :- ① {f, g, k} ② {f, g, h}, ③ {d, e, f} ④ {b, c, e, f}

⑤ {a, c, e, f}.

fund. circuits :- ① g h k ② a b c ③ a b d h k f ④ a b d e

let fund. c/s branches a b d e → chord.

{a, c, e, f} {b, c, e, f} {d, e, f} ← fund. cutset.

With respect to a given spanning tree of a graph a chord that determines a fundamental circuit, occurs in every fundamental cutset associated with the branches present into that circuit.

~~ex fund. c/set.~~

with respect to a given spanning tree of a graph a branch that determines a fundamental cutset is present in every fundamental circuit associated with the chords present ~~in~~ ~~th~~ into that fundamental cut-set.

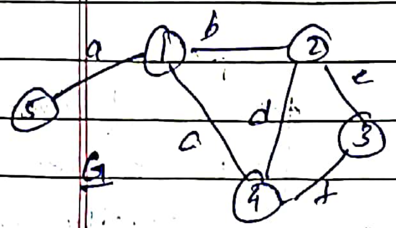
ex fund. c/set:- {f, g, k}
chord → branch

{a, b, d, h, k, f} {g, h, k, f} fund. c/s

Q. ☆ Connectivity - denotes wheater the graph is connected or not.

A. 1) Edge connectivity 2) Vertex connectivity.

☆ Edge connectivity :-
Minimum no. of edges whose removal will make the graph disconnected.



→ to find edge connectivity, identify the smallest degree vertex.

cut set = {g}, {c, d, e}. {c, d, f}, ...

minimum no. of edge = 1

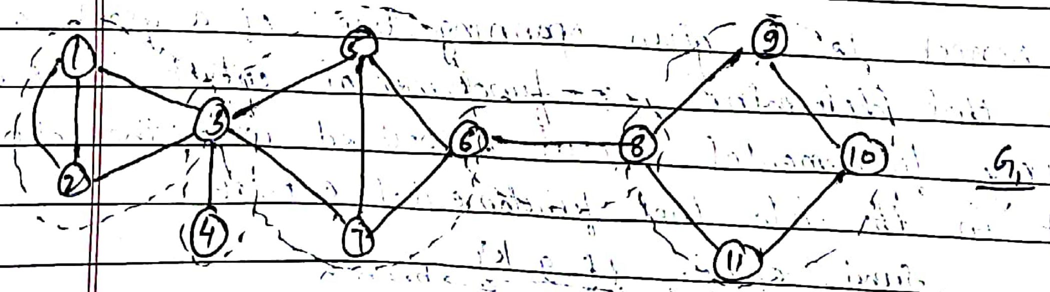
∴ edge connectivity = 1

☆ Vertex connectivity
Minimum number of vertices whose removal will make the graph disconnected.
Here, vertex connectivity is 1, since by removing ① we make the graph disconnect.

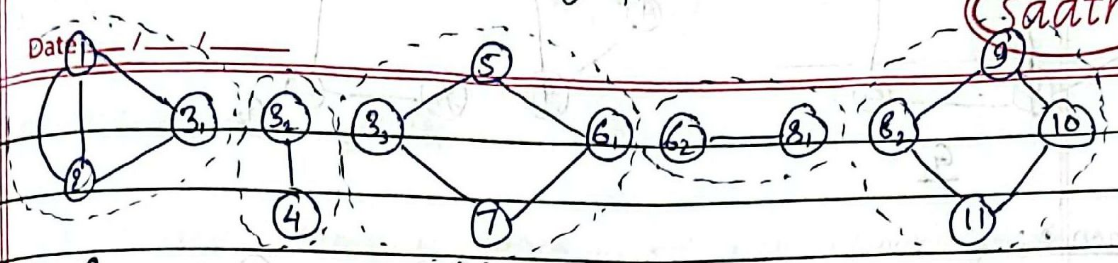
Separability
1) Separable
if vertex connectivity of a graph is exactly 1 then separable.
→ the deleted vertex is called cut vertex/point (Articulation point)

2) Non separable
if vertex connectivity is > 1 the non separable.

☆ 1- Isomorphism



Operation 1: Split the cut vertex into two, to produce 2 disjoint sub graphs.



G_{12}

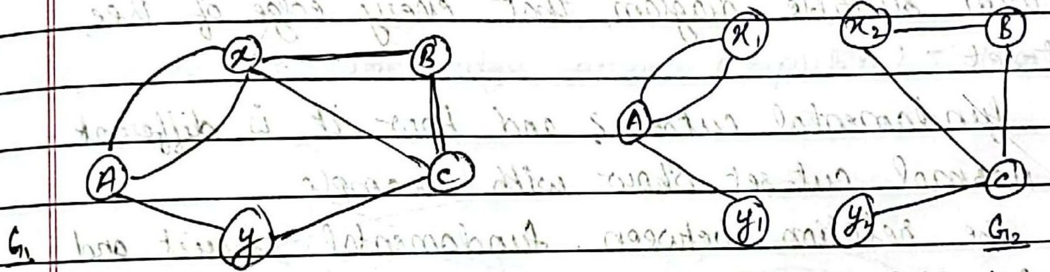
after simultaneous operation 1.

graph G_1 & G_2 are 1-isomorphism.

15/03/2023.

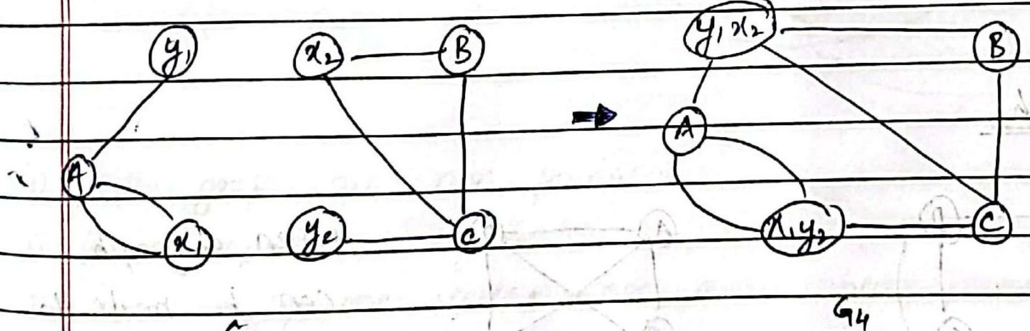
2-Isomorphism.

Non-separable graph G
& Vertex connectivity = 2.



Operation 1: Split the cut vertex into two.
(split operation)

Operation 2: Join the cut vertices in such a way, that x_1 is connected with y_2 and x_2 is connected to y_1
(Twist operation)

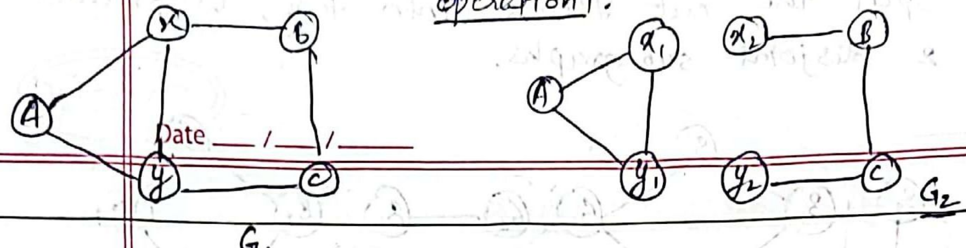


G_1 & G_4 are 2-isomorphic and phenomenon is known as two isomorphism.

Q

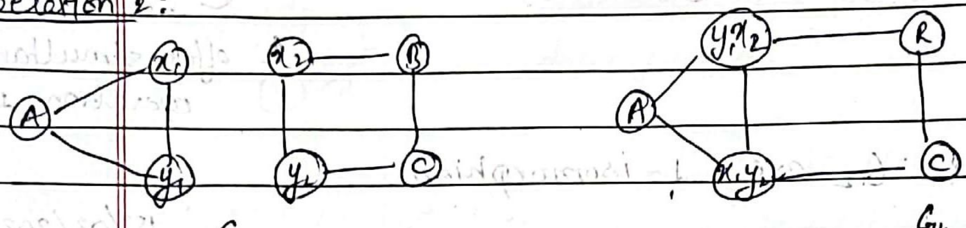
A.

Operation 1:



G_1 G_2

Operation 2:



G_3 G_4

Chapter. 4

Q1 Define a cut set with suitable diagram and mention its properties.

Q2 Explain with suitable diagram, that every edge of tree is a cut-set.

Q3 What is fundamental cutset? and how it is different from a normal cut-set. Show with example.

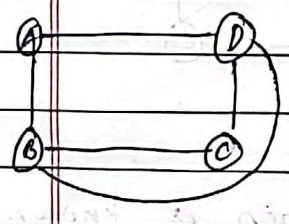
Q4 Find out the relation between, fundamental circuit and fundamental cutset with proper example.

Q5 What do you mean by connectivity and seperability?

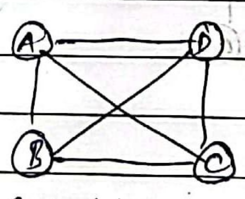
Q.6 What is an articulation point or cut vertex?

Q7 Explain 1- isomorphism and 2- isomorphism with suitable diagram.

★ Planar Graph.



Planar graph



Embedding

When a graph can be made to draw in a plane.

Planar graph - If there is no edge crossover in a ~~sign~~ single plane.

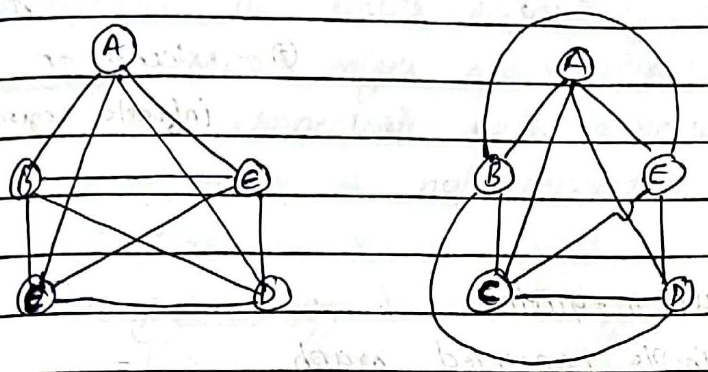
Non-planar graph - If there is at least one edge crossover in ~~sine~~

Embedding - It is the technique by which a graph can be drawn on a single plane/surface.

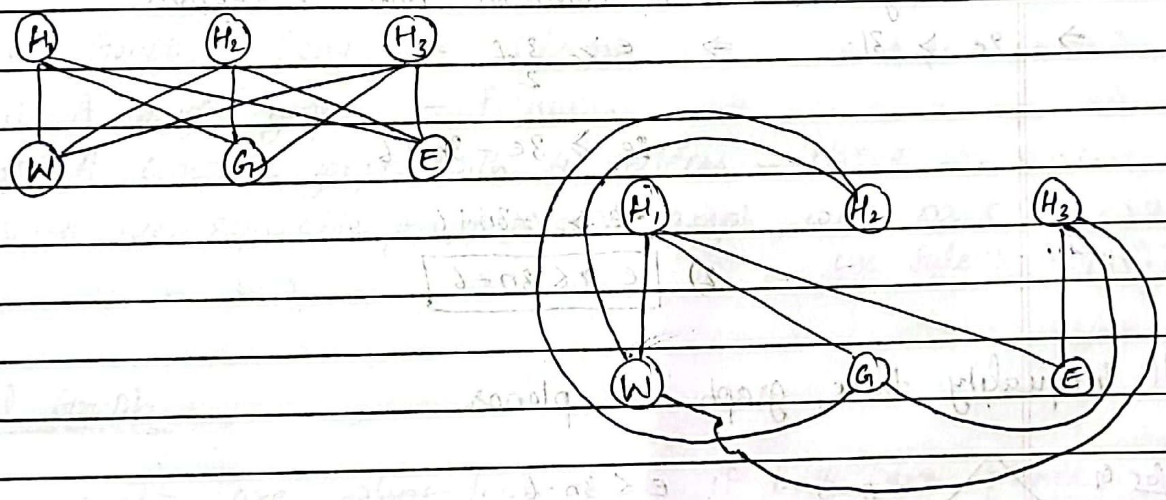
Date __/__/__

Kuratowski's 2 graph.

1. Five vertex complete graph → Non-planar graph.



2. Six vertex & nine edge graph. (Bipartite) → Non-planar

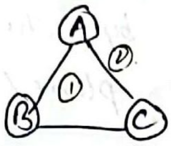


1) Both graph are non-planar.
2) Both graph are regular.
3) Removal of one vertex can make them planar.
4) Removal of one edge can make them planar.

Q ~~A~~ Region / Face

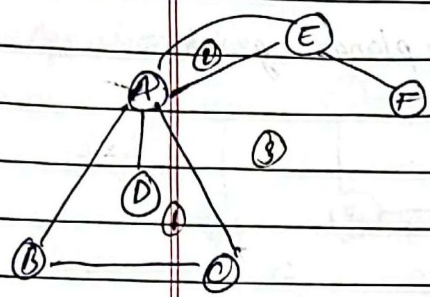
A $\boxed{f = e-n+2}$

euler's equation



$f = e-n+2$
$= 3-3+2$
$= 2.$

Region - bounded area of graph.



$f = 7-6+2$
$= 3.$

① & ② - internal or finite region

③ - external or infinite region.

Two inequalities from euler's equation

1) $e \geq \frac{3}{2} f$ / simple connected graph

→ you need at least 3 edge to form boundary

→ one edge will be common b/w 2 regions.

$\Rightarrow 2e \geq 3f \Rightarrow e \geq \frac{3}{2} f$

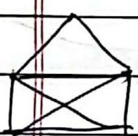
$2e \geq 3e - 3n + 6.$

$-e \geq -3n+6$

2) $\boxed{e \leq 3n-6}$

⇒ # If inequality true, graph is planar.

for G



$n=5$
$e=10$

$e \leq 3n-6$
$10 \leq 3\times5-6.$
$10 \leq 9.$ false
Hence, non-planar

for graph that need at least 4 edge to form boundary

$2e \geq 3f \Rightarrow 2e \geq 4f$

$\Rightarrow e \geq 2f$

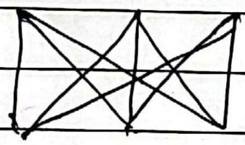
$\Rightarrow e \geq 2(e-n+2)$

$e \geq 2e - 2n + 4$

$\boxed{e \leq 2n-4}$

$9 \leq 2\times6-4$

$9 \leq 8$ false



$n=6$
$e=9$

STEPS TO DETECT PLANARITY

1) If a disconnected graph G is having K-components check the planarity of each of the k components.

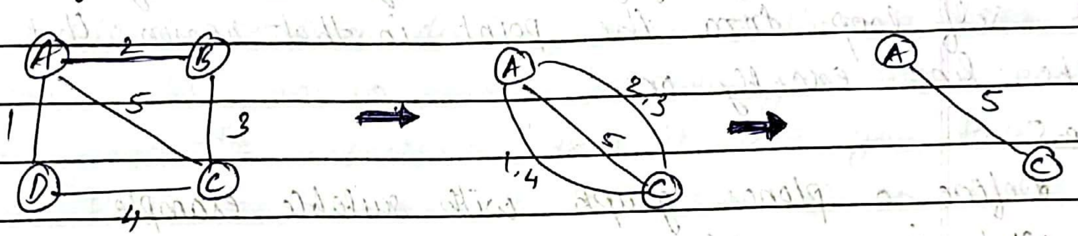
→ if, any one of the components is non-planar then the graph is also non-planar.

2) Remove all self loop from given graph.

3) Remove all parallel edges by keeping only one edge.

4) Remove all series edges by merging them.

Series edge - two edges are called series edge, if they are connected with a vertex whose degree is 2 or who is not connected with any other 3rd edge.



Resultant Graph :- (any one of them)

1) A single graph — Planar

2) A complete graph with 4 vertices — Planar

3) A non-separable ~~graph~~ simple graph with $n \geq 5$ & $e \geq 7$ use Euler's inequality.

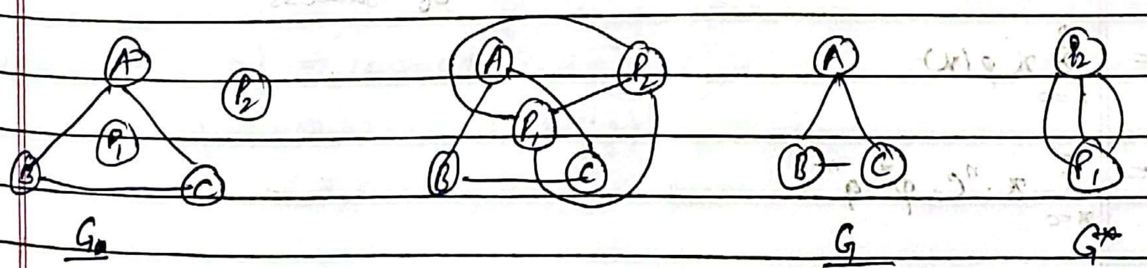
★ Dual Graph

→ Two graph are ~~plana~~ dual graph if both are planer.

to find dual graph :-

1) Take a point or vertex inside every region

2) Join the vertice in such a way that the joining line intersects every common edge exactly once.



Dual Graph to each other.

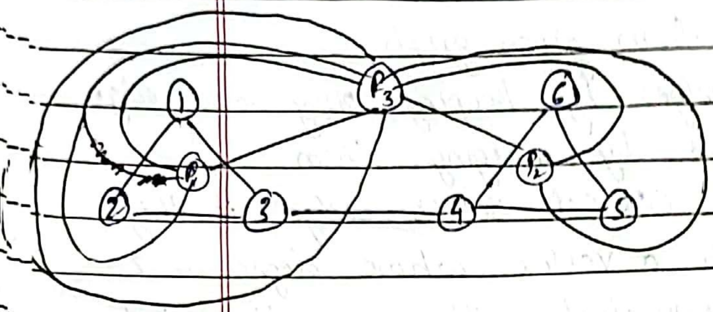
Page No.

Q) Properties :-

A. $e = e^*$

$n = f^*$

$f = n^*$



$e = 7 \quad e^* = 7$

$f = 3 \quad f^* = 6$

$n = 6 \quad n^* = 3$

3) If there's an edge lying completely in one region draw a self loop from the point in that region that intersec the line exactly once.

Chap-5

Q1. Define a planar graph with suitable example.

Q2 What is embedding.

Q3 What are the properties of K_5 and $K_{3,3}$

Q4 What is a region and infinite region. Show with proper diagram.

Q5 Using Euler's inequality check the planarity of K_5 & $K_{3,3}$.

Q6 Mention the steps to detect planarity with the resultant graph.

Maths.

★ Binomial Distribution (Discrete)

$$B(n, p) = {}^nC_x \, p^x q^{n-x}$$

$n \rightarrow$ no. of trials.

$p \rightarrow$ prob. of success.

$q \Rightarrow 1-p \rightarrow$ prob. of fail.

$x =$ no. of success.

$$\text{Mean} = \sum_{x=0}^{n} x\, p(x)$$

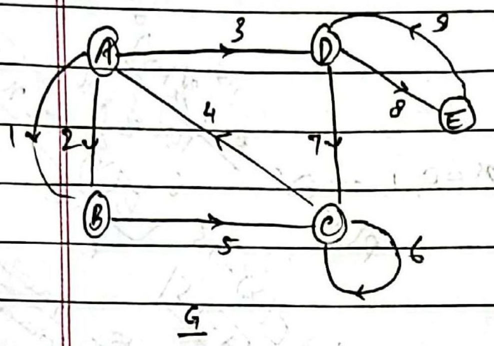
$$= \sum_{x=0}^{n} x \cdot {}^nC_x \, p^x q^{n-x}$$

$$= {}^nC_1 \, p q^{n-1} + 2 \, {}^nC_2 \, p^2 q^{n-2} + 3 \, {}^nC_3 \, p^3 q^{n-3} + \dots + n \cdot p^n$$

Q 20/03/2023
A

Date __/__/__

Directed Graph



for undirected graph :-

edge $\xrightarrow{\text{mapped}}$ some unordered pair of vertice.

$\Rightarrow 8 \rightarrow (D, E)$ or (E, D)

for directed graph :-

edge $\xrightarrow{\text{mapped}}$ some ordered pair of vertices.

$\Rightarrow 8 \rightarrow (D, E)$

$\Rightarrow 6 \rightarrow (C, C)$

Definition :- (Pg-194 onwards)
set of edges and vertices which has some ordered pair or vertices

Directed graph :-
1) self loop.
2) Parallel edges. (direction must be same)
3) Degree (v)

In-degree (∂^-)   Out degree (∂^+)

$\partial^-(A) = 1$ $\partial^+(A) = 3$. $\partial^-(B) = 2$ $\partial^+(B) = 1$ $\partial^-(E) = 1$ $\partial^+(E)$

$\partial^-(C) = 3$ $\partial^+(C) = 2$ $\partial^-(D) = 2$ $\partial^+(D) = 2$

$\Sigma \partial^-(v_i) = 9$ $\Sigma \partial^+(v_i) = 9$.

$\Rightarrow$ Sum of indegree and out-degree are equal.

Handshaking dilemma

$$\boxed{\Sigma \partial^+(v_i) + \Sigma \partial^-(v_i) = 2e}$$

Q What is simple digraph?

Q

Page No.

✱ Walk, Path and Circuit

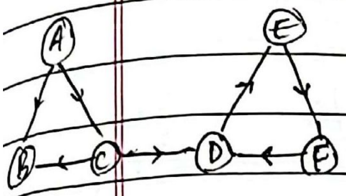
Date ___/___/___

If undirected :-

P(A, C) = A-4C → Semi-path.

A 2 B 5 C. → Directed path.

✱ Connected graph. → Strongly connected.
→ Weakly connected.



→ If in any pair of vertices has a path which is only semi-path then it is weakly connected.

→ If directed path is available between any pair of vertices then it is strongly connected graph.

Q) What is directed graph? Explain with proper diagram.

Q) Define self-loop, parallel edge, In-degree and out-degree from the directed graph with suitable example.

Q) Explain Handshaking dilemma with proper example.

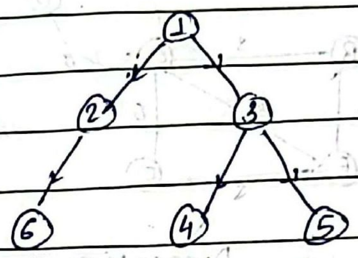
Q) What is an isomorphic digraph?

Q) Define a complete symmetric digraph.

Q) Define directed walk, semi-walk, directed path, sem-path, semi-circuit, directed circuit, strongly connected and weakly connected digraph.

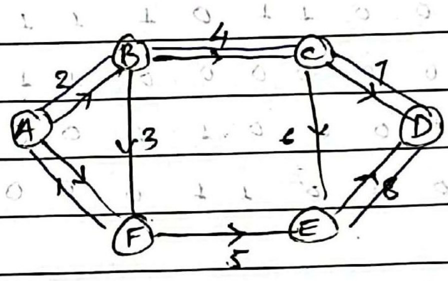
Q What is an Arborescence?

It is a directed ~~tree~~ Swathi where root's in-degree is 0.



Or it is directed tree provided exactly one vertex having its degree zero known as its root.

✱ Fundamental circuit :



Branches = {1, 2, 4, 7, 8}

Chords = {3, 5, 6}

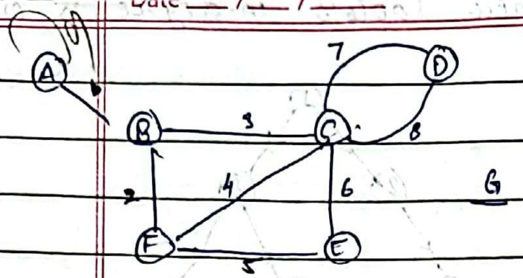
{1, 2, 3}, {1, 2, 4, 7, 8, 5}, {6, 7, 8}

Semi-circuit, Semi-circuit, Semi-circuit

(If direction C-D changes then, it will become directed circuit)

Q Matrix [Pg-137] ch-7

A ☆ Incident Matrix.



$A_{ij} = [a_{ij}]$ Dimension ⇒ $n \times e$.

= 1, if j^{th} edge is incident on its i^{th} vertex

= 0, otherwise

	1	2	3	4	5	6	7	8
A	1	0	0	0	0	0	0	0
B	1	1	1	0	0	0	0	0
C	0	0	1	1	0	1	1	1
D	0	0	0	0	0	0	1	1
E	0	0	0	0	1	1	0	0
F	0	1	0	1	1	0	0	0

A =

→ parallel edges will give identical coloumns.

→ no. of 1 in every row will denotes degree of that vertice.

→ every edge is incident exactly on two vertices.

Dimension ⇒ $(n-1) \times e$.

Reduced Incidence Matrix (A_f)

↳ we can delete any one row.

$A_f =$

	1	2	3	4	5	6	7	8
A	1	0	0	0	0	0	0	0
B	1	1	1	0	0	0	0	0
~~D~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~
E	0	0	0	0	1	1	0	0
F	0	1	0	1	1	0	0	0

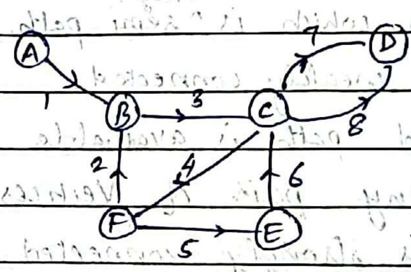
we can reconstruct incident matrix from reduced incident matrix.

→ Incidence matrix does not consider a self loop.

Q why self loop are not considered in I.M.?

→ If self loop is considered degree of vertex = 2

☆ Incidence Matrix in Digraph (Ch-9, Pg-215)



$A_{ij} = [a_{ij}]$

= 1, if j^{th} edge is incident out from i^{th} vertex.

= -1, if j^{th} edge is incident in to i^{th} vertex

= 0, otherwise.

Dimension ⇒ $n \times e$

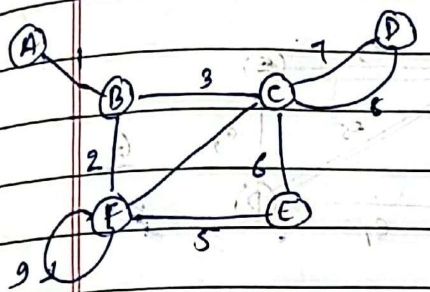
	1	2	3	4	5	6	7	8
A	1	0	0	0	0	0	0	0
B	-1	-1	1	0	0	0	0	0
C	0	0	-1	1	0	-1	1	1
D	0	0	0	0	0	0	-1	-1
E	0	0	0	0	-1	1	0	0
F	0	1	0	-1	1	0	0	0

↳ no. of + will denote out-degree
— " — — " — in degree

↳ In every colbumn there's one +1 & one −1.

☆ Circuit Matrix

Pg-142 & also properties.



B_{ij} → no. of edges.
↳ no. of ckt

$B_{ij} = [b_{ij}]$

= 1, if i^{th} ckt includes j^{th} edge.

= 0, otherwise

$C_1 = \{2, 3, 4\}$ $C_5 = \{9\}$

$C_2 = \{4, 5, 6\}$

$C_3 = \{7, 8\}$

$C_4 = \{2, 3, 6, 5\}$.

$$B_{ij} = \begin{array}{c|cccccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline C_1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ C_2 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ C_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ C_4 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ C_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

↳ In any row, no. of 1's denote no. of edges.

Points to :– (Pg. 144)

1) $AB^T = BA^T = 0$ (mod 2)

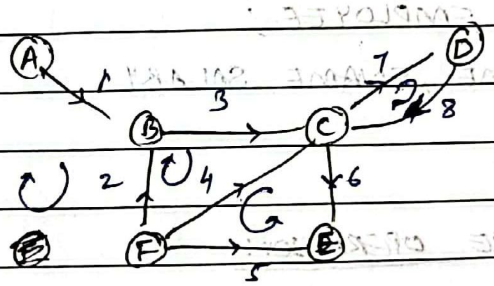
☆ Circuit Matrix in Directed Graph

(Pg-216)

$B_{ij} = [b_{ij}]$

= +1, if the ith cut includes jth edge and their orientation is same

= −1, if the ith cut includes jth edge and their orientation is opposite.



$C_1 = \{2, 3, 4\}$

$C_2 = \{4, 5, 6\}$.

$C_3 = \{7, 8\}$

$C_4 = \{2, 3, 6, 5\}$

$$B = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & +1 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & +1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 \\ 0 & +1 & +1 & 0 & -1 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

Page No.

Q

A

Date ___/___/___

★ SELECT OPERATION

SELECT * FROM Employee
WHERE DNO = 4 ;
$\sigma_{DNO = 4}$ (EMPLOYEE)

SELECT * From Employee
WHERE SALARY > 30000 ;
$\sigma_{SALARY > 30000}$ (EMPLOYEE)

★ PROJECT OPERATION

SELECT LNAME, FNAME, SALARY
FROM EMPLOYEE ;
$\pi_{LNAME, FNAME, SALARY}$

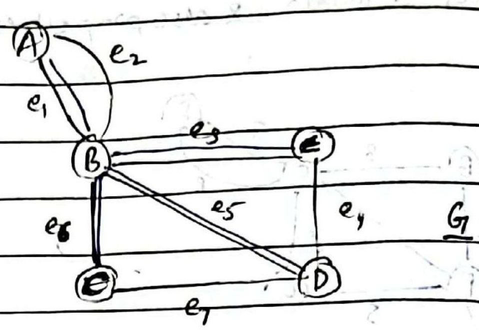
★ RENAME OPERATION

★ Relational Integrity Constraints

→ Key constraints
→ Entity integrity
→ Referential integrity

22/03/2023 GT

FUNDAMENTAL CIRCUIT MATRIX



S.T

branches — $\{e_1, e_3, e_5, e_6\}$

chords — $\{e_2, e_4, e_7\}$

$\{e_1, e_2\}$, $\{e_3, e_4, e_5\}$, $\{e_6, e_5, e_7\}$

$B_f = [b_{ij}]$

$= 1$, if j^{th} edge is included in i^{th} ckt

$= 0$, otherwise.

$$
\begin{array}{c|ccc|cccc|}
 & e_2 & e_4 & e_7 & e_1 & e_3 & e_5 & e_6 \\
c_1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
c_2 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
c_3 & 0 & 0 & 1 & 0 & 0 & 1 & 1
\end{array}
$$

(B_f = the matrix above; columns e_2, e_4, e_7: chords; columns e_1, e_3, e_5, e_6: branches)

$= [\, I_\mu \; \vdots \; B_t \,]$

Identity matrix

$t = (e-n+1) \times (n-1)$

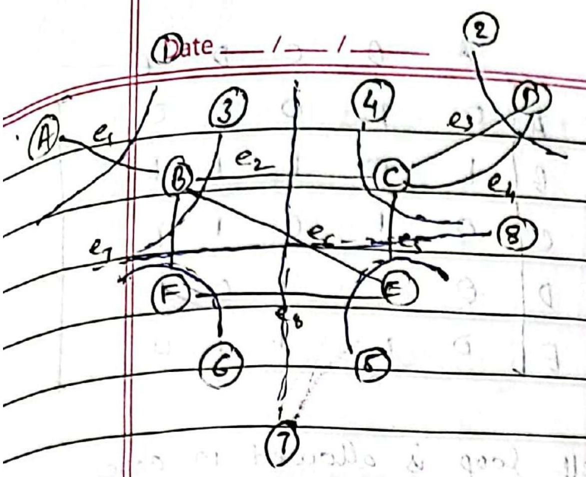
$\mu = (e-n+1) \times (e-n+1)$

Here, $7 - 5 + 1 = 3$

DIY — Fundamental circuit matrix on digraph.

* CUT-SET MATRIX.

Date: __ / __ / __



$C = [C_{ij}]$

$= 1$, if j^{th} edge is included in the i^{th} c/set

$= 0$, otherwise

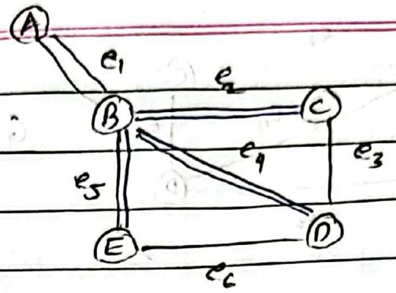
rows → no. of c/sets

col^{m}s → no. of edges

$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \end{array}$$

* cut set matrix for digraph will be same.

★ FUNDAMENTAL CUT-SET MATRIX



S.T

branches – $\{e_1, e_2, e_4, e_5\}$.

chords – $\{e_3, e_6\}$.

Cutsets.

$e_1 \rightarrow \{e_1\}$.

$e_2 \rightarrow \{e_2, e_3\}$.

$e_4 \rightarrow \{e_3, e_4, e_6\}$.

$e_5 \rightarrow \{e_5, e_6\}$

rows = no. of c/sets

col^{m}s = no. of edges

$C_f = [C_{ij}]$

$= 1$, if j^{th} edge is included in i^{th} c/set.

$= 0$, otherwise.

$$C_f = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc|cccc} e_3 & e_6 & e_1 & e_2 & e_4 & e_5 \end{array} \\ \left[\begin{array}{cc|cccc} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

* $[\,C_c \mid I_4\,]$

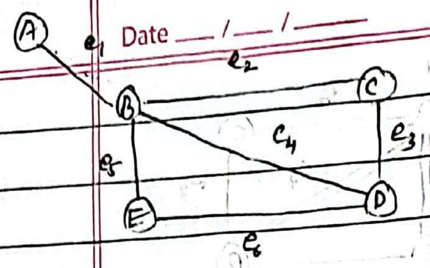
C_c: row × col → no. of cables → $(n-1) \times (e-n+1)$

I_4: row × col → $(n-1) \times (n-1)$

☆ PATH MATRIX: $[P(v_i, v_j)]$

Q

A



$$X = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 0 & 0 & 0 \\ B & 1 & 0 & 1 & 1 & 1 \\ C & 0 & 1 & 0 & 1 & 1 \\ D & 0 & 1 & 1 & 0 & 1 \\ E & 0 & 1 & 1 & 1 & 0 \end{array}$$

$P(A, D)$

P_1 : A e_1 B e_4 D

P_2 : A e_1 B e_2 C e_3 D

P_3 : A e_1 B e_5 E e_6 D

* $P(v_i, v_j)$ or

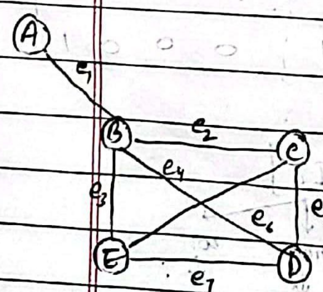
$P(x, y) = [P_{ij}]$

= 1, if j^{th} edge is included in the i^{th} path

0, otherwise

$P(A,D)$	e_1	e_2	e_3	e_4	e_5	e_6
1	1	0	0	1	0	0
2	1	1	1	0	0	0
3	1	0	0	0	1	1

☆ ADJACENCY MATRIX:



$X = [x_{ij}]$

sq. matrix of n

= 1, if there is an edge bet^n i^{th} & j^{th} vertex

= 0, otherwise

* self loop is allowed in case of adjacency matrix but not in case of incident matrix.

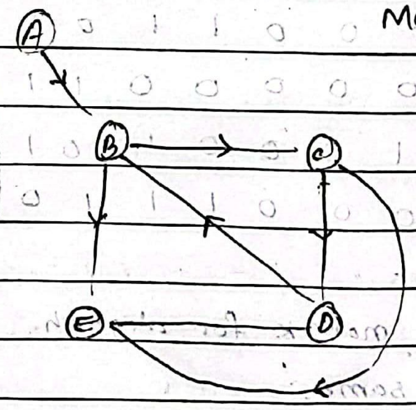
* parallel edges are not allowed in case of adjacency matrix.

Asg^n

Q. Why self loops are not allowed in incident matrix?

Q. Why parallel edges are not allowed in adjacency matrix?

Directed Graph (Adjacency Matrix)



$X = [x_{ij}]$

= 1, if there is an edge directed from i^{th} vertex to j^{th} vertex.

0, otherwise.

$$X = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

—+—+—+—+—+—+—+—+—+—+—

MATHS ———————— 25/03/23.

*** Echelon form of a Matrix**

$$\sim \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix} \quad R_3 \to R_3 - 5R_1$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix} \quad R_3 \to \frac{1}{4} R_3$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -3 & -1 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

Here this is the required :

Let $x_1 = (1, 1, -1)$
$x_2 = (2, 3, -5)$
$x_3 = (2, -1, 4)$

Let k_1, k_2, k_3 are scalars such that

$$k_1 x_1 + k_2 x_2 + k_3 x_3 = 0$$

$$= k_1(1,1,-1) + k_2(2,3,-5) + k_3(2,-1,4)$$

$$= (k_1 + 2k_2 + 2k_3,\ k_1 + 3k_2 - k_3,\ -k_1 - 5k_2 + 4k_3)$$

$$\Rightarrow k_1 + 2k_2 + 2k_3 = 0$$
$$k_1 + 3k_2 - k_3 = 0$$
$$-k_1 - 5k_2 + 4k_3 = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & -1 \\ 1 & -5 & 4 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The coeff matrix

$$= \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & -1 \\ -1 & -5 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -3 \\ 0 & -3 & 6 \end{bmatrix} \quad R_2 \to R_2 - R_1,\ R_3 \to R_3 + R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & -3 \end{bmatrix} \quad R_3 \to R_3 + 3R_2$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore k_1 + 2k_2 + 2k_3 = 0$$
$$k_2 - 3k_3 = 0$$
$$-3k_3 = 0$$

$$\Rightarrow k_1 = k_2 = k_3 = 0$$

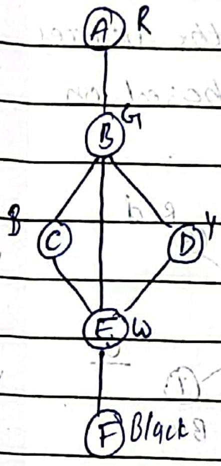
$\therefore x_1, x_2, x_3$ are linearly independent.

GT - 27/03/2023

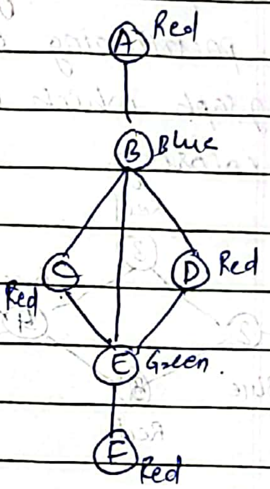
Colouring

Proper colouring.

If the graph is coloured in such a way that no two adjacent vertices get the same colour, then this is known as colouring or proper colouring.



G_1



G_2

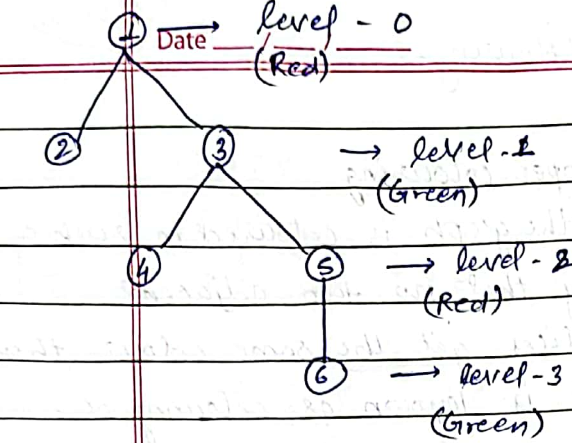
Chromatic number = 3.

Chromatic number → It is the minimum no. of colour to colour a graph properly.

Properties :-

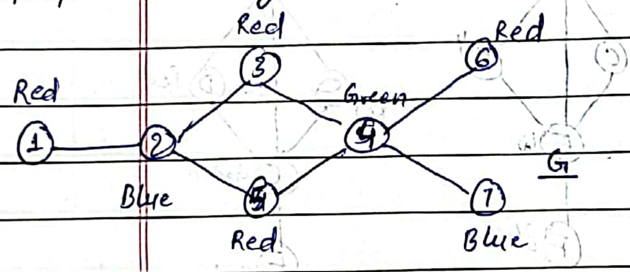
1) A graph with all isolated vertices is 1 chromatic / mono chromatic.
2) A graph with one or more edges is at least 2-chromatic except self loop.
3) A complete graph with n-vertices is n-chromatic.
4) A graph having one circuit with $n \geq 3$ is 2 chromatic if n is even and 3 chromatic if n is odd

5) Every tree with two or more vertices is 2-chromatic.



★ Chromatic partitioning/co

It is partitioning of the vertices of a graph which is based on proper coloring



K(chromatic no.) = 3

V = { 1, 2, 3, 4, 5, 6, 7 }

V_1 = { 1, 3, 5, 6 } → Red
V_2 = { 2, 7 } → Blue
V_3 = { 4 } → Green

Chromatic partitioning

Property:-

1) The vertices in one partition is having the same colour.

2) The vertices in one partition are not adjacent.

Based on these properties.

1) Independent set.

2) Maximal independent set.

1) Independent set

It is the set of vertices made in chromatic partitioning, where none of the vertices are adjacent.

2) Maximal independent.

It is a independent ~~sent~~ set, where no vertex can be added without destroying its independence property.

↳ none of the vertices are adjacent

Independent set	Maximal independent set
{4} + 1	✗
{3}	✓
V_1 = { 1, 3, 5, 6 }	→ Maximal I.S.
V_2 = { 2, 7 }	→ Maximal I-S.
V_3 = { 4 }	→ Not maximal I.S.

3) Largest maximal independent set

It is the maximal independent set which has largest no. of vertices ex: V_1

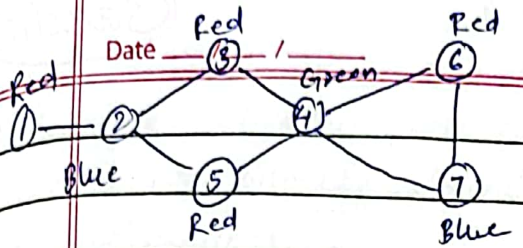
4) Independence number: It the no. of vertices in the largest maximal independent set.

$\beta(G) = 4$

Q what is independence number?

A write all from independent set.

1) Domination set.



It is the set of vertices where either all vertices are taken or some of the vertices are taken in such a way that ~~most~~ rest of the vertices will be adjacent to them.

V = {1, 2, 3, 4, 5, 6, 7}

V_1 = {2, 6}. (1, 3, 4, 5, 7 are adjacent to 2 or 6)

Domination property - all of the vertices are adjacent.

V_1 = {1, 3, 5, 6} → Dominating set

V_2 = {2, 7} → Dominating set.

V_3 = {4} → not dominating set.

2) Minimal dominating set -

It is a dominating set from where no vertex can be removed without destroying its domination property.

V_1 = {1, 3, 5, 6} → Minimal dominating

V_2 = {2, 7} → minimal dominating set

V_3 = {4} → Not minimal dominating

3) Smallest minimal dominating set.

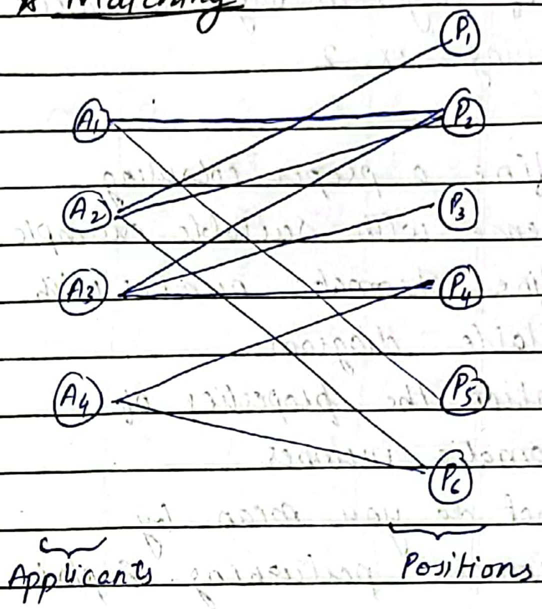
It is the minimal dominating having the smallest no. of vertices. ex. V_2

4) Domination number.

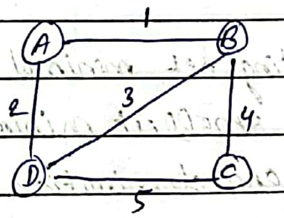
It is the no. of vertices in the smallest ~~small~~ minimal dominating set.

$\alpha(G) = 02.$

★ Matching



It is the set of edges, where no two edges are adjacent.



e_1 = {1, 5} - Red

e_2 = {2, 4} - Blue

e_3 = {3} - Green.

2) Maximal matching

It is the matching where no edge can be added without destroying its matching property.

e_1 = {1, 5} - maximal matching

e_2 = {2, 4} - maximal matching

e_3 = {3} - maximal matching

3) Largest maximal matching.
It is the maximal matching which has ~~maximal matching~~ largest no. of edges.

4) Matching number:
No. of edges in largest maximal matching. ex. 2

Q. Define a proper colouring scheme with suitable example.
Q. Define chromatic number with suitable diagram.
Q. Mention the properties of chromatic number.
Q. What do you mean by chromatic partitioning. Explain
Q. Explain independent set, maximal independent, largest maximal independent set and independent number.
Q. Explain dominating set, minimal dominating set, smallest minimal dominating set and domination number.
Q. Diff. b/w independence number and domination number.
Q. Explain matching, maximal matching, largest maximal matching and matching number.

$\Rightarrow (-3-\lambda)[1-\lambda-\lambda+\lambda^2-16]=0$

$\Rightarrow (-3-\lambda)(\lambda^2-2\lambda-15)=0$

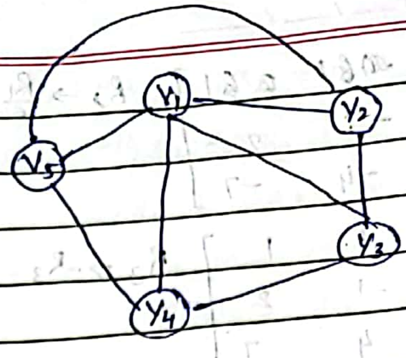
Date ___/___/___

$\Rightarrow (-3-\lambda)(\lambda^2-5\lambda+3\lambda-15)=0$

$\Rightarrow (-3-\lambda)[\lambda(\lambda-5)+3(\lambda-5)]=0$

$\Rightarrow \lambda = -3, -3, 5$

4.



4-5 marks (Imp.)

★ **Chromatic Polynomial** $P_n(\lambda)$

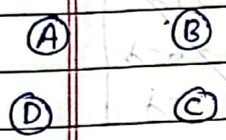
no. of vertices — n; no. of colours — λ

$$\boxed{P_n(\lambda) = \sum_{i=1}^{n} c_i \begin{bmatrix} \lambda \\ i \end{bmatrix}}$$

$P_n(\lambda) \rightarrow$ how many ways G can be coloured.

$$= c_1 \frac{\lambda}{\lfloor 1} + c_2 \frac{\lambda(\lambda-1)}{\lfloor 2} + c_3 \frac{\lambda(\lambda-1)(\lambda-2)}{\lfloor 3} + \dots + c_n \frac{\lambda(\lambda-1)(\lambda-2)\dots(\lambda-n+1)}{\lfloor n}$$

1. NULL G

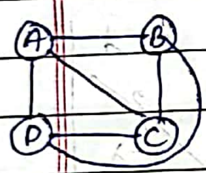


$P_n(\lambda) = \lambda^n$

$\boxed{P_n(\lambda) = \lambda^4}$

$$= c_1 \frac{\lambda}{\lfloor 1} + c_2 \frac{\lambda(\lambda-1)}{\lfloor 2} + c_3 \frac{\lambda(\lambda-1)(\lambda-2)}{\lfloor 3} + c_4 \frac{\lambda(\lambda-1)(\lambda-2)(\lambda-3)}{\lfloor 4} + c_5 \frac{\lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4)}{\lfloor 5}$$

2. COMPLETE G



$P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)$

In general,

$P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)\dots(\lambda-n+1)$

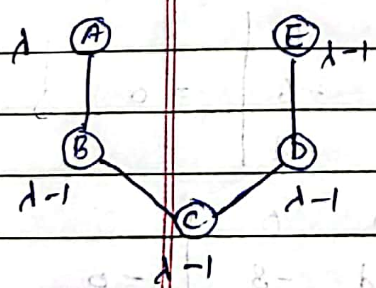
$\Rightarrow P_n(\lambda) =$

Here, $c_1 = 0$, $c_2 = 0$, $c_3 = \lfloor 3$, $c_4 = \lfloor 4 \times 2$, $c_5 = \lfloor 5$

$\Rightarrow P_n(\lambda) = 0 + 0 + \lambda(\lambda-1)(\lambda-2) + 2\lambda(\lambda-1)(\lambda-2)(\lambda-3) + \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4)$

$= \lambda(\lambda-1)(\lambda-2)[1+2(\lambda-3) + (\lambda-3)(\lambda-4)]$

3. PATH G

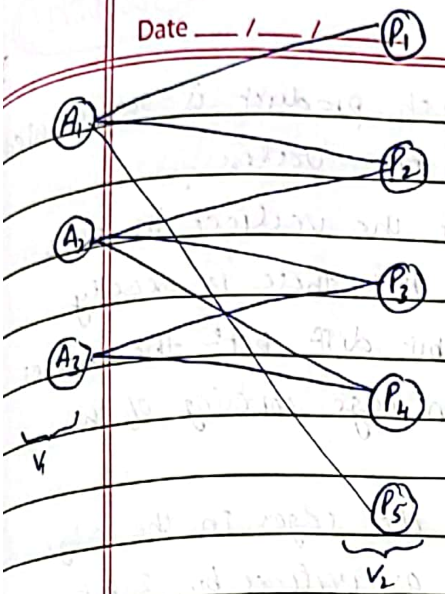


$P_n(\lambda) = \lambda(\lambda-1)^4$

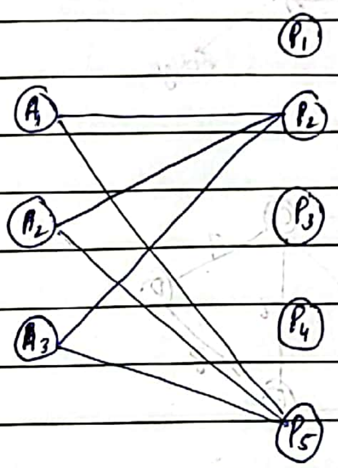
In general

$\boxed{P_n(\lambda) = \lambda(\lambda-1)^{n-1}}$

Date __/__/__



According to Hall's theorem
→ complete matching exists
if $\delta(G) \le 0$
highest value in last column
$\delta(G) = 1$.



Hall's Theorem

A complete matching of V_1 into V_2 in a bipartite graph exists if every subset of r vertices in V_1 is collectively adjacent to r or more vertices in V_2 for all values of r.

Value of r	V_1	V_2	P − q
r = 1	$\{A_1\}$	$\{P_1 P_2 P_5\}$	−2
	$\{A_2\}$	$\{P_1 P_3 P_4\}$	−2
	$\{A_3\}$	$\{P_3 P_4\}$	−1
r = 2	$\{A_1, A_2\}$	$\{P_1 P_2 P_3 P_4 P_5\}$	−3
	$\{A_2, A_3\}$	$\{P_2, P_3, P_4\}$	−1
	$\{A_3, \{A_1\}$	$\{P_1 P_2 P_3 P_4 P_5\}$	−3
r = 3	$\{A_1, A_2, A_3\}$	$\{P_1 P_2 P_3 P_4 P_5 \cancel{P}\}$	−2

Value of r	V_1	V_2	P − q
r = 1	$\{A_1\}$	$\{P_2, P_5\}$	−1
	$\{A_2\}$	$\{P_2, P_5\}$	−1
	$\{A_3\}$	$\{P_2, P_5\}$	−1
r = 2	$\{A_1, A_2\}$	$\{P_2, P_5\}$	0
	$\{A_1, A_3\}$	$\{P_2, P_5\}$	0
	$\{A_3 A_1\}$	$\{P_2 P_5\}$	0
r = 3	$\{A_1 A_2 A_3\}$	$\{P_2 P_5\}$	1

P = no. of vertices in V_1
q = no. of vertices in V_2

Largest value in last column = −1.
⇒ deficiency $\delta(G)$

Here $\delta(G) = 1$
Complete matching does not exist.
Maximal matching = No. of vertices in V_1 − $\delta(G)$
= 3 − 1
= 2.

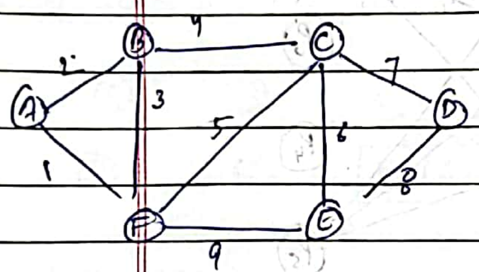
Date ___/___/______

★ Covering

1. Covering
2. ~~Maximal~~ Minimal covering
3. ~~Largest maximal~~ Smallest minimal covering
4. Covering No.

Covering → Vertex, edge

Q.

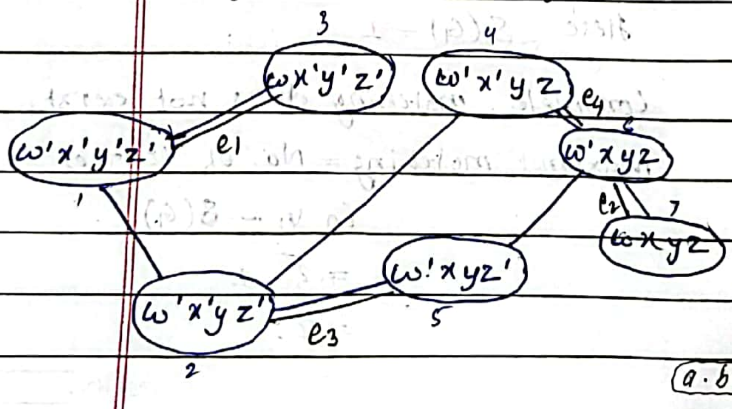


Vertex cov = {A, C, E F}

edge cov = {1, 2, 4, 7, 8}

★ Edge covering

$F = w'x'y'z' + w'x'yz' + wx'y'z' + w'x'yz + w'xyz' + w'xyz + wxyz$

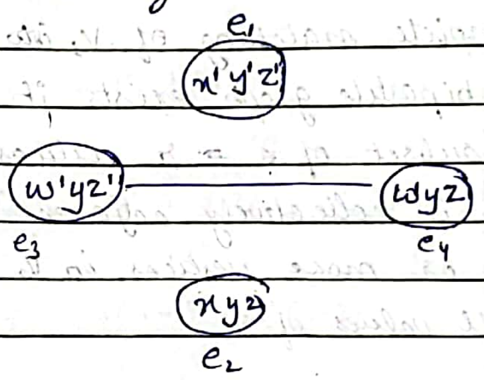


Step 1: Each product is represented by one vertex.

2: Connect the vertices in such a way that there is exactly one bit diff betn the vertices

3: Find an edge covering of the graph.

4: Convert the edges in the edge covering as vertices by 3-bit similarity



$\boxed{F = xyz + x'y'z' + w'y}$

∴ minimised equation of given equation.

Ques

Four colour conjecture.

End term: 5 × 10 mks

1, 2, 4, 5

mult color cube x. prof's eqn

matrix tree theorem

1-isomorphism, 2-isomorphism

directed graph (assignment)

planar graph (dual graph)

(a.bt = 0) todays class (4-5 questions) Page No.

All questions from ...